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Joint Scheduling of a Multi-Objective Stochastic Complex Production Problem Via an Intelligent Real-Time Predictive Maintenance Model

Abstract. *Condition-based maintenance (CBM) is a new generation of maintenance methodologies that help production system managers follow the real-time state of their system and enhance its reliability. CBM determines the right time and type of maintenance activities. However, it has less portion of the production research area. This research tries to fill a part of this gap by developing a joint production and maintenance model. This model introduces a multi-objective real-time version of a flexible job shop scheduling problem (FJSP) that manages maintenance affairs through the CBM mechanism. The proposed integrated model includes various stochastic items, such as maintenance activation times, maintenance durations, number of different types of maintenance, and real-time shocking process to get close to a real-world environment. The proposed stochastic objective functions are complementation time (C_{max}), maintenance costs (MC), and the average system degradation level (ASDL). To solve the developed NP-Hard model, two multi-objective simulation-based optimizations (MSBO) algorithms, called non-dominated sorting genetic algorithm (NSGAII) and multi-objective evolutionary algorithm based on decomposition (MOEAD) are developed. The visualized shocking process alongside other graphs completes the introduction of the behavior of the real-time model and the proposing solving algorithms to the considered research area.*

Keywords: *Condition-based maintenance (CBM), shocking process, flexible production model, multi-objective simulation-based optimization (SBO), real-time modelling.*

JEL Classification: L62, L63, C61, C63, D81, M11.

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1. Introduction

Machine availability is a standard classical assumption in most production and scheduling problems. In real-world problems, one can find lots of situations in which a machine breaks down or needs to be repaired or maintained. Besides, maintenance activities have a considerable portion of the system cost, or they have direct effects on system reliability. Of course, there is a well-developed environment of concepts in maintenance, particularly in the field of maintenance modeling and optimization (Cho & Parlar, 1991).

Over the years, various studies have highlighted the importance of maintenance in production systems. Maintenance costs and their influence on system reliability have been emphasized in different works (Ahmad & Kamaruddin, 2012). In particular, new generations of tools, such as radio frequency identification (RFID), various sensors, micro-electro-mechanical systems (MEMS), wireless telecommunication, supervisory control and data acquisition (SCADA), and product-embedded information devices (PEID) present new potential for conducting maintenance affairs (Shin & Jun, 2015). Therefore, opening the door of this land to production-planning problems is so of interest.

Condition-based maintenance (CBM) is one of those attractive philosophies in the rich area of maintenance and reliability considering the new technologies. CBM is an effective tool for balancing maintenance costs, system reliability, or other system performance items. It determines the proper maintenance requirements by monitoring the degradation of the machine or system over its design life cycle (Shin & Jun, 2015).

The rest of this section reviews the literature on the proposed problem and introduces the gaps. Details of these studies are summarized in Table 1.

1.1 Integration of maintenance and FJSP

A developed version of the scheduling problem is FJSP. Zribi and Borne (2005) regarded the machines' unavailability as a result of PM activities. Gao et al. (2006) merged FJSP with PM to create a model where maintenance jobs have a non-fixed duration that is decided upon throughout the scheduling process. Wang and Yu (2010) studied two maintenance policies that suggested flexible activities in a time window or fixed activities determined beforehand. Moreover, they develop FJSP by maintaining resource constraints. Moradi et al. (2011) optimized a bi-objective FSJP with unavailability and makespan objective functions jointed by PM. Dalfard and Mohammadi (2012) also studied integrated FJSP and PM. Li and Pan (2012) studied the same integration that optimized *Cmax*, *TWL*, and *CWL*. Rahmati et al. (2013) developed two multi-objective evolutionary algorithms to address the flexible job shop scheduling problem. Li et al. (2014) continued the study on integrated FJSP and PM. Mokhtari and Dadgar (2015) proposed time-varying failure rates in an FJSP with PM considerations in which PM durations are fixed. Ahmadi et al. (2016) investigated FJSP random machine breakdown while taking simulation into account. Khoukhi et al. (2017) proposed a dual ant colony approach for FJSP with PM. Zandieh et al. (2017) and Rahmati et al. (2018) developed a single objective version of the FJSP with CBM considerations. Wang et al. (2021) also introduced a multi-objective meta-heuristic optimization algorithm based on a multi-region division sampling strategy for a FJSP developed by PM and the transportation process. Liu et al. (2022) also investigated a FJSP to minimize total energy consumption with preventive maintenance. Zhang et al. (2023) proposed a multi-objective scheduling problem for the remanufacturing system (RMS) to lower completion time and energy. The problem included concurrent disassembly/reassembly workstations and FJSP reprocessing shops. Fan et al. (2024) proposed an improved tuna swarm optimization (GCLNTSO) algorithm to solve the dynamic and FJSP with random machine breakdowns. Zhang et al. (2024) addressed a multi-objective FJSP with preventive maintenance and transportation operations, accounting for sequence-dependent setup times. Chen et al. (2025) proposed an integrated DQN-based approach that improves scheduling efficiency and system reliability in FJSP. Trivedi and Gupta (2025) used simulation to integrate preventive maintenance in flexible job shop scheduling for improved efficiency.

1.2 Gap analysis and the objectives of the research

According to the studies mentioned in the previous subsection and Table 1, a slight part of the literature focused on CBM, and they are mostly considered periodic or PM strategies in their deterministic model. In Table 1, the abbreviations are defined in the last row of the first part of the table.

Moreover, based on the existing literature and as far as we know, the related joint models of production and CBM have not proposed mathematical modeling. They are also almost a single objective and have not considered maintenance cost affairs in their models.

To address the identified research gaps, the main objective of this study is to develop a real-time multi-objective stochastic framework that integrates flexible job shop scheduling with condition-based maintenance decisions. The proposed model aims to capture real-world manufacturing dynamics by incorporating real-time degradation monitoring, stochastic shock processes, and both preventive and corrective maintenance actions. In addition, the study seeks to simultaneously optimize key performance criteria, including completion time (C_{max}), maintenance cost (*MC*), and average system degradation level (*ASDL*), under uncertainty. Finally, two simulation-based multi-objective optimization algorithms, namely NSGAII and MOEAD, are employed to solve the resulting NP-hard problem.

These algorithms are specifically designed to handle the stochastic and real-time nature of the proposed framework. In addition, the proposed framework incorporates several realistic assumptions that are rarely considered simultaneously in the literature. For example, machine breakdowns may occur between inspection periods, and different types of maintenance activities, including preventive and corrective maintenance, are considered with stochastic durations. Additionally, innovative real-time visualization techniques are introduced to better illustrate the behavior of the proposed model.

The structure outlined below serves as a guide for the remainder of the paper. In Section 2, a detailed discussion of the proposed model is provided, where its key aspects are covered. Moving on to Section 3, the development of uniform operators for the different components of the simulation agent, as well as the algorithms that have been designed to address the problems, is carried out. Section 4 presents an explanation of the performance of both the model and the method, using numerical examples. Finally, the paper draws to a close in Section 5, where the conclusions are summarized.

Table 1. Literature review of FJSP and maintenance integration

Ref. #	Year	Scheduling Types	Objectives		Types of Maintenance			Solving Methodologies			Degradation
			Single	Multi	CM	PM	CBM	Meta-heuristics	Exact	SBO	
Zribi & Borne	2005	FJSP	C_{max}			√		Hybrid GA			
Gao et al.	2006	FJSP		C_{max}, TWL, CWL		√		GA			
Wang & Yu	2010	FJSP	C_{max}								
Moradi et al.	2011	FJSP		C_{max} Unavailability		√		NSGAII			
Dalfard & Mohammadi	2012	FJSP		C_{max} TL		√		SA, GA			
Li & Pan	2012	FJSP		C_{max}, TWL, CWL		√		TS			
Rahmati et al.	2013	FJSP		C_{max}, TWL, CWL				NSGA-II, NRGA,			

Ref. #	Year	Scheduling Types	Objectives		Types of Maintenance			Solving Methodologies			Degradation
			Single	Multi	CM	PM	CBM	Meta-heuristics	Exact	SBO	
Li et al.	2014	FJSP		C_{max}, *TWL*, *CWL*		√		DABC			
Mokhtari, & Dadgar	2015	FJSP	C_{max}			√		SA		√	√
Ahmadi et al.	2016	FJSP		C_{max} *Stability*	√			NSGAII NRGA			
Zandieh et al.	2017	FJSP	C_{max}				√	ICA		√	
Rahmati et al.	2018	FJSP	C_{max}		√	√	√	BBO		√	√
Wang et al.	2021	FJSP		C_{max}, *TEC*		√		MDSS-MOGA-DE			
Liu et al.	2022	FJSP	*TEC*			√		NDEHO			
Zhang et al	2023	FJSP		*CT ,EC*				IGWO			
Fan et al.	2024	FJSP		*CT,CI*				GCLNTSO			√
Zhang et al.	2024	FJSP	*CT*			√		ABC, LOA			
Chen et al.	2025	FJSP	*Cmax*		√	√		DQN-MI			√
Trivedi & Gupta	2025	FJSP		*MEC, RPW, OPMS*		√				√	
Our Work		FJSP		C_{max},*Cost*, *ASDL*	√	√	√	NSGAII MOEAD		√	√

Objective Functions:
Cmax = Maximum completion time (Makespan);

Ref. #	Year	Scheduling Types	Objectives		Types of Maintenance			Solving Methodologies			Degradation
			Single	Multi	CM	PM	CBM	Meta-heuristics	Exact	SBO	

TWL = Total workload;
CWL = Critical workload;
TL = Total load;
TEC = Total energy consumption;
CT = Completion time;
EC = Energy consumption;
CI = Carbon emissions index;
MEC = Maintenance energy cost;
RPW = Remaining processing workload;
OPMS = Overall production maintenance score;
MC = Maintenance cost;
ASDL = Average system degradation level.

Source: Authors' processing.

2. Proposed joint problem

This study tries to fill a part of the mentioned gap, explained in the previous section. To do so, it will model a joint real-time problem of CBM and a flexible production model, namely FJSP. Therefore, the two preliminary concepts are first discussed in two separate subsections. The integrated model will then be developed.

2.1 The Joint CBM-FJSP

FJSP is a production scheduling problem with n jobs$J(J_i, i \in \{1,2,\dots,n\})$, each job $i(J_1,\dots,J_n)$ is consisted of n_i operations $O\left(O_{ij}, j \in \{1,2,\dots,n_i\}\right)$, that are intended to be handled by m machines$M(M_k, k \in \{1,2,\dots,m\})$. FJPS is known as a popular *NP-hard* problem even in its simple version.

CBM approach is a popular maintenance method that is known as an alternative to classical scheduled items such as preventive maintenance (PM). It focuses on the actual condition of the systems and can present a tradeoff between maintenance cost and system performance in so many cases.

In this paper, the CBM concept is mimicked in a real-time process in which the degradation level (*DL*) of the system is monitored permanently. In case the *DL* exceeds a preset lower critical limit (*L*), the maintenance controller activates proper PM activities. Besides, if the *DL* of the machine reaches the upper control limit (*H*), a failure or breakdown occurs, and corrective maintenance is done. Fig.1 illustrates the critical levels mentioned and their related zone. Additionally, this figure shows an example of degradation based on shocks applied to the machinery and the kinds of relevant maintenance decisions made in each zone.

Various numbers of *S* values indicate the machine degradation shock times during the simulation procedure in this figure. For example, in this sample figure, the machine experiences seven shocks, denoted by *S1* through *S7* on the horizontal axis. Additionally, the M_j values on this axis indicate the machine's j^{th} maintenance activity's duration. The machine's degradation level is still less than *L* following shocks *S1* through *S3*, thus no repair is carried out. Following that, machine degradation leaps to the bound of preventive maintenance *L* because of the fourth stochastic shock (*S4*). As a result, the PM maintenance activity is identified at *2T*, the inspection time. The deterioration level in *M1* is improved and recovered by the PM maintenance action.

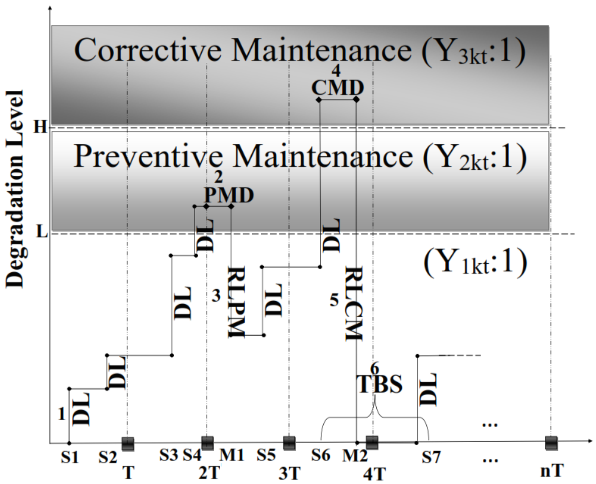

Figure 1. A sample of the associated stochastic events and stochastic degradation
Source: Authors' own creation.

The machine operates in this state of degradation until *S5* occurs. Since shock *S5*'s degradation level is lower than L's, no maintenance activity is needed. However, *S6* further degrades the machine than H, so corrective maintenance is necessary. Corrective maintenance differs from PM in two key ways of 1) it takes place between inspection intervals that disrupt the machine's functionality, and 2) it improves the machine's degradation to a new level or degradation zero in *M2*.

Each number in Fig.2 represents a stochastic event that occurs at that moment. Table 2 summarizes the type and distribution of this stochastic event. This table also presents the degradation (*Deg*) update scheme in row numbers 3 and 5. The following assumptions are also considered in the proposed model.

Table 2. The descriptions for the numbers presented in Fig.1

Event Number	Event Name	Symbol	Distribution and updating scheme	
1	Shock or degradation level	*DL*	*exponential distribution*	$DL \sim Exp(\eta)$
2	PM duration	*PMD*	*lognormal distribution*	$PMD \sim \log normal(\mu_{PM}, \sigma_{PM})$
3	Improving or recovery level through PM	*RLPM*	*lognormal distribution*	$RLPM \sim \log normal(\mu_{PM'}, \sigma_{PM'})$ $Deg_{new} = Deg_{old} - RLPM; L \leq Deg_{old} < H$
4	CM duration	*CMD*	*lognormal distribution*	$CMD \sim \log normal(\mu_{CM}, \sigma_{CM})$
5	Improving or recovery level through CM	*RLCM*		$Deg_{new} = Deg_{old} - RLCM; \quad Deg_{old} \geq H$
6	Stochastic time between two shocks	*TBS*	*exponential distribution*	*TBS~Exp(λ)*

Source: Calculation made by authors.

The following assumptions are also considered in the proposed model.

- The task's processes follow a preset and fixed order.
- Priorities for jobs are equal.
- The operations of various jobs are not restricted in terms of priority.
- At time zero, machines are available.
- Work starts at time zero.
- Machine setup time is disregarded.
- There is very little time between operations
- Every machine can do a single operation at a specific moment.
- The values of the degradations are stochastic
- Operations may be stopped during the process based on the degradation level, but jobs cannot be resumed.
- The PM duration of the machines follows a lognormal distribution, but with different parameters.
- The CM duration of the machines follows a lognormal distribution, but with different parameters.
- The degradation after PM improves at a random rate and is not fixed at the starting point.
- The degradation's improving degree after CM is set to its starting value.
- There is a stochastic interval between two shocks.

- Error-free inspection system reports and displays the system's actual state of degradation.

2.2 The proposed model

This section formulates the proposed model according to the mentioned concepts and assumptions in previous subsections and the following definitions in the Nomenclature. This model is a developed version of the simple pure real-time FJSP introduced by Demir & Isleyen (2013).

Nomenclature

Indexes

t, r = index of time period
i, h = index of jobs $(1, \dots, n)$
j, g = index of operations $(1, \dots, J_i)$
k = index of machines $(1, \dots, m)$
q = index of number of PM maintenance
p = index of number of CM maintenance

Parameters

n = Total number of jobs
m = Total number of machine
J_i = Total number of operations of job i
a_{kij} = Describe the capable machine set M_{ij} *is assigned to operation* O_{ij}
P_{kij} = Processing time of O_{ij} *if performed* On machine k
M = A large number
E_k = *the set of operations which can be performed on machine k*
C_{PM0K} = Fixed Cost of PM on machine k
C_{CM0K} = Fixed Cost of CM on machine k
$V_{ijk}: \begin{cases} 1, & if\ O_{ij}\ performed\ on\ machine\ k \\ 0, & otherwise \end{cases}$
L = Lower maintenance level
H = Upper maintenance level

Variables

$Z_{ijhgk}: \begin{cases} 1, & if\ O_{ij}\ precedes\ operation\ O_{hg}\ on\ machine\ k \\ 0, & otherwise \end{cases}$

$W_{ijk1}: \begin{cases} 1, & if\ O_{ij}\ is\ processed\ on\ machine\ k\ during\ period\ t \\ 0, & otherwise \end{cases}$

$Y_{lkt}: \begin{cases} 1, & if\ machine\ k\ during\ period\ t\ is\ in\ l\ state \\ 0, & otherwise \end{cases}$

TMC = Total Maintenance Cost
C_{max} = Makespan
$ASDL$ = Average system degradation level
$N_{PM_k}(T)$ = Number of PM on machine k during T
C_{PM1K} = Variable Cost of PM on machine k
PMD_{kij} = Duration of PM related to O_{ij} on machine k
$N_{CM_k}(T)$ = Number of CM on machine k during T
C_{CM1K} = Variable Cost of CM on machine k
CMD_{kij} = Duration of CM related to O_{ij} on machine k
$Deg_k(t)$ = Degradation of machine k during t
$RLPM(t)$ = Recovery level due to PM in t
$RLCM(t)$ = Recovery level due to CM in t
WT_{kij} = Wasted time of O_{ij} due to break of maintenance
S_e = e^{th} shock time

$$Min\ C_{max} = Max\left(W_{ijkt} * t\right)\ \forall i, t, j = J_i, k \tag{1}$$

$$Min\ TMC = C_{PM0}N_{PM}(T) + \left(\sum_{q=0}^{N_{PM_k}(T)} C_{PM1} \times PMD_{kij}\right) + C_{CM0}N_{CM}(T) + \left(\sum_{q=0}^{N_{CM_k}(T)} C_{CM1} \times CMD_{kij}\right)\quad \forall k \tag{2}$$

$$Min\ ADSL = \left\{Mean\left(\left\{Mean\left(Deg_k(t)\right)\right\}_T\right)\right\}_M \tag{3}$$

$$Deg_k(t) \leq L - M * (1 - Y_{1kt});\ \ \forall k, t \tag{4}$$
$$Deg_k(t) > L + M * (1 - Y_{2kt});\ \ \forall k, t \tag{5}$$
$$Deg_k(t) \leq H - M * (1 - Y_{2kt});\ \ \forall k, t \tag{6}$$
$$Deg_k(t) > H + M * (1 - Y_{3kt});\ \ \forall k, t \tag{7}$$
$$\sum_{l=1}^{3} Y_{lkt} = 1; \qquad \forall k, t \tag{8}$$
$$\sum_t W_{ijkt} = V_{ijk} * \left(P_{kij} + PMD_{kij} + CMD_{kij} + WT_{kij} * max\{(1 - Y_{lkt}); sign\left(\sum_t W_{ijk(t-1)}\, Y_{2kt}\right); sign\left(\sum_t W_{ijk(t-1)}\, Y_{3kt}\right)\}\right) \quad \forall i, j, \forall k \in Mij \tag{9}$$
$$PMD_{kij} = \sum_t W_{ijk(t-1)}\, Y_{2kt} \quad \forall i, j, \forall k \in Mij \tag{10}$$
$$CMD_{kij} = \sum_t W_{ijk(t-1)}\, Y_{3kt} \quad \forall i, j, \forall k \in Mij \tag{11}$$
$$\sum_i \sum_j W_{ijkt} \leq 1 \quad \forall k, t \tag{12}$$
$$\sum_{k \in Mij} V_{ijk} = 1 \quad \forall i, j \tag{13}$$
$$P_{kij} * W_{hgkt} \ \leq \sum_{r=1}^{t-1} W_{ijkr} + M * \left(1 - Z_{ijhgk}\right) \quad \forall i \leq h, \forall j, g, t = 2, \forall k \in Mij \cap Mhg \tag{14}$$
$$P_{khg} * W_{ijkt} \ \leq \sum_{r=1}^{t-1} W_{hgkr} + M * \left(Z_{ijhgk}\right) \quad \forall i \leq h, \forall j, g, t = 2, \forall k \in Mij \cap Mhg \tag{15}$$
$$\sum_{k \in Mij} P_{kij} * V_{ijk} * \sum_{h \epsilon Mij+1} W_{ij+1ht} \leq \sum_{r=1}^{t-1} \sum_{k \in Mij} W_{ijkr} * V_{ijk} \quad k \epsilon Mij + 1, h \epsilon Mij + 1, \forall i, j, t = 1, \dots, J_{i-1}, \forall t = 2, \dots, T \tag{16}$$
$$P_{kij} * \left(W_{ijkt} - W_{ijkt+1}\right) + \sum_{r=t+2}^{T} W_{ijkr} \leq P_{kij} \quad \forall i, j, k, t = 1, \dots, T - 2 \tag{17}$$
$$W_{ijkt} \in \{0,1\} \quad \forall i, j, k, t \tag{18}$$
$$Z_{ijhgk} \in \{0,1\} \quad \forall i \leq h, \forall j, g, t = 2, \forall k \in Mij \cap Mhg \tag{19}$$
$$V_{ijk} \in \{0,1\} \quad \forall i, j, k \tag{20}$$
$$Y_{lkt} \in \{0,1\} \quad \forall l, k, t \tag{21}$$

In this model, Equation (1) formulates the stochastic complementation time of each solution. Equation (2) calculates the stochastic maintenance cost terms, including the total fixed cost and the total variable cost due to the duration of the maintenance activities. Besides, Equation (3) discusses the third objective function of the model denotes the average reliability of the system during the whole schedule period. The other equations, presented in the model, formulate the real-time constraints of the model.

Equations (4) and (7) determine the condition of the system according to the real-time level of the degradation. In case no maintenance is required, Y_{1kt} gets 1, and for PM or CM, Y_{2kt} or Y_{3kt} becomes 1, respectively. According to (8), these three conditions, presented in Fig. 2, are not consistent, and at that moment, only one of them can occur. Equation (9) formulates the duration of each operation's processing on its assigned machine, considering the main processing time P_{kij}, waste time after the break of the job due to maintenance requirements (WT_{kij}) and PM (PMD_{kij}) or CM (CMD_{kij}), if needed. The sign part gets 0 in case Y_{1kt} is 1 and gets 1 when PMD_{kij} or CMD_{kij} are active. The PMD_{kij} and CMD_{kij} in this equation are calculated according to (10) and (11), respectively.

Equation (12) ensures that at each moment, each machine can only process one operation, while (13) guarantees that one machine is assigned to each operation. Equations (14) and (15) control the precedence constraint of the operations of different jobs, and (16) checks the same rule for the operations of a specific job. Equation (17) ensures that no other event happens during a complete process for an operation, considering what happens for that operation in (9). The other equations determine the types of variables.

3. Multi-Objective Simulation-Based Solving Algorithms

This part prepares and integrates the materials of the simulation process and multi-objective algorithms consistent with the structure of the proposed real-time stochastic model. In this paper, two MOSBOs, NAGSII and MOEAD, are developed to solve the model. Since both algorithms are population-based evolutionary algorithms, first the solution structure and required neighborhood schemes of the algorithms are discussed. Then, the total and brief logic of the algorithms is presented. Finally, the simulation agents of the MOSBOs are described in detail

3.1 Neighborhood designs and solution structure

This paper implements two vectors as long as the total number of operations (TNOP) for creating a chromosome or solution scheme. In this scheme, the machine operating sequence is represented by the first vector, and the assignment vector is displayed by the second. Figure 2 illustrates a sample structure of the solution.

Sequence Vector								
O_{31}	O_{32}	O_{21}	O_{33}	O_{11}	O_{22}	O_{12}	O_{23}	O_{13}
Machine Assignment Vector								
M_3	**M_2**	**M_4**	**M_2**	**M_1**	**M_1**	**M_3**	**M_1**	**M_2**

Figure 2. The solution structure

Source: Authors' own creation.

The sequencing vector (SV) applies to a random hybrid strategy (RHS) of swap, reversion, and insertion. This hybrid strategy is illustrated in Figure 3, schematically.

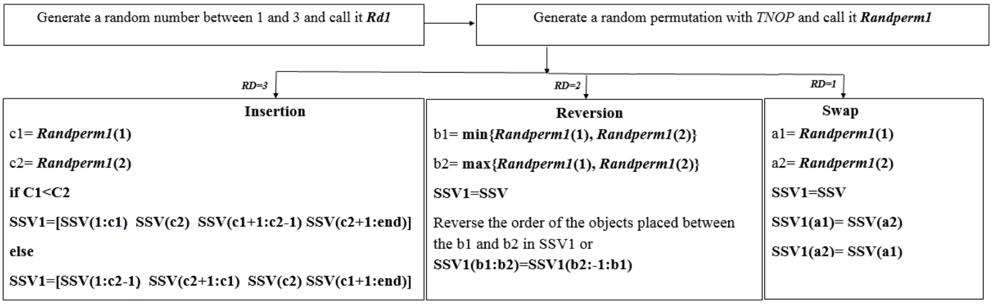

Figure 3. The proposed hybrid SV neighborhood structure operator
Source: Authors' own creation.

The assignment vector (AV) is updated through the machine changing operator (MCO) from the capable set of each operation, as Figure 4.

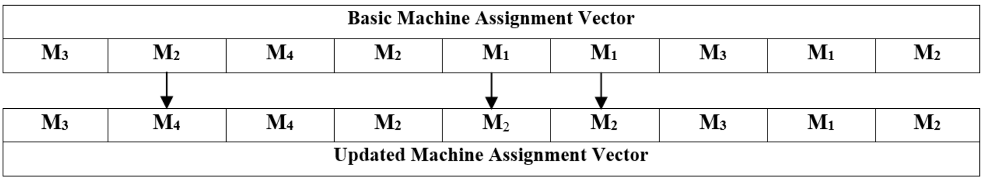

Figure 4. The proposed AV neighborhood structure operator
Source: Authors' own creation.

3.2 The NSGAII

The heart of the evolution process of NSGAII is the genetic algorithm (GA) (Deb et al., 2000). GA has three operators, including reproduction, mutation, and crossover, for exploring the search space. Since the reproduction operator is a copy of the elite chromosome to the next generation, it generally does not need a specific structure. For mutation operators, in this paper, RHS and MCO are implemented to create neighborhood structures for sequencing and assignment, respectively. The crossover part implements improved precedence operation crossover (IPOX) for SV and multipoint preservative crossover (MPX) for MV (Rahmati et al., 2012). The other operators are the NSGAII regular to conduct Pareto-based evolution.

3.3 The MOEAD

MOEAD implements a specific decomposition scheme in which first the multi-objective optimization problem is broken down into several distinct sub-problems of single goal optimization, and then, through a population-based approach, the optimization process is conducted on the sub-problems simultaneously (Zhang and Li, 2007). This paper creates new solutions from the neighbors through the flowchart of Figure 5.

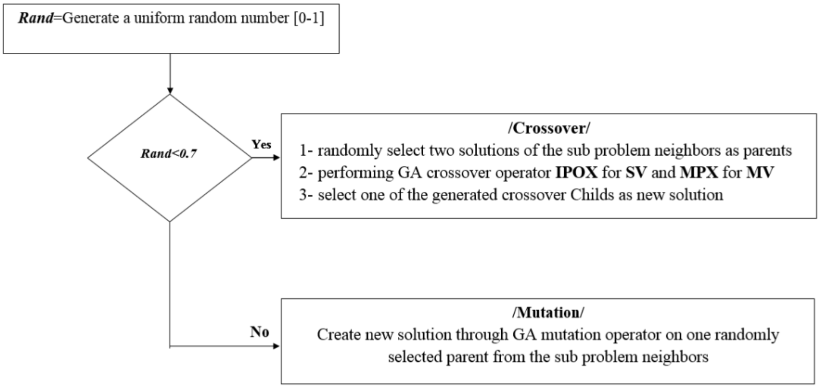

Figure 5. The operator for MOEAD's proposed neighborhood structure
Source: Authors' own creation.

3.4 The simulation agent of the algorithm

The proposed simulation agent of the MSBO simulates the real-time stochastic concept of the proposed model. The proposed model contains different stochastic components, such as *RL*, *PMD*, *RLPM*, *CMD*, *RLCM,* or *TBS,* to reflect a realistic version of the CBM. The model's states are dynamically altered by these variables. Stochastic optimization issues are generally divided into two classes: the control class and the parametric class. Additionally, they go by the names static and dynamic class, respectively. For every state, a collection of static parameters is part of static optimization. On the other hand, solutions vary depending on dynamic conditions in control optimization. MSBO is a member of the control or dynamic class and fully covers the optimization process. Due to the stochastic dynamic character of the topic in this research, the simulation technique mimics the dynamic approach Figure 6 how the suggested MSBO is organized as a flow chart of Figure 6.

To report a more robust solution to the optimization algorithm, this MSBO conducts a loop of simulation runs (*Numsim*) to obtain the average value of the objective functions of solutions. The notation *dt* in this Figure denotes the sample time of the simulation, and *VT* and *LVT* represent the predetermined length between visit times and the obtained last visit time, respectively. This flow chart starts with a solution from the optimization process and produces the simulated version of the objective functions. The machines and jobs updating functions and the maintenance decision function are implemented from Rahmati et. al. (2018).

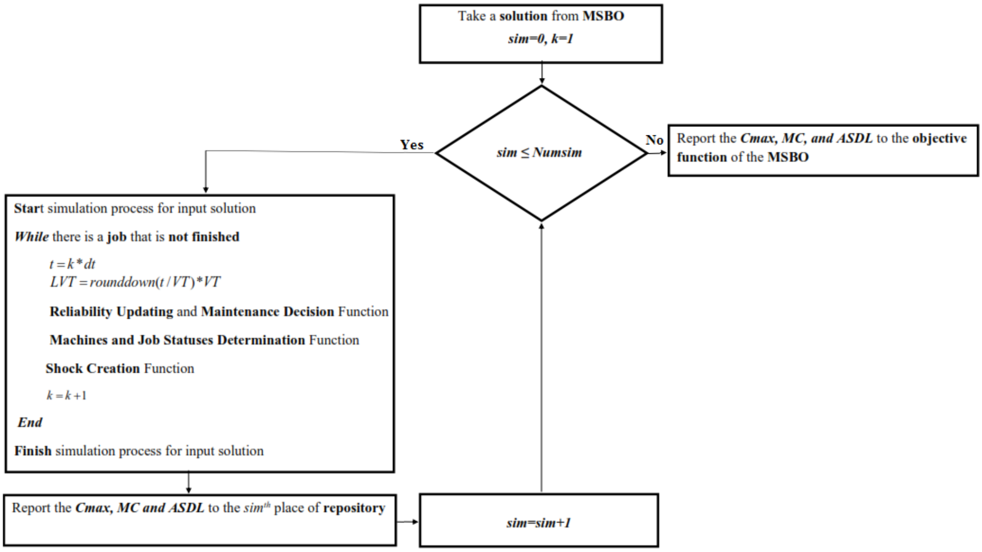

Figure 6. The complete pseudo-code of the suggested MSBO simulation portion
Source: Authors' own creation.

4. Computational Results

Computational results discuss the proposed model and the solving methodologies in this section through different numerical examples implemented from Rahmati et al. (2018). According to these metrics, two types of analysis, i.e., non-statistical and statistical tests, are conducted. The non-statistical part compares the rare values of metrics and presents them in a table and Fig. platforms. The statistical part includes numerous hypothesis tests and related graphs. Hypothesis part run t-tests, and the graphs are boxplots. The Gant Chart alongside the deterioration plot illustrates the feasible solutions obtained for the proposed stochastic real-time CBM model. The Gantt Chart also includes different active maintenance activities.

4.1 Algorithm Outputs

The algorithms' outputs on various metrics are shown in Table 3. The directions in the first row of this table indicate the maximum and minimum direction of each measure. These guidelines indicate that the best algorithm is indicated in the last row of the table for each statistic. Considering the values that are highlighted in all metrics, NSGAII is superior in the number of solutions (*NOS*) and diversity (*D*), while MOEAD wins the *Time*, *SNS*, and *MID* metrics. From a theoretical perspective, this behavior can be explained by the population-based nature of NSGAII, which enhances solution diversity under high uncertainty in stochastic CBM environments. In contrast, MOEAD's decomposition strategy improves local convergence but limits global exploration capability in highly dynamic degradation systems. However, Table 4 changes the obtained view from

Table 3. Based on the quantity of dominating solutions (NDS) metric in this table, most of the solutions of MOEAD are dominated by NSGAII. This dominance pattern indicates that in stochastic and real-time scheduling environments, global Pareto dominance is strongly influenced by algorithmic exploration ability rather than purely decomposition efficiency. These results are illustrated in Fig.7 schematically. Figure 9 presents the state of the solutions of the algorithms in their Pareto fronts. According to this Figure, NSGAII prepares more numerous and varied sets of solutions for decision-makers. This may indicate that stochastic degradation increases solution space complexity, making diversity preservation a critical factor in obtaining high-quality Pareto fronts.

Table 3. Comparing the outcomes of MOEAD with the suggested NSGAII

	NSGAII					MOEAD				
	Time↓	**NOS↑**	**MID↓**	**SNS↓**	**Diversity↑**	**Time↓**	**NOS↑**	**MID↑**	**SNS↓**	**Diversity↑**
FJSCBM1	1123.12	34	10008.16	10748.40	36979.58	412.93	2	833.77	315.18	450.38
FJSCBM2	28518.55	50	604315.70	1487234.29	8945067.44	1099.70	6	3739.54	182.59	443.45
FJSCBM3	17305.02	37	274546.19	788318.47	3497777.64	644.42	2	904.99	0.14	2.21
FJSCBM4	2687.57	50	12689.41	17540.38	87792.35	880.78	6	2238.28	508.07	1355.95
FJSCBM5	6250.67	43	39779.62	45308.35	215998.56	1346.68	6	4890.75	586.81	1766.35
FJSCBM6	3483.81	50	37097.04	105949.90	630642.08	589.55	2	407.36	0.11	1.45
FJSCBM7	9997.09	50	62531.63	150890.87	759609.26	1432.72	4	6131.26	123.38	273.28
FJSCBM8	2235.08	50	6783.16	6434.68	23661.25	1214.06	5	3188.46	193.11	479.79
FJSCBM9	18662.51	50	67731.40	152558.69	897592.77	2828.71	6	10385.37	576.95	1715.04
FJSCBM10	2976.51	47	12788.24	2178.73	9373.33	2913.84	4	7801.30	375.61	906.66
Sum	93239.93	461	1128270.55	2767162.77	15104494.27	13363.41	43	40521.07	2861.96	7394.55

Source: Calculation made by authors.

Table 4. Comparison of the domination of the solutions of the algorithms

	NSGAII			MOEAD		
	NOS↑	**NDS↓**	**FNS↑**	**NOS**	**NDS↓**	**FNS↑**
FJSCBM1	34	0	34	2	2	0
FJSCBM2	50	0	50	6	6	0
FJSCBM3	37	0	37	2	2	0
FJSCBM4	50	0	50	6	6	0
FJSCBM5	43	0	43	6	3	3
FJSCBM6	50	6	44	2	0	2
FJSCBM7	50	0	50	4	2	2
FJSCBM8	50	0	50	5	5	0
FJSCBM9	50	0	50	6	6	0
FJSCBM10	47	0	47	4	3	1
Sum	**461**	**6**	**455**	43	35	8

Source: Calculation made by authors.

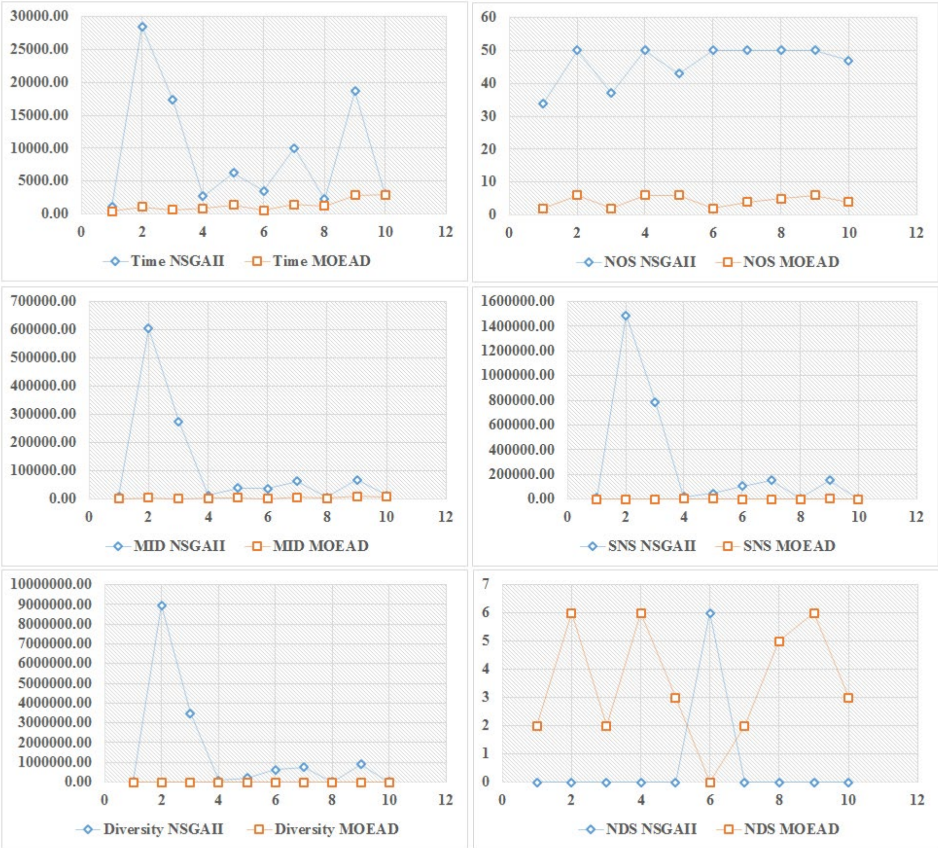

Figure 7. A multi-objective output metric for each of the test problems, 1 to 10
Source: Calculation made by authors.

Table 5 presents the *t*-test outputs for different metrics. According to comparing the *P*-values of these hypothesis tests with the considered significant level 0.05, which is also visualized in Figure 8, in Time, *NOS*, and *NDS* algorithms have significant differences. These statistically significant differences confirm that the observed performance variations are not incidental but are rooted in structural differences between the algorithms under stochastic CBM conditions.

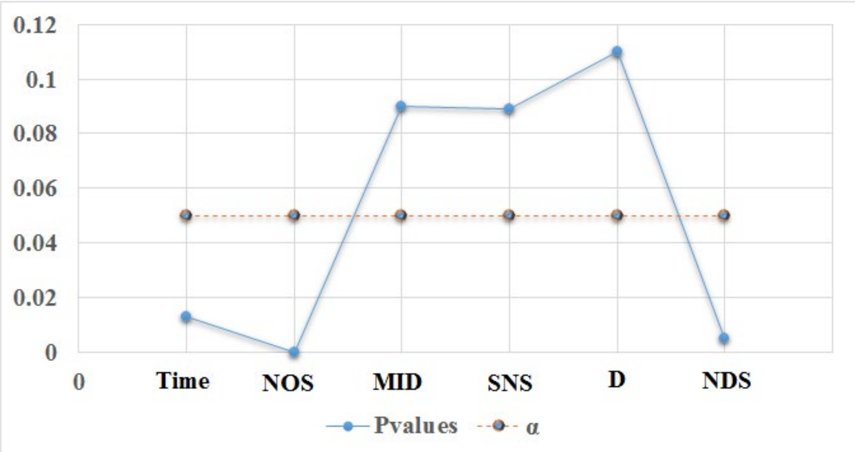

Figure 8. Comparing the measures' *P*-values with the significance level (0.05)
Source: Calculation made by authors.

Pareto fronts for the three test problems related to test problem 10 is presented in 9. To provide a comprehensive visual picture of the fronts, these figures show the main front in three-dimensional space and its projection in several two-dimensional planes. Additionally, a single sample solution is chosen for each of these Pareto fronts of Figure 9, and the evolution of the Gantt chart is shown with the sample solution's real-time degradation updates in Figure 10.

Table 5. Comparing suggested algorithms statistically

	Metric Name	*P*-value	T-test Result
1	*Time (T)*	0.013	$H_0 : \mu_{Time,NSGAII}$
2	*NOS*	0.00	$H_0 : \mu_{NOS,NSGAII} \neq \mu_{NOS,MOEAD}$
3	*MID*	0.09	$H_0 : \mu_{MID,NSGAII} = \mu_{MID,MOEAD}$
4	*SNS*	0.089	$H_0 : \mu_{SNS,NSGAII} = \mu_{SNS,MOEAD}$
5	*D*	0.11	$H_0 : \mu_{Diversity,NSGAII} = \mu_{Diversity,MOEAD}$
6	*NDS*	0.005	$H_0 : \mu_{NDS,NSGAII} \neq \mu_{NDS,MOEAD}$

Source: Calculation made by authors.

4.2 Discussion

The developed algorithms and the suggested stochastic multi-objective model are discussed in this section. Our CBM problem assumes two determining levels, *L* and H. In this study, these levels are set to be *20* and *40*, respectively. This Figure shows that to make the suggested CBM feasible, more than six stochastic components are considered. On the primary selected problem, FJSCBM10, these elements and variables are also displayed in Figure 9. The real-time degradation level ($DL(t)$) is demonstrated through the evolution of the Gantt chart for a sample solution derived from the Pareto front, as shown in Figure 10. Number (1), or *RL*, and Number (6), or *TBS*, in Figure 10 show a series of shocks and degradations brought on by Machine 3's activation of operation 6.2, respectively. However, the corresponding values are given collectively for a particular operation based on the tiny values of these variables, as they are displayed in a figure with this scale separately. The way that Figures 2 and 8 function determines how *DL* is regulated. *TBS* shock timings are produced by Figure 10. Additionally, the (3) values illustrate how the PM (*RLPM*) affects machine 3's deterioration level and how long the PM lasts, as indicated by Number (2). When the deterioration level rises below the *L* level, PM occurs. From a theoretical CBM perspective, this validates the effectiveness of threshold-based maintenance policies (L and H levels), where preventive actions are triggered adaptively to control stochastic degradation propagation in real-time systems. The evolution of the Gantt chart reflects the real-time degradation level (*DL(t)*) for a sample solution selected from the Pareto front, as depicted in Figure 9, highlighting the dynamic changes in system performance over time.

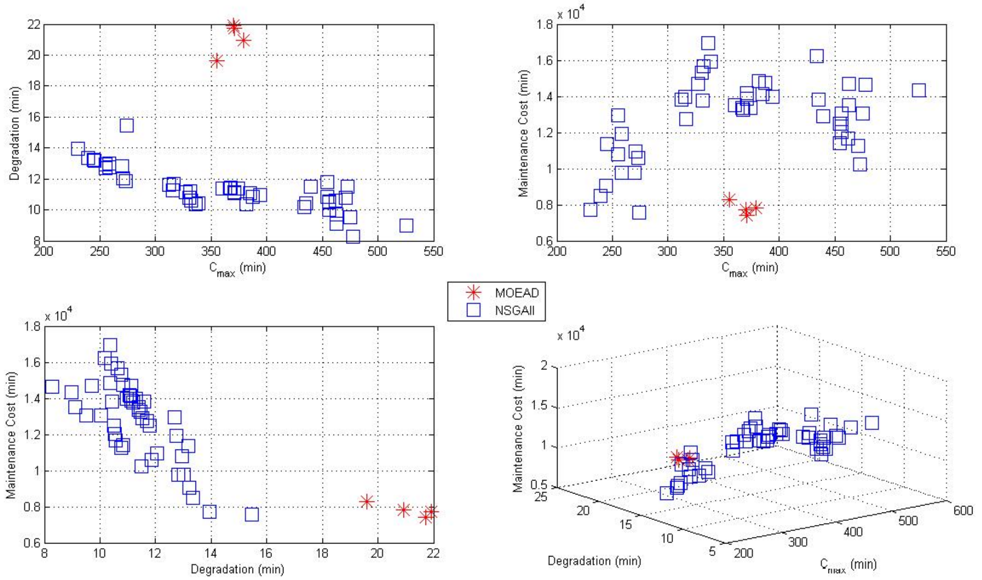

Figure 9. An example Pareto front of the algorithms in FJSCBM10
Source: Calculation made by authors.

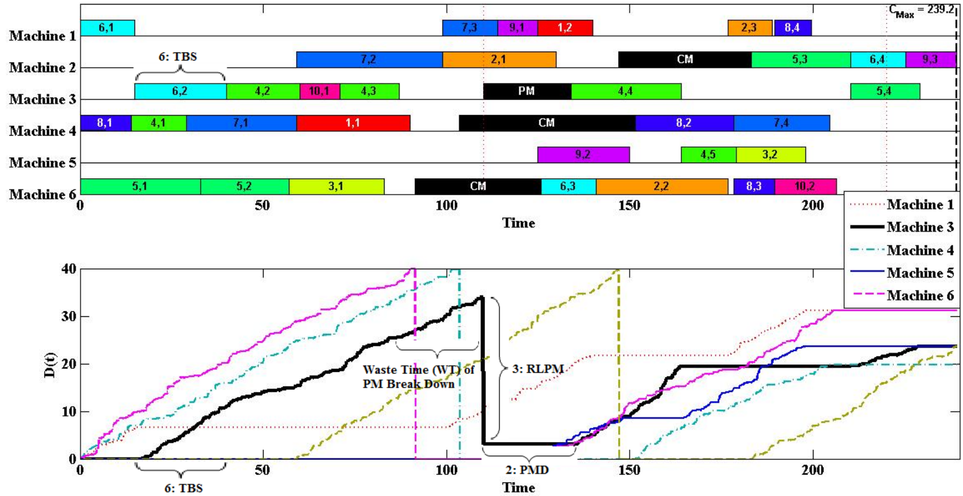

Figure 10. Gantt chart showing real-time degradation level ($DL(t)$) for a Pareto front solution

Source: Calculation made by authors.

With the help of the simulation algorithm's autonomous detection engine, the WT in this figure illustrates the wasted time in relation to the repair requirement. In the degradation Figure, it indicates that operation $O_{4,4}$ had been initiated and degraded. However, machine *3* needed PM because its *DL* was higher than *L*. Consequently, the maintenance activities commenced, and operation $O_{4,4}$ was interrupted. Naturally, since the jobs in our situation cannot be resumed, they are begun over after their maintenance operations. In summary, these numbers demonstrate that an algorithm can be developed to govern a process both intelligently and independently.

This Gantt chart illustrates the progression of the real-time degradation level ($DL(t)$) for a selected sample solution from the Pareto front, highlighting dynamic changes in system performance over time, as previously detailed in Figure 9. Based on both statistical tests and computational performance results, NSGAII exhibits more robust performance in highly stochastic and real-time CBM environments due to its stronger exploration capability, while MOEAD remains competitive in scenarios where convergence efficiency is prioritized.

5. Conclusions

This paper mimics the real-world maintenance approach, namely CBM, and proposes a joint multi-objective maintenance and production problem. According to a rich literature review done in the paper, this subject has not been addressed. This study developed a multi-objective real-time stochastic model consisting of the problem under investigation. Additionally, it created and suggested an intelligent, self-governing CBM operator that continuously assesses the degree of deterioration and chooses the appropriate kind of maintenance action that is needed. The proposed problem encompasses at least six stochastic variables to get close to the right sort of real problem. The proposed stochastic objective functions of the model were C_{max}, maintenance costs, and the average system degradation level. Then, because of the stochastic nature of the model, two multi-objective simulation-based algorithms, called NAGSII and MOEAD, were developed to conduct the optimization process. The necessary operators for the suggested algorithms were well explained and provided with enough examples. Additionally, several creative and unique visualization approaches directly demonstrated the suggested logics of the real-time stochastic model. Lastly, a variety of multi-objective measures and statistical hypothesis tests were used to compare the algorithms' capabilities. These findings indicate that NSGAII is a more suggestive algorithm. Future work of this research may investigate the development of the mentioned problem with terms like queuing theory or other resources such as humans to make the model more realistic. Developing other stochastic techniques or reinforcement learning methods to handle the problem is also of interest.

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