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## **A Nested Game Model for Real-Time Pricing of Smart Grid under Dual-Carbon Target**

**Abstract.** *With the advancement of China's dual-carbon goal strategy, large-scale integration of renewable energy into power grid has posed significant challenges. To address these challenges effectively, it is crucial to encourage users to actively adjust their electricity consumption according to price signals and prioritise renewable energy. This can not only ensure the stability of the power grid but also improve economic efficiency. A smart grid system, consisting of an aggregated power supplier and multiple residential users, is considered. In this system, all users form a population, and the supplier offers electricity to users in two forms: traditional and renewable energy power. The supplier is responsible for setting prices and incentives, while users independently make decisions regarding their power demand based on the information provided by the supplier. Different from the existing literature, this study also incorporates the guilt and discomfort cost, which reflects users' psychological responses to different types of power. As a result, a nested game model is established. Specifically, a Stackelberg game is played between the supplier and the user population, and an evolutionary game occurs within the user population. To solve this model, two iterative algorithms are developed. Moreover, numerical analysis is carried out to verify the rationality of the model and the effectiveness of the algorithms. The results indicate that renewable power is consistently cheaper. Both traditional and renewable power can achieve supply-demand equilibrium quickly. Additionally, the residential demand for renewable power accounts for 17.26% of total residential demand, which can help achieve China's binding targets for renewable energy consumption.*

**Keywords:** smart grid, Stackelberg game, evolutionary game, discomfort cost, guilt cost.

**JEL Classification:** C7, D5.

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### **1. Introduction**

In the context of the gradual depletion of traditional energy sources and the ongoing transformation of the global climate, energy issues have attracted significant attention all over the world. In response to these challenges, China set ambitious targets in September 2020 for achieving 'carbon peak' by 2030 and 'carbon neutrality' by 2060. To this end, in June 2022, nine government departments jointly released

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the “14th Five-Year Plan for Renewable Energy Development,” which aims to increase the proportion of renewable energy generation in the nation's total electricity consumption to over 50% by 2025. Building on this, in May 2024, the State Council issued the “Energy Conservation and Carbon Reduction Action Plan for 2024-2025,” aiming to strengthen the promotion of energy conservation and carbon reduction efforts. Subsequently, in July, the National Development and Reform Commission and the National Energy Administration jointly released the “Notice on the Responsibility Weight of Renewable Energy Electricity Consumption and Related Matters for 2024”, which clearly stated that the responsibility weight of renewable energy electricity consumption in 2024 is a binding indicator. It is evident that for a considerable period in the future, the coordinated supply of traditional and renewable energy will be an inevitable trend. Therefore, how to effectively promote the consumption of renewable energy power and ensure the coordinated supply of these two types of electricity are key issues that need to be addressed urgently (Zhang & Sun, 2024; Etanya et al., 2025). Traditional energy power, generated from resources such as coal, oil, and natural gas among others, provides a stable output that can be adjusted to meet demand. However, it consumes non-renewable resources and emits substantial amounts of CO<sub>2</sub>, conflicting with China's dual-carbon goals. In contrast, renewable energy electricity, or green electricity, produces minimal carbon emissions but is unstable and weather dependent (Gan et al., 2025; Iris & Lam, 2021). Due to the difficulty of large-scale storage and the requirement for real-time balance between supply and demand in power systems, achieving a complete balance solely through supply-side regulation capabilities is challenging (Gao, 2022; Yang et al., 2023; Wang et al., 2024; Luo & Gao, 2025). In the context of smart grid environments, demand-side management has attracted considerable attention (Gelazenskaskas et al., 2014; Chiu et al., 2017; Gao, 2022; Tamaki et al., 2024), which involves guiding users to actively adjust their electricity consumption through pricing adjustments or incentive measures. And one of the key strategies for DSM is demand response (Vishal & Safak, 2022; Mohammad & Madeleine, 2024; Hua et al., 2024), which involves guiding users to actively adjust their electricity consumption through pricing adjustments or incentive measures. Furthermore, price responsiveness lies at the core of demand response. Existing pricing mechanisms mainly include fixed pricing, time-of-use pricing, peak pricing, and real-time pricing (RTP). Among these, RTP is widely regarded as one of the most effective and economical pricing mechanisms, and is considered the ideal pricing mechanism for future smart grid environments (Luo et al., 2023).

Existing research on RTP primarily focuses on two methodologies: social welfare maximisation and game theory. The former, inspired by Samadi et al. (2010), employs optimisation techniques to obtain RTP, considering the social aspects of electricity (Li et al., 2024; Qu et al., 2024; Yang et al., 2023; Luo et al., 2023). The latter, viewing electricity as a commodity, utilises game models to establish supply-demand equilibrium (Mohsenian-Ran et al., 2010; Ozge & Ummuhan, 2024; Dipti et al., 2016; Wang et al., 2024; Dai et al., 2021; Ma et al., 2024; Cai et al., 2014; Li et al., 2023; Hu et al., 2023). In smart grids, where RTP is largely governed by

supply-demand dynamics, the application of game theory in analysing strategic interactions among various entities offers significant adaptability. Given the hierarchical nature of electricity markets, investigating RTP through Stackelberg game frameworks has emerged as a new research priority (Dipti et al., 2016; Wang et al., 2014; Dai et al., 2021; Ma et al., 2024; Cai et al., 2014; Li et al., 2023; Hu et al., 2023). However, most existing studies concentrate on systems with a single supplier and multiple electricity users. Although studies (Cai et al., 2014; Li et al., 2023; Hu et al., 2023) considered multiple suppliers and users, they all assumed that suppliers sell the identical electricity and engage in non-cooperative game play. In fact, the relationship between traditional and renewable energy power suppliers is more complex than mere competition. To establish a novel energy system that is clear, low-carbon, safe, controllable, flexible, efficient, intelligent, friendly, open, and interactive, it is crucial to coordinate the supply of both power types. Suppliers should induce users to prioritise renewable energy through price signals and incentive measures (Dai et al., 2021; Wang et al., 2024), achieving the balance of supply and demand while maximising their own utility. Additionally, existing literature has relatively little consideration for users' psychological factors (Zhang & Sun, 2024; Wang et al., 2024) such as national identity and pride, which can influence their electricity consumption choices. Furthermore, few studies considered residential users' bounded rationality and homogeneity (Cai et al., 2014; Li et al., 2023; Gao et al., 2021), which is closer to the actual situation. And in this paper, we try to address these issues.

It should be clarified that among all the literature, our study is most closely related to (Cai et al., 2014; Li et al., 2023), with four main differences: (1) Electricity is divided into two types based on its sources: traditional and renewable energy power; (2) An aggregated power supplier is introduced to coordinate power supply, rather than assuming non-cooperative behaviour among power suppliers; (3) Psychological factors are taken into consideration; (4) An incentive factor for the consumption of renewable energy is introduced on the supply side.

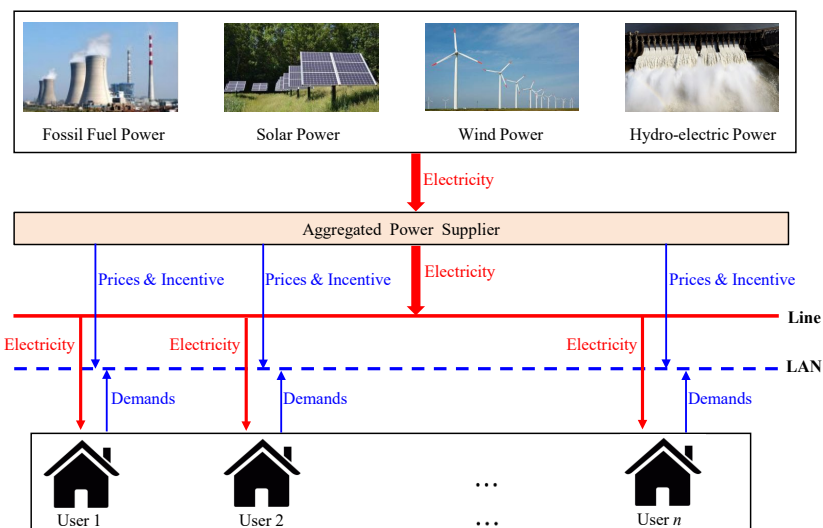
The structure of this paper is as follows: First, the system model is presented in the second section. The third section explores the evolutionary game among user population and provides an iterative algorithm for identifying the evolutionary equilibrium. The fourth section examines the Stackelberg game between the aggregated power supplier and user population, offering the proof of existence and an iterative algorithm for the Stackelberg equilibrium. The fifth section involves numerical simulations to validate the model's rationality and the algorithm's feasibility. The final section concludes the study.

## **2. System model**

Consider a smart grid system that comprises an aggregated power supplier and  $n$  residential users. The supplier distributes electricity via power lines. Simultaneously, local area networks (LANs) facilitate two-way, real-time communication between the supplier and each user through smart meters. The supplier consolidates electricity from diverse generators and distributes it in the form

of both traditional and renewable energy sources. By setting different electricity prices and offering various incentives, the supplier aims to guide users in consuming electricity rationally and prioritising the use of renewable energy. This approach can promote the achievement of dual-carbon goals while also maximising the supplier's own interests. It is assumed that each user can only select one type of electricity for consumption, with the objective of maximising their personal welfare. A power cycle is divided into  $T$  time slots, with no coupling between each slot. Without loss of generality, we take an arbitrary time slot  $t \in \{1, 2, \dots, T\}$  as an example and omit  $t$  for brevity.

At the beginning of the period, the supplier sets the prices for both types of power and announces them to the users through LANs. Recognising the binding targets for renewable energy consumption and its output instability, the supplier provides dynamic incentives to users for consuming renewable energy. It is assumed that residential users living in the same smart community are homogeneous, have similar preferences for electricity, and exhibit bounded rationality, aligning with the characteristics of populations in evolutionary games. Upon receiving the announced prices and incentives, each user independently selects one type of electricity to maximise their personal welfare. Concurrently, each user feedback their electricity demands to the supplier through LANs. The supplier then updates the electricity prices based on the received demand from the user population, announces them, and guides generators until supply and demand are balanced. This strategic interaction between the supplier and the user population constitutes a Stackelberg game, wherein the supplier serves as the leader and the user population as the follower. All users constitute a population that reaches equilibrium through evolutionary game dynamics. The system framework can be illustrated in Figure 1.



**Figure 1. System Framework**

*Source:* The images (Users and power plants) are from the web, others are drawn by authors based on PowerPoint.

## 2.1 Aggregated Power Supplier

The aggregated power supplier offers both traditional and renewable energy power, with prices denoted as  $p_{trad}$  and  $p_{renew}$  respectively. Electricity, being a quasi-public good, exhibits notable social characteristics. The prices satisfy  $p_{trad} \in [p_{trad}^{min}, p_{trad}^{max}]$  and  $p_{renew} \in [p_{renew}^{min}, p_{renew}^{max}]$ , where  $p_{trad}^{min}$  and  $p_{renew}^{min}$  are the minimum prices that can only cover the cost of power generation, and  $p_{trad}^{max}$  and  $p_{renew}^{max}$  are the maximum electricity prices set by the government, respectively. The supply quantities of these two types of electricity are denoted as  $L_{trad}$  and  $L_{renew}$ , respectively, satisfying  $L_{trad} \in [L_{trad}^{min}, L_{trad}^{max}]$ ,  $L_{renew} \in [L_{renew}^{min}, L_{renew}^{max}]$ , where  $L_{trad}^{min}$  and  $L_{renew}^{min}$  are the minimum electricity supplies to meet the basic power demand of users, and  $L_{trad}^{max}$  and  $L_{renew}^{max}$  represent the rated power generation capacity.

The supplier's revenue is derived from the electricity charges ( $F$ ) collected from residential users. Expenditures include costs ( $C_1$ ) paid to traditional electricity generators for generation, maintenance costs ( $C_2$ ) for renewable energy generation equipment and incentives ( $C_3$ ) provided to users for consuming renewable energy. Specifically,

$$F = p_{trad}D_{trad} + p_{renew}D_{renew},$$

where  $D_{trad}$  and  $D_{renew}$  are expressed by (6) and (9), respectively.

$$C_1 = aL_{trad}^2 + bL_{trad} + c, C_2 = \theta L_{renew}^2 + \eta L_{renew}, C_3 = \varsigma D_{renew}$$

where  $a, \theta, \varsigma > 0$ ,  $b, c, \eta \geq 0$  are constants. In summary, the welfare of the supplier can be expressed as

$$W_1 = F - C_1 - C_2 - C_3. \quad (1)$$

## 2.2 Users

As the educational level of residents rises, their sense of patriotism and social responsibility tends to increase. They may feel proud when their actions align with national strategic goals, and guilty when they conflict with these goals. It is widely believed that the benefits residential users derive from consuming electricity stem from the satisfaction it brings. However, the costs associated with electricity consumption encompass not only the electricity bills for both types of power but also the guilt incurred from using traditional energy sources and the discomfort resulting from the instability of renewable energy supply.

The residents' satisfaction from electricity consumption can be represented by the quadratic utility function  $U$  (Gao, 2022; Luo et al., 2023; Zhang & Sun, 2024), which is a non-decreasing and concave function. Specifically, the function  $U(x, w)$  is defined as follows:

$$U(x, w) = \begin{cases} wx - \frac{\alpha}{2}x^2, & 0 \leq x \leq \frac{w}{\alpha} \\ \frac{w^2}{2\alpha}, & x > \frac{w}{\alpha} \end{cases}$$

where  $x$  is the amount of users' electricity consumption,  $w$  reflects the user's electricity consumption preference and  $\alpha$  is a pre-determined parameter. Obviously, the user's utility remains unchanged if  $x > \frac{w}{\alpha}$ , i.e. the user's electricity consumption preferences are primarily reflected within  $0 \leq x \leq \frac{w}{\alpha}$ . Based on this, Equation (2) is adopted to depict the  $i$ th user's utility:

$$U(x_i, w_i) = w_i x_i - \frac{\alpha}{2} x_i^2, \quad x_i^{\min} \leq x_i \leq x_i^{\max} = \frac{w_i}{\alpha} \quad (2)$$

where  $x_i^{\min}$  and  $x_i^{\max}$  denote the  $i$ th user's minimum and maximum electricity consumption, respectively.

To encourage users to prioritise the consumption of renewable energy, the supplier offers dynamic incentives, denoted as  $\zeta x_{i,\text{renew}}$ , where  $\zeta > 0$  is the incentive factor, and  $x_{i,\text{renew}}$  is the amount of renewable energy consumed by the  $i$ th user.

The electricity bill expenses of the users correspond to the electricity income of the supplier, i.e.,  $F$ .

When users consume traditional energy power, considering the carbon emissions and the depletion of non-renewable resources during power generation, a sense of guilt is incurred (Zhang & Sun, 2024), denoted as  $C_{i,g}$ ,

$$C_{i,g}(x_{i,\text{trad}}, g_i) = g_i x_{i,\text{trad}}^2$$

where  $g_i > 0$  is the  $i$ th user's guilt coefficient, which varies according to the user's level of environmental awareness, and  $x_{i,\text{trad}}$  denotes the  $i$ th user's traditional energy power consumption.

When users consume renewable energy power, a certain discomfort cost is incurred due to the instability of renewable energy power generation. Let  $C_{i,\text{uncom}}$  represent the  $i$ th user's discomfort cost resulting from the consumption of renewable energy power. Assume  $C_{i,\text{uncom}}$  is proportional to  $x_{i,\text{renew}}$  (the  $i$ th user's consumption of renewable energy) and  $\sigma_{\text{renew}}$  (the degree of fluctuation of renewable energy electricity), we can express  $C_{i,\text{uncom}}$  as follows,

$$C_{i,\text{uncom}}(x_{i,\text{renew}}) = \tau_i x_{i,\text{renew}} \sigma_{\text{renew}}$$

where  $\tau_i > 0$  is the  $i$ th user's discomfort coefficient, and  $\sigma_{\text{renew}} > 0$  denotes the standard deviation of renewable energy power. With the development of new energy storage technologies and the widespread use of new energy storage devices, this part of the cost is expected to diminish.

In summary, the welfare  $W_{2,i}^{\text{trad}}$  and  $W_{2,i}^{\text{renew}}$  of the  $i$ th user from consuming traditional and renewable energy power can be expressed as follows:

$$W_{2,i}^{\text{trad}} = w_{i,\text{trad}} x_{i,\text{trad}} - \frac{\alpha}{2} x_{i,\text{trad}}^2 - p_{\text{trad}} x_{i,\text{trad}} - g_i x_{i,\text{trad}}^2 \quad (3)$$

$$W_{2,i}^{\text{renew}} = w_{i,\text{renew}} x_{i,\text{renew}} - \frac{\alpha}{2} x_{i,\text{renew}}^2 + \zeta \cdot x_{i,\text{renew}} - p_{\text{trad}} x_{i,\text{renew}} - \tau_i x_{i,\text{renew}} \sigma_{\text{renew}}. \quad (4)$$

### 3. Evolutionary game among the user population

Considering the homogeneity and bounded rationality of users living in the same smart community, all users in the same smart community are treated as a single population, and evolutionary game analysis is employed. Suppose that all individuals within the population adopt the same strategy. The participants are all users  $i \in I$ , and the population is the set of all users denoted as  $I = \{1, 2, \dots, n\}$ . The strategy involves the probability of choosing traditional energy power,  $y^{trad}$ , and the probability of choosing renewable energy power,  $y^{renew}$ , where  $0 \leq y^{trad} \leq 1$ ,  $0 \leq y^{renew} \leq 1$  and  $y^{trad} + y^{renew} = 1$ . The payoff is the welfare of the user population when choosing the two types of power, which are denoted as  $W_2^{trad}$  and  $W_2^{renew}$ , respectively. After receiving the electricity prices and incentive measures provided by the supplier, each user selects one type independently adjusting and determining their own behaviour.

#### 3.1 Replication Dynamics

Let  $x_{i,trad}^*$  be the optimal amount of traditional energy power purchased by  $i$ th user, then

$$x_{i,trad}^* = \underset{x_{i,trad}}{\operatorname{argmax}} W_{2,i}^{trad} = \begin{cases} x_i^{min}, & \frac{w_{i,trad} - p_{trad}}{\alpha + 2g_i} \leq x_i^{min} \\ \frac{w_{i,trad} - p_{trad}}{\alpha + 2g_i}, & x_i^{min} \leq \frac{w_{i,trad} - p_{trad}}{\alpha + 2g_i} \leq x_i^{max} \\ x_i^{max}, & \frac{w_{i,trad} - p_{trad}}{\alpha + 2g_i} \geq x_i^{max} \end{cases} \quad (5)$$

Let  $D_{trad}$  denote the total demand for traditional energy power of the user population, then

$$D_{trad} = y_{trad} \sum_{i=1}^n x_{i,trad}^*. \quad (6)$$

According to the characteristics of Stackelberg game, the supplier is the leader and holds the first-mover advantage. After observing the actions of the supplier, users determine their own power demand. Consequently, in the evolutionary game within the user population,  $p_{trad}$  and  $L_{trad}$  are constants. The welfare of the user population will be discussed in two cases below.

① When  $L_{trad} \geq D_{trad}$ , the traditional energy power demand of the user population is met, the welfare of the population can be expressed as

$$\begin{aligned} \pi_{trad} &= \sum_{i=1}^n W_{2,i}^{trad} = \sum_{i=1}^n \left[ (w_{i,trad} - p_{trad}) x_{i,trad}^* - \left( \frac{1}{2} \alpha + g_i \right) (x_{i,trad}^*)^2 \right] \\ &= \frac{1}{2} \sum_{i=1}^n (\alpha + 2g_i) (x_{i,trad}^*)^2 \end{aligned}$$

②when  $L_{trad} < D_{trad}$ , the traditional energy power demand of the user population cannot be met, and the  $i$ th user can only obtain  $\frac{L_{trad}}{D_{trad}}x_{i,trad}^*$  unit of traditional energy power. In this scenario, the welfare of the user population can be quantified as

$$\begin{aligned}\pi_{trad} &= \sum_{i=1}^n \left[ (w_{i,trad} - p_{trad}) \frac{L_{trad}}{D_{trad}} x_{i,trad}^* - \left( \frac{1}{2} \alpha + g_i \right) \left( \frac{L_{trad}}{D_{trad}} x_{i,trad}^* \right)^2 \right] \\ &= \left[ \frac{L_{trad}}{D_{trad}} - \frac{1}{2} \left( \frac{L_{trad}}{D_{trad}} \right)^2 \right] \sum_{i=1}^n (\alpha + 2g_i) (x_{i,trad}^*)^2\end{aligned}$$

To sum up,

$$\pi_{trad} = \begin{cases} \frac{1}{2} \sum_{i=1}^n (\alpha + 2g_i) (x_{i,trad}^*)^2, & L_{trad} \geq D_{trad} \\ \left[ \frac{L_{trad}}{D_{trad}} - \frac{1}{2} \left( \frac{L_{trad}}{D_{trad}} \right)^2 \right] \sum_{i=1}^n (\alpha + 2g_i) (x_{i,trad}^*)^2, & L_{trad} < D_{trad} \end{cases} \quad (7)$$

Similarly, the optimal renewable energy consumption for the  $i$ th user can be determined as

$$\begin{aligned}x_{i,renew}^* &= \operatorname{argmax}_{x_{i,renew}} W_{2,i}^{renew} \\ &= \begin{cases} x_i^{min}, & \frac{w_{i,renew} + \varsigma - p_{renew} - \tau \sigma_{renew}}{\alpha} \leq x_i^{min} \\ \frac{w_{i,renew} + \varsigma - p_{renew} - \tau \sigma_{renew}}{\alpha}, & x_i^{min} \leq \frac{w_{i,renew} + \varsigma - p_{renew} - \tau \sigma_{renew}}{\alpha} \leq x_i^{max} \\ x_i^{max}, & \frac{w_{i,renew} + \varsigma - p_{renew} - \tau \sigma_{renew}}{\alpha} \geq x_i^{max} \end{cases} \quad (8)\end{aligned}$$

The demand of the user population for renewable energy power  $D_{renew}$  is

$$D_{renew} = y_{renew} \sum_{i=1}^n x_{i,renew}^*. \quad (9)$$

The welfare of the user population is

$$\pi_{renew} = \begin{cases} \frac{1}{2} \sum_{i=1}^n \alpha (x_{i,renew}^*)^2, & L_{renew} \geq D_{renew} \\ \left[ \frac{L_{renew}}{D_{renew}} - \frac{1}{2} \left( \frac{L_{renew}}{D_{renew}} \right)^2 \right] \sum_{i=1}^n \alpha (x_{i,trad}^*)^2, & L_{renew} < D_{renew} \end{cases} \quad (10)$$

Thus, the replication dynamic equation of the user population can be formulated as a differential equation as follows

$$\frac{dy_{trad}}{dt} = y_{trad}(\pi_{trad} - \bar{\pi}) = y_{trad}(1 - y_{trad})(\pi_{trad} - \pi_{renew}), \quad (11)$$

where  $\bar{\pi} = y_{trad}\pi_{trad} + y_{renew}\pi_{renew}$  represents the average welfare obtained by the user population from consuming both traditional and renewable energy power. Clearly, if the welfare derived from consuming traditional energy power exceeds that from consuming renewable energy power, the likelihood of the population favouring traditional energy power will rise, aligning with real-world scenarios.



### 3.2 Evolutionary equilibrium

Evolutionary equilibrium is a stability condition that maintains the population strategy unchanged. According to equation (11), when the probability of the user population choosing traditional energy power is either 0 or 1, or when the welfare of the user population choosing traditional energy power is indistinguishable from that of choosing renewable energy power, i.e.,  $\pi_{trad} = \pi_{renew} = \bar{\pi}$ , the state of the user population  $\{y_{trad}, y_{renew}\}$  will remain unchanged, and an equilibrium is obtained. Denote the equilibrium state as  $y^* = [y_{trad}^*, y_{renew}^*]$ , then we have  $\frac{dy_{trad}}{dt}|_{y^*} = 0$ .

Next, we will employ the Lyapunov method to ascertain whether the equilibrium point  $y^*$  is an Evolutionary Stable Strategy (ESS).

**Lemma 1.** (Cai et al., 2014) If  $V(x)$ , a scalar function of the state  $x$ , has a continuous first derivative and satisfies the following conditions:

- $V(x)$  is positive definite;
- $\dot{V}(x)$  is negative definite;
- $V(x) \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ ,

then the equilibrium at the origin is globally asymptotically stable.

**Theorem 1.** The replicator dynamics equation (11) converges to the evolutionary equilibrium point  $y^* = [y_{trad}^*, y_{renew}^*]$ .

*Proof:* Denote  $e_{trad} = y_{trad}^* - y_{trad}$  and let  $V = \frac{1}{2}(e_{trad})^2$ , then  $V$  is continuous and differentiable, and it also possesses the following properties:

- a) It is positive definite. For any  $t$ , it is evident that  $V \geq 0$ ;
- b) Its derivative is negative definite. Notice that

$$\begin{aligned} \frac{dv}{dt} &= e_{trad} \frac{de_{trad}}{dt} = -(y_{trad}^* - y_{trad}) \frac{dy_{trad}}{dt} \\ &= -(y_{trad}^* - y_{trad}) y_{trad} (1 - y_{trad}) (\pi_{trad} - \pi_{renew}). \end{aligned}$$

Without loss of generality, let  $\pi_{trad} > \pi_{renew}$ . From equation (11),  $y_{trad}^* > y_{trad}$ , thus  $\frac{dv}{dt} < 0$ , i.e.  $\dot{V}$  is negative definite;

- c) If  $e_{trad} \rightarrow \infty$ , then  $V \rightarrow \infty$ .

By Lemma 1, the dynamic system (11) is stable, and  $y^* = [y_{trad}^*, y_{renew}^*]$  is an ESS. ■

Theoretically, to identify an ESS, one can first calculate the Jacobian matrix of the replicator dynamics equation and then judge the stability by examining the signs of the eigenvalues of the Jacobian matrix at the stable point. Considering the complexity of the replicator dynamics equation (11), an iterative algorithm is provided here.

#### Algorithm 1 Iterative Algorithm for the Evolutionary Game

**step 0** (Initialisation) Receive the electricity prices  $p_{trad}, p_{renew}$ , incentive factor  $\varsigma$  and power supply  $L_{trad}, L_{renew}$  released by the supplier. Randomly generate  $y_{trad}$ , which represents the proportion of the user population choosing traditional energy power. Then, the proportion of the user population choosing the

renewable energy power is  $y_{renew} = 1 - y_{trad}$ . Given the allowable error  $\varepsilon$  and the iteration step size  $\lambda$ , the algorithm will proceed as follows.

**step 1** (User-side distributed algorithm) Each user determines  $x_{i,trad}^*$  according to equation (5) and  $x_{i,renew}^*$  according to equation (8).

**step 2** (power demand of user population) Determine  $D_{trad}$  according to equation (6) and  $D_{renew}$  according to equation (9).

**step 3** (welfare of user population) Determine  $\pi_{trad}$  according to equation (7) and  $\pi_{renew}$  according to equation (10).

**step 4** (Judgment step) Determine if any of the equations  $0 \leq y_{trad} \leq \varepsilon$ ,  $0 \leq 1 - y_{trad} \leq \varepsilon$ , or  $|\pi_{trad} - \pi_{renew}| \leq \varepsilon$  is valid.

If yes, output  $D_{trad}$ ,  $D_{renew}$ ,  $y_{trad}$ ,  $y_{renew}$ , and end the algorithm.

Otherwise, update  $y_{trad}$  according to the following formula and go to step 2.

$$y_{trad} \leftarrow y_{trad} + \lambda y_{trad} (1 - y_{trad}) (\pi_{trad} - \pi_{renew}),$$

where  $\lambda > 0$  is the step size.

#### 4. Stackelberg game between the supplier and user population

The power supplier consolidates the management of power plants that supply power to the smart community. By designing  $p_{trad}$  and  $p_{renew}$  and incentive measures based on the received power output  $L_{trad}$  and  $L_{renew}$ , the power supplier aims to maximise  $W_1$  while ensuring supply-demand balance. The corresponding optimisation problem is as follows:

$$\max_{p_{trad}, p_{renew}} W_1. \quad (12)$$

Apparently, equation (12) can be decomposed into the following two optimisation problems, according to different sources of power:

$$\begin{aligned} \max_{p_{trad}} W_1^{trad} &= p_{trad} D_{trad} - (a L_{trad}^2 + b L_{trad} + c) \\ \text{s. t.} \quad & p_{trad}^{min} \leq p_{trad} \leq p_{trad}^{max} \end{aligned} \quad (13)$$

and

$$\begin{aligned} \max_{p_{renew}} W_1^{renew} &= p_{renew} D_{renew} - (\theta L_{renew}^2 + \eta L_{renew}) - \varsigma D_{renew} \\ \text{s. t.} \quad & p_{renew}^{min} \leq p_{renew} \leq p_{renew}^{max} \end{aligned} \quad (14)$$

where  $W_1^{trad}$  is the supplier's welfare from selling traditional energy power, and  $W_1^{renew}$  is the supplier's welfare from selling renewable energy power.

In solving problem (13),  $L_{trad}$ , the amount of traditional energy received by the supplier, is known, and should be treated as a constant. Based on the characteristics of Stackelberg game, the supplier makes decisions based on the optimal response of users (given by Equation (5)). Thus problem (13) is a strictly concave function with respect to the decision variables. Furthermore, the constraint set is a bounded, closed, and convex set. Hence, problem (13) is a convex optimisation problem with a unique global optimal solution. By the same token, problem (14) is also a convex optimisation problem and has a unique global optimal solution.

In solving problems (13) and (14), since the supplier cannot obtain users' private information and can only acquire the power demand of the user population through LANs, traditional convex optimisation techniques cannot be applied directly. Based on the supply and demand theory of microeconomics, the power prices can be adjusted according to the following formula:

$$p_{trad} = p_{trad} + \lambda_{trad}(D_{trad} - L_{trad}), \quad (15)$$

$$p_{renew} = p_{renew} + \lambda_{renew}(D_{renew} - L_{renew}) \quad (16)$$

where  $\lambda_{trad}, \lambda_{renew} > 0$  are the power price adjustment coefficients.

In the actual operation of the power grid, the aggregated power supplier, on the one hand, broadcasts the updated prices to the users, and on the other hand, calculates the electricity demand based on market clearing conditions, which is then used to guide the power generation activities of electricity generators. In problem (13), specifically, the electricity price  $p_{trad}$  is treated as a constant, with  $D_{trad}$  set equal to  $L_{trad}$ . By solving the following equation, the optimal power supply  $L_{trad}^*$  can be obtained:

$$\begin{aligned} \max_{L_{trad}} W_1^{trad} &= p_{trad}L_{trad} - (aL_{trad}^2 + bL_{trad} + c) \\ \text{s.t.} \quad &L_{trad}^{min} \leq L_{trad} \leq L_{trad}^{max} \end{aligned}$$

The solution yields the following results:

$$L_{trad}^* = \begin{cases} L_{trad}^{min}, & \frac{p_{trad}-b}{2a} < L_{trad}^{min} \\ \frac{p_{trad}-b}{2a}, & L_{trad}^{min} \leq \frac{p_{trad}-b}{2a} \leq L_{trad}^{max} \\ L_{trad}^{max}, & \frac{p_{trad}-b}{2a} \geq L_{trad}^{max} \end{cases} \quad (17)$$

In problem (14), treating  $p_{renew}$  as a constant and setting  $D_{renew} = L_{renew}$ , the optimal power supply  $L_{renew}^*$  can be obtained by solving the following equation:

$$\begin{aligned} \max_{L_{renew}} W_1^{renew} &= p_{renew}L_{renew} - (\theta L_{renew}^2 + \eta L_{renew}) - \varsigma D_{renew} \\ \text{s.t.} \quad &L_{renew}^{min} \leq L_{renew} \leq L_{renew}^{max} \end{aligned}$$

where

$$L_{renew}^* = \begin{cases} L_{renew}^{min}, & \frac{p_{renew}-\eta-\varsigma}{2\theta} < L_{renew}^{min} \\ \frac{p_{renew}-\eta-\varsigma}{2\theta}, & L_{renew}^{min} \leq \frac{p_{renew}-\eta-\varsigma}{2\theta} \leq L_{renew}^{max} \\ L_{renew}^{max}, & \frac{p_{renew}-\eta-\varsigma}{2\theta} > L_{renew}^{max} \end{cases} \quad (18)$$

**Theorem 2.** (Existence of equilibrium) The game that describes the strategic interaction between the supplier and user population has a Stackelberg equilibrium.

*Proof:* In the upper level, the supplier determines  $p_{trad}$  and  $p_{renew}$  by solving problems (13) and (14). Since these problems are convex optimisation problems, they possess a unique optimal solution. Consequently, a unique equilibrium exists between the supplier and the users.

Based on the power prices published by the supplier, according to Theorem 1, there exists a unique ESS for the population of users in the lower level. Consequently, a unique equilibrium exists between the supplier and the user population.

Theorem 2 establishes the existence of the Stackelberg equilibrium but does not provide a specific solution method. Consequently, an iterative algorithm grounded in the strategic interaction between the supplier and the user population is presented as follows:

**Algorithm 2** Iterative algorithm based on the Stackelberg game

**Step 0 (Initialisation)** The supplier receives  $L_{trad}$  and  $L_{renew}$ , estimates the total demand  $D_{trad}$  and  $D_{renew}$  based on historical data. Set the incentive factor  $\varsigma$  for users to consume renewable energy power, along with the electricity prices  $p_{trad}$  and  $p_{renew}$  based on the condition of supply and demand balance in the power market. Given an error limit  $\varepsilon$  for the algorithm. The parameters involved in the model are specified, including  $a, b, c, \theta, \eta, \alpha, w, g, x_i^{min}, x_i^{max}, L_{trad}^{min}, L_{trad}^{max}, L_{renew}^{min}, L_{renew}^{max}, p_{trad}^{min}, p_{trad}^{max}, p_{renew}^{min}$  and  $p_{renew}^{max}$ . Let the algorithm iteration counter  $s = 0$ . Announcements of  $\varsigma, p_{trad}$  and  $p_{renew}$  are made to the users through the smart transmission system.

**step 1** (Users' side)

Execute Algorithm 1 and return  $D_{trad}$  and  $D_{renew}$ .

**Step 2** (Aggregated Power Supplier Side)

Receive  $D_{trad}$  and  $D_{renew}$ .

Check if both  $0 \leq L_{trad} - D_{trad} < \varepsilon$  and  $0 \leq L_{renew} - D_{renew} < \varepsilon$  are valid.

If yes, output  $p_{trad}, p_{renew}, D_{trad}, D_{renew}, L_{trad}, L_{renew}$ , and end the process.

Otherwise, update the electricity prices according to Equations (15) and (16), and announce the updated prices to the users. Then, guide the power generators to supply electricity according to Equations (17) and (18). Go to Step 1.

## 5. Numerical simulations

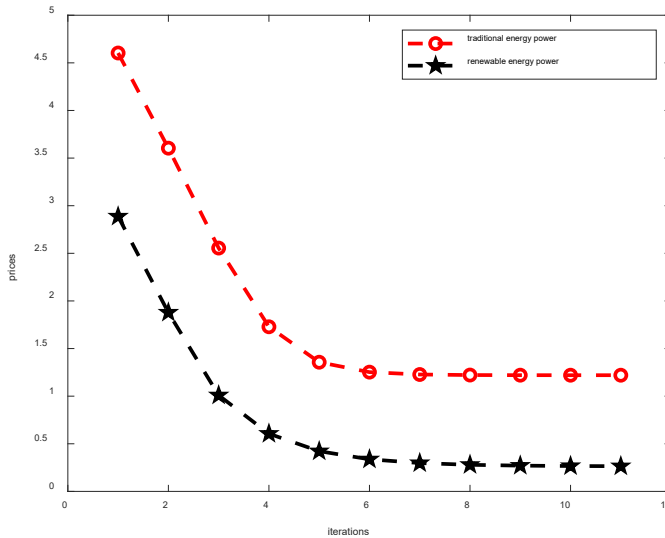
To verify the feasibility of the model and the effectiveness of the algorithm, this section presents a numerical simulation case. A smart grid system comprising an aggregated power supplier and 50 residential users is considered. Again, taking a single time slot as an example. The parameter configurations are detailed in Table 1.

**Table 1. Parameter values**

parameters	values	parameters	values	parameters	values
$a$	0.01	$\theta$	0.01	$p_{trad}^{min}$	0.3
$b$	0	$\eta$	0	$p_{trad}^{max}$	5
$c$	0	$\varsigma$	0.01	$p_{renew}^{min}$	0
$p_{renew}^{max}$	4.5	$\sigma$	40	$L_{trad}^{min}$	50
$L_{trad}^{max}$	150	$L_{renew}^{min}$	0	$L_{renew}^{max}$	190
$\lambda$	0.01	$\lambda_{renew}$	0.01	$g$	[0.08,0.16]
$\lambda_{trad}$	0.01	$\varepsilon$	0.1	$\tau$	U [0.01,0.03]
$\alpha$	0.5	$w_{trad}$	U [1,3]	$w_{renew}$	U [1.5,4]
$x_i^{min}$	1	$x_i^{max}$	3		

*Source:* Some of the parameters were adopted from literature, others were self-set.

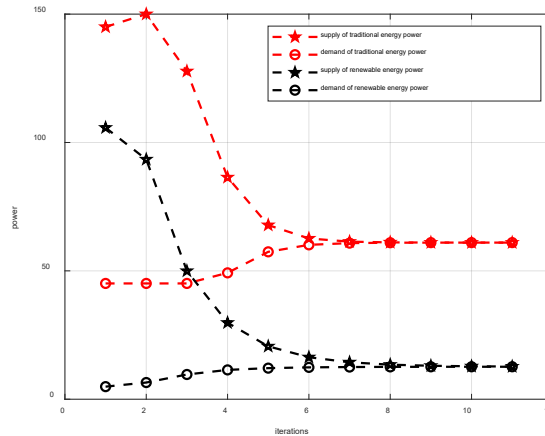
Figure 2 illustrates the evolution of the prices for the two types of power. The algorithm achieves convergence after 11 iterations, at which point the price for traditional energy power is determined to be 1.2201, while the price for renewable energy power is significantly lower at 0.2647. This considerable difference in pricing serves as a strong incentive for consumers to favour the consumption of renewable energy, thereby promoting its priority consumption.



**Figure 2. Evolution of power prices during a certain time slot**

*Source:* Figure 2 is obtained by authors based on Matlab 2021a.

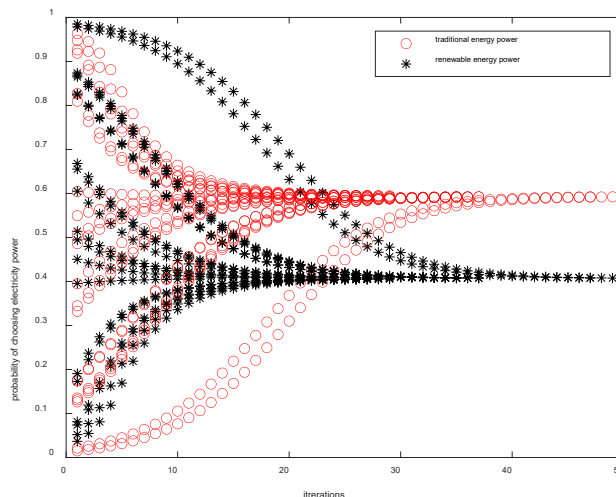
Figure 3 depicts the dynamics of supply and demand for the two types of power. Both types of power swiftly achieved a balance between supply and demand. The total demand for traditional energy power from the residential user population is 61.0043, while the total demand for renewable energy power is 12.7347. The demand for renewable energy power accounts for 17.26%, which achieves China's binding targets of renewable energy consumption.



**Figure 3. Evolution of supply and demand for the two types of electricity**

Source: Figure 3 is obtained by authors based on Matlab 2021a.

Figure 4 displays the evolutionary outcomes of the user population with 20 different randomly generated initial probabilities, given  $p_{trad} = 1.22$ ,  $p_{renew} = 0.26$ ,  $L_{trad} = 61.00$  and  $L_{renew} = 12.73$ . As shown in Figure 4, the probabilities of the user population selecting the two types of power are not sensitive to the initial values and are only related to the prices and supply quantities of the two types of power. And as depicted in Figure 4, the user population swiftly attains the evolutionary equilibrium state through mutual imitation and learning, with a probability of 0.5953 for choosing traditional energy power and 0.4047 for selecting renewable energy power.



**Figure 4. Evolutionary Game Results of User Population**

Source: Figure 4 is obtained by authors based on Matlab 2021a.

## 6. Conclusions

In an integrated grid with multiple energy sources, guiding users to prioritise the use of renewable energy through pricing mechanisms and incentive strategies can effectively enhance renewable energy usage, improve energy efficiency, and bolster social welfare while maintaining the balance of supply and demand within the power system. This paper employs a nested game model to analyse the strategic interaction between the supply side and the demand side by providing dynamic incentives for the consumption of renewable energy power, considering both economic costs and emotional factors on the demand side. The findings indicate that the two types of electricity swiftly reach a supply-demand balance, with the price of renewable energy consistently lower than that of traditional energy, thereby effectively promoting the consumption of renewable energy.

However, the coordinated supply of renewable and traditional energy electricity will persist until renewable energy becomes the mainstream source of electricity. In fact, with the large-scale deployment of energy storage facilities, the instability of renewable energy output will be mitigated. As the educational level of residents increases, their sense of social responsibility and patriotic sentiment will also be further enhanced, leading to active consumption of renewable energy electricity. Of course, in the process of constructing the new energy system, appropriate incentive measures formulated by the supply side based on the received electricity situation can effectively promote the timely and efficient consumption of renewable energy electricity, reduce carbon emissions, and promote the realisation of the low-carbon goal.

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