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Mixed or Special Live Streaming? Evolutionary Game Analysis of Merchants' Channel Selection in Live Streaming Commerce

Abstract. *The advent of social e-commerce platforms has promoted an emerging mode of trade known as live streaming commerce, in which merchants face a basic channel decision: to sell in a mixed session shared with other merchants, or in a special session of their own. The mixed channel splits the anchor's fee but also splits the viewer's attention; the special channel costs more but leaves the broadcast room undivided. We model the choice between the two as an evolutionary game between two competing merchant populations, derive the replicator dynamics and analyse the stability of the resulting equilibria. The dynamics are shown to depend on the payoff parameters only through each merchant's net advantage of mixed over special streaming and through the co-presence externality created when the rival enters the same session. Within the stability conditions derived, the sign of that externality determines the outcome: negative externalities drive the merchants apart, one to each channel; positive externalities drive them onto a common channel, with the initial distribution of choices, rather than the fee gap, selecting which one. Simulation confirms the analysis and quantifies how each parameter shifts convergence.*

Keywords: *live streaming commerce, channel selection, replicator dynamics, evolutionary game, merchant decision-making.*

JEL Classification: C72, C61, L81, M31.

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1. Introduction

In recent years, social e-commerce platforms have given rise to live streaming commerce. In China, social media platforms such as TikTok and Kwai have become central channels through which merchants promote and sell their products; by December 2022 China's live streaming shopping audience had reached 515 million, or 48.2% of all Chinese netizens (Li et al., 2023). Traditional platforms including Taobao, JD.com and Pinduoduo have followed, encouraging merchants and anchors to sell through live sessions.

Live streaming commerce is a form of webcasting in which anchors demonstrate products through real-time interaction and viewers buy directly. Unlike static text or photo displays (Sun et al., 2019), the format raises participation and trust (Zhang et al., 2022) and encourages impulse purchase.

Research on live streaming commerce has developed along three lines. The first asks what makes viewers buy. Perceived streamer expertise and social capital raise purchase intention (Xu et al., 2022); interpersonal interaction inside the broadcast room builds the swift *guanxi* that converts viewing into buying (Chen et al., 2022); and the roles a streamer performs shape how a session sells (Zhang et al., 2022). This line takes the broadcast room as given. It explains why a session converts, but not which session a merchant should be in.

The second line studies the operating decisions that follow adoption: joint pricing and streaming effort (Liu, Xu, et al., 2022), pricing under anticipated regret (Liu, Hua, et al., 2022) and under impulse buying (Jiang and Cai, 2021), and the horizon of retailer-agency cooperation (Mao et al., 2022). A related group asks whether and in what form the channel should be opened at all: Zhang and Tang (2023) and Liu et al. (2024) derive when a manufacturer or an e-seller should introduce live streaming; Ji et al. (2023) and Wang et al. (2024) compare selling formats and distribution contracts; Ye et al. (2024) and Zhen et al. (2024) analyse influencer selection and streamer effort; Xiao et al. (2024) value effort against sales commitments. These models are vertical: one seller bargains with a streamer, an agency or a platform, and rivals enter only through a reduced-form demand term.

The third line, closest to this paper, treats the streaming decision as strategic between competing sellers. Huang et al. (2024) ask whether and when competing retailers should introduce a live stream, and Lu and Duan (2024) derive streaming strategies for two vertically differentiated firms. In both, the decision is binary adoption under full rationality and the rivals' broadcasts remain physically separate. Once adoption is settled, the operative question is no longer whether to stream but whether to stream beside a rival in one session or alone, a decision whose payoff depends on who else is in the room.

Merchants revise this choice repeatedly, observe rivals imperfectly and imitate what appears to work, so bounded rationality is the natural behavioural assumption and evolutionary game theory the standard tool for it. The tool is well established in operations and e-commerce settings, including pollution control (Fan et al., 2021), procurement outsourcing (Niu et al., 2019), platform innovation (Zhao, 2024) and

supplier-retailer preservation (He et al., 2025). We adopt it for the same reason and claim no methodological novelty; what is new is the payoff structure and what it implies.

What none of this work models is the externality of co-presence in a single session. Two merchants in one mixed session share the anchor's fixed fee and the traffic the room attracts, but they also share the viewer's attention and budget; a merchant in a special session carries the whole fee and owns the room. Whether co-presence is worth more or less than solo presence is therefore a property of the product category rather than of the fee schedule, and, within the stability conditions derived below, it is this property that governs the outcome.

Our contributions are threefold. First, we formalise mixed versus special live streaming as a game between two competing merchant populations and show that the whole dynamic is driven by two composite quantities per merchant: the net advantage of mixed over special streaming against a rival who streams alone, and the co-presence externality, that is, the change in that advantage when the rival joins the same session. Second, we show that within those conditions the sign pattern of the two externalities, rather than the fee differential, selects the regime: both negative, the merchants anti-coordinate and separate, so no channel standard emerges and an agency cannot durably fill one session with direct rivals; both positive, they coordinate, but on either channel, and the initial distribution of choices selects which; opposite in sign, no stable strategy exists. Third, we index the two regions of attraction by the areas of the quadrilaterals they delimit, and translate the comparative statics into slot-pricing and session-curation rules for merchants, streamer agencies and platforms.

The remainder of the paper is organised as follows. Section 2 describes the channel selection problem. Section 3 develops the evolutionary game model and analyses stability under the different conditions. Section 4 simulates the model and examines the factors driving the evolution of equilibrium. Section 5 concludes and sets out implications and future research.

2. Problem

We study two competing merchants, O and Y, choosing between mixed and special live streaming anchors. A mixed session shares costs but sharpens competition and lowers profit; the order in which brands appear, say L'Oréal before Estée Lauder, already shifts sales between them. A special session buys exclusive exposure at a higher fee, yet does not remove rivalry or traffic spillovers. The question is when the two merchants' strategies reach a stable equilibrium. Notation and assumptions appear in Table 1.

Table 1. Notations

Notation	Description
R_s	Merchant O benefits when merchant O and merchant Y select mixed live streaming
G_s	Merchant Y benefits when merchant O and merchant Y select mixed live streaming
R_p	Merchant O benefits when merchant Y selects special live streaming, and merchant O selects mixed live streaming
R_e	Merchant O benefits when merchant O selects special live streaming
G_p	Merchant Y benefits when merchant O selects special live streaming, and merchant Y selects mixed live streaming
G_e	Merchant Y benefits when merchant Y selects special live streaming
K_p	Costs of Merchant O/Y selecting mixed live streaming
K_e	Costs of Merchant O/Y selecting special live streaming
x	Proportion of population O choosing mixed live streaming
y	Proportion of population Y choosing mixed live streaming

Source: Authors' processing.

Assumption 1: Merchants O and Y are boundedly rational, and their channel choices are limited to “mixed live streaming” and “special live streaming”.

Assumption 2: Merchants O and Y face identical anchor fees within the same channel type. However, the cost of choosing the special live streaming channel exceeds that of the mixed channel, i.e., $K_e > K_p$.

Assumption 3: When merchants O and Y both choose mixed live streaming they share one anchor and one session, and the benefit of a joint session may differ from that of a solo one.

Assumption 4: Initially, merchant O chooses a proportion x for mixed live streaming, with the remaining proportion $1 - x$ allocated to special live streaming, where $0 \leq x \leq 1$. Similarly, merchant Y selects a proportion y for mixed live streaming, with $1 - y$ for special live streaming, where $0 \leq y \leq 1$.

3. Model

3.1 Evolutionary Game Model

Because the two merchants reach their channel choices through repeated play under bounded rationality, we model the process as an evolutionary game. O and Y denote two populations of competing merchants, and x and y are the shares within

each that stream mixed; "merchant O" is shorthand for a merchant drawn from population O. The payoff matrix is shown in Table 2.

Table 2. Merchant's gaming payment matrix

Merchant O	Merchant Y	
	Mixed live streaming(y)	Special live streaming($1-y$)
Mixed live streaming(x)	$R_s - K_p, G_s - K_p$	$R_p - K_p, G_e - K_e$
Special live streaming($1-x$)	$R_e - K_e, G_p - K_p$	$R_e - K_e, G_e - K_e$

Source: Authors' processing.

In Table 2, we can get the utility of merchant O selecting "mixed live streaming" (U_p), the utility of selecting "special live streaming" (U_e), and the average utility ($\bar{U}$) under the two live streaming channels which are shown in equations (1)-(4).

$$U_p = y(R_s - K_p) + (1-y)(R_p - K_p) \quad (1)$$

$$U_e = y(R_e - K_e) + (1-y)(R_e - K_e) \quad (2)$$

$$\bar{U} = xU_p + (1-x)U_e \quad (3)$$

$$\bar{U} = (1-x)(R_e - K_e) + x(-K_p + R_p + y(R_s - R_p)) \quad (4)$$

Similarly, the utility of merchant Y selecting "mixed live streaming" (V_p), the utility selecting "special live streaming" (V_e) and the average utility ($\bar{V}$) under the two live channels are shown in equations (5)-(8).

$$V_p = x(G_s - K_p) + (1-x)(G_p - K_p) \quad (5)$$

$$V_e = x(G_e - K_e) + (1-x)(G_e - K_e) \quad (6)$$

$$\bar{V} = yV_p + (1-y)V_e \quad (7)$$

$$\bar{V} = (1-y)(G_e - K_e) + y(G_p - K_p + x(G_s - G_p)) \quad (8)$$

Therefore, the replicated dynamic equation $U(x)$ for Merchant O selecting "mixed live streaming" is shown in equations (9) and (10).

$$U(x) = \frac{dx}{dt} = x(U_p - \bar{U}) \quad (9)$$

$$U(x) = (1-x)x(-K_p + K_e + R_p - R_e + y(R_s - R_p)) \quad (10)$$

The replication dynamic equations $V(y)$ are also shown in equations (11) and (12) for the merchant Y who selects "mixed live streaming".

$$V(y) = \frac{dy}{dt} = y(V_p - \bar{V}) \quad (11)$$

$$V(y) = (1-y)y(-G_e - K_p + K_e + G_p + x(G_s - G_p)) \quad (12)$$

The two replicated dynamic equations (10) and (12) mentioned earlier are consolidated into an equation set, denoted as $U(x) = 0$ and $V(y) = 0$, yielding five equilibrium points: O (0,0), A (1,0), B (1,1), C (0,1) and $E(\frac{-G_p+G_e+K_p-K_e}{G_s-G_p}, \frac{K_p-K_e-R_p+R_e}{R_s-R_p})$.

Remark 1. Equations (10) and (12) involve the eight payoff parameters only through four composite quantities: $\alpha_O = K_e - K_p + R_p - R_e$ and $\beta_O = R_s - R_p$ for merchant O, and $\alpha_Y = K_e - K_p + G_p - G_e$ and $\beta_Y = G_s - G_p$ for merchant Y. Here α is a merchant's net advantage of mixed over special live streaming when the rival streams alone, and β is the co-presence externality, the change in that advantage when the rival joins the same session. Parameter sets sharing the same (α, β) generate identical trajectories, so absolute revenue and fee levels are immaterial; the four conditions of Section 3.2 are sign restrictions on these differences.

By the evolutionary game model of merchants' live channel selection, the stability analysis of the five equilibrium points involves evaluating the determinant of the Jacobian matrix J , denoted as $\det(J)$, and the trace of the matrix, $\text{tr}(J)$. If $\det(J) > 0$ and $\text{tr}(J) < 0$ for a specific equilibrium point, then that equilibrium point is considered an Evolutionarily Stable Strategy (ESS) (Niu et al., 2019). The Jacobian matrix J can be obtained using equations (13)-(17).

$$J = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial U(x)}{\partial x} & \frac{\partial U(x)}{\partial y} \\ \frac{\partial V(y)}{\partial x} & \frac{\partial V(y)}{\partial y} \end{pmatrix} \quad (13)$$

$$\frac{\partial U(x)}{\partial x} = (1 - 2x) (-K_p + K_e + R_p - R_e + y(R_s - R_p)) \quad (14)$$

$$\frac{\partial U(x)}{\partial y} = (R_s - R_p)(1 - x)x \quad (15)$$

$$\frac{\partial V(y)}{\partial x} = (G_s - G_p)(1 - y)y \quad (16)$$

$$\frac{\partial V(y)}{\partial y} = (-G_e - K_p + K_e + G_p + x(G_s - G_p))(1 - 2y) \quad (17)$$

Since each initial point and evolved point only holds significance within the two-dimensional space $V = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$, the following conditions must be satisfied: $0 \leq \frac{-G_p + G_e + K_p - K_e}{G_s - G_p} \leq 1$, $0 \leq \frac{K_p - K_e - R_p + R_e}{R_s - R_p} \leq 1$. The values of each equilibrium point (0,0), (0,1), (1,0), (1,1) and (x_o, y_o) in the Jacobian matrix under different conditions are presented in Table 3 below. Here, $p = \frac{(-G_p + G_e + K_p - K_e)(R_s - R_p)(G_s - G_e - K_p + K_e)}{(G_s - G_p)^2}$ and $q = \frac{(K_p - K_e - R_p + R_e)(G_s - G_p)(R_s - K_p + K_e - R_e)}{(R_s - R_p)^2}$.

Table 3. Values of the Jacobi matrix at the equilibrium point

Equilibrium point	a_{11}	a_{12}	a_{21}	a_{22}
(0,0)	$-K_p + K_e + R_p - R_e$	0	0	$-G_e - K_p + K_e + G_p$
(0,1)	$-K_p + K_e + R_s - R_e$	0	0	$-G_p + G_e + K_p - K_e$

Equilibrium point	a_{11}	a_{12}	a_{21}	a_{22}
(1,0)	$K_p - K_e - R_p + R_e$	0	0	$G_s - G_e - K_p + K_e$
(1,1)	$K_p - K_e - R_s + R_e$	0	0	$-(G_s - G_e - K_p + K_e)$
(x_o, y_o)	0	p	q	0

Source: Authors' processing.

3.2 Stability Analysis

The stability of each equilibrium under the different conditions in Table 3 supports the following propositions.

Proposition 1. Condition 1: $G_p - G_e - K_p + K_e < 0$, $K_p - K_e - R_p + R_e < 0$, $K_p - K_e - R_s + R_e > 0$, $G_s - G_e - K_p + K_e > 0$; Condition 2: $G_p - G_e - K_p + K_e > 0$, $K_p - K_e - R_p + R_e > 0$, $K_p - K_e - R_s + R_e < 0$, $G_s - G_e - K_p + K_e < 0$. Under condition 1 or condition 2, the model has no evolutionary stable point, i.e., there is no evolutionary stable strategy.

Proof: Under condition 1 or condition 2, the stability analysis results of the Jacobian matrix are presented in Table 4. It is observed that no equilibrium point simultaneously satisfies $\det(J) > 0$ and $\text{tr}(J) < 0$. Therefore, no equilibrium point meets the criteria for an ESS, and Proposition 1 is proven.

Table 4. Stability analysis results under conditions 1 or 2

Equilibrium point	$\det(J)$	$\text{tr}(J)$	Local stability
(0,0)	—	Unknown	Saddle point
(0,1)	—	Unknown	Saddle point
(1,0)	—	Unknown	Saddle point
(1,1)	—	Unknown	Saddle point
(x_o, y_o)	+	0	Center

Source: Authors' processing.

Under conditions 1 or 2 the points (0,0), (0,1), (1,0) and (1,1) are saddle points, so no equilibrium solution exists. Taking condition 2, the decision process splits in two: (a) when merchant Y selects mixed live streaming, merchant O does the same, because its relative net payment is positive; (b) when merchant O selects mixed live streaming, merchant Y finds special live streaming more profitable. The two sides therefore chase each other and no equilibrium is reached (Fan et al., 2021).

Proposition 2. Condition 3: $G_p - G_e - K_p + K_e > 0$, $K_p - K_e - R_p + R_e < 0$, $K_p - K_e - R_s + R_e > 0$, and $G_s - G_e - K_p + K_e < 0$. Under condition 3, there are two ESS points in the model, ESS1 (0,1) and ESS2 (1,0).

Proof: Under condition 3, the results of the stability analysis of the Jacobian matrix are shown in Table 5. It is observed that there exist equilibrium points for

which $\det(J) > 0$ and $\text{tr}(J) < 0$. Specifically, the points (0,1) and (1,0) satisfy these criteria, indicating that they are ESS. Therefore, Proposition 2 is proven.

Table 5. Stability analysis results under condition 3

Equilibrium point	$\det(J)$	$\text{tr}(J)$	Local stability
(0,0)	+	+	Point of instability
(0,1)	+	−	ESS1
(1,0)	+	−	ESS2
(1,1)	+	+	Point of instability
(x_0, y_0)	−	0	Saddle point

Source: Authors' processing.

From condition 3, it can be deduced that $G_s < G_p$ and $R_s < R_p$. It implies that when the joint benefit to merchant O and merchant Y selecting mixed live streaming is less than the benefit to each of them selecting mixed live streaming alone, their channel selections diverge: one merchant takes mixed live streaming while the other takes special live streaming. The evolution of merchants under this condition is illustrated in Figure 1(b).

Proposition 3. Condition 4: $G_p - G_e - K_p + K_e < 0$, $K_p - K_e - R_p + R_e > 0$, $K_p - K_e - R_s + R_e < 0$ and $G_s - G_e - K_p + K_e > 0$. Under condition 4, the model contains two ESS points, denoted as equilibrium points ESS3 (0,0) and ESS4 (1,1).

Proof: Under condition 4, the results of the stability analysis of the Jacobian matrix are shown in Table 6. The analysis indicates that there exist equilibrium points where $\det(J) > 0$ and $\text{tr}(J) < 0$. Thus, the equilibrium points (0,0) and (1,1) are identified as ESS, Proposition 3 is proved.

Table 6. Stability analysis results under condition 4

Equilibrium point	$\det(J)$	$\text{tr}(J)$	Local stability
(0,0)	+	−	ESS3
(0,1)	+	+	Point of instability
(1,0)	+	+	Point of instability
(1,1)	+	−	ESS4
(x_0, y_0)	−	0	Saddle point

Source: Authors' processing.

Under Condition 4, since $G_s > G_p$ and $R_s > R_p$, both merchants gain more from mixed live streaming together than alone. If merchant Y chooses the special channel, merchant O's relative net payment for mixed streaming is negative and O follows it there, and symmetrically for merchant Y. Both merchants therefore settle on the same channel, at (0,0) or at (1,1), as illustrated in Figure 1(c).

Propositions 1 to 3 admit a single reading through Remark 1. Each of the four conditions of Section 3.2 requires $\alpha_i(\alpha_i + \beta_i) < 0$ for both merchant populations, which is what places the interior equilibrium strictly inside the unit square; given

that, the four differ only in the signs of the two co-presence externalities. Both negative: a rival in one's own session destroys more attention than the shared fee saves, and the merchants anti-coordinate, exactly one taking the mixed channel (Proposition 2). Both positive: co-presence pools traffic and broadens the assortment on offer, and they coordinate on a common channel (Proposition 3). Opposite in sign: one merchant is always chasing and the other always fleeing, and no stable strategy exists (Proposition 1). When α_i and $\alpha_i + \beta_i$ instead share a sign, that population has a dominant channel and the game is trivial. The fee gap $K_e - K_p$ enters only through α ; within a regime it moves the interior point, and hence which equilibrium is reached from a given starting state, and it changes the regime only when the gap moves α past 0 or past $-\beta$, at which point the interior equilibrium leaves the unit square and one channel becomes dominant. What decides whether competing merchants share a broadcast room is thus the sign of the co-presence externality rather than the price of a special session.

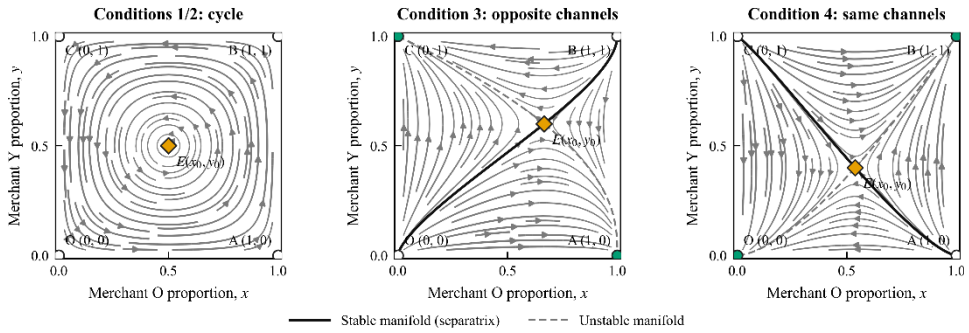


Figure 1. Dynamic evolution process

Source: Authors' own creation.

Under conditions 3 and 4 the ESS is not unique, so the final ESS of the system must be discussed case by case; these are referred to as Case 1 and Case 2 respectively.

In Case 1 (Figure 1(b)), an initial state in region BCOE converges to (0,1), merchant O taking the special channel and Y the mixed one; an initial state in BAOE converges to (1,0). The direction depends on the saddle point (x_0, y_0) : the closer it lies to the upper left, the larger the BAOE area and the more likely convergence to (1,0). The two areas are given in (18) and (19). In Figures 1(b) and 1(c) the solid black curve through E is the stable manifold of the saddle point and the exact boundary between the two regions of attraction, while the grey dashed curve is the unstable manifold. Equations (18) to (20) approximate that boundary by straight-sided quadrilaterals, which is the form used throughout this section.

$$S_{BCOE} = \frac{1}{2} \left(1 + \frac{K_p + G_e - K_e - G_p}{G_s - G_p} - \frac{R_e + K_p - R_p - K_e}{R_s - R_p} \right) \quad (18)$$

$$S_{BAOE} = \frac{1}{2} \left(1 + \frac{R_e + K_p - R_p - K_e}{R_s - R_p} - \frac{K_p + G_e - K_e - G_p}{G_s - G_p} \right) \quad (19)$$

From the equations above, it's evident that S_{BCOE} and S_{BAOE} are diametrically opposed, so we analyse only one. In this paper, we opt to investigate the parameters involved in S_{BCOE} . In equation (18), it becomes apparent that G_s and R_s , G_p and R_p , as well as G_e and R_e , exhibit opposite signs and hence contrasting effects. For brevity we select G_s , R_p and G_e for analysis, together with the costs of mixed live streaming K_p and special live streaming K_e . The outcomes are as follows:

(1) Gain G_s : A higher gain to merchant Y when both stream mixed enlarges the BCOE area and makes convergence to C (0,1) more likely, where O takes the special channel and Y the mixed one.

(2) Gain R_p : Taking the derivative of S_{BCOE} with respect to R_p yields $\frac{R_s - R_e - K_p + K_e}{2(R_s - R_p)^2} < 0$, so a higher gain to merchant O from streaming mixed alone shrinks the BCOE area and makes convergence to A (1,0) more likely, where O takes the mixed channel and Y the special one.

(3) Gain G_e : The derivative of S_{BCOE} with respect to G_e is $\frac{1}{2(G_s - G_p)} < 0$: the larger the gain to merchant Y from selecting the special live streaming, the smaller the BCOE area and the more likely convergence to A (1,0).

(4) Cost K_p : As the derivative of S_{BCOE} with respect to K_p is $\frac{1}{2} \left(\frac{1}{G_s - G_p} - \frac{1}{R_s - R_p} \right)$, given $G_s < G_p$ and $R_s < R_p$, when $G_s - G_p > R_s - R_p$, the size of S_{BCOE} varies inversely with K_p . Therefore, the larger K_p is, the more easily the system converges to A (1,0); conversely, when $G_s - G_p < R_s - R_p$, the system converges to C (0,1).

(5) Cost K_e : Since S_{BCOE} derives $\frac{1}{2} \left(\frac{1}{R_s - R_p} - \frac{1}{G_s - G_p} \right)$ from K_e , the size of S_{BCOE} rises with K_e when $G_s - G_p > R_s - R_p$. Therefore, the larger K_e is, the more easily the system converges to C (0,1); conversely, when $G_s - G_p < R_s - R_p$, the system converges to A (1,0).

In Case 2 (Figure 1(c)), an initial state in region AOCE converges to O (0,0), both merchants choosing the special channel; an initial state in ABCE converges to B (1,1), both choosing the mixed channel. The area of ABCE is given by equation (20).

$$S_{ABCE} = 1 - \frac{1}{2} \left(\frac{-G_p + G_e + K_p - K_e}{G_s - G_p} + \frac{-R_p + R_e + K_p - K_e}{R_s - R_p} \right) \quad (20)$$

This expression is structurally symmetrical to that of S_{BCOE} in Case 1, with the same five parameters- G_s , R_p , G_e , K_p , and K_e -governing the size and direction of convergence. Their effects mirror Case 1 with reversed tendencies, so we omit the repeated derivations and note that increases in G_s , R_p , and K_e generally enlarge the ABCE area, promoting convergence to (1,1), while increases in G_e and K_p reduce the area, encouraging convergence to (0,0).

4. Simulation Analysis

We integrate the replicator dynamics numerically for merchants O and Y, first to check the evolutionary outcomes of channel selection in both cases and then to trace the effect of G_s , R_s , G_e , K_p and K_e on the equilibrium reached.

4.1 Basic Simulation Analysis

Firstly, we set the initial proportions $x = 0.5$ and $y = 0.5$.

Parameter values are dimensionless index points rather than monetary amounts. By Remark 1 the trajectories depend only on the composites α and β , so the levels carry no separate meaning and only the differences are interpreted. The baseline sets used in Section 4.1 satisfy the sign conditions strictly: Case 1 satisfies Condition 3 and Case 2 satisfies Condition 4, which makes each case a test of its proposition rather than an illustration. K_e exceeds K_p because a special session is not shared, and the baseline ratio $K_e / K_p = 2$ is a normalisation rather than an estimate; Section 4.2 varies it between 1.1 and 2.9. The sweeps in Tables 7 and 8 move the interior equilibrium, and with it the boundary between the two regions of attraction, while the initial state is held at $(0.5, 0.5)$; a few endpoints also reach or cross a condition boundary. Most of the reversals reported below therefore occur while the sign conditions still hold, when the moving boundary passes the fixed initial state. In the G_s sweep of Table 7, for instance, the interior point moves from $x_0 = 0.4$ at $G_s = 40$ to $x_0 = 0.67$ at $G_s = 60$ and passes the initial value 0.5 at $G_s = 50$, which is where the outcome switches. The thresholds visible in Figures 4 to 7 are thus properties of the model rather than artefacts of the values chosen.

Case 1: When $G_s < G_p$, $R_s < R_p$, set $R_s = 40$, $G_s = 40$ or 60 , $R_p = 90$, $R_e = 70$, $G_p = 90$, $G_e = 80$, $K_p = 10$, $K_e = 20$. As shown in Figure 2, for a given G_s , the proportions x and y of merchants O and Y choosing mixed streaming stabilise at opposite extremes (0 or 1), leading to a stable outcome of either {mixed, special} or {special, mixed}, consistent with Proposition 2.

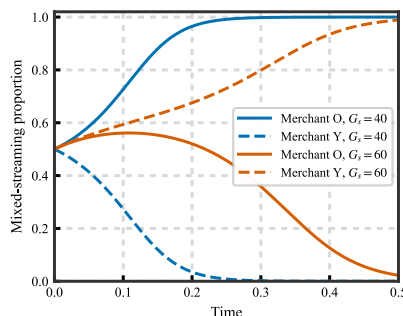


Figure 2. Stability simulation analysis under Case 1

Source: Authors' own creation.

Case 2: When $G_s > G_p$, $R_s > R_p$, we take $R_s = 90$, $G_s = 80$ or 100 , $R_p = 40$, $R_e = 70$, $G_p = 35$, $G_e = 80$, $K_p = 10$, $K_e = 20$. As shown in Figure 3, for a given G_s , over time the proportions x and y of merchants O and Y choosing mixed live streaming both converge to the same stable value, either 0 or 1. Thus, the system evolves to either {mixed, mixed} or {special, special}, consistent with Proposition 3.

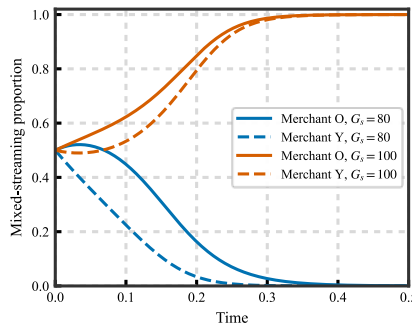


Figure 3. Stability simulation analysis under Case 2

Source: Authors' own creation.

4.2 Effects of Different Parameters on the Evolution

Parameter changes affect the two merchants differently across contexts. To examine these effects we vary one parameter at a time over several levels, with the initial mixed streaming proportions of merchants O and Y both set to 0.5. Since pairs of parameters (G_s and R_s , G_p and R_p , G_e and R_e) act alike, we take one from each pair- G_s , R_p , and G_e -for analysis, together with the mixed streaming cost K_p and the special streaming cost K_e .

First, we analyse equilibrium evolution in Case 1 ($G_s < G_p$ and $R_s < R_p$) using the values in Table 7, which satisfy the Case 1 conditions. Here $K_p - 1$ and $K_p - 2$ represent K_p effects under $G_s - G_p > R_s - R_p$ and $G_s - G_p < R_s - R_p$, respectively; $K_e - 1$ and $K_e - 2$ are defined similarly for K_e . To compare the K_p and K_e effects, we vary G_s and R_s , G_p and R_p , and G_e and R_e in pairs.

Table 7. Parameter values in Case 1

	G_s	G_p	R_s	R_p	G_e	R_e	K_p	K_e
G_s	40,50, 60,70	90	40	90	80	70	10	20
R_p	60	90	40	80,90, 100,110	80	70	10	20
G_e	45	90	40	90	60,70, 80,90	70	10	20

	G_s	G_p	R_s	R_p	G_e	R_e	K_p	K_e
$K_p - 1$	65	90	40	100	80	70	7,11, 15,19	20
$K_p - 2$	40	100	60	75	90	80	7,11, 15,19	20
$K_e - 1$	60	90	40	90	80	70	10	11,14, 17,20
$K_e - 2$	40	90	60	90	70	80	10	11,14, 17,20

Source: Authors' processing.

Figures 4 and 5 show that in Case 1 merchants O and Y always converge to different channels, whatever the parameter values, though their tendencies shift with the parameters.

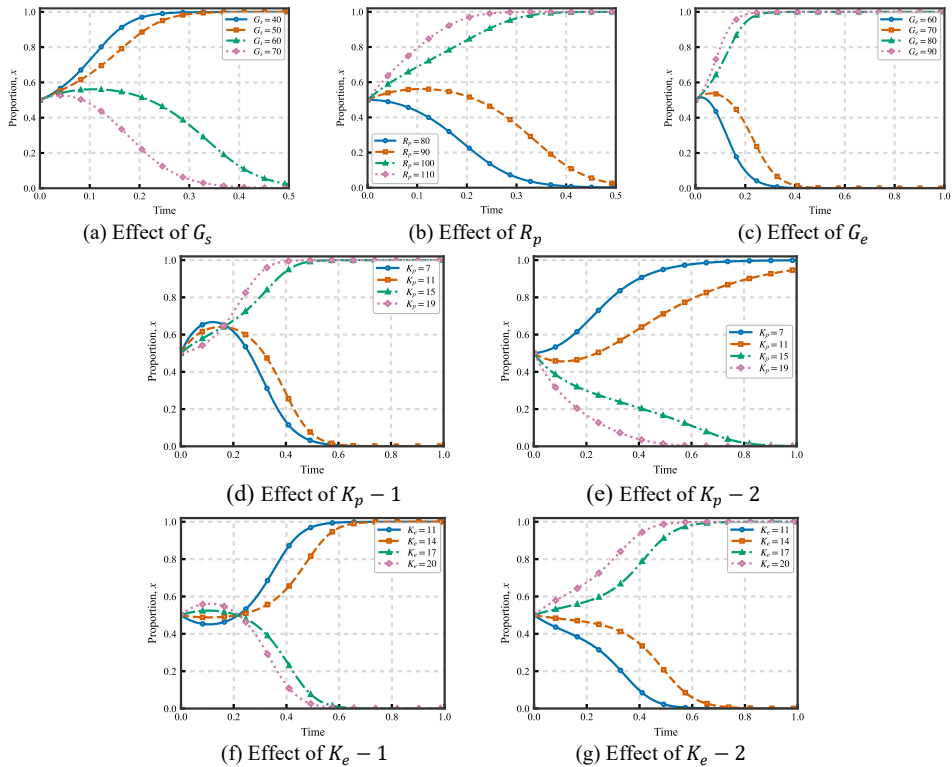


Figure 4. Parameter effects on the equilibrium evolution of merchant O, Case 1

Source: Authors' own creation.

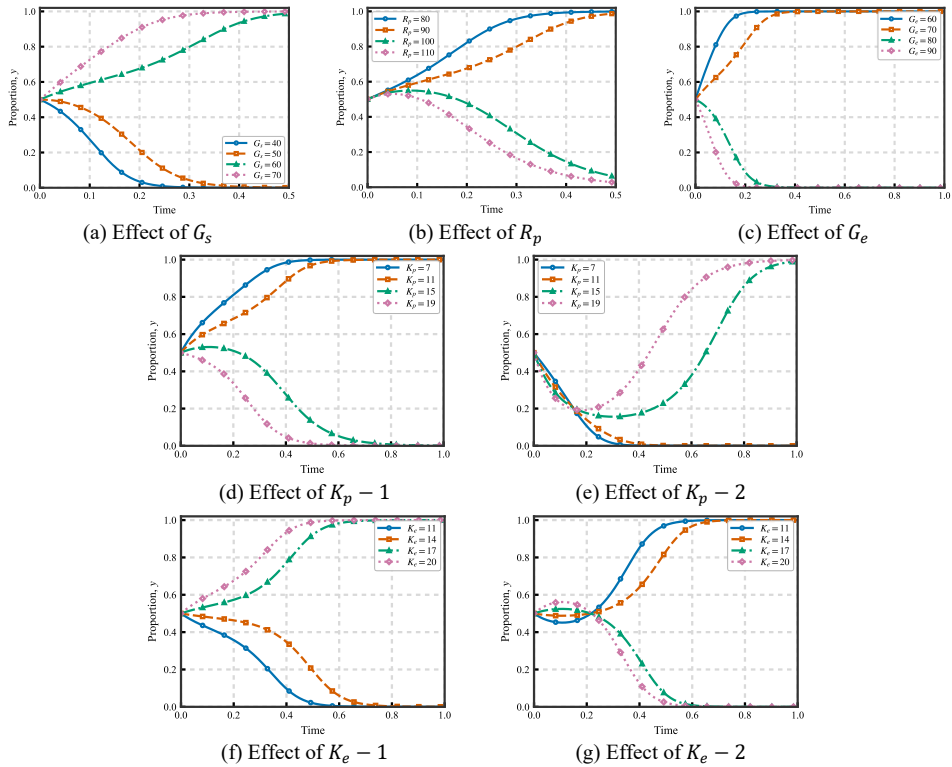


Figure 5. Parameter effects on the equilibrium evolution of merchant Y, Case 1

Source: Authors' own creation.

Take Figure 4(a) and 5(a). At $G_s = 40$ merchant Y gains little from a shared session and moves to the special channel while merchant O keeps the mixed one; at $G_s = 50$ the same outcome arrives more slowly. At $G_s = 60$ and 70 the ranking reverses. Larger G_s thus accelerates convergence towards whichever channel each merchant prefers.

In Case 1, an increase in G_s over a limited range does not change the direction of evolution, so G_s has a negative incentive effect. Beyond that range, merchant O and merchant Y evolve in opposite directions and G_s has a positive incentive effect on the system's evolution. For the remaining parameters in Figure 4 and Figure 5, as R_p and G_e , merchant O gradually shifts to mixed live streaming from special live streaming, which is opposite to the effect of G_s , consistent with the calculations in Section 3.2. Regarding the effects of K_p and K_e , the results reverse across conditions, again as calculated.

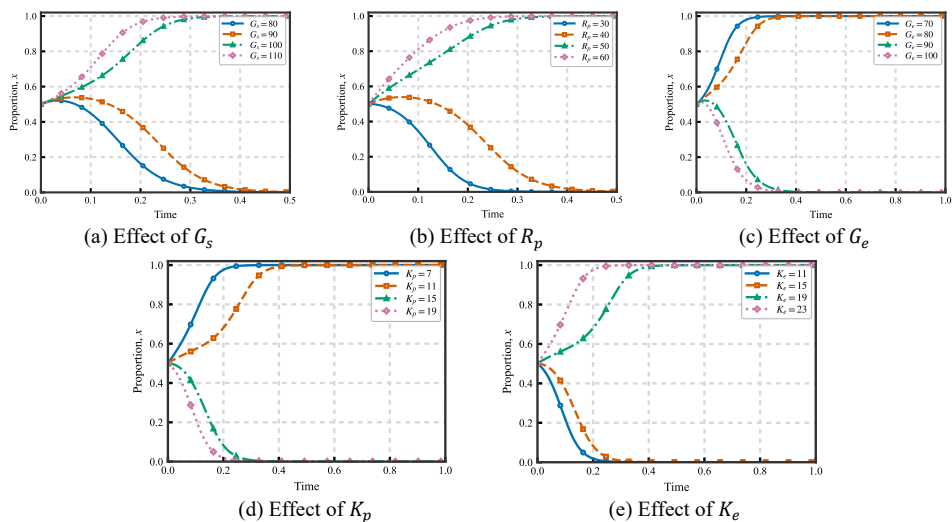
We now turn to Case 2, where $G_s > G_p$ and $R_s > R_p$, the initial x and y remaining 0.5. Table 8 describes the different parameter values in detail.

Table 8. Parameter values in Case 2

	G_s	G_p	R_s	R_p	G_e	R_e	K_p	K_e
G_s	80,90, 100,110	35	90	40	80	70	10	20
R_p	90	35	90	30,40, 50,60	80	70	10	20
G_e	100	35	90	40	70,80, 90,100	70	10	20
K_p	100	35	90	40	80	70	7,11, 15,19	20
K_e	100	35	90	40	80	70	10	11,15, 19,23

Source: Authors' processing.

Figures 6(a) and 7(a) vary G_s over 80, 90, 100 and 110 in Case 2, other parameters held fixed. At $G_s = 80$ the joint mixed payoff exceeds the solo mixed payoff, yet both merchants settle on special live streaming and engage anchors to promote their own brands exclusively; the fee is higher but exposure is undivided. Raising G_s to 90 leaves that outcome intact and only slows the approach of x and y to zero.

**Figure 6. Parameter effects on the equilibrium evolution of merchant O, Case 2**

Source: Authors' own creation.

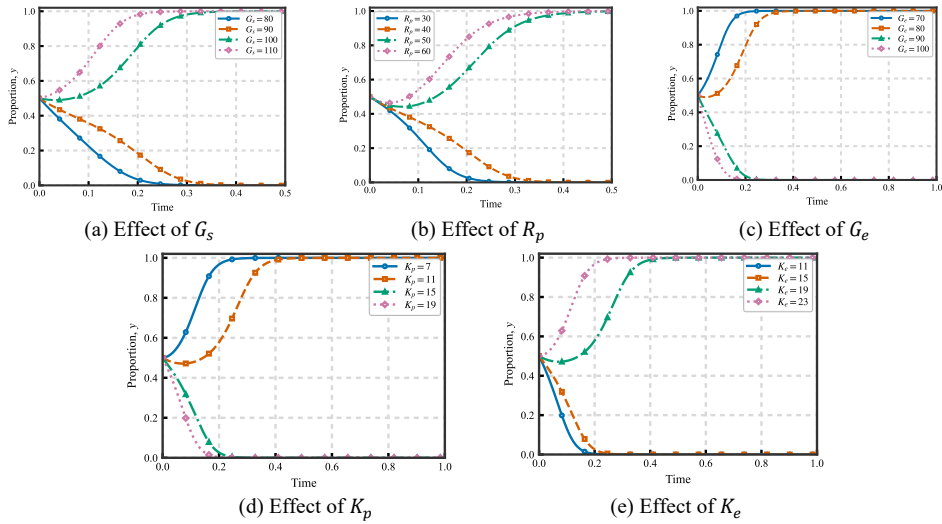


Figure 7. Parameter effects on the equilibrium evolution of merchant Y, Case 2

Source: Authors' own creation.

As G_s increases to 100 and 110, both merchants shift from special to mixed live streaming: within this parameter range the gains from a shared assortment and a shared fee outweigh the loss of exclusive attention.

In Case 2, moderate increases in G_s leave the direction of evolution unchanged; beyond a threshold G_s reverses it. Higher G_s also favours merchant Y's choice of mixed streaming. Similar patterns hold for R_p , G_e , K_p , and K_e .

5. Conclusions and Implications

We have modelled merchants' choice between mixed and special live streaming as an evolutionary game between two competing populations. The replicator dynamics yield five equilibria whose stability, as Remark 1 makes explicit, is governed by two composite quantities per merchant: the net advantage of mixed over special streaming against a rival who streams alone, and the co-presence externality created when that rival joins the same session. Within the conditions set out in Section 3.2, the sign pattern of the two externalities distinguishes three regimes: separation when both are negative, coordination on a common channel when both are positive, and perpetual cycling when they differ in sign. Simulation reproduces the three regimes and shows that a parameter change moves the interior equilibrium, and with it the boundary between the two regions of attraction; the outcome reverses when that moving boundary passes the initial state, and at a few endpoints the sweep also reaches a condition boundary.

These results complement rather than repeat the existing channel literature. Zhang and Tang (2023) and Liu et al. (2024) establish when a single seller should open a live streaming channel, and Huang et al. (2024) and Lu and Duan (2024)

extend the adopt-or-not decision to competing sellers; in all of them the rival affects demand but never enters the seller's own broadcast. Our setting begins where theirs ends, and the comparative statics differ in kind. In an adoption model a lower streaming cost makes the channel more attractive to both firms. Here a lower mixed session fee K_p raises the likelihood of a mixed outcome for one merchant and lowers it for the other whenever the two externalities differ in magnitude, because K_p enters the basin boundary through $1/(G_s - G_p) - 1/(R_s - R_p)$. A uniform fee cut therefore has no uniform effect.

Three rules follow, each conditional on the sign of the co-presence externality in the category concerned. For merchants, that sign rather than the fee schedule is the operative diagnostic: where it is negative, as it plausibly is where assortments are homogeneous and switching costs low, the model gives separation, and following a rival into its session is a move the dynamics will undo; where it is positive, co-presence pools traffic and the merchants should expect to share a channel. For streamer agencies, the same diagnostic works as a curation rule: within the interior-regime conditions a shared session is stable only for brands whose externality is positive, and a fee discount moves the interior equilibrium without changing that sign, so a modest discount will not by itself retain direct rivals; a large enough change in the fee can, however, carry the system into a dominance region. For platforms, the coordination regime is path dependent, since both {mixed, mixed} and {special, special} are stable and the initial distribution of choices selects between them; a subsidy that moves an early share of merchants across the interior point can therefore select the standard, although the model prices neither the subsidy nor its cost.

The anchor fee is treated as exogenous and identical for both merchants within a channel. Endogenising it, allowing merchants to differ in brand strength, and admitting traditional e-commerce channels alongside the two live streaming channels are natural extensions, as is incorporating logistics and demand risk.

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