Ting LIU, PhD Candiate
liutingcyl@163.com
Shandong University of Finance and Economics, China

Fangxia ZHAO, PhD (corresponding author)
zfx88666@163.com
School of Management Engineering, Capital University of Economics and Business, China

Dynamic Evolution of Travel Mode Choice: A Game-Theoretic Analysis with Shared Mobility Considerations

Abstract. *Understanding the evolution of travel mode choice behaviour is critical for urban transportation planning and policy-making. This study develops an evolutionary game model to analyse how travellers dynamically shift between shared cars, public transit, and private vehicles under changing economic and policy conditions. Using replicated dynamic equations and stability analysis, we identify equilibrium points and their stability under different scenarios. Numerical simulations reveal that government subsidies for shared cars, car-sharing travel costs, and public transit fares significantly influence the evolution of travel mode choices. Our findings indicate that strategic subsidies can effectively shift travellers toward shared mobility, while excessive public transit costs can drive users toward private vehicle use. The results provide a theoretical foundation for policymakers to optimise transportation policies and encourage sustainable urban mobility. Future research will explore real-time behavioural adaptations and machine learning-based travel demand predictions.*

Keywords: *travel mode choice, evolutionary game theory, stability analysis, car-sharing.*

JEL Classification: R14, R15, R48.

Received: 28 March 2025	**Revised: 6 September 2025**	**Accepted: 10 September 2025**

1. Introduction

Travelers' mode choice behaviour is a central issue in urban transportation management, as it fundamentally shapes the structure of urban mobility and informs the development of effective traffic management policies. Rapid urban economic growth and accelerated urbanisation have led to a surge in transportation demand and an expansion in the diversity of available travel modes. Traditional policies have predominantly focused on promoting public transport to alleviate urban congestion and environmental pressures (Jefferson, 1996; Wardman, 2004; Suryani et al., 2020; Huang et al., 2022; Wang and Huang, 2023, Tu, Geng and Zhang, 2023). However, the emergence of shared cars has introduced new dynamics, impacted the usage of public transit and private cars, and prompted some travellers to shift toward shared car travel (Shaheen, Cohen and Farrar, 2019). This shift necessitates a deeper understanding of how travellers adapt their choices in response to new mobility options.

DOI: 10.24818/18423264/59.3.25.05

Traditional static models, rooted in random utility theory and utility maximisation, have been widely used to analyse travel mode choices. However, these models often assume static decision-making processes, overlooking the dynamic and adaptive nature of traveller behaviour under conditions of incomplete information, differing values, and changing traffic environments. Travellers exhibit bounded rationality, estimating the convenience and cost of various modes based on empirical utility, and their choices evolve dynamically toward a stable equilibrium. Evolutionary game theory offers a robust framework to model this dynamic process, capturing the preference phenomena and evolutionary trends in travel mode choices (Wu et al., 2019; Li et al., 2020; Zhu and Li, 2023; Xu, Tan and Zhang, 2024; Lisowski, 2023; Cao et al., 2023). This study focuses on urban medium-distance travel and employs evolutionary game theory to analyse the dynamic choice process among shared cars, public transit, and private cars following the entry of shared car enterprises. The contributions of this study are as follows:

(1) Proposing an evolutionary dynamics approach to study the choice evolution of shared cars, public transit, and private cars based on evolutionary game theory.

(2) Identifying equilibrium points of the evolutionary dynamic model and analysing their stability conditions through simulation.

(3) Examining the role of government subsidies in influencing travellers' preferences toward shared cars.

(4) Simulating and analysing the impact of shared car and public transit travel costs on the final evolution outcomes.

The paper is organised as follows: Section 2 reviews relevant literature on shared cars and travel mode choice. Section 3 introduces the evolutionary dynamics model. Section 4 analyses equilibrium points and their stability. Section 5 presents numerical results to evaluate the model and its equilibria. Section 6 summarises the findings and discusses their implications for urban transportation policy.

2. Literature review

The study of travel mode choice has evolved significantly, with early research focusing on macro-level transport mode structures and transitioning to micro-level individual behaviour analysis. Initial models, such as the transfer curve method, relied on extensive survey data to establish relationships between mode share rates and influencing factors, producing transfer curve graphs (Shaheen, Cohen and Farrar, 2019). With advances in probability theory and the refinement of travel units, research shifted toward individual travel characteristics. Since the 1970s, McFadden and others introduced utility theory from economics into mode choice modelling, developing non-aggregate models based on stochastic utility and utility maximisation principles (Mcfadden, 1974; Mcfadden and Train, 2000). These disaggregate models, notably the Logit model, incorporate variables such as travel behaviour, traveller attributes, and environmental factors to predict mode choices (Horowitz, 1980; Ortuzar, 1983; Mark and Uncles, 1987; Wen and Koppelman, 2001; Hoogendoorn and Bovy, 2005; Ashiabor, Baik and Trani, 2007). Over time, the Logit model evolved into a comprehensive system, including the Multinomial Logit (MNL), Mixed Logit (ML), and Nested Logit (NL) models, addressing various choice scenarios (Bhat, 2001; Kalouptsidis and Psaraki, 2010; Bhatta and Larsen, 2011; Chang and Lu, 2013; Murray-Truite et al., 2014; Paulssen et al., 2014; Ye et al., 2017; Chen, 2020; Liu, Li and Fan, 2022; Rodrigues, 2022). To overcome limitations like the Independence from Irrelevant Alternatives (IIA) property in Logit models, the Probit model and Dogit model were developed to account for flexible choice attributes (Horowitz, 1980; Ortuzar, 1983; Hoogendoorn and Bovy, 2005; Lewis, 1972; Ai and Norton, 2003; Gaudry and Dagenais, 1979; Gaudry and Wills, 1979; Gaudry, 1981).

Recent studies have advanced the application of disaggregate models. Ye et al. (2017) proposed a statistically rigorous method to test distributional assumptions in the random components of utility functions for MNL and Multiple Discrete-Continuous Extreme Value (MDCEV) models. Rodrigues (2022) introduced an Amortised Variational Inference approach to scale Bayesian inference in mixed MNL models for large datasets, leveraging GPU-accelerated computation. Kalouptsidis and Psaraki (2010) explored approximate computation of choice probabilities in mixed Logit models using Taylor expansions and high-order moments of random coefficients. Paulssen et al. (2014) investigated the influence of personal values on travel mode choice using the ML model. Chen (2020) applied the mixed Logit model to analyse factors influencing residents' mode choices in a specific community, while Liu, Li and Fan (2022) used it to examine cyclist injury severity in daytime and nighttime crashes.

With the rise of shared mobility, recent literature has explored its impact on travel behaviour. Shaheen and Cohen analysed the role of shared mobility in reducing private car use, highlighting its potential to complement public transit (Shaheen and Cohen, 2020). Coenegrachts et al. (2024) examined the substitution effects of car-sharing on public transport in European cities, finding significant shifts in mode choice under specific cost and convenience scenarios. Zhang et al. (2022) used machine learning to predict mode choice shifts with the integration of shared cars, emphasising the role of real-time data in model accuracy. Additionally, García-Melero et al. (2021) applied a hybrid choice model to study the impact of shared mobility on urban traffic patterns, incorporating latent variables like environmental consciousness.

The advent of computer technology has also spurred the use of machine learning in mode choice research, offering data-driven insights into complex traveller behaviours (Cheng et al. 2019; Zhao et al., 2020; Kashifi et al., 2022; Xia, Chen and Zimmermann, 2023). However, traditional models often assume static decision-making, which fails to capture the dynamic adaptation process of travellers under changing conditions. Evolutionary game theory provides a dynamic framework to model this process, analysing how preferences evolve and stabilise over time (Wu et al., 2019; Li et al., 2020; Zhu and Li, 2023; Xu et al., 2024; Lisowski, 2023; Cao et al., 2023). Recent studies, such as those of Xue et al. (2025), have applied evolutionary game theory to study competition between shared and traditional transport modes, highlighting the role of subsidies and pricing strategies. Similarly, Xue et al. (2025) explored the dynamic evolution of mode choices in the context of shared mobility, emphasising the impact of policy interventions.

This study builds on these advancements by applying evolutionary game theory to model the dynamic choice process among shared cars, public transit, and private cars, addressing the gap in understanding how shared mobility influences urban travel behaviour under bounded rationality.

3. Evolutionary dynamics model

This paper focuses on the study of medium-distance travel mode choice behaviour, considering shared car, public transit, and private car travel, not considering short-distance travel such as walking and cycling, and not considering long-distance travel such as trains, planes, and ships. In the process of medium-distance travel mode choice, travellers choose different travel mode according to the principle of maximising their benefits. The parameters of the model are shown in the Table 1.

Table1. Description of parameters

E_S	the benefit of car-sharing travel
E_B	the benefit of public transit
E_P	the benefit of private car
S	the government subsidies for car-sharing travel
C_S	the car-sharing travel cost under the condition of free-flow
C_B	the public transit travel cost under the condition of free-flow
C_P	the private car travel cost under the condition of free-flow
T	the benefit loss of public transit travel due to congestion in the vehicle
F	the benefit loss of private car travel due to road congestion
D	the benefit loss of public transit travel due to road congestion

Source: Authors' own creation.

All the parameters in the table1 are positive, and meet the condition that benefit is greater than loss, the following inequality holds.

$$E_S + S > C_S \tag{1}$$

$$E_B > C_B + D \tag{2}$$

$$E_B > C_B + T \tag{3}$$

$$E_P > C_B + F \tag{4}$$

Let the choice probability of shared car, public transport and private car is $x, y, z \in [0,1]$, and $x + y + z = 1$. The benefit matrix of travel mode choice is shown in Table 2.

Table 2. Benefit matrix

	Shared car	Public transit	Private car
Shared car	$E_S + S - C_S$, $E_S + S - C_S$	$E_S + S - C_S$, $E_B - C_B$	$E_S + S - C_S$, $E_P - C_P$
Public transit	$E_B - C_B$, $E_S + S - C_S$	$E_B - C_B - T$, $E_B - C_B - T$	$E_B - C_B - D$, $E_P - C_P$
Private car	$E_P - C_P$, $E_S + S - C_S$	$E_P - C_P$, $E_B - C_B - D$	$E_B - C_B - F$, $E_B - C_B - F$

Source: Authors' own creation.

Let E_1, E_2 and E_3 are the expected benefit of shared car, public transport and private car, respectively. $\overline{E}$ is the mean expected benefit. The following formula holds:

$$E_1 = x\left(E_S + S - C_S\right) + y\left(E_S + S - C_S\right) + z\left(E_S + S - C_S\right) = E_S + S - C_S \tag{5}$$

$$\begin{aligned} E_2 &= x\left(E_B - C_B\right) + y\left(E_B - C_B - T\right) + z\left(E_B - C_B - D\right) \\ &= \left(x+y\right)D - Ty + E_B - C_B - D \end{aligned} \tag{6}$$

$$E_3 = x\left(E_P - C_P\right) + y\left(E_P - C_P\right) + z\left(E_P - C_P - F\right) = \left(x+y\right)F + E_P - C_P - F \tag{7}$$

$$\begin{aligned} \overline{E} &= xE_1 + yE_2 + zE_3 \\ &= \left(D - T\right)y^2 + \left(E_B - C_B - D\right)y + \left(E_S + S - C_S\right)x + Dxy \\ &\quad - F\left(x+y\right)^2 - \left(x+y\right)\left(E_P - C_P - 2F\right) + E_P - C_P - F \end{aligned} \tag{8}$$

The mutation equation is as follows:

$$\begin{aligned} F_1 &= \frac{dx}{dt} = x\left(E_1 - \overline{E}\right) \\ &= x\left[\left(T-D\right)y^2 - \left(E_B - C_B - D\right)y - \left(E_S + S - C_S\right)x - Dxy + F\left(x+y\right)^2 \right. \\ &\quad \left. + \left(x+y\right)\left(E_P - C_P - 2F\right) + E_S + S - C_s - E_p + C_P + F\right] \end{aligned} \tag{9}$$

$$\begin{aligned} F_2 &= \frac{dy}{dt} = y\left(E_2 - \overline{E}\right) \\ &= y\left[\left(T-D\right)y^2 - \left(E_B - C_B - D + T\right)y - \left(E_S + S - C_S\right)x - Dxy + F\left(x+y\right)^2 \right. \\ &\quad \left. + \left(x+y\right)\left(E_P - C_P - 2F + D\right) + E_B - C_B - D - E_p + C_P + F\right] \end{aligned} \tag{10}$$

4. Equilibrium points and stability

According to the stability theory of differential equations, let $F_1 = 0$ and $F_2 = 0$, and get the equilibrium point, and further analyse the stability of the evolution model.

Case 1: When $y = 0$, the following formula holds:

$$\begin{cases} F_1 = x\left(x-1\right)\left(Fx + E_P - C_P - F - E_S - S + C_S\right) = 0 \\ F_2 = 0 \end{cases} \tag{11}$$

The equilibrium points is $\left(0,0,1\right)$, $\left(1,0,0\right)$ and ($\dfrac{C_P + F + E_S + S - C_S - E_P}{F}$,0, $1 - \dfrac{C_P + F + E_S + S - C_S - E_P}{F}$).

Case 2: when $x = 0$, the following formula holds:

$$\begin{cases} F_1 = 0 \\ F_2 = y\left(y-1\right)\left[\left(F + T - D\right)y + E_P - C_P - F + D - E_B C_B\right] = 0 \end{cases} \tag{12}$$

The equilibrium points is $\left(0,0,1\right)$, $\left(1,0,0\right)$ and (0, $\dfrac{C_P + F - D + E_B - C_B - E_P}{F + T - D}$, $1 - \dfrac{C_P + F - D + E_B - C_B - E_P}{F + T - D}$).

The Jacobian matrix is used to judge the stability at the equilibrium point. The Jacobian matrix is as follows:

$$J(x,y)=\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} \tag{13}$$

$$\begin{aligned}\frac{\partial F_1}{\partial x} &= 2Fx^2+(T+D)y^2-(E_B-D-C_B)y-x(2E_s+2S-2C_s-E_P+C_P+2F) \\ &-2xy(D-F)+F(x+y)^2+(x+y)(E_P-C_P-2F)+E_s+S-C_s-E_P+C_P+F\end{aligned} \tag{14}$$

$$\frac{\partial F_1}{\partial y}=2xy(T+F-D)+(2F-D)x^2-(E_B-D-C_B-E_P+C_P+2F)x \tag{15}$$

$$\frac{\partial F_2}{\partial x}=(2F-D)y^2+2Fxy-(E_S+S-C_S-E_P+C_P+2F-D)y \tag{16}$$

$$\begin{aligned}\frac{\partial F_2}{\partial y} &= (2F+3T-3D)y^2-(2E_B-3D-2C_B+2T-E_P+C_P+2F)y-x(E_s+S-C_s) \\ &-2xy(D-F)+F(x+y)^2+(x+y)(E_P-C_P-2F+D)+E_B-D-C_B-E_P+C_P+F\end{aligned} \tag{17}$$

Equilibrium point 1: $(0,0,1)$, the Jacobian matrix is as follows:

$$J(x,y)=\begin{pmatrix} E_S+S-C_S-E_P+C_P+F & 0 \\ 0 & E_B-D-C_B-E_P+C_P+F \end{pmatrix} \tag{18}$$

When $E_s+S+C_P+F<C_S+E_P$ and $E_B+C_P+F<C_B+D+E_P$, the eigen value of Jacobian matrix is negative, and it is asymptotically stable.

Equilibrium point 2: $(0,1,0)$, the Jacobian matrix is as follows:

$$J(x,y)=\begin{pmatrix} T-E_B+C_B+E_S+S-C_S & 0 \\ 0 & \mathrm{T}+C_B+E_P-E_B-C_P \end{pmatrix} \tag{19}$$

When $T+C_B+E_S+S<C_S+E_B$ and $T+C_B+E_P<E_B+C_P$, the eigenvalue of Jacobian matrix is negative, and it is asymptotically stable.

Equilibrium point 3: when $F>0$ and $D>0$, the equilibrium point is $(\frac{C_P+F+E_S+S-C_S-E_P}{F},0,1-\frac{C_P+F+E_S+S-C_S-E_P}{F})$, the Jacobian matrix is as follows:

$$J(x,y)=\begin{pmatrix} \frac{C_P+F+E_S+S-C_S-E_P}{F} & *_1 \\ 0 & \frac{D(C_P+F+E_S+S-C_S-E_P)+F(C_S+E_B-E_S-S-C_B-D)}{F} \end{pmatrix} \tag{20}$$

Where $*_1$ has no influence on the positive and negative judgment of eigenvalue of the matrix. When $C_S+E_P-F<C_P+E_S+S<C_S+E_P$, $C_S+E_B<E_S+S+C_B+D$, and $D(C_P+F+E_S+S-C_S-E_P)+F(C_S+E_B-E_S-S-C_B-D)<0$, the eigenvalue of Jacobian matrix is negative, and it is asymptotically stable.

Equilibrium point 4: when $F+T>D$, the equilibrium point is (1,0,0), the Jacobian matrix is as follows:

$$J(x,y)=\begin{pmatrix} C_S+E_P-C_P-E_S-S & \mathrm{C}_B+E_P-E_B-C_P \\ 0 & \mathrm{C}_S+E_B-E_S-C_B-S \end{pmatrix} \tag{21}$$

When $C_S+E_P<C_P+E_S+S$ and $C_S+E_B<E_S+C_B+S$, the eigenvalue of Jacobian matrix is negative, and it is asymptotically stable.

Equilibrium point 5: when $F+T>D$, the equilibrium point is (0, $\frac{C_P+F-D+E_B-C_B-E_P}{F+T-D}$, $1-\frac{C_P+F-D+E_B-C_B-E_P}{F+T-D}$), the Jacobian matrix is as follows:

$$J(x,y)=\begin{pmatrix} \frac{F(C_B+E_P+D-C_P-F-E_B)+(F+Y-D)(F+E_S+S+C_P-C_S-E_P)}{F+Y-D} & 0 \\ *_2 & \frac{(C_P+F+E_B-D-C_B-E_B)(E_B+C_P-E_P-C_B-T)}{F+Y-D} \end{pmatrix} \tag{22}$$

Where $*_2$ has no influence on the positive and negative judgment of eigenvalue of the matrix. When $C_B+E_P+D-F<C_P+E_B<C_B+E_P+T$, $C_P+F+E_S+S+C_P<C_S+E_P$ and $(F+T-D)(F+E_S+S+C_P-C_S-E_P)+F(C_B+E_P+D-F-E_B)<0$, the eigenvalue of Jacobian matrix is negative, and it is asymptotically stable. We summarise all the equilibrium points as shown in the Table 3.

Table 3. Equilibrium points and its satisfying conditions

Equilibrium points	Satisfying conditions	Stability
$(0,0,1)$	$E_S+S+C_P+F<C_S+E_P$ $E_B+C_P+F<C_B+D+E_P$	Asymptotically stable
$(0,1,0)$	$T+C_B+E_S+S<C_S+E_B$ $T+C_B+E_P<E_B+C_P$	Asymptotically stable
($\frac{C_P+F+E_S+S-C_S-E_P}{F}$, 0 $1-\frac{C_P+F+E_S+S-C_S-E_P}{F}$)	$F>0, D>0$ $C_S+E_P-F<C_P+E_S+S<C_S+E_P$ $C_S+E_B<E_S+S+C_B+D$ $D(C_P+F+E_S+S-C_S-E_P)$ $+F(C_S+E_B-E_S-S-C_B-D)<0$	Asymptotically stable
$(1,0,0)$	$C_S+E_P<C_P+E_S+S$ $C_S+E_B<E_S+C_B+S$	Asymptotically stable
(0, $\frac{C_P+F-D+E_B-C_B-E_P}{F+T-D}$, $1-\frac{C_P+F-D+E_B-C_B-E_P}{F+T-D}$)	$F+T>D$ $C_B+E_P+D-F<C_P+E_B<C_B+E_P+T$ $C_P+F+E_S+S+C_P<C_S+E_P$	Asymptotically stable

Equilibrium points	Satisfying conditions	Stability
	$(F+T-D)(F+E_S+S+C_P-C_S-E_P)$ $+F(C_B+E_P+D-F-E_B)<0$	

Source: Authors' own creation.

5. Case study

In this part, firstly, the equilibrium point and stability are simulated. Secondly, the government's subsidies for shared car S, the car-sharing travel costs of free flow C_S, and the public transit travel costs of free flow C_B are simulated and analysed.

In this study, we analyse the dynamic evolution of travel mode choices (shared cars, public transit, and private cars) using an evolutionary game theory approach, based on a dataset collected from Beijing in 2024. The dataset comprises travel behaviour survey, capturing their mode choices, socioeconomic characteristics, and perceptions of travel costs and benefits. Travel cost parameters (e.g., C_S, C_B, C_P) were derived from local car-sharing platforms, public transit fare schedules, and average fuel and parking costs, while benefits (e.g. E_S, E_B, E_P) were estimated based on traveller-reported convenience, comfort, and time savings. The dataset and methods are detailed below, followed by simulations of equilibrium points, stability, and the impacts of government subsidies, car-sharing travel costs, and public transit travel costs. The results are further discussed to highlight their implications for urban traffic management and policy.

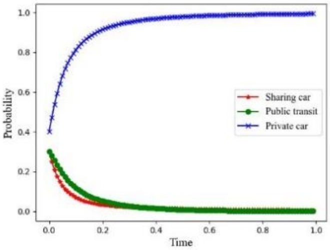
5.1 Equilibrium points and stability

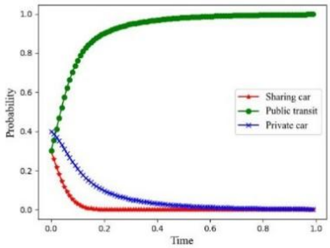
We simulated the results for the five equilibrium points in Table 3. Let the initial value is (0.3, 0.3, 0.4) and take values according to the conditions that the equilibrium point needs to meet, the parameter values are shown in Table 4. These simulated parameter values were calibrated to reflect realistic scenarios in urban transportation systems. For example, in Equilibrium Point 1, where private cars dominate, we set high private car benefits and low costs to simulate a city where private car ownership is heavily subsidised or where public transit and shared cars are underdeveloped. Conversely, in Equilibrium Point 4, we set high shared car subsidies and high private car costs, representing a policy environment that strongly encourages shared mobility. These scenarios enable us to explore how different policy interventions shape travel mode dynamics.

Table 4. Parameter values of equilibrium points

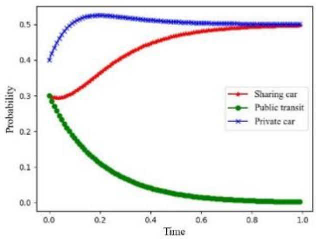
Equilibrium points	E_S	E_B	E_P	C_S	C_B	C_P	S	T	F	D
Equilibrium point 1	50	120	180	35	75	100	4	6	60	30
Equilibrium point 2	50	140	180	25	50	100	20	6	60	30
Equilibrium point 3	50	120	180	20	60	100	20	6	60	30
Equilibrium point 4	50	120	180	20	60	100	55	6	60	30
Equilibrium point 5	50	120	180	35	60	100	3	6	60	30

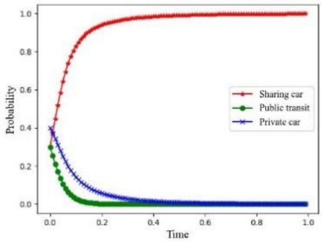
Source: Authors' own creation.



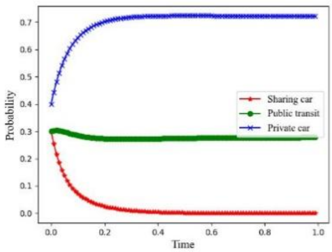


(a) Equilibrium point 1 (b) Equilibrium point 2





(c) Equilibrium point 3 (d) Equilibrium point 4

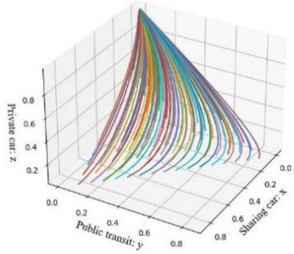


(e) Equilibrium point 5

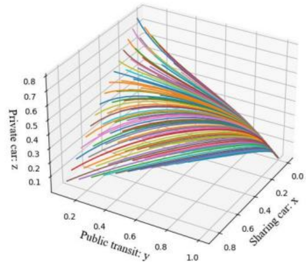
Figure 1. Evolution result of equilibrium points

Source: Authors' own creation.

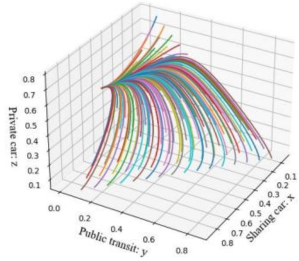
Figure 1 shows the process of travel mode choice evolution. As can be seen from this figure, equilibrium 1: with the increase of time, the probability of choosing shared cars and public transit gradually approaches 0, and the probability of choosing private cars infinitely approaches 1; equilibrium 2: with the increase of time, the probability of choosing shared car and private car gradually approaches 0, and the probability of choosing public transit infinitely approaches 1; equilibrium 3:with the increase of time, the probability of choosing shared cars and private car gradually approaches 0.5, and the probability of choosing public transit infinitely approaches 0; equilibrium 4: with the increase of time, the probability of choosing public transit and private car gradually approaches 0, and the probability of choosing shared car infinitely approaches 1; equilibrium 5: with the increase of time, the probabilities of choosing shared cars, public transit and private car approach 0, 0.28 and 0.72, respectively. These simulation results illustrate how different parameter settings lead to distinct stable states in travel mode choices. In Equilibrium Point 1, private cars dominate due to their perceived benefits (e.g., flexibility, comfort) outweighing those of shared cars and public transit, possibly due to low operating costs or high private car subsidies. Conversely, in Equilibrium Point 4, shared cars become the preferred mode when government subsidies significantly reduce their effective costs, making them more attractive than public transit and private cars. Equilibrium Point 3 represents a mixed strategy where shared cars and private cars have equal choice probabilities, indicating a balance between their benefits and costs. Equilibrium Point 2 shows public transit as the dominant choice, likely occurring when public transit is heavily subsidised or shared cars and private cars are relatively expensive or inconvenient. Finally, Equilibrium Point 5 describes a scenario where private cars remain the most frequently chosen mode, but public transit retains a significant share, suggesting that while private cars are preferred, public transit remains a viable option for some travellers.



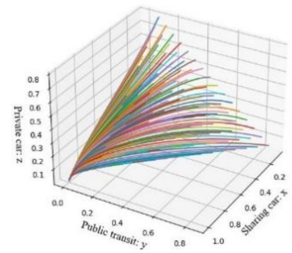
(a) Equilibrium point 1



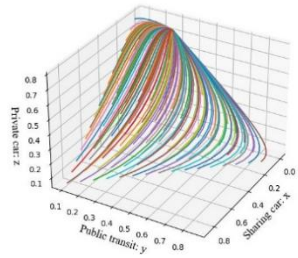
(b) Equilibrium point 2



(c) Equilibrium point 3



(d) Equilibrium point 4



(e) Equilibrium point 5

Figure 2. Evolution results of equilibrium points with the different initial values

Source: Authors' own creation.

Figure 2 shows the evolution results of different initial points. It can be seen from the figure that the initial points have no effect on the final evolution results for five equilibrium points. These results highlight the sensitivity of travel mode choices to relative benefits and costs. For instance, Equilibrium 1 suggests that without sufficient incentives, private cars remain the preferred mode due to their high convenience and autonomy. Equilibrium 4, however, shows that strategic subsidies can shift preferences toward shared cars, aligning with sustainability goals. The robustness across initial conditions underscores the model's applicability to diverse urban contexts.

5.2 The simulation analysis of car-sharing subsidy

When the government's subsidy S for travel using shared cars is 0, 10, 20 and 30, the evolution of travellers' choice of three types of travel modes is analysed. The values of other parameters are shown in Table 5.

Table 5. Parameter values of car-sharing subsidy analysis

Parameters	E_S	E_B	E_P	C_S	C_B	C_P	T	F	D
Value (Yuan)	50	120	180	20	60	100	6	60	30

Source: Authors' own creation.

Let the initial value is (0.3, 0.3, 0.4), the evolution result with the different car-sharing subsidy is shown in Figure 3. It can be seen from the analysis in the figure that, as S increases, the probability of travellers choosing shared cars gradually increases. The probability of choosing public transit gradually decreases, and finally converges to 0. The probability of choosing private cars also gradually decreases, but there is still some probability of choosing private cars for travel.

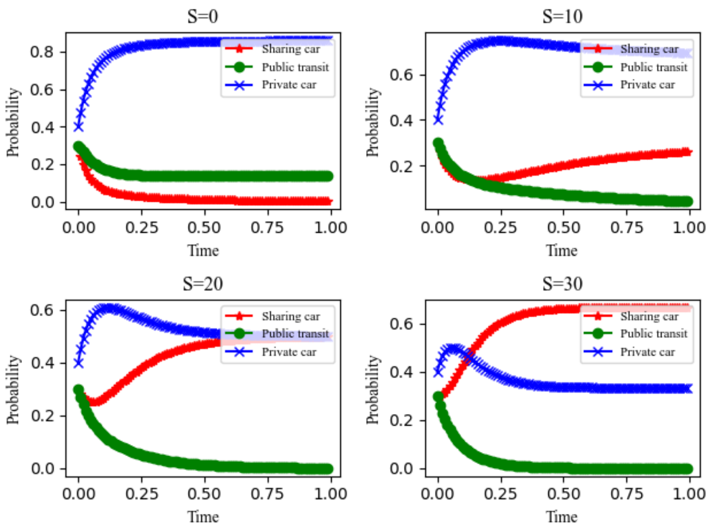


Figure 3. Evolution results with the different car-sharing subsidy
Source: Authors' own creation.

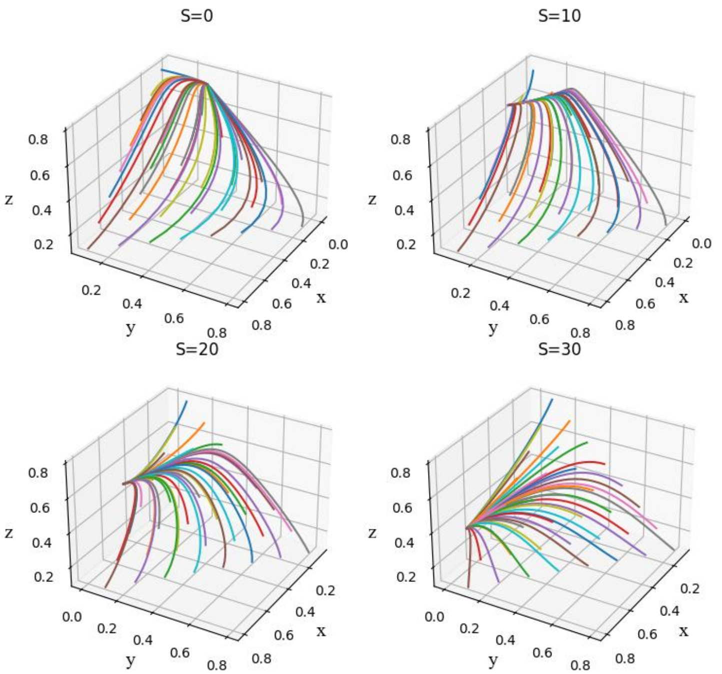


Figure 4. Evolution result of different car-sharing subsidy with different initial values
Source: Authors' own creation.

Figure 4 shows the evolution result of different car-sharing subsidy with different initial values. It can be seen from the figure that the car-sharing subsidy has an effect on the final evolution results, and different car-sharing subsidy eventually converge to the different stable points. It is shown that the government's support for the use of shared cars plays an important role in the direction of travellers to choose the travel mode of shared cars. The government provides appropriate subsidies to stimulate users to transfer to using shared cars, which is also conducive to reducing the probability of using private cars. At the same time, the subsidy should not be too high, which will reduce the probability of public transit choice.

5.3 The simulation analysis of car-sharing travel cost

When the car-sharing travel cost under the condition of free-flow C_S is 0, 15, 25 and 35, the evolution of travellers' choice of three types of travel modes is analysed. The values of other parameters are shown in Table 6.

Table 6. Parameter values of car-sharing travel cost analysis

Parameters	E_S	E_B	E_P	C_B	C_P	S	T	F	D
Value (Yuan)	50	120	180	60	100	20	6	60	30

Source: Authors' own creation.

Let the initial value is (0.3, 0.3, 0.4), the evolution result with the different car-sharing travel cost is shown in Figure 5. It can be seen from the analysis in the figure that as E_S increases, the probability of travellers choosing private car gradually increases. The probability of choosing shared car gradually decreases, and finally converges to 0. The probability of choosing public transit also gradually decreases, but there is still small probability.

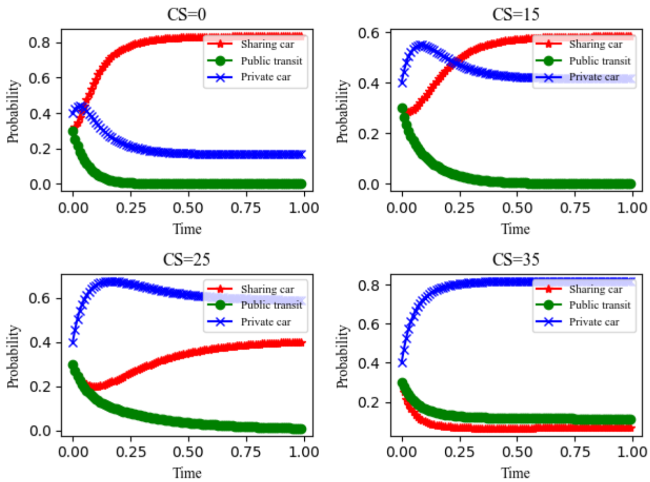


Figure 5. Evolution results with the different car-sharing travel cost
Source: Authors' own creation.

Figure 6 shows the evolution result of different car-sharing travel costs with different initial values. It can be seen from the figure that the car-sharing travel cost shave effect on the final evolution results, and different car-sharing travel costs eventually converge to the different stable points. This shows that, if the enterprise wants to stimulate users' use of sharing car, it needs to reasonably set the travel cost, so that some travellers using private cars and public transit will shift to choose shared cars for travel.

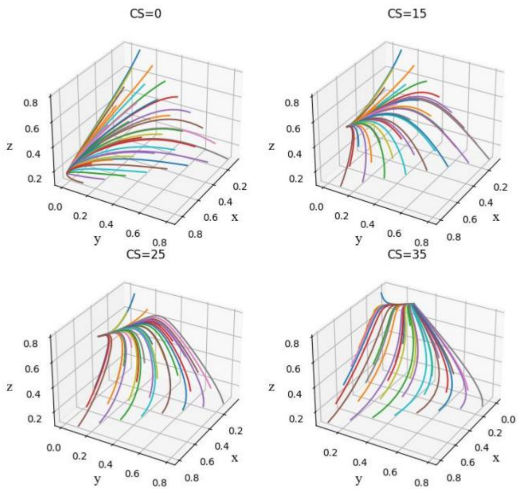


Figure 6. Evolution result of different car-sharing travel cost with different initial values

Source: Authors' own creation.

5.4 The simulation analysis of public transit travel cost

When the public transit travel cost under the condition of free-flow C_B is 40, 45, 50 and 55, the evolution of travellers' choice of three types of travel modes is analysed. The values of other parameters are shown in the following Table 7.

Table 7. Parameter values of public transit travel cost analysis

Parameters	E_S	E_B	E_P	C_S	C_P	S	T	F	D
Value (Yuan)	50	120	180	20	100	20	6	60	30

Source: Authors' own creation.

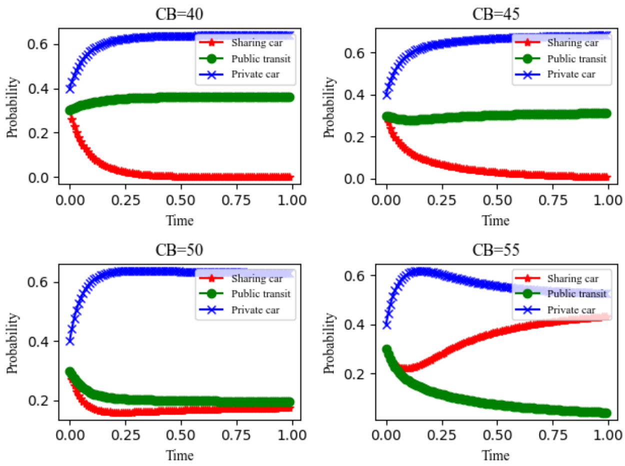


Figure 7. Evolution result with the different public transit travel cost

Source: Authors' own creation.

Let the initial value is (0.3, 0.3, 0.4), the evolution result with the different public transit travel cost is shown in Figure 7. It can be seen from the analysis in the figure that as C_B increases, the probability of travellers choosing public transit car gradually decreases. The probability of choosing sharing car gradually increases. The probability of choosing private also gradually decreases.

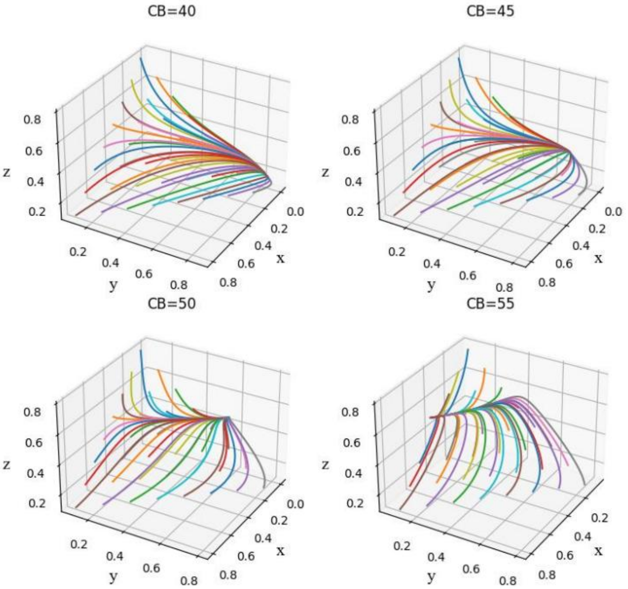


Figure 8. Evolution result of different public transit travel cost with different initial values
Source: Authors' own creation.

Figure 8 shows the evolution result of different public transit travel cost with different initial values. It can be seen from the figure that the public transit travel cost has an effect on the final evolution results, and different public transit travel cost eventually converge to the different stable points. Therefore, it can be concluded that at the charging standard of the public transit can affect the proportion of shared cars. The high charges of public transit will make travellers change to car-sharing mode.

6. Conclusions

This study applies evolutionary game theory to model urban travel mode choice dynamics, examining interactions between shared cars, public transit, and private vehicles. Through stability analysis and numerical simulations, we identify five equilibrium points that illustrate how travellers adjust their choices over time in response to policy interventions and economic conditions. Key findings indicate that government subsidies for shared cars can effectively drive mode shifts, with a statistically significant reduction in private vehicle reliance. However, excessive public transit fares may act as a deterrent to users, creating a potential risk of increased private car usage due to the lack of viable alternatives. These insights highlight the importance of balanced policy interventions, where strategic subsidies and optimal pricing structures can enhance urban mobility efficiency and sustainability.

Despite its contributions, this study has limitations. The model does not account for real-time behavioural variations or external shocks, such as economic downturns or fuel price fluctuations. Future research should explore integrating machine learning-based demand forecasting and real-time data analytics to enhance the predictive accuracy of travel mode evolution models. Additionally, incorporating multi-modal transportation interactions (e.g., micromobility, ride-hailing) could provide a more comprehensive understanding of emerging mobility trends in smart cities.

By offering quantitative insights into travel mode evolution, this research provides a theoretical and practical foundation for urban planners and policymakers to develop more effective, data-driven transportation strategies that promote sustainable mobility and reduce urban congestion.

References

[1] Ai, C., Norton, E.C. (2003), *Interaction Terms in Logit and Probit Models*. *Economics Letters*, 80(1), 123-129.

[2] Coenegrachts, E., Vanelslander, T., Verhetsel, A., Beckers, J. (2024), *Analyzing Shared Mobility Markets in Europe: A Comparative Analysis of Shared Mobility Schemes across 311 European Cities. Journal of Transport Geography*, 118, 103918. https://doi.org/10.1016/j.jtrangeo.2024.103918.

[3] Ashiabor, S., Baik, H., Trani, A. (2007), *Logit Models for Forecasting Nationwide Intercity Travel Demand in the United States. Transportation Research Record: Journal of the Transportation Research Board*, 1-12.

[4] Bhat, C.R. (2001), *Quasi-random maximum simulated likelihood estimation of the mixed multinomial logit model. Transportation Research Part B: Methodological*, 35(7), 677-693.

[5] Bhatta, B.P., Larsen, O.I. (2011), *Errors in variables in multinomial choice modeling: A simulation study applied to a multinomial logit model of travel mode choice. Transport Policy*, 18(2), 326-335.

[6] Cao, G.M., Zhao, X.X., Gao, H.H., Tang, M.C. (2023), *A game theory analysis of intelligent transformation and sales mode choice of the logistics service provider. Advances in Production Engineering & Management,* 18(3), 327-344.

[7] Chang, M.S., Lu, P.R. (2013), *A Multinomial Logit Model of Mode and Arrival Time Choices for Planned Special Events. Journal of the Eastern Asia Society for Transportation Studies*, 10, 710-727.

[8] Chen, P. (2020), *Choice Behavior of Residents' Travel mode based on Mixed Logit Model: A Case Study of Riverside Garden Community*. *World Scientific Research Journal*, 6(1), 125-131.

[9] Cheng, L., Chen, X., De Vos, J., Lai, X., Witlox, F. (2019), *Applying a random forest method approach to model travel mode choice behavior. Travel Behaviour and Society*, 14, 1-10.

[10] Gaudry, M.J.I., Dagenais, M.G. (1979), *The dogit mode. Transportation Research Part B: Methodological*, 13(2), 105-111.

[11] Gaudry, M.J.I., Wills, M.J. (1979), *Testing the dogit model with aggregate time-series and cross-sectional travel data. Transportation Research Part B: Methodological*, 13(2), 155-166.

[12] Gaudry, M.J.I. (1981), *The inverse power transformation logit and dogit mode choice models. Transportation Research Part B: Methodological*, 15(2), 97-103.

[13] Hoogendoorn, S.P., Bovy, P.H.L. (2005), *Pedestrian Travel Behavior Modeling. Networks and Spatial Economics*, 5(2), 193-216.

[14] Horowitz, J. (1980), *The Accuracy of the Multinomial Logit Model as an Approximation to the Multinomial Probit Model of Travel Demand. Transportation Research Part B: Methodological,* 14(4), 331-341.

[15] Huang, Q.L., Wang, W., Liang, X.J., Xu, L., Niu, X.Y., Yang, X.Y. (2022), L*ast-mile delivery optimization considering the demand of market distribution methods: A case studies using Adaptive Large Neighborhood Search algorithm. Advances in Production Engineering & Management,* 17(3), 350-366.

[16] Jefferson, C. (1996), *Improving access by public transport. Landscape and Urban Planning*, 35(2-3), 173-179.

[17] Ortúzar, J. D., Willumsen, L. G. (2011), *Modelling Transport* (4th ed.). Chichester: John Wiley & Sons, Ltd., https://www.wiley.com/en-gb/Modelling+Transport%2C+4th+Edition-p-9781119993520.

[18] Kalouptsidis, N., Psaraki, V. (2010), *Approximations of choice probabilities in mixed logit models. European Journal of Operational Research*, 200(2), 529-535.

[19] Kashifi, M.T., Jamal, A., Kashefi, M.S., Almoshaogeh, M., Rahman, S.M. (2022), *Predicting the travel mode choice with interpretable machine learning techniques: A comparative study. Travel Behaviour and Society*, 29, 279-296.

[20] Lewis, J.A. (1972), *Probit Analysis. Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 21(2), 210-211.

[21] Li, Q., Zhang, Z., Li, K., Wei, Z., Zhang, J. (2020), *Evolutionary dynamics of traveling behavior in social networks. Physica A: Statistical Mechanics and its Applications*, 545, 123664.

[22] Xue, Y., Bao, G., Tan, C., Chen, H., Liu, J., He, T., Qiu, Y., Zhang, B., Li, J., Guan, H. (2025), *Dynamic Evolutionary Game on Travel Mode Choices Among Buses, Ride-Sharing Vehicles, and Driving Alone in Shared Bus Lane Scenarios. Sustainability*, 17(5), 2101, https://doi.org/10.3390/su17052101.

[23] Lisowski, J. (2023), *Computer Simulation of a Game Control Model in a Complex Maritime Traffic Environment. International Journal of Simulation Modelling,* 22(3), 416-425.

[24] Liu, S., Li, Y., Fan, W. (2022), *Mixed logit model based diagnostic analysis of bicycle-vehicle crashes at daytime and nighttime. International Journal of Transportation Science and Technology*, 11(4), 738-751.

[25] Mark, D., Uncles, M. (1987), *Discrete Choice Analysis: Theory and Application to Travel Demand. Journal of the Operational Research Society,* 38(4), 370-371.

[26] Mcfadden, D. (1974), *Conditional Logit Analysis of Qualitative Choice Behavior. Academic Press*, 1, 107-113.

[27] Mcfadden, D., Train, K. (2000), *Mixed MNL models for discrete response. Journal of Applied Econometrics,* 15(5), 447-470.

[28] Murray-Tuite, P., Wernstedt, K., Yin, W.H. (2014), *Behavioral shifts after a fatal rapid transit accident: A multinomial logit model. Transportation Research Part F: Traffic Psychology and Behaviour*, 24, 218-230.

[29] Ortuzar, J.D.D. (1983), *Nested Logit Models for Mixed-mode Travel in Urban Corridors. Transportation Research Part A: General,* 17(4), 283-299.

[30] Paulssen, M., Temme, D., Vir, A., Walker, J.L. (2014), *Values, attitudes and travel behavior: a hierarchical latent variable mixed logit model of travel mode choice. Transportation & Logistics Management*, 41(4), 873-888.

[31] Rodrigues, F. (2022), *Scaling Bayesian inference of mixed multinomial logit models to large datasets. Transportation Research Part B: Methodological*, 158, 1-17.

[32] Shaheen, S., Cohen, A., Farrar, E. (2019), *Chapter five – Carsharing's impact and future. In: Fishman, E. (Ed.), Advances in Transport Policy and Planning*, 4, 87-120, https://www.sciencedirect.com/science/article/abs/pii/S2543000919300356.

[33] Shaheen, S., Cohen, A. (2020), *Shared mobility and urban transport: Opportunities and challenges. Transport Reviews*, 40(6), 763-784.

[34] Xue, Y., Bao, G., Tan, C., Chen, H., Liu, J., He, T., Qiu, Y., Zhang, B., Li, J., Guan, H. (2025), *Dynamic Evolutionary Game on Travel Mode Choices Among Buses, Ride-Sharing Vehicles, and Driving Alone in Shared Bus Lane Scenarios. Sustainability*, 17(5), 2101, https://doi.org/10.3390/su17052101.

[35] Suryani, E., Hendrawan, R.A., Adipraja, P.F.E., Indraswari, R. (2020), S*ystem dynamics simulation model for urban transportation planning: a case study. International Journal of Simulation Modelling*, 19(1), 5-16.

[36] Tu, Q., Geng, G., Zhang, Q. (2023), *Multi-Step Subway Passenger Flow Prediction under Large Events Using Website Data. Tehnički vjesnik,* 30(5), 1585-1593.

[37] Wang, W., Huang, Q. (2023), *Research on Urban End-Delivery Paths Considering the Consumer's Delivery Time Demand. Tehnički vjesnik,* 30(4), 1209-1216.

[38] García-Melero, G., Sainz-González, R., Coto-Millán, P., Valencia-Vásquez, A. (2021), *Sustainable Mobility Policy Analysis Using Hybrid Choice Models: Is It the Right Choice? Sustainability*, 13(5), 2993, https://doi.org/10.3390/su13052993.

[39] Wardman, M. (2004), *Public transport values of time. Transport Policy,* 11(4), 363-377.

[40] Wen, C.H., Koppelman, F.S. (2001), *The generalized nested logit model. Transportation Research Part B: Methodological*, 35(7), 627-641.

[41] Wu, X.H., He, M.L., Cao, C.C., Shi, Y.J. (2019), *Evolution trend analysis of urban residents' low-carbon travel development based on multidimensional game theory. Journal of Central South University*, 2, 3388-3396.

[42] Xia, Y., Chen, H., Zimmermann, R. (2023), *A Random Effect Bayesian Neural Network (RE-BNN) for travel mode choice analysis across multiple regions. Travel Behaviour and Society*, 30, 118-134.

[43] Xu, M.Q., Tan, Q.J., Zhang, Y.Y. (2024), *An evolutionary game analysis of digital decision making in manufacturing enterprises under reward and punishment mechanism. Economic Computation and Economic Cybernetics Studies and Research*, 58(1), 52-69.

[44] Ye, X., Garikapati, V.M., You, D., Pendyala, R.M. (2017), *A practical method to test the validity of the standard Gumbel distribution in logit-based multinomial choice models of travel behavior. Transportation Research Part B: Methodological*, 106, 173-192.

[45] Zhang, W., Zhang, Y. (2022), *Machine learning approaches for predicting mode choice in shared mobility systems. Journal of Transport Geography*, 103, 103389.

[46] Zhao, X., Yan, X., Yu, A., Hentenryck, P.V. (2020), *Prediction and behavioral analysis of travel mode choice: A comparison of machine learning and logit models. Travel Behaviour and Society*, 20, 22-35.

[47] Zhu, D. Li, H.C. (2023), *Research on the mechanism of government incentive measures to rail transit station renewal based on top mode- an evolutionary game analysis. Economic Computation and Economic Cybernetics Studies and Research*, 57(3), 23-42.