

Serkan AKBAŞ, PhD (corresponding author)

serkanakbas@ktu.edu.tr

Karadeniz Technical University, Trabzon, Turkey

Türkan ERBAY DALKILIÇ, PhD

terbay@ktu.edu.tr

Karadeniz Technical University, Trabzon, Turkey

Yeşim AKBAŞ, PhD

yesimyeginoglu@ktu.edu.tr

Karadeniz Technical University, Trabzon, Turkey

Portfolio Optimization with a Multi-Objective Weighted Fuzzy Goal Programming Approach

Abstract. *Technological developments and increasing complexity in financial markets have highlighted the need to consider various financial criteria in addition to risk and return in portfolio management. This study proposes a portfolio selection model integrating the CRITIC method with Multi-Objective Weighted Fuzzy Goal Programming. Criterion weights are determined using CRITIC by considering the variability of each criterion and the correlations among criteria. The criteria are then formulated as fuzzy goals, and membership functions are constructed using target and tolerance values to represent uncertainty. The CRITIC weights are incorporated into the fuzzy goal programming model, while different values of the θ parameter are used to generate alternative portfolio allocations. The proposed approach is applied to NASDAQ-100 stocks and compared with CVaR, Mean-Variance, and ASM methods. Performance is evaluated using portfolio returns, Sharpe and Sortino ratios, and rolling-window analysis. The results show that the proposed method provides favorable performance compared with the other portfolio optimization methods in terms of both returns and risk-adjusted performance. Overall, the findings indicate that the proposed model provides an effective and applicable approach to multi-criteria portfolio selection under uncertainty.*

Keywords: *portfolio optimization, fuzzy logic, goal programming, CRITIC method, Multi-Criteria Decision Making.*

JEL Classification: C44, C61, G11.

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1. Introduction

Portfolio selection has emerged as one of the keys focuses on modern finance, driven by advancements in the economic world. It involves creating the optimal investment mix based on specific constraints and the investor's expectations. Both traditional and modern portfolio strategies are employed in constructing this

investment set. Markowitz (1952), the pioneer of the modern portfolio approach, added a mathematical dimension to portfolio selection with his development of the mean-variance model, known as the basis of modern portfolio theory. There are many studies on portfolio management by researchers in literature. In today's evolving and liberalizing financial markets, a variety of sophisticated techniques are employed to construct portfolios that aim to maximize returns within acceptable risk parameters for investors.

Genç and Çelik used the Konno-Yamazaki model to optimize their portfolios in their studies (Genç & Çelik, 2009). Stummer et al. were the first to introduce the Multi-Criteria Decision Support System, which identifies the set of Pareto-efficient solutions and then enables the decision maker to interactively filter and explore these solutions in different ways (Stummer, Kiesling, & Gutjahr, 2009). Ceren and Köksalan investigated the effects of simultaneous evaluation of different criteria on portfolio optimization by using various combinations of expected return, variance, liquidity and CVaR criteria (Ceren & Köksalan, 2014). He et al., presented a new multi-task learning model that evaluates portfolio risk under the Mean-VaR decision-making framework by combining estimated returns and residual data to create a robust portfolio, in their study (He, Hong, Xu, & Li, 2024).

The complexity of the portfolio selection process due to technological developments in financial markets requires decision makers to quickly determine the most appropriate solution method. To cope with such situations, methods based on Multi-Criteria Decision Making (MCDM) techniques have begun to be used. MCDM is a sophisticated approach used to address complex problems by evaluating alternatives based on multiple conflicting criteria. MCDM methods are effectively applied in various fields such as environmental management (Fatemi & Rezaei-Moghaddam, 2019), network planning (Arregui, Salvador, & Vázquez, 2017), natural resource management (Peri & Tal, 2020) and economics (Jana, Roy, & Mazumder, 2009). Since the purpose of the portfolio selection process is to maximize the return on investment, minimize the risk and bring the specific financial ratios to optimal levels, portfolio selection is one of the areas where MCDM methods are effectively used. There are many studies in literature where MCDM methods and portfolio selection processes are used together (Liu, Zhang, & Xu, 2012; Mansour, Cherif, & Abdelfattah, 2019; Marasović & Babić, 2011; Meghwani & Thakur, 2017). Li et al. used three criteria, such as terminal wealth, potential risk value and skewness in their proposed portfolio selection model and designed a novel intelligent hybrid algorithm to solve this model (Li, Wu, Lu, Wang, & Hu, 2021). Jiang et al. focused on the remanufacturing technology portfolio and used the AHP method to weight the six criteria they determined while creating the portfolio (Jiang, Zhang, & Sutherland, 2011). Ho et al. proposed an MCDM model for portfolio selection that includes Decision Making Experimentation and Evaluation Laboratory (DEMATEL), Analytical Network Process (ANP) and VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Jerry Ho, Tsai, Tzeng, & Fang, 2011).

One of the techniques developed by Charnes et al. is goal programming, which provides effective solutions for multi-objective models that consider more than one objective (Charnes, Cooper, & Ferguson, 1955). The first goal programming model in finance was developed by Lee and Lerro (Lee & Lerro, 1973). MCDM techniques are used to determine the objective function weights of a portfolio selection problem to be modeled as a goal programming problem. DEMATEL, Entropy, SWARA and Criteria Importance Through Intercriteria Correlation (CRITIC) methods are the most common methods used for weighting. In their study, Deng and Chen proposed to use the Entropy method to determine the criteria weights in the portfolio selection problem, which they considered as an MCDM problem under intuitive fuzzy conditions when no previous data is available (Deng & Chen, 2020). Comparisons based on consideration of more than one financial ratio can be handled by the MCDM. Lu et al. calculated a combined weight value by merging the weights derived from the CRITIC method with those obtained from the entropy weight method, as part of developing a comprehensive evaluation index system for selecting agricultural machinery (Lu, Zhao, Zhou, & Wei, 2022). In their study, Krishnan et al. developed the Distance Correlation Based CRITIC (D-CRITIC) method, which is a modified version of the CRITIC method (Krishnan, Kasim, Hamid, & Ghazali, 2021).

Since financial markets are instantly affected by social and political changes, there is uncertainty in the portfolio selection process. This uncertainty in financial markets also causes uncertainty in risk and return criteria that are effective in portfolio selection. In such uncertainties, the fuzzy logic approach, which is an effective method for portfolio optimization, is preferred. Fuzzy logic, first introduced by Zadeh in 1965 (Zadeh, 1965), is used to control processes in complex and uncertain situations. Akbaş and Dalkılıç, proposed a new model that uses the trapezoidal fuzzy numbers for criteria weighting in portfolio selection model as an alternative to the Enea and Piazza's portfolio selection model, which uses the triangular fuzzy numbers (Akbaş & Dalkılıç Erbay, 2019). Garcia et al. developed a multi-objective model that evaluates portfolio performance by considering return, risk, and liquidity. In their model, both return and liquidity are represented as trapezoidal fuzzy numbers (Garcia, González-Bueno, Guijarro, Oliver, & Tamošiūnienė, 2020). In their study, Akbaş and Dalkılıç developed a two-stage model for portfolio selection by considering investment data and expert opinions using trapezoidal fuzzy numbers (Akbaş & Erbay Dalkılıç, 2021). Mehlawat et al. proposed two fuzzy multi-objective portfolio selection models using liquidity, entropy and risk as objectives (Mehlawat, Gupta, Kumar, Yadav, & Aggarwal, 2020). In their study, Yaman and Dalkılıç proposed a hybrid algorithm that combines a nonlinear neural network and a genetic algorithm to address the mean-variance cardinality constraint portfolio optimization problem (Yaman & Erbay Dalkılıç, 2021).

In portfolio selection processes, the situation where the goal parameters related to the criteria are not definite brings about the situation where the right-side values related to the constraints of the model to be established in the form of goal

programming for the solution are fuzzy. Thus, the problem becomes a fuzzy goal programming problem. There are many suggestions in the literature for modeling and solving the fuzzy goal programming problem. In the studies of Gupta and Bhattacharjee, the problem of fuzzy goal programming with different importance levels was considered to determine the desired and realistic solutions for each goal, and in order to obtain efficient solutions, the weighted fuzzy goal programming method was proposed, where the weights were connected to the fuzzy operator in the constraint and only the deviation variables were introduced in the goal constraint (Gupta & Bhattacharjee, 2012). In their studies, Bilbao et al. proposed a Goal Programming model that includes uncertain investor goals regarding the ratios of both high and low risk assets to select an optimal portfolio and applied it to a real portfolio selection problem (Bilbao, Arenas, Jiménez, Gladish, & Rodríguez, 2006). Jawad et al. developed an algorithm by transforming the Fuzzy AHP (FAHP) into Spherical Fuzzy AHP (SFAHP), taking advantage of AHP and The Spherical Fuzzy Sets (SFS), in their study (Jawad, Naz, & Muqaddus, 2024).

In their studies, Parra et al. formulated a goal problem where objectives and constraints are fuzzy for the optimum portfolio selection process by considering unclear return, risk and liquidity criteria from the investor's perspective (Arenas Parra, Bilbao Terol, & Rodríguez Uría, 2001). Messaoudi et al. proposed a fuzzy chance constrained goal programming model to solve a multi-attribute financial portfolio selection problem under randomness and fuzziness and applied it to the Tunisian stock market during the period between July 2003 and December 2007 (Messaoudi, Aouni, & Rebai, 2017). The model presented by Jimenez et al. is a model that can be applied to portfolio selection problems by using the fuzzy goal programming approach and the standard goal programming approach together, allowing fuzzy and crisp goals to be included in the same problem (Jimenez, Bilbao-Terol, & Arenas-Parra, 2018).

This study proposes a novel portfolio selection method based on Multi-Objective Weighted Fuzzy Goal Programming (MOWFGP) to assist investors planning to invest in financial markets in making effective portfolio selection decisions. The proposed method offers an integrated solution process that enables the data-driven determination of the weights of the criteria considered in portfolio selection and the inclusion of uncertainties in market conditions into the model using a fuzzy goal programming approach. The method essentially consists of two stages. In the first stage, the criteria to be used in the portfolio selection problem are determined and a dataset related to these criteria is created. Then, the weights (w_j) of the criteria are calculated using the CRITIC method proposed by Diakoulaki et al. (Diakoulaki, Mavrotas, & Papayannakis, 1995). The CRITIC method offers a more objective approach compared to subjective weighting methods, as it determines the weights directly from the data by considering the comparability of the criteria and the correlation relationships between the criteria without requiring the subjective evaluations of the decision-maker. In the second stage, the obtained criterion weights are used as the objective function coefficients within the MOWFGP model of the portfolio selection problem. In this context, the medians of the observed values

related to the criteria are determined as target values, and tolerance values are calculated to reflect the uncertainty of the targets in the model. For targets that need to be maximized, the tolerance values are determined by considering the difference between the maximum value and the median value of the relevant criterion, and for targets that need to be minimized, the difference between the median value and the minimum value. Thus, instead of assuming that the targets are definitely met, a fuzzy structure is created that allows deviations from the targets within the determined tolerance ranges. An important feature of the proposed method is the systematic modification of the right-hand side values of the fuzzy targets through the θ parameter. By taking different values of θ in the range $[0,1]$, different fuzzy target programming models are created, and alternative portfolio distributions are obtained by solving each model. Thanks to this process, a series of alternative portfolio solutions that take into account the uncertainty levels of the targets in different ways are obtained, and the scenario that provides the maximum return among these solutions is determined as the optimum portfolio solution. To evaluate the effectiveness of the proposed algorithm, the developed solution process was applied to stocks included in the National Association of Securities Dealers Automated Quotations (NASDAQ-100) index. The resulting portfolio allocations were compared with the Conditional Value at Risk (CVaR), Mean-Variance, and ASM methods, which are widely used in the literature. The experimental results show that the proposed method can be an effective alternative to classical portfolio optimization methods because it considers not only return and risk criteria but also different financial ratios that may affect portfolio selection; determines criterion weights based on data; and includes market uncertainties in the model using a fuzzy goal programming approach. Furthermore, the ability to obtain alternative portfolios corresponding to different investment conditions by systematically changing the θ parameter allows the proposed method to be adapted to the different risk-return preferences of investors.

The rest of the article is as follows: Chapter 2 deals with portfolio optimization. Chapter 3 discusses the CRITIC method in detail. Chapter 4 introduces FLP and MOWFGP, and Chapter 5 presents a step-by-step solution algorithm using the proposed Fuzzy Goal Programming model for portfolio selection. Chapter 6 includes a numerical example where the proposed algorithm is applied and compared with existing methods. Finally, Chapter 7 presents the conclusions.

2. Portfolio Optimization

In financial terms, a portfolio can be defined as a collection of stocks and other investment instruments purchased by a person or institution. Portfolio management can be defined as the process of determining the financial assets that should be included in the portfolio and applying certain methods to determine the proportions in which these assets should be included in the portfolio to obtain the highest expected return with the lowest risk (Baker & Filbeck, 2013).

In the finance literature, one of the two basic approaches that allow successful portfolio creation is the Traditional Portfolio Approach, which is based on the diversification of securities, and the other is the Modern Portfolio Approach, which is based on mathematical foundations. The modern portfolio approach addresses the portfolio that characterizes securities rather than individually. According to the investment approach of Harry Markowitz, the founder of the Modern Portfolio Approach; The investor should make a prediction that will increase the expected return as much as possible and reduce the variance of the returns, in other words, the risk (Markowitz, 1952). According to this approach, creating an optimal portfolio from securities is possible through a mathematical modeling and solution process. Such a portfolio optimization is one of the most common applications of the modern portfolio approach. The most used modern approaches for portfolio optimization are Mean-Variance, CVaR and Active-set method (ASM). The mean-variance model developed by Markowitz enabled the quantification of risk and defined the portfolio selection process as a linear programming problem by combining an objective function that will minimize the risk to obtain the ratio of assets in the portfolio, constraints that will ensure that the expected return of the portfolio is equal to a certain value and that the sum of the ratios of the selected assets in the portfolio is 1.

CVaR is a risk measure used for risk management in financial markets. This measure calculates the average loss that can occur beyond a certain confidence interval. CVaR expresses the average loss beyond the VaR at a certain confidence level. CVaR is a useful measure since it is a convex risk measure in optimization problems and can be applied in portfolio optimization processes due to this feature.

One of the methods used to reduce portfolio risk and provide the best asset allocation in line with certain return goals is ASM. This method is an effective iterative method for solving optimization problems that include inequality constraints. And with this feature, it is frequently used in solving portfolio optimization problems that can be expressed as constrained linear or nonlinear optimization problems. The Markowitz optimization model, which will allow investors to obtain a certain return level by minimizing their risk, can be solved with the ASM method and portfolio distributions can obtain optimally.

In addition to portfolio returns, risk-adjusted performance measures provide a more comprehensive assessment of portfolio performance by considering the return achieved relative to the risk undertaken. Among these measures, the Sharpe and Sortino ratios are widely used to evaluate portfolio performance from different perspectives of risk. The Sharpe ratio (SR) measures the excess return of a portfolio relative to its total risk and is calculated using Eq. (1).

$$SR = (R_p - R_f) / \sigma_p, \quad (1)$$

where R_p denotes the portfolio return, R_f denotes the risk-free rate, and σ_p represents the standard deviation of portfolio returns. A higher Sharpe ratio indicates a higher excess return per unit of total risk. Unlike the Sharpe ratio, which considers

total return variability, the Sortino ratio focuses specifically on downside risk by considering unfavorable deviations below a minimum acceptable return. Therefore, it does not penalize positive return fluctuations and provides an assessment of portfolio performance relative to downside risk. The Sortino ratio (SoR) is calculated using Eq. (2).

$$SoR = (R_p - MAR) / \sigma_d, \quad (2)$$

where R_p denotes the portfolio return, MAR represents the minimum acceptable return, and σ_d denotes the downside deviation of portfolio returns below the minimum acceptable return. A higher Sortino ratio indicates better performance relative to downside risk. Unlike the Sharpe ratio, which penalizes both positive and negative deviations through total volatility, the Sortino ratio focuses specifically on unfavorable deviations below the minimum acceptable return (Bruni, Cesarone, Scozzari, & Tardella, 2017).

3. CRITIC Method

Since the model to be created for the portfolio selection process will be a weighted multi-objective programming model, the problem of determining the weights of the criteria to be used in the problem is encountered. This problem can be solved by using one of the weighting methods such as the DEMATEL, Entropy, SWARA, CRITIC method, which are multi-criteria decision-making techniques defined in literature. These methods can be classified as subjective methods that are directly based on the decision maker's opinion and objective methods that do not require decision makers' evaluations by using the data in the decision matrix. In this study, the CRITIC method, which calculates the weight coefficients of the deviation variables related to the criteria to be included in the objective function of the portfolio optimization problem to be modelled as a goal programming problem, by using the numerical parameters of the data without requiring decision maker information, was used. The values given by the decision matrix regarding the criteria are normalized using Eq. (3) and Eq. (4) for benefit and cost-oriented criteria, respectively.

$$r_{ij} = \frac{x_{ij}}{\max(x_{ij})} \quad (3)$$

$$r_{ij} = \frac{\min(x_{ij})}{x_{ij}} \quad (4)$$

By calculating the Pearson correlation coefficients between the normalized criterion values, the measures of conflict values between the j^{th} criterion and other criteria are obtained with Eq. (5).

$$\sum_{k=1}^n (1 - l_{kj}) \quad (5)$$

C_j , the amount of information about the j^{th} criterion and is expressed by Eq. (6).

$$C_j = \sigma_j \sum_{k=1}^n (1 - l_{kj}) \quad (6)$$

here, l_{kj} represents the correlation coefficients between the criteria, σ_j represents the variance value within the criteria. The weights of the criteria are calculated using the information amounts C_j as given in Eq. (7).

$$w_j = \frac{C_j}{\sum_{j=1}^n C_j} \quad (7)$$

4. Multi-Objective Weighted Fuzzy Goal Programming

4.1 Fuzzy Linear Programming

In linear programming problems, the coefficients related to the objective function, the right-hand side values or both parameters may not consist of exact numbers. In such cases, the linear programming problem is modeled as a FLP problem by creating membership functions related to the mentioned parameters. In case the parameters are fuzzy, the decision variable values are obtained with the solution approaches in the literature. Since the situation of the multi-objective weighted programming problem to be established in this study having fuzzy right-hand side values is considered, Verdegay's approach was used as the solution method. The general model of the linear programming problem with fuzzy right-hand side values is given in Eq. (8).

$$\max z = cx$$

$$(Ax)_i \leq \tilde{b}_i, i = 1, 2, \dots, m \quad (8)$$

$$x \geq 0$$

The membership functions related to the fuzzy constraints of the general model given in Eq. (8) are defined as given in Eq. (9) and Eq. (10), respectively, depending on whether the fuzzy constraints are in the form of $\leq$ and $\geq$.

$$\mu_{i(\leq)}(x) = \begin{cases} 1 & \text{if } (Ax)_i < b_i \\ 1 - [(Ax)_i - b_i]/p_i & \text{if } b_i \leq (Ax)_i \leq b_i + p_i \\ 0 & \text{if } (Ax)_i > b_i + p_i \end{cases} \quad (9)$$

$$\mu_{i(\geq)}(x) = \begin{cases} 1 & \text{if } (Ax)_i > b_i \\ 1 + [(Ax)_i - b_i]/p_i & \text{if } b_i - p_i \leq (Ax)_i \leq b_i \\ 0 & \text{if } (Ax)_i < b_i - p_i \end{cases} \quad (10)$$

The membership function for the constraint to be maximized given by Eq. (9) is as given in Figure 1(a) and the membership function for the constraint to be minimized given by Eq. (10) is as given in Figure 1(b).

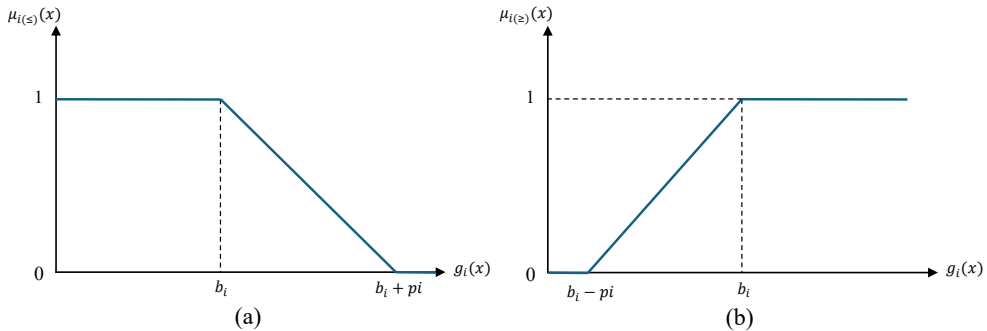


Figure 1. The Membership Functions of Constraints

Source: Illustration by authors.

When the membership functions given in Eq. (9) and Eq. (10) are substituted into the model given in Eq. (8), the problem is modeled as a parametric programming problem as given in Eq. (11).

$$\max z = cx$$

$$(Ax)_i \leq b_i + (\theta \times p_i), i = 1, 2, \dots, m \quad (11)$$

$$x \geq 0 \text{ and } \theta \in [0, 1]$$

here, p_i is the tolerance value for the right-side parameter b_i . Thus, the fuzzy linear programming problem given by Eq. (11) is equivalent to a strict parametric linear programming problem by using the forms of the membership functions of the fuzzy constraints given in Eq. (9) and Eq. (10).

4.2 Multi-Objective Weighted Fuzzy Goal Programming

In the MOWFGP problem, fuzzy goals are defined in the form of $G_k(x) \gtrsim b_k$ [for maximizing $G_k(x)$] and $G_k(x) \lesssim b_k$ [for minimizing $G_k(x)$] where, the signs " $\gtrsim$ " and " $\lesssim$ " correspond to the fuzzy version of " $\leq$ " and " $\geq$ ".

b_k represents the highest and lowest levels that the k^{th} goal can attain. Thus, the MOWFGP problem is modeled as given in Eq. (12).

$$\min \sum_{k=1}^n w_k (d_k^{\mp}) \quad (12)$$

$$G_k(x) + d_k^- - d_k^+ = b_k + (\theta \times p_k), \quad k = 1, \dots, l$$

$$G_k(x) + d_k^- - d_k^+ = b_k - (\theta \times p_k), \quad k = (l + 1), \dots, n$$

$$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b_{k+1}, x \geq 0 \text{ and } \theta \in [0,1]$$

$$\sum_{k=1}^n w_k = 1$$

here, $d_k^{\pm}$ corresponds to the deviation variables, p_k to the tolerance value, and w_k to the weights.

5. Proposed Algorithm

In this study, an algorithm that allows the use of the weighted fuzzy goal programming problem in portfolio selection is proposed. The modeled goal programming is formulated to provide a solution to the situation where the right-hand side values of the constraints of the problem are fuzzy. The coefficients of the objective function of the fuzzy goal programming model are obtained by the critic method. The steps of the proposed algorithm are as follows:

Algorithm 1: CRITIC-Based MOWFGP for Portfolio Selection

Input: Decision matrix $X = [x_{ij}]$, Criterion types (maximization/minimization)

Output: Optimal portfolio allocation x^* , Optimal θ value θ^* , Maximum portfolio return R^*

- 1: Construct the decision matrix X based on the portfolio selection criteria.
- 2: Normalize X according to the criterion types using Eqs. (3) and (4).
- 3: Calculate the correlation coefficients using Eq. (5).
- 4: Calculate the information amount C_j for each criterion using Eq. (6).
- 5: Calculate the CRITIC weights w_j using Eq. (7).
- 6: Determine the right-hand-side value b_k of each criterion as the median of its observations.
- 7: Determine the tolerance value p_k for each criterion
- 8: **for** k : 1 to n
- 9: if criterion k is to be maximized then
- 10: $p_k = \max(x_{ik}) - b_k$
- 11: else
- 12: $p_k = b_k - \min(x_{ik})$
- 13: end if
- 14: end
- 15: **for** $\theta = 0$ to 1 with an increment of 0.1 do
- 16: **for** $k = 1, \dots, n$ do
- 17: if criterion k is to be maximized then
- 18: $b_k(\theta) \leftarrow b_k + \theta p_k$
- 19: else

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20:          $b_k(\theta) \leftarrow b_k - \theta p_k$ 
21:     end if
22: end for
23: Formulate and solve the MOWFGP model using Eq. (12).
24: Obtain the portfolio allocation  $x(\theta)$  and calculate the portfolio return  $R(\theta)$ .
25: end for
26: Select  $x^*, \theta^*$ , and  $R^*$  corresponding to the highest  $R(\theta)$ .
27: return  $x^*, \theta^*, R^*$ .

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6. Numerical Example

In this section, to examine the effectiveness of the proposed method, the portfolio allocation was optimized using the financial data determined as the criterion for 100 stocks in the NASDAQ-100 index between 01.01.2021-31.12.2023, and the portfolio allocations were determined and compared with the CVaR, Mean Variance and ASM methods available in literature.

6.1 Data

The 42-month return rates of the stocks listed on the NASDAQ-100 index were obtained from <https://tr.investing.com/>. The average return rates for the first 36 months, the risk values calculated from the return rates, and the values for Earnings Per Share (EPS), Gross Profit Margin (GPM), Return on Assets (ROA), Book Value Per Share (BVPS), and Earnings Yield (EY) ratios, which were also obtained from the same source, are provided in Table 1. Table 1 also includes the maximization/minimization status of the criteria used during the process. The financial ratio data for the 6 months after the 36-month period was used as test data to examine the effectiveness of the methods.

Table 1. Criteria Values Corresponding to assets for the NASDAQ-100

		Return	Risk	EPS	GPM	ROA	BVPS	EY
	Assets	max	min	max	max	max	max	max
x_1	AXON	0.0282	0.1264	3.9400	59.8000	8.2000	27.7900	0.4000
x_2	ADSK	-0.0014	0.1003	5.0800	92.0000	11.3000	12.1700	1.9000
x_3	TEAM	0.0113	0.1466	-1.5100	81.5000	-8.7000	3.9600	-0.6000
x_4	AZN	0.0099	0.0564	4.1900	82.6000	6.5000	26.2700	4.9000
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{97}	WBD	-0.0135	0.1719	-4.5900	41.6000	-9.8000	14.3200	-14.6000
x_{98}	WDAY	0.0090	0.1016	6.1200	75.6000	10.7000	32.6800	2.5000
x_{99}	XEL	-0.0004	0.0574	3.3700	45.4000	2.8000	34.7100	4.9000
x_{100}	ZS	0.0128	0.1468	-0.2400	78.0000	-0.9000	9.3700	-0.1000

Source: <https://tr.investing.com/>.

6.2 Obtaining the Weights for the Model with the CRITIC Method

To obtain the coefficients of the objective function of the portfolio optimization problem to be modeled in the form of fuzzy goal programming using the CRITIC method, the normalized values regarding the criteria determined using Eq. (3) and Eq. (4) are given in Table 2.

Table 2. Normalized Values for the Criteria Related to NASDAQ100 assets

	Assets	Return	Risk	EPS	GPM	ROA	BVPS	EY
x_1	AXON	0.5572	0.3740	0.0266	0.6479	0.0976	0.1023	0.0377
x_2	ADSK	-0.0284	0.4715	0.0343	0.9967	0.1345	0.0448	0.1792
x_3	TEAM	0.2239	0.3225	-0.0102	0.8830	-0.1036	0.0146	-0.0566
x_4	AZN	0.1952	0.8382	0.0283	0.8949	0.0774	0.0967	0.4623
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{97}	WBD	-0.2676	0.2749	-0.0310	0.4507	-0.1167	0.0527	-1.3774
x_{98}	WDAY	0.1773	0.4651	0.0414	0.8191	0.1274	0.1203	0.2358
x_{99}	XEL	-0.0086	0.8241	0.0228	0.4919	0.0333	0.1278	0.4623
x_{100}	ZS	0.2542	0.3219	-0.0016	0.8451	-0.0107	0.0345	-0.0094
	Total	25.7486	52.1439	5.3080	62.3554	12.3690	10.8079	28.0755
	Average	0.2575	0.5214	0.0531	0.6236	0.1237	0.1081	0.2808
	Std. Dev.	0.2339	0.1767	0.1118	0.2060	0.1406	0.1468	0.3108

Source: Calculation made by authors.

The correlation between normalized criterion values is calculated using the Pearson correlation coefficient and is given in Table 3.

Table 3. Correlation Matrix for Criteria

	Return	Risk	EPS	GPM	ROA	BVPS	EY
Return	1.0000	-0.2721	0.0887	0.0822	0.3383	-0.0047	0.0043
Risk	-0.2721	1.0000	0.0365	-0.1906	-0.0437	0.1102	0.3627
EPS	0.0887	0.0365	1.0000	0.0883	0.2284	-0.0383	0.1921
GPM	0.0822	-0.1906	0.0883	1.0000	0.0967	-0.0551	-0.1260
ROA	0.3383	-0.0437	0.2284	0.0967	1.0000	-0.1130	0.2634
BVPS	-0.0047	0.1102	-0.0383	-0.0551	-0.1130	1.0000	0.1811
EY	0.0043	0.3627	0.1921	-0.1260	0.2634	0.1811	1.0000

Source: Calculation made by authors.

The measures of conflict values between the j^{th} criterion and other criteria were calculated with Eq. (5) and are given in Table 4.

Table 4. Matrix of the Measures of Conflict Values for Criteria

	Return	Risk	EPS	GPM	ROA	BVPS	EY
Return	0.0000	1.2721	0.9113	0.9178	0.6617	1.0047	0.9957
Risk	1.2721	0.0000	0.9635	1.1906	1.0437	0.8898	0.6373
EPS	0.9113	0.9635	0.0000	0.9117	0.7716	1.0383	0.8079
GPM	0.9178	1.1906	0.9117	0.0000	0.9033	1.0551	1.1260
ROA	0.6617	1.0437	0.7716	0.9033	0.0000	1.1130	0.7366
BVPS	1.0047	0.8898	1.0383	1.0551	1.1130	0.0000	0.8189

	Return	Risk	EPS	GPM	ROA	BVPS	EY
EY	0.9957	0.6373	0.8079	1.1260	0.7366	0.8189	0.0000
Total	5.7634	5.9971	5.4043	6.1045	5.2298	5.9198	5.1224

Source: Calculation made by authors.

The information amount C_j calculated with Eq. (7) and the w_j weights calculated with Eq. (8) are as given in Table 5.

Table 5. C_j and w_j values

	Return	Risk	EPS	GPM	ROA	BVPS	EY	Total
C_j	1.3480	1.0597	0.6042	1.2574	0.7351	0.8693	1.5919	7.4656
w_j	0.1806	0.1419	0.0809	0.1684	0.0985	0.1164	0.2132	

Source: Calculation made by authors.

6.3 Mathematical Model of the Problem

While creating the mathematical model of the problem, the fuzzy goals to be converted into constraints are given in Table 6. The median values calculated for the criteria determined to be used in the portfolio distribution were used as the right-side value for the goal constraints to be created. In this study, since the situation of the goals on the constraints being uncertain is addressed, the constraint inequalities related to the goals are expressed as fuzzy.

Table 6. Constraints on Fuzzy Goals for Criteria

Goals	Criteria	Constraints on fuzzy goals for criteria
Goal 1 (Max)	Return	$0.0282x_1 - 0.0014x_2 + \dots - 0.0004x_{99} + 0.0128x_{100} \gtrsim 0.0114$
Goal 2 (Min)	Risk	$0.1264x_1 + 0.1003x_2 + \dots + 0.0574x_{99} + 0.1468x_{100} \lesssim 0.0935$
Goal 3 (Max)	EPS	$3.9400x_1 + 5.0800x_2 + \dots + 3.3700x_{99} - 0.2400x_{100} \gtrsim 4.2200$
Goal 4 (Max)	GPM	$59.80x_1 + 92.0000x_2 + \dots + 45.4000x_{99} + 78.0000x_{100} \gtrsim 57.2500$
Goal 5 (Max)	ROA	$8.2000x_1 + 11.3000x_2 + \dots + 2.8000x_{99} - 0.9000x_{100} \gtrsim 8.5500$
Goal 6 (Max)	BVPS	$27.7900x_1 + 12.1700x_2 + \dots + 34.7100x_{99} + 9.3700x_{100} \gtrsim 20.1850$
Goal 7 (Max)	EY	$0.4000x_1 + 1.900x_2 + \dots + 4.9000x_{99} - 0.1000x_{100} \gtrsim 3.1000$

Source: Calculation made by authors.

The median values calculated for the criteria determined to be used in the portfolio distribution were used as the right-side value for the goal constraints to be created. In this study, since the situation of the goals on the constraints being uncertain is addressed, the constraint inequalities related to the goals are expressed as fuzzy. To convert the fuzzy goals into deterministic constraints in the solution phase, the tolerance values to be added to the right-side values determined as fuzzy are determined.

Tolerance values are determined as the difference between the maximum and median values of the criterion for fuzzy goals to be maximized, and as the difference between the median and minimum values for fuzzy goals to be minimized. Tolerance values for the problem are as given in Table 7.

Table 7. Tolerance Amounts for Fuzzy Goals

	Return	Risk	EPS	GPM	ROA	BVPS	EY
Assets	max	min	max	max	max	max	max
Max	0.0505	0.2896	147.9500	92.3000	84.0000	271.7000	10.6000
Min	-0.0285	0.0473	-21.0300	12.7000	-25.6000	-108.9700	-14.6000
Median	0.0114	0.0935	4.2200	57.2500	8.5500	20.1850	3.1000
Tolerance	0.0391	0.0462	143.7300	35.0500	75.4500	251.5150	7.5000

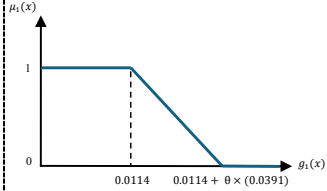
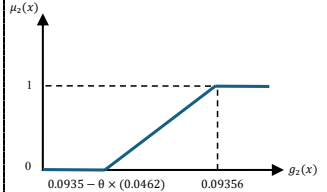
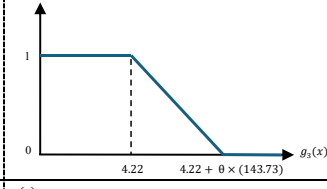
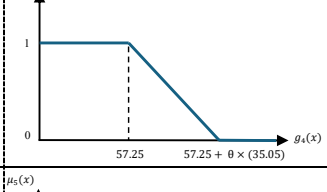
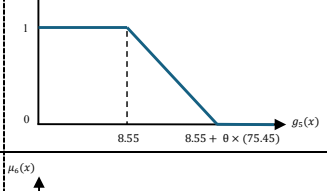
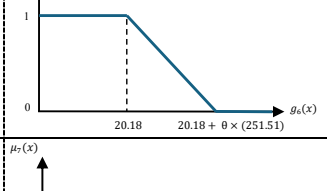
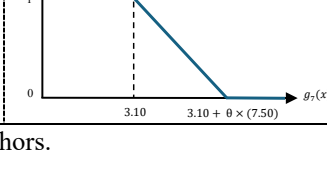
Source: Calculation made by authors.

By using the tolerance values p_k and the θ parameter, the goals expressed as fuzzy constraints were transformed into goal constraints, as given in Table 8.

To standardize the goal constraints, the constraints in the form of inequality were converted to the form of equality by adding the deviation variables expressed as $d_j^\pm$ to the constraints. The objective function of the problem is created by including the negative deviations related to the Return, EPS, GPM, ROA, BVPS and EY rates that are desired to be maximized and the positive deviations related to the risk rates that are desired to be minimized. In the objective function created to minimize the unwanted deviation variables in total, the w_j weights obtained with the CRITIC method and given in Table 5 were used as coefficients. Each criterion is expressed as a goal constraint using the right-side values converted to deterministic form using the tolerance values given in Table 7. With the addition of the constraint expressing that the x variable values corresponding to the portfolio distribution must be 1 in total, the weighted MOWFGP problem is modeled as given in P₁.

$$\begin{aligned}
 P_1: \min & 0.1806d_1^- + 0.1419d_2^+ + 0.0809d_3^- + 0.1684d_4^- + 0.0985d_5^- + 0.11164d_6^- + 0.2132d_7^- \\
 & 0.0282x_1 - 0.0014x_2 + \dots - 0.0004x_{99} + 0.0128x_{100} + d_1^- - d_1^+ = 0.0114 + \theta \times (0.0391) \\
 & 0.1264x_1 + 0.1003x_2 + \dots + 0.0574x_{99} + 0.1468x_{100} + d_2^- - d_2^+ = 0.0935 - \theta \times (0.0462) \\
 & 3.94x_1 + 5.08x_2 + \dots + 3.37x_{99} - 0.24x_{100} + d_3^- - d_3^+ = 4.22 + \theta \times (143.73) \\
 & 59.8x_1 + 92x_2 + \dots + 45.4x_{99} + 78x_{100} + d_4^- - d_4^+ = 57.25 + \theta \times (35.05) \\
 & 8.2x_1 + 11.3x_2 + \dots + 2.8x_{99} - 0.9x_{100} + d_5^- - d_5^+ = 8.55 + \theta \times (75.45) \\
 & 27.79x_1 + 12.17x_2 + \dots + 34.71x_{99} + 9.37x_{100} + d_6^- - d_6^+ = 20.185 + \theta \times (251.515) \\
 & 0.4x_1 + 1.9x_2 + \dots + 4.9x_{99} - 0.1x_{100} + d_7^- - d_7^+ = 3.1 + \theta \times (7.50) \\
 & x_1 + x_2 + \dots + x_{99} + x_{100} = 1 \\
 & x_i \geq 0, \quad i = 1, \dots, 100 \\
 & \theta \in [0, 1] \\
 & d_j^-, d_j^+ \geq 0, \quad j = 1, \dots, 7.
 \end{aligned}$$

Table 8. Constraints on Goals and Membership Functions

Constraints on goals for criteria	Membership functions
$0.0282x_1 - 0.0014x_2 + \dots - 0.0004x_{99} + 0.0128x_{100} \geq 0.0114 + \theta \times (0.0391)$	
$0.1264x_1 + 0.1003x_2 + \dots + 0.0574x_{99} + 0.1468x_{100} \leq 0.0935 - \theta \times (0.0462)$	
$3.9400x_1 + 5.0800x_2 + \dots + 3.3700x_{99} - 0.2400x_{100} \geq 4.2200 + \theta \times (143.7300)$	
$59.80x_1 + 92.0000x_2 + \dots + 45.4000x_{99} + 78.0000x_{100} \geq 57.2500 + \theta \times (35.0500)$	
$8.2000x_1 + 11.3000x_2 + \dots + 2.8000x_{99} - 0.9000x_{100} \geq 8.5500 + \theta \times (75.4500)$	
$27.7900x_1 + 12.1700x_2 + \dots + 34.7100x_{99} + 9.3700x_{100} \geq 20.1850 + \theta \times (251.5150)$	
$0.4000x_1 + 1.900x_2 + \dots + 4.9000x_{99} - 0.1000x_{100} \geq 3.1000 + \theta \times (7.5000)$	

Source: Calculation made by authors.

The variables expressed here as d_j^-, d_j^+ are deviation variables used to standardize the goal constraints. In addition, by adding θ to the right-side values as a multiplier of the tolerance values, the problem turns into a parametric linear programming problem.

The portfolio distributions obtained by solving the goal programming problems corresponding to the changing right-side values for the values of θ increasing by 0.1 in the range $[0,1]$ are given in Table 9.

Table 9. Portfolio Distributions Correspond to Different Values of θ

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
AZN	0	0.2004	0.2519	0	0	0	0	0	0	0	0
ASML	0.0673	0.1692	0	0	0	0	0	0	0	0	0
AMAT	0.0473	0	0	0	0	0	0	0	0	0	0
AAPL	0	0	0.0910	0	0	0	0	0	0	0	0
BIIB	0	0	0	0.1478	0	0	0	0	0	0	0
BKNG	0	0.0004	0.1240	0.1950	0.2544	0.3195	0.2643	0.1982	0.1321	0.0661	0
CSCO	0.3486	0	0	0	0	0	0	0	0	0	0
CCEP	0.2316	0	0	0	0	0	0	0	0	0	0
DDOG	0.1774	0	0	0	0	0	0	0	0	0	0
KLAC	0	0.4895	0	0	0	0	0	0	0	0	0
META	0	0.0294	0	0	0	0	0	0	0	0	0
MU	0.0846	0	0	0	0	0	0	0	0	0	0
NVDA	0	0	0.1481	0.2422	0.2010	0.0153	0	0	0	0	0
PCAR	0	0.0539	0.0692	0	0	0	0	0	0	0	0
PDD	0	0	0	0.0539	0	0	0	0	0	0	0
PEP	0	0	0.0451	0	0	0	0	0	0	0	0
REGN	0	0.0571	0.2706	0.3611	0.5446	0.6651	0.7357	0.8018	0.8679	0.9339	1.0000
SNPS	0.0431	0	0	0	0	0	0	0	0	0	0

Source: Calculation made by authors.

The returns provided by the portfolio distributions given in Table 9 between January 1, 2024, and June 30, 2024 and the averages of these returns are as given in Table 10. Here, since the average return reaches its maximum value when $\theta = 0.4$, the portfolio distribution corresponding to this value was used to compare with the results obtained by other methods.

Table 10. 6-Month Return Amounts (%) and Averages (%) for Different Values of θ

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Jan. 24	2.23	4.26	4.95	6.89	8.58	4.90	5.10	5.66	6.22	6.78	7.34
Feb. 24	2.17	9.58	4.29	5.72	6.81	1.73	1.53	1.76	2.00	2.23	2.47
Mar. 24	3.26	3.21	4.59	3.75	3.82	1.44	0.94	0.61	0.29	-0.04	-0.37
Apr. 24	-2.60	-1.49	-1.28	-4.34	-6.18	-6.58	-6.77	-6.94	-7.12	-7.29	-7.46
May.24	0.24	8.16	9.76	13.73	13.27	10.10	9.88	9.92	9.96	10.01	10.05
Jun. 24	5.24	5.70	4.80	6.49	7.73	6.57	6.61	6.77	6.92	7.08	7.23
Average Return	1.76	4.90	4.52	5.37	5.67	3.03	2.88	2.96	3.05	3.13	3.21

Source: Calculation made by authors.

The results regarding the portfolio distribution obtained from CVaR, Mean Variance, ASM and the method proposed in the study are given in Table 11.

Table 11. Portfolio Distributions Obtained from Different Methods

	CvaR	MV	ASM	Proposed Method
AMZN	0.2902	0.1979	0	0
BIIB	0.1349	0.1398	0	0
BKNG	0	0	0	0.2544
AVGO	0	0	0.0641	0
CSCO	0.1319	0.1337	0	0
CEG	0	0	0.1934	0
CSGP	0.0568	0.0611	0	0
DDOG	0	0.0405	0	0
DXCM	0.0277	0.1178	0	0
FANG	0.0376	0.0419	0.0380	0
FAST	0.2148	0.1543	0	0
GEHC	0	0	0.2560	0
NVDA	0	0	0	0.2010
ORLY	0	0	0.3678	0
PANW	0	0	0.0807	0
REGN	0	0	0	0.5446
TXN	0.1061	0.1130	0	0

Source: Calculation made by authors.

The monthly portfolio returns and their average values for the period from January 1, 2024, to June 30, 2024, corresponding to the portfolio allocations presented in Table 11, are reported in Table 12. To complement the return-based comparison with a risk-adjusted performance assessment, the Sharpe and Sortino ratios were also calculated using the realized monthly portfolio returns. Higher Sharpe and Sortino ratios indicate better risk-adjusted performance.

Table 12. Out-of-Sample Returns (%) and Risk-Sdjusted Performance of the Portfolio Optimization Methods from January to June 2024

	CvaR	MV	ASM	Proposed Method
Jan. 24	0.06	-0.65	3.88	8.58
Feb. 24	4.72	2.84	16.63	6.81
Mar. 24	4.10	5.29	2.94	3.82
Apr. 24	-4.59	-4.28	-7.66	-6.18
May.24	0.19	-0.79	2.25	13.27
Jun. 24	2.01	1.73	4.56	7.73
Average Return	1.08	0.69	3.77	5.67
Sharpe Ratio	0.194	0.079	0.432	0.799
Sortino Ratio	0.319	0.130	1.012	1.945

Source: Calculation made by authors.

The comparative display of the average expected returns for the six-month period obtained from the methods is given in Figure 2.

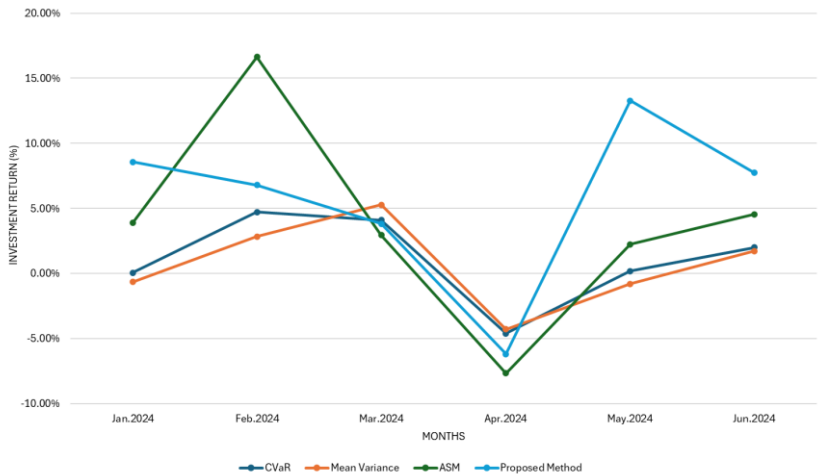


Figure 2. Average Returns Obtained from Methods by Month

Source: Illustration by authors.

As shown in Table 12, the proposed method achieved the highest average return over the January–June 2024 evaluation period, with an average return of 5.67%, followed by ASM (3.77%), CVaR (1.08%), and Mean-Variance (0.69%). At the monthly level, the proposed method achieved the highest return in January, May, and June, while ASM achieved the highest return in February and Mean-Variance in March and April.

In addition to its superior average return, the proposed method also exhibited the best risk-adjusted performance. It achieved the highest Sharpe ratio (0.799), followed by ASM (0.432), CVaR (0.194), and Mean-Variance (0.079). Similarly, the proposed method achieved the highest Sortino ratio (1.945), followed by ASM (1.012), CVaR (0.319), and Mean-Variance (0.130). Overall, these findings show that the proposed method provided the highest average return while also achieving the best risk-adjusted performance among the compared methods over the six-month evaluation period, considering both total return variability and downside risk.

To provide a more robust assessment of the out-of-sample performance of the proposed approach across different periods, an additional rolling-window analysis was conducted. In each window, 36 months of data were used as the portfolio formation period, and the performance of the resulting portfolio was evaluated over the subsequent three-month period, which was not used in the portfolio formation process. After each evaluation, both the 36-month portfolio formation period and the three-month out-of-sample test period were shifted forward by three months. The portfolio formation and out-of-sample test periods for the four rolling windows are presented in Table 13.

Table 13. Rolling-window Structure used for out-of-sample Performance Evaluation

Window	Portfolio formation period (36 months)	Out-of-sample test period (3 months)
1	01.01.2021–31.12.2023	01.01.2024–31.03.2024
2	01.04.2021–31.03.2024	01.04.2024–30.06.2024
3	01.07.2021–30.06.2024	01.07.2024–30.09.2024
4	01.10.2021–30.09.2024	01.10.2024–31.12.2024

Source: Authors' processing.

For each window, the CRITIC criterion weights were recalculated using only the data from the corresponding 36-month portfolio formation period. The MOWGP model was then solved using these recalculated weights to obtain window-specific portfolio allocations. Thus, no information from the subsequent three-month test period was used in the portfolio formation process. The performance of the portfolio obtained in each window was evaluated exclusively using the realized data from the corresponding three-month out-of-sample period. This rolling-window structure allows the performance of the proposed approach to be examined across different time periods rather than relying on a single portfolio formation and evaluation period. For each 36-month portfolio formation period, the proposed method was applied for θ values ranging from 0 to 1 in increments of 0.1, resulting in a separate portfolio allocation for each θ value. The realized returns of these portfolios were then evaluated over the corresponding subsequent three-month period, and the θ value yielding the highest average return over this period was selected. Accordingly, the selected θ values were 0.2, 0.3, 0, and 0.2 for P1, P2, P3, and P4, respectively. The returns associated with these selected portfolios are presented in Table 14. For each method, the table reports the monthly realized returns over the corresponding three-month evaluation period, together with the average return for each window.

Table 14. Monthly and Sverage out-of-sample Returns (%) of the Portfolio Optimization Methods across the Rolling Windows

Window	Month	CVaR	MV	ASM	Proposed Method
P1	Jan. 24	0.06	-0.65	3.88	7.31
	Feb. 24	4.72	2.84	16.63	13.89
	Mar. 24	4.10	5.29	2.94	3.71
	Average	2.96	2.49	7.82	8.30
P2	Apr. 24	-4.18	-4.99	-8.74	-4.34
	May. 24	0.14	-3.12	0.16	13.73
	Jun. 24	0.18	0.20	6.10	6.49
	Average	-1.29	-2.64	-0.83	5.29
P3	Jul. 24	3.58	3.03	-0.59	-1.60
	Aug. 24	1.56	1.67	1.91	9.32
	Sep. 24	0.14	-0.80	2.04	2.59
	Average	1.76	1.30	1.12	3.44
P4	Oct. 24	6.20	2.81	0.16	-4.89
	Nov. 24	19.47	6.30	4.72	-1.31
	Dec. 24	-8.90	-7.67	-14.22	-5.39
	Average	5.59	0.48	-3.11	-3.87

Source: Calculation made by authors.

As shown in Table 14, the proposed method achieved the highest average out-of-sample return in the first three rolling windows. In P1, proposed method generated an average return of 8.30%, compared with 7.82% for ASM, 2.96% for CVaR, and 2.49% for MV. The superiority of proposed method was particularly pronounced in P2, where it achieved a positive average return of 5.29%, while CVaR, MV, and ASM yielded average returns of -1.29%, -2.64%, and -0.83%, respectively. proposed method also recorded the highest average return in P3 at 3.44%, followed by CVaR (1.76%), MV (1.30%), and ASM (1.12%). In P4, however, proposed method recorded an average return of -3.87% and did not outperform the benchmark methods. CVaR achieved the highest average return in this window (5.59%), followed by MV (0.48%) and ASM (-3.11%). Overall, proposed method achieved the highest average out-of-sample return in three of the four rolling windows, indicating that its relative performance advantage was maintained across most of the evaluated periods, although this advantage was not observed in the final window.

In addition to the window-specific analysis, the overall out-of-sample performance of the portfolio optimization methods was evaluated over the entire 12-month period from January to December 2024. For this purpose, the monthly out-of-sample returns obtained across the four consecutive rolling windows were combined. The arithmetic mean of these 12 monthly returns was calculated to assess the overall return performance, while the Sharpe and Sortino ratios were used to evaluate risk-adjusted performance. The results are presented in Table 15.

Table 15. Overall out-of-sample Return and Risk-Adjusted Performance of the Portfolio Optimization Methods over the 2024 Rolling-window Evaluation Period

Method	Average Monthly Return (%)	Sharpe Ratio	Sortino Ratio
CvaR	2.26	0.272	0.620
MV	0.41	0.001	0.001
ASM	1.25	0.112	0.170
Proposed Method	3.29	0.417	0.880

Source: Calculation made by authors.

As shown in Table 15, the proposed method achieved the highest average monthly out-of-sample return (3.293%) over the 12-month evaluation period, followed by CVaR (2.256%), ASM (1.249%), and Mean-Variance (0.409%). The proposed method also achieved the highest Sharpe ratio (0.417) and Sortino ratio (0.880). These results indicate that the proposed method provided the best overall performance among the compared approaches in terms of both average return and risk-adjusted performance over the 2024 rolling-window evaluation period.

7. Conclusions

In this study, the portfolio optimization problem was modeled using a MOWFGP approach to support investors in determining appropriate investment strategies under uncertain financial market conditions. In the proposed model,

Return, Risk, EPS, GPM, ROA, P/B, BVPS, and EY were determined as portfolio selection criteria and formulated as fuzzy goals. In the first stage, the weights of these criteria were calculated using the CRITIC method by considering the variability of each criterion and the correlation relationships among the criteria. In the second stage, the median values of the criteria were defined as target values, and tolerance values were determined to represent the uncertainty associated with these targets. Based on the target and tolerance values, fuzzy membership functions were constructed for the portfolio selection criteria, and the criterion weights obtained from the CRITIC method were incorporated into the MOWFGP model. Finally, a solution algorithm was developed in which the θ parameter was systematically varied within the range $[0,1]$ to generate different fuzzy target levels and obtain alternative portfolio allocations. To evaluate the effectiveness of the proposed approach in portfolio selection, the method was applied to stocks included in the NASDAQ-100 index. In the initial analysis, 36 months of data were used for portfolio formation, and the realized performance of the resulting portfolio was evaluated over the subsequent six-month period. The performance of the proposed method was compared with CVaR, Mean-Variance, and ASM in terms of portfolio returns and risk-adjusted performance measures, including the Sharpe and Sortino ratios. In addition, a rolling-window analysis was conducted to examine the performance of the proposed approach across different time periods. In each rolling window, a 36-month portfolio formation period was followed by a three-month evaluation period, and both periods were shifted forward by three months in each successive window. This rolling-window structure enabled the performance of the proposed approach to be evaluated across different time periods rather than relying on a single portfolio formation and evaluation period, thereby providing a more robust assessment of its performance over time.

The empirical results demonstrate the performance of the proposed method from both return-based and risk-adjusted perspectives. In the initial six-month evaluation period, the proposed method with $\theta=0.4$ achieved the highest average return of 5.67%, compared with 3.77% for ASM, 1.08% for CVaR, and 0.69% for Mean-Variance. The proposed method also achieved the highest Sharpe ratio (0.799) and Sortino ratio (1.945), indicating favorable risk-adjusted performance in terms of both total return variability and downside risk. The rolling-window analysis further enabled the performance of the methods to be evaluated across different time periods. The proposed method achieved the highest average return in three of the four rolling windows, with average returns of 8.30%, 5.29%, and 3.44% in P1, P2, and P3, respectively. In P4, the highest average return was achieved by the CVaR method. When the monthly returns obtained across the four rolling windows were considered over the entire 12-month evaluation period, the proposed method achieved the highest average monthly return of 3.29%, compared with 2.26% for CVaR, 1.25% for ASM, and 0.41% for Mean-Variance. In addition, the proposed method achieved the highest overall Sharpe ratio (0.417) and Sortino ratio (0.880) among the compared methods.

Overall, the findings indicate that incorporating multiple financial criteria in addition to return and risk, determining their relative importance through the CRITIC method, and representing uncertainty through fuzzy goals provide a useful framework for portfolio selection. The proposed approach yielded favorable results in terms of both return and risk-adjusted performance across the evaluated periods. Furthermore, the use of the θ parameter enables the generation of alternative portfolio allocations under different fuzzy target levels, thereby providing flexibility in the portfolio selection process. In future studies, a larger number of financial ratios affecting portfolio selection may be incorporated into the model, thereby extending the proposed approach to encompass a broader set of criteria. In addition, fuzzy criterion weights may be considered to represent uncertainty not only in the portfolio goals but also in the relative importance of the criteria. The proposed approach may also be evaluated over longer investment horizons, different financial markets, and different market conditions to further examine its performance and applicability.

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