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Cost-Volume-Profit Analysis of Intensive Courses in a Higher Education Institute: A Bi-objective Pure Integer Programming Model

Abstract. *In this paper, we address the bi-objective cost-volume-profit problem in a higher education institute in order to maximise the session number of intensive courses as well as the profit of all intensive courses during the semester. In this problem, the intensive courses are predefined in the higher educational system and three main sets including a set of several groups of students to be trained in the classroom (1-2 students, 3-5 students, 6-9 students), a set of teacher's degree (master, PhD), and a set of groups of courses (basic/general courses, specialised/main courses, laboratory/workshop courses) are defined in order to classified the parameters, variables, and constraints concerning to the problem. To solve the problem, a bi-objective pure integer programming (PIP) model is extended, and the ε-constraint multi-objective algorithm is employed to tackle PIP. For this aim, the proposed approach is illustrated by providing 7 real cases. Then, the proposed algorithm on all instances is run, and an optimum Pareto front (non-dominated solutions) is found for each case. Consequently, the computational experiments state that our proposed model and method create an optimised pattern that is applied as a novel approach of the institute to solve the cost-volume-profit problem.*

Keywords: *cost-volume-profit, bi-objective pure integer programming model, intensive courses, ε-constraint multi-objective, higher education institute.*

JEL Classification: C61.

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1. Introduction

Some academic systems such as private colleges and higher education institutes may be in peril of closure because of financial pressures, and on the other hand, the student's enrolment in such institutes is declining (Eide, 2018), since the recession has a significant effect on reducing the ability of students to enrol the private system (Long, 2014). Consequently, these critical conditions lead the private academies to find ways to prohibit bankruptcy. Since the decline of students in higher education institutes worldwide is inevitable, most institutes seek solutions to control their costs

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in order to adapt to the current situation. The revenue of academic systems is provided by students as customers, and the major part of their costs is related to the core of the system, i.e., lecturers and professors. In addition, intensive courses or short-time terms in most private academies allow students to learn efficiently in a short time (Ningrum, 2022; Kamola, 2025), and, as we know, the intensive courses impose less cost on the systems. Therefore, the cost-volume-profit problem is addressed to optimise the cost/profit as well as the number of intensive courses. The following contributions to the literature are offered:

- The cost-volume-profit problem is addressed in a higher education institute, and a novel bi-objective pure integer programming model corresponding to the educational regulations of the institute is extended.
- To characterise the problem under case conditions, a novel data classification based on sets of students, classes, and teachers is defined for the parameters, variables, and constraints.
- Also, an ε-constraint bi-objective algorithm is employed to tackle PIP, and moreover, a new pattern is introduced and analysed for managing the intensive courses in higher education institutes.

The next sections are described as follows. In Section 2, we present several research studies related to intensive courses in educational institutes. In Section 3, a bi-objective pure integer programming model for the intensive courses of the higher education institute is extended. In Section 4, real cases as illustrative data are provided, and the results are analysed, and lastly, the conclusion and the suggestions for future research are presented in Section 5.

2. Literature review

In a study (Wijayanti & Prasetyo, 2021), the cost-volume-profit problem in organic fertiliser production is addressed, and linear programming is constructed to optimise the use of resources as well as profit. Moreover, the BEP variable is analysed according to the results. In another study (Maurya et al, 2015), the linear programming model is extended to optimise the profit of an Ethiopian chemical company. The strength of these researches is to establish the mathematical models for optimising profit in several industries. In different research (Kutnjak et al., 2022), the adaptation of educational institutions to new conditions is addressed, and the essentiality of learning sessions through intensive courses is discussed. Therefore, the experiences of international intensive courses in teaching digital technology are presented. In a study (Ningrum, 2022), an Intensive English Course (IEC) programme is studied. In this survey, the effectiveness of the IEC programme is assessed. The results indicate that the programme is effective in improving the students' performance, as well as teaching methods. In another study (Kamola, 2025), the intensive courses are addressed, and the importance of these courses is investigated. The author concludes that the IEC programme helps students learn the language effectively in a short period of time. Although in these studies, intensive courses are considered as a key parameter for reducing the costs and increasing the profit, there is no quantitative approach to optimise the criteria such as profit, and hence, the lack of a mathematical model is con of these studies. There are few qualitative studies that focus on improving the educational quality and innovation in these systems (Purkayastha, 2024; Umami et al., 2024; Elert and Henrekson, 2025). On the other hand, in a study (Torrieri et al., 2025) the shortage of accommodations for students is addressed, and both profit and quality in the educational system are reviewed; however, there is no quantitative tool to optimise profit as well as the quality. Although theoretical models are proposed to find interactions between non-profit and for-profit institutes or to analyse the literature related to the economics and education, there is no study that formulates mathematical models to optimise the profit as well as quality of education in higher education institutes. In our study, in addition to the profit objective, the satisfaction of students and teachers is optimised. The satisfaction has a direct impact on quality in educational systems. To the best of our knowledge, the cost-volume-profit problem has not been studied for higher education institutes in any research as we studied and no study has extended a bi-objective pure integer programming as well as the ε-constraint approach to solve the problem. The conceptual framework of the problem is shown in Figure 1.

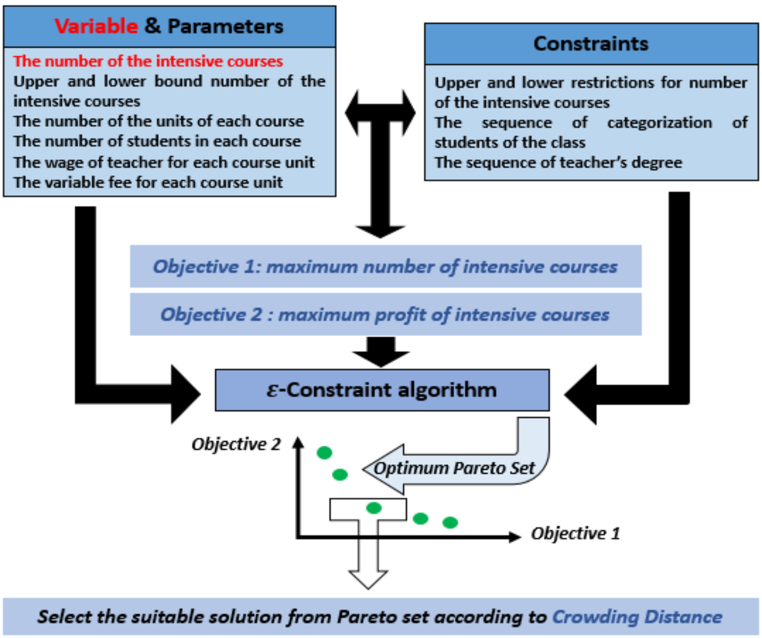

Figure 1. Conceptual framework of optimised pattern for intensive courses

Source: Authors' own creation.

3. Pure integer programming model for the intensive courses of the higher education institute

In the addressed problem, the intensive courses are predefined in the higher educational system according to the educational approvals. There are three main sets: (1) a set of several groups of students to be trained in the classroom (1-2 students, 3-5 students, 6-9 students), (2) a set of teacher's degree (master, PhD), (3) a set of groups of courses (basic/general courses, specialised/main courses, laboratory/workshop courses). To solve the problem, a bi-objective pure integer programming (PIP) model is extended, and an ε-constraint multi-objective approach is employed to tackle PIP. All notations and the parameters of the model are described in Table 1.

Table 1. The Sets and parameters of the pure integer programming model

Notation	Description
Sets	
$i \in \{1,2,3\}$	The categorisation of the students of the classroom (CSC) i.e., 1-2 students, 3-5 students, 6-9 students
$j \in \{1,2\}$	Teacher's degree (TD) in the institute i.e., Master, PhD
$k \in \{1,2,3\}$	The categorisation of the courses (CC) i.e., basic/general courses, specialised/main courses, laboratory/workshop courses
Parameters	
P_{ijk}	The variable fee per course unit based on the categorisation of the courses
N_{ijk}	The number of units related to the course
S_{ijk}	The number of students who enrolled for the course
C_{ijk}	The wage per course unit for each teacher
U_{ijk}	The upper bound number of the intensive course corresponding to the ith CSC, jth TD, kth CC
L_{ijk}	The lower bound number of the intensive course corresponding to the ith CSC, jth TD, kth CC
Variable	
x_{ijk}	The number of the intensive course corresponding to the ith CSC, jth TD, kth CC

Source: Authors' processing.

The pure integer programming model is presented as follows:

$$Max\ Z_1 = \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} x_{ijk} \tag{1}$$

$$Max\ Z_2 = \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} S_{ijk}.P_{ijk}.N_{ijk} - \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} C_{ijk}.N_{ijk}.x_{ijk} \tag{2}$$

$s.t.$

$$x_{ijk} \le U_{ijk} \qquad \forall i \in \{1,2,3\}, j \in \{1,2\}, k \in \{1,2,3\} \tag{3}$$

$$x_{ijk} \ge L_{ijk} \qquad \forall i \in \{1,2,3\}, j \in \{1,2\}, k \in \{1,2,3\} \tag{4}$$

$x_{ijk} \leq x_{(i+1),jk} \quad \forall i \in \{1,2\}, j \in \{1,2\}, k \in \{1,2,3\}$ (5)

$x_{i,(j+1),k} \leq x_{ijk} \quad \forall i \in \{1,2,3\}, j \in \{1\}, k \in \{1,2,3\}$ (6)

$x_{ijk} \in \mathbb{Z} \quad \forall i \in \{1,2,3\}, j \in \{1,2\}, k \in \{1,2,3\}$ (7)

In the above model, equation (1) states the first objective function i.e. the maximum number of intensive course sessions during the semester. Equation (2) expresses the second objective, i.e. the maximum profit of all intensive courses, so that the total revenue is obtained according to the first term and the total cost is calculated according to the second term. Constraints (3) and (4) guarantee that the number of intensive course sessions must be less than the upper bound and more than the lower bound. Constraint (5) ensures that the sequence of the CSCs is met; for instance, the number of intensive course sessions corresponding to the first group (1-2 students) must be less than those of the second group (3-5 students) and those of the second group must be less than those of the third group (6-9 students) for each course by each teacher. Constraint (6) specifies that the number of intensive course sessions corresponding to the PhD teachers must be less than that of the master teachers for each CSC and each course. Constraint (7) reflects the integer decision variables. In the above model, constraints (5) and (6) are presented according to the restrictions and regulations of the higher education institute, and these inequities are called logical constraints. It must be noted that the first objective has a direct impact on the satisfaction of both student and teacher since increasing the number of intensive course sessions leads to improving the knowledge of the student as well as the income of the teacher and, on the contrary, decreasing the number of intensive course sessions consequences the less knowledge for student as well as the less income for teacher, and this situation causes dissatisfaction of system.

Since the problem is modelled as multi-objective mathematical programming, and on the other hand, the first and the second objectives are in contrast to each other, this is necessary to employ a multi-objective approach to solve this model. Although profit is a very important value for the institution, the satisfaction of students as well as teachers is vital for the survival of the institution and, therefore, the first objective function is as important as the profit in order to maintain the institution in the long term. The ε-constraint is applied to solve the bi-objective PIP model. In this approach, the institution determines several desired profits as constraints, and then the optimal number of intensive course sessions is obtained according to the epsilon-constraints. Therefore, the prime model is modified so that the desired profit value is specified as a constraint by decision makers ($Profit_{min}$), and then the bi-objective model is converted to the single objective PIP model as follows:

$$Max\ Z = \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} x_{ijk} \tag{8}$$

$s.t.$

$$\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} S_{ijk}.P_{ijk}.N_{ijk} - \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} C_{ijk}.N_{ijk}.x_{ijk} \geq Profit_{min} \tag{9}$$

$$x_{ijk} \leq U_{ijk} \qquad \forall i \in \{1,2,3\}, j \in \{1,2\}, k \in \{1,2,3\} \qquad (10)$$
$$x_{ijk} \geq L_{ijk} \qquad \forall i \in \{1,2,3\}, j \in \{1,2\}, k \in \{1,2,3\} \qquad (11)$$
$$x_{ijk} \leq x_{(i+1),jk} \qquad \forall i \in \{1,2\}, j \in \{1,2\}, k \in \{1,2,3\} \qquad (12)$$
$$x_{i,(j+1),k} \leq x_{ijk} \qquad \forall i \in \{1,2,3\}, j \in \{1\}, k \in \{1,2,3\} \qquad (13)$$
$$x_{ijk} \in \mathbb{Z} \qquad \forall i \in \{1,2,3\}, j \in \{1,2\}, k \in \{1,2,3\} \qquad (14)$$

As it is shown, the logical constraints are considered in the modified model, and therefore, this model is called the first PIP model. Differently, the above model was rewritten without the logical constraints, and a new small model was constructed without constraints (12,13), and this is called the second PIP model. The first and second PIP models are presented to compare the results of the optimised pattern and to analyse the objective values using Pareto criteria in different situations: (i) when the restrictions of the institute are considered in the problem, (ii) when the restrictions of the institute are removed from the problem.

4. Results and Discussion

4.1 Illustrative Examples

To assess the proposed approach, six real cases concerning seven semesters in a higher education institute are collected. It must be noted that all data from the autumn of 2021 to the winter of 2024 are extracted from the educational and financial databases of the institute. To collect data, firstly, a list of all intensive courses (classes with less than 10 students) related to each semester is provided, and then checklists are completed according to the required parameters as shown in Table 2. It must be noted that 137 records or intensive courses for the 1st semester (2021), 133 records for the 2nd semester (2021), 150 records for the 1st semester (2022), 179 records for the 2nd semester (2022), 275 records for the 1st semester (2023), 285 records for the 2nd semester (2023), 257 records for the 1st semester (2024) were collected. A record sample based on fundamental parameters is displayed in Table 3. The model was coded in GAMS software and ran on an Intel Core (TM) Duo CPU T2450, 2.00 GHz computer with 1 GB of RAM.

Table 2. The fundamental parameters for data collection for each semester

Ro	Parameter Description
1	The number of units related to the course
2	The number of students who enrolled in the course
3	The categorisation of the students of the classroom (i) – CSC
4	Teacher’s degree in institute (j) – TD
5	The categorisation of the courses (k) – CC
6	The identification according to the i,j,k indexes (ijk) – (ID)
7	The approved number of sessions of the intensive course according to the
8	The variable fee per course unit based on the identification
9	The wage per course unit for each teacher
10	The overhead cost of the institute

Source: Authors’ processing.

Table 3. The fundamental parameters for data collection for each semester

Units	Students	CSC	TD	CC	ID	Sessions	Fee	Wage	Overhead
3	5	2	1	2	2.1.2	12	1704828	25572420	0.9

Source: Authors' processing.

It must be noted that the classification structure according to the course identification has been proposed and approved by the higher educational council in the institute, and so, this structure has been applied in this study. The classification structure for some groups is shown in Table 4. The categorisation of the students of the classroom (1-2 students, 3-5 students, 6-9 students) entitled CSC (i) is shown in the second column. The third column represents the teacher's degree in the institute (master, PhD) using the TD (j) title. The categorisation of the courses (basic/general courses, specialised/main courses, laboratory/workshop courses) is indicated in the fourth column, namely CC (k), and the last column entitled ID displays the code of each variable that has been defined according to the indexes and sets of PIP models.

Table 4. The classification structure

CSC (i)	TD (j)	CC (k)	ID
1-2 (1)	MSc (1)	basic/general (1)	1.1.1
1-2 (1)	PhD (2)	specialised/main (2)	1.2.2
3-5 (2)	MSc (1)	laboratory/workshop (3)	2.1.3

Source: Authors' processing.

4.2 Evaluation of the proposed algorithm's performance in all instances

The proposed algorithm to find the optimum solution in this study is the epsilon (ε) constraint, and this method is an approach that is employed in multi-objective optimisation to find Pareto optimal solutions, in this algorithm, one objective is considered as the primary objective function and is optimised while the other objective functions are considered as constraints with specific upper bounds (epsilon values). This method is preferred over the other multi-objective methods, such as sum-weighted and absolute priority methods, since in the epsilon constraint method, the Pareto fronts are generated, and direct control over the spacing of the solutions on the Pareto front is handled.

As presented in Equations (8-9), ε-constraint is applied to convert the bi-objective to the single-objective model so that, the first objective i.e., the maximum number of intensive course sessions during the semester is considered as the main objective and the second objective i.e., the maximum profit of all intensive courses is considered as constraint greater than a threshold as the desired profit of the institute. To compute this desired value (a feasible profit) in the institute for all intensive courses based on Eq. (2), the total income of the variable fee for the classes under 10 students is calculated according to the data collected and $\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} S_{ijk}.P_{ijk}.N_{ijk}$, and then, the overall cost of the institute as well as the total wage of the teachers corresponding to the intensive courses

($\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3} C_{ijk}.N_{ijk}.x_{ijk}$) are subtracted from the total income, considering the least approved number of intensive course sessions.

Table 5. The results of the Pareto front solutions for instance 1

ID / Solution	1*	2*	3*	4*	5*	6*	7*	8*
3.1.1	14	15	15	15	15	15	15	15
1.1.2	6	7	11	12	12	12	12	12
2.1.2	12	12	12	12	12	12	12	12
3.1.2	14	14	14	14	14	14	14	14
1.2.2	4	4	4	6	8	10	11	12
2.2.2	9	9	9	9	9	10	11	12
3.2.2	13	13	13	13	13	13	13	13
1.1.3	4	10	10	10	10	10	10	10
2.1.3	9	10	10	10	10	10	10	10
3.1.3	10	10	10	10	10	10	10	10
1.2.3	3	10	10	10	10	10	10	10
2.2.3	7	10	10	10	10	10	10	10
3.2.3	9	10	10	10	10	10	10	10
f_1: NICS	114	134	138	141	143	146	148	150
f_2: Profit	$1.3e^8$	$1.1e^8$	$0.9e^8$	$0.7e^8$	$0.5e^8$	$0.3e^8$	$0.1e^8$	0

Source: Authors' processing.

This feasible profit is taken into account as the maximum profit obtained based on the least number of intensive course sessions. Moreover, this value is considered as maximum, and zero is considered as minimum profit on the right-hand side related to the constraint in order to obtain optimum Pareto front solutions. The details of the Pareto solution related to instance 1, including the value of decision variables and both objectives, are displayed in Table 5. Also, a sample of the Pareto front related to instance 3 is shown in Figure 2. The horizontal axis shows the first objective, and the vertical axis presents the second objective.

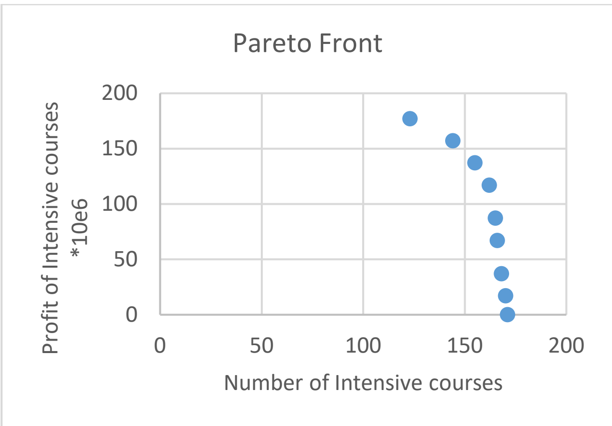

Figure 2. Pareto Front of Instance 3 in the first PIP model
Source: Authors' own creation.

The computational results of the proposed algorithm using the first and second PIP models for all instances are presented in Tables 6 and 7, respectively. The content of these tables gives us the value of the objectives concerning the optimum Pareto front of each semester.

Table 6. The results of the Pareto front solutions of the ε-constraint method using the first PIP model for the instances (semesters 1 to 7)

S	Instance 1		S	Instance 2		S	Instance 3	
	f_1	f_2		f_1	f_2		f_1	f_2
1*	114	136663402	1*	126	79951643	1*	123	177157033
2*	134	116663402	2*	129	69951643	2*	144	157157033
3*	138	96663402	3*	144	59951643	3*	155	137157033
4*	141	76663402	4*	149	49951643	4*	162	117157033
5*	143	56663402	5*	153	39951643	5*	165	87157033
6*	146	36663402	6*	158	29951643	6*	166	67157033
7*	148	16663402	7*	160	19951643	7*	168	37157033
8*	150	0	8*	163	0	8*	170	17157033
						9*	171	0
S	**Instance 4**		**S**	**Instance 5**		**S**	**Instance 6**	
	f_1	f_2		f_1	f_2		f_1	f_2
1*	141	156328861	1*	136	321527457	1*	135	309970594
2*	144	136328861	2*	159	251527457	2*	137	259970594
3*	165	116328861	3*	163	201527457	3*	157	209970594
4*	173	96328861	4*	174	151527457	4*	163	159970594
5*	180	76328861	5*	180	101527457	5*	167	109970594
6*	184	56328861	6*	184	51527457	6*	169	59970594
7*	186	36328861	7*	186	11527457	7*	171	29970594
8*	188	16328861	8*	186	0	8*	171	0
9*	190	0						
S	**Instance 7**		**..S**	**…Instance 7**		**..S**	**…Instance 7**	
	f_1	f_2		f_1	f_2		f_1	f_2
1*	127	737092827	4*	156	437092827	7*	165	137092827
2*	148	637092827	5*	160	337092827	8*	169	0
3*	154	537092827	6*	162	237092827			

Source: Authors' processing.

The S denotes the number of each solution in the optimum Pareto front, and the first and second objective values are notated with f_1 and f_2. As this is indicated, the second objective for both models are same, but the values of the first objective in the second model are better than those of the first model.

Table 7. The results of the Pareto front solutions of the ε-constraint method using the second PIP model for the instances (semesters 1 to 7)

S	Instance 1		S	Instance 2		S	Instance 3	
	f_1	f_2		f_1	f_2		f_1	f_2
1*	114	136663402	1*	124	79951643	1*	123	177157033
2*	134	116663402	2*	140	69951643	2*	146	157157033
3*	138	96663402	3*	147	59951643	3*	155	137157033
4*	142	76663402	4*	153	49951643	4*	163	117157033
5*	144	56663402	5*	158	39951643	5*	168	87157033
6*	147	36663402	6*	163	29951643	6*	169	67157033
7*	149	16663402	7*	166	19951643	7*	171	37157033
8*	152	0	8*	170	0	8*	172	17157033
						9*	174	0
S	Instance 4		S	Instance 5		S	Instance 6	
	f_1	f_2		f_1	f_2		f_1	f_2
1*	141	156328861	1*	145	321527457	1*	131	309970594
2*	159	136328861	2*	163	251527457	2*	149	259970594
3*	163	116328861	3*	167	201527457	3*	157	209970594
4*	181	96328861	4*	184	151527457	4*	163	159970594
5*	187	76328861	5*	188	101527457	5*	168	109970594
6*	192	56328861	6*	193	51527457	6*	173	59970594
7*	195	36328861	7*	196	11527457	7*	176	29970594
8*	197	16328861	8*	197	0	8*	178	0
9*	198	0						
S	Instance 7			…Instance 7			…Instance 7	
	f_1	f_2	..S	f_1	f_2	..S	f_1	f_2
1*	123	737092827	4*	163	437092827	7*	171	137092827
2*	148	637092827	5*	167	337092827	8*	173	0
3*	156	537092827	6*	169	237092827			

Source: Authors' processing.

To compare the generated solutions of the first and second models of all instances, this is necessary to compare their optimum Pareto solution, and for the comparison of two Pareto sets that are found by two different multi-objective algorithms, there are many performance indicators. In this part, the most important measures, such as spacing metric (Schott, 1995), hyper-volume, and coverage of two sets (Zitzler, 1999), are taken into account for the analysis of the results. These performance measures are used to compare the performance of algorithms; nevertheless, our purpose for using the multi-objective metrics in this analysis is to compare two Pareto sets of two different models.

- The Spacing metric (SM) is used to evaluate the uniformity of the spread of the solutions in the non-dominated set, and this measure is calculated as follows:

$$S = \sqrt{\frac{\sum_{i=1}^{N-1}(d_i - \bar{d})^2}{(N-1)}} \quad (15)$$

where d_i denotes the Euclidean distance between the consecutive solutions of the Pareto set, $\bar{d}$ is the mean of all distances, and N is the number of solutions in the Pareto set. It is clear that the smaller the SM, the better the spacing of the Pareto set.

- The hyper-volume (HV) of the Pareto set S is defined as follows:

$$HV(S,r) = \lambda(\cup_{x \in S} H(F(x),r)) \quad (16)$$

where r is a reference point that is dominated by all solutions in S, $\lambda(.)$ is the Lebesgue measure, and $H(a,b)$ represents the hypercube with the body diagonal ab. It is obvious that the solution sets with higher metric values are better because they dominate a larger region of the objective space.

- The Coverage of two sets metric (CT) assesses the coverage of each non-dominated set by another set. The function C maps an ordered pair (A,B) to the interval [0,1] as follows:

$$C(A,B) = \frac{|\{b \in B | \exists\, a \in A: a \succcurlyeq b\}|}{|B|} \quad (17)$$

the value $C(A,B) = 1$ means that all solutions in Pareto set B are dominated by A. On the contrary, $C(A,B) = 0$, shows none of the solutions in B are dominated by A.

The optimum Pareto solutions of the models are compared, and the results of the comparison corresponding to the three measures (SM, CT, and HV) are displayed in Table 8.

Table 8. The comparison of the models according to the SM, CT, and HV measures for the instances (semesters 1 to 7)

Models	Problems	SM	CT	HV
1	Instance 1	0.148	0	18786183496
	Instance 2	0.097	0.125	11782262880
	Instance 3	**0.106**	0	27856695610
	Instance 4	0.102	0.111	26529825868
	Instance 5	0.140	0	52824107002
	Instance 6	0.154	0.125	48144971574
	Instance 7	0.127	0.125	1.1332E+11
2	Instance 1	**0.137**	**0.625**	**18862846898**
	Instance 2	**0.078**	**0.875**	**12161972738**
	Instance 3	0.108	**0.667**	**28171009676**
	Instance 4	**0.088**	**0.778**	**27576785617**
	Instance 5	**0.122**	**1**	**55229381572**
	Instance 6	**0.098**	**0.625**	**48864824544**
	Instance 7	**0.126**	**0.75**	**1.16043E+11**

Source: Authors' processing.

The results indicate that the Pareto set of the second model on all instances, except for instance 3 is better than the Pareto set of the first model although the SM for the solution of instance 3 in the first model is better than the second model; however, the HV and CT for the solution of instance 3 in the second model are better than those of the first model. Consequently, the Pareto set of the first model is dominated by the second model, and in other words, if the restrictions of the institute are not considered to assign the intensive courses, more sessions will be held with the same profit as the first model.

The ANOVA test results for the performance of bi-objective models as a response variable are displayed in Table 9. According to the P-value for both the effect of the model and the instance, it is indicated that these factors affect the response significantly. Moreover, the rejection of the H_0 hypothesis concerning the HV and CT metrics verifies that significant difference between the two models' objectives, and therefore, the statistical results of the HV indicator show that the Pareto set of the second model is better than that of the first model, significantly.

Table 9. ANOVA results for bi-objective models

Hypothesis	$H_0: \mu(HV)_{Model1} = \mu(HV)_{Model2}$ $H_1: Otherwise$				
Source of variation	**DF**	**SS**	**MS**	**F**	***P*-value**
Model (**HV**)	1	4.19744E+18	4.19744E+18	7.55	0.033
Problem	6	1.46312E+22	2.43853E+21	4388.23	0.000
Error	6	3.33418E+18	5.55697E+17		
Total	13	1.46387E+22			
Result	Reject H_0				
Hypothesis	$H_0: \mu(CT)_{Model1} = \mu(CT)_{Model2}$ $H_1: Otherwise$				
Source of variation	**DF**	**SS**	**MS**	**F**	***P*-value**
Model (**CT**)	1	1.66911	1.66911	137.84	0.000
Problem	6	0.06915	0.01152	0.95	0.523
Error	6	0.07265	0.01211		
Total	13	1.81091			
Result	Reject H_0				
Hypothesis	$H_0: \mu(SM)_{Model1} = \mu(SM)_{Model2}$ $H_1: Otherwise$				
Source of variation	**DF**	**SS**	**MS**	**F**	***P*-value**
Model (**SM**)	1	0.000978	0.000978	5.36	0.060
Problem	6	0.004940	0.000823	4.52	0.045
Error	6	0.001094	0.000182		
Total	13	0.007012			
Result	Accept H_0				

Source: Authors' processing.

Furthermore, the statistical results of the CT indicator indicate that the Pareto set of the second model is significantly better than that of the first model, significantly. However, there is no significant difference between the Pareto sets of the two models according to the results of the SM measure. In Figure 3, the comparison of the results of the CT indicator for the two models is shown, and the

Pareto of the second model outperforms the first model. This analysis shows that the second model without sequencing constraints can be promising if some restrictions in the institutes are not taken into account.

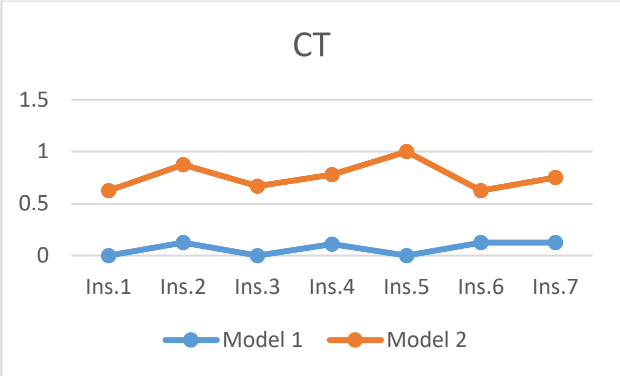

Figure 3. Comparison of two models based on CT-metric for all instances
Source: Authors' own creation.

To select one of the solutions of the Pareto set, the crowding distance (CD) assignment is employed. This criterion is obtained for each point of the Pareto set as follows:

$$d_i^m = |I_{i+1}^m - I_{i-1}^m| / (f_m^{max} - f_m^{min}) \quad (18)$$

$$cd_i = \sum_{i=1}^{m} d_i^m \quad (19)$$

where d_i^m is the distance of solution (i) from the neighbors, I_{i+1}^m and I_{i+1}^m denote the value of mth objective for two neighbors of solution (i), f_m^{max} and f_m^{min} are the maximum and minimum values of mth objective, and cd_i denotes the crowding distance of solution (i).

Table 10. Selected solutions for each semester based on the crowding distance

Instance	CD	S	f_1	f_2
Ins.1	0.959357	2*	134	116663402
Ins.2	0.790692	3*	144	59951643
Ins.3	1.439708	2*	144	157157033
Ins.4	0.923843	3*	165	116328861
Ins.5	0.913219	2*	159	251527457
Ins.6	1.044833	3*	157	209970594
Ins.7	0.914193	2*	148	637092827

Source: Authors' processing.

It is obvious that the point with the largest CD is the most important solution among all points of the Pareto set because the point that has more distance from its neighbours; occupies the largest region of solution. Therefore, the most important solution of each Pareto set concerning each semester was determined according to the CD assignment. Table 10 shows the selected solution for each semester.

4.3 Discussion

In this section, the changes in the number of intensive course sessions concerning each variable or group versus total profit changes for both proposed PIP models are discussed according to the ANOVA test. The results of the changes are presented in Table 11. The first column shows ID groups or decision variables, and the second column presents the mean of the cost coefficients as a parameter. To obtain these data, the cost coefficient of each decision variable in the second objective on all instances for each PIP model was normalised, and then the averages of the coefficients for instances were obtained. The next two columns display the percentage of changes related to the profit for the first PIP model, and the last two columns show the same percentage for the second PIP model.

Table 11. The changes in the number of intensive course sessions versus profit changes

ID	Parameter	Model 1		Model 2	
		A*	B**	A	B
1.1.1	0.01796211	3.1428571	4.5714286	4.4285714	3.4285714
2.1.1	0.02075302	3.7142857	1.2857143	1.7142857	3.4285714
3.1.1	0.02193304	0.4285714	0.2857143	0.4285714	0.5714286
1.2.1	0	0	0	0	0
2.2.1	0.00105963	1.1428571	0	1.1428571	0
3.2.1	0.00105386	0.4285714	0	0.4285714	0
1.1.2	0.17144861	0.7142857	2.1428571	0.7142857	2
2.1.2	0.2231102	0	0.7142857	0	0.4285714
3.1.2	0.25682633	0	0.2857143	0	0.1428571
1.2.2	0.03188342	1	2.8571429	3	4.7142857
2.2.2	0.05801933	0.4285714	3.2857143	0	4.5714286
3.2.2	0.08049724	0.1428571	0.8571429	0	1.4285714
1.1.3	0.02515994	4.2857143	0.4285714	4.2857143	1.7142857
2.1.3	0.02411975	0.7142857	0	0.4285714	0.5714286
3.1.3	0.03360778	0	0	0	0
1.2.3	0.00842594	5.4285714	0.4285714	5.7142857	0
2.2.3	0.00871388	3	0	2.5714286	0
3.2.3	0.01542593	0.8571429	0	0.5714286	0.2857143
Sum	**1**	**25.428571**	**17.142857**	**25.428571**	**23.285714**

* The decrease from 100% to 71.55% of the desired profit
** The decrease from 71.55% to 0% of the desired profit
Source: Authors' processing.

According to the averages of normalised profits for each solution of Pareto sets of instances, two percentage intervals including 'A' (0-28.45%) and 'B' (28.45-100%) were determined. In other words, 'A' denotes decreasing the desired profit to 71.55% of its value, and 'B' denotes decreasing from 71.55% of the desired profit to 0. The data inside columns A and B present the changes in the average number of intensive course sessions versus the decrease in profit percentage.

As indicated in model 1, after deteriorating the profit to 71.55% of its value, the average number of intensive course sessions in groups of ‘1.1.1’, ‘2.1.1’, ‘1.1.3’, ‘1.2.3’, and ‘2.2.3’ is increased (3.14), (3.71), (4.28), (5.28), and (3) respectively, while there is no changes in the number of intensive course sessions for specialised groups such as ‘2.1.2’, and ‘3.1.2’ and besides, the mean of number of intensive course sessions of specialised groups such as ‘1.1.2’, ‘2.2.2’, and ‘3.2.2’ has slight enhance (0.71), (0.43), and (0.14) respectively. Therefore, these results indicate that most categories of workshop/laboratory and general/basic courses with a number of students between 1 to 5 have a high increase versus the dwindle of profit to 71.55% of its value and so this is logical since the relative cost coefficients of these groups are very low in comparison with specialised courses group. As is seen, not only in these results but also in the first example that is displayed in Table 5 after decreasing the profit by less than 20% of its value, the number of intensive course sessions of the first and third groups of courses is increased to its maximum value. Besides, after the slow dwindling of profit to less than 71.55% of its value, the mean of the number of intensive course sessions in groups of ‘1.1.2’, ‘2.1.2’, ‘3.1.2’, ‘1.2.2’, ‘2.2.2’, and ‘3.2.2’ is increased (2.14), (0.71), (0.28), (2.85), (3.28) and (0.85), slowly. Moreover, it is indicated the changes in the number of intensive sessions for courses with master’s degree teachers are less than changes in the number of intensive sessions for courses with Ph.D. degree teachers in the second group because the relative cost coefficients of the first teacher’s group are very high in comparison with the latter group. On the other hand, results in model 2 indicate that there is no significant difference between session number changes of the first and third groups in models 1 and 2 versus a deterioration of the profit to 71.55% of its value. In the second group, the results of model 1 are different from model 2. For example, the total changes of ‘1.2.2’ versus the decrease in profit from 100% to 0% of its value from model 1 to model 2 is (+3.86). The total changes of ‘2.2.2’ versus the decrease of profit from 100% to 0% of its value from model 1 to model 2 is (+0.86). The total change of ‘3.2.2’ versus the decrease of profit from 100% to 0% of its value from model 1 to model 2 is (+0.43). However, the total change of ‘1.1.2’ versus the decrease in profit from 100% to 0% of its value from model 1 to model 2 is (-0.14). The total change of ‘2.1.2’ versus the decrease of profit from 100% to 0% of its value from model 1 to model 2 is (-0.28). The total change of ‘3.1.2’ versus the decrease of profit from 100% to 0% of its value from model 1 to model 2 is (-0.14). Therefore, analysis of the number of intensive course sessions changes of the second group versus the gradual decrease of the profit to 0% demonstrates that the changes in sessions number for courses with Ph.D. degree teachers versus the changes in sessions number for courses with master’s degree teachers has been enhanced in comparison with model 1. Consequently, after removing two logical constraints of the institute from the first model, it is observed that the sessions intensive number of groups with lower cost coefficient will be increased to its maximum value without any order, and it is suggested that the intensive courses, especially in the second group, are distributed among PhD’s degree and master’s degree teachers, uniformly for the main model so that all groups have had fair sessions number during semester.

5. Conclusions

In this paper, the bi-objective cost-volume-profit problem in a higher education institute is studied, and a novel pattern consisting of model and method is proposed to optimise cost-volume-profit for the intensive courses in the institute. In the first place, a bi-objective pure integer programming (PIP) model is extended so that the session number of intensive courses as the first objective and the profit of all intensive courses during the semester as the second objective is maximised. In this PIP model, three main sets are defined in order to categorise the parameters, variables, and constraints concerning the problem. The first set introduces several groups of students to be trained in the classroom (1-2 students, 3-5 students, 6-9 students), the second set presents the teacher's degree (master, Ph.D.), and the last set represents the groups of courses (basic/general courses, specialised/main courses, laboratory/workshop courses). Then, to solve the bi-objective mathematical model, the ε-constraint multi-objective approach is applied, and 7 real instances are provided to illustrate the proposed method. Data of each instance corresponds to the extracted information of the semester and includes the variable fee per course unit based on the categorisation of the courses, the number of units related to the course, the number of students who enrolled for the course, the wage per course unit for each teacher, and upper and lower bound number of the intensive course corresponding each categorised data. This information is collected from the educational and financial database of the institute. Finally, the proposed algorithm on all instances is run, and an optimum Pareto front including non-dominated solutions is found for each semester and the optimum and the most important solution is selected from the optimum Pareto front based on the crowding distance criterion. On the other hand, the sequencing constraints related to the regulations of the institute in the main PIP are removed, and the second PIP model is solved using ε-constraint. The results indicate that the Pareto set obtained by solving the second model is not dominated by the Pareto set of the first model according to the SM, CT, and HV metrics. Consequently, the new model can be promising if some restrictions in the institutes are not considered. The result analysis indicates that our proposed pattern can be applied as a novel approach of institute for the cost-volume-profit problem.

Lastly, some directions as opportunities for future work in this scope are suggested. According to the importance of education in Internet banking (Aleca et al, 2025), applying the optimised pattern is proposed for intensive courses in Internet banking. Some MADM, such as AHP or ANP, can be applied to obtain weights for each decision variable in the first objective or the categorisation of the intensive course, and the results of the weighted model can be compared to the current simple model. Moreover, some metaheuristic algorithms can be developed to solve this problem for very large sizes.

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