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Distributed Real-Time Pricing for Smart Grid with Multiple Sellers and Integrated with Renewable Energy and Storage Devices

Abstract. *In this paper, a distributed real-time pricing algorithm based on social welfare maximisation is proposed to institute electricity buying-back schemes for the smart grid that contains multiple providers and integrated with renewable energy (RE) and storage devices. In the proposed model, a profit function is introduced to encourage people to use more RE, and the depreciation of the storage capacity is considered. By dual decomposition, the primal multiseller-multibuyer problem is decoupled into a set of single-buyer and single-seller-single-time-slot subproblems, through which the relationship between prices of electricity and Lagrangian multipliers is derived. Then, a distributed algorithm is further designed to obtain the optimal solution. The strong duality of the original problem is also demonstrated. With this approach, subproblems are solved by each user and utility company, respectively, which ensures privacy and system scalability. Numerical results show that the proposed method has good performance in reducing peak-time loading and balancing system energy distribution.*

Keywords: *real-time pricing; multiple sellers and multiple users; renewable energy and storage devices; dual decomposition; distributed algorithm.*

JEL Classification: D6.

1. Introduction

Currently, there are increasing renewable energy (RE) and storage devices incorporated into power grids, which is beneficial for grid dependability and resiliency (Miner et al., 2012; Tan et al., 2015). However, due to the uncontrollability of RE and the uncertainty of demand, the power grid faces great challenges in energy supply and demand.

However, as reform and deregulation in the power industry proceed, there are more than one utility company is emerging in the power market (Dai et al., 2017; Deng et al., 2015). How to devise a real-time demand response for the multiseller-multibuyer smart grid with RE and storage devices has become a critical problem.

Dynamic pricing is one of the most essential demand-side management techniques to encourage users to consume energy more carefully and wisely, helping

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the smart grid operator better alleviate peak-time loading and balance energy provisioning. With the integration of bidirectional communication and advanced control technologies for power systems, real-time pricing (RTP) has become the most direct and efficient dynamic pricing (Hussain & Gao, 2018; Namerikawa et al., 2015; Siano & Sarno, 2016).

Social welfare maximisation is a basic model of RTP. Samadi et al. formulated RTP scheme into a social welfare maximisation model and designed distributed gradient algorithm to get RTP(Goudarzi et al., 2011; Samadi et al., 2010). In Nguyen et al. (2017), an alternating direction method of multipliers (ADMM) approach was used to solve the social welfare maximisation model. In Li et al. (2021) and Wang & Gao (2019), Karush–Kuhn–Tucker (KKT) conditions were used to obtain RTP. (Gao, 2022)investigated the relationship between the shadow price and the Lagrange multiplier for a non-smooth optimisation problem. In Li & Gao (2023), the startup cost of the supply side was incorporated into the pricing framework, and the convex hull method was used to calculate the convex hull price.

Besides, Qian et al. (2013) proposed a RTP scheme that reduces the peak-to-average load ratio by solving a two-stage optimisation problem. Kobayashi et al. (2014), Tao & Gao (2020), and Zhu et al. (2018) employed a game or dynamic stochastic process to study RTP.

The existing literature used to assume that there is only one provider in the smart grid. In reality, there are increasing numbers of providers emerging in the power market. Meanwhile, a large amount of RE and storage devices incorporated into power grids make the existing RTP infeasible. Deng et al. (2015) proposed a distributed real-time demand response algorithm for smart grid with multisellers. Kamyab et al. (2016) addressed the interaction among multiple utility companies and multiple customers in smart grid and formulated the DR problem into two noncooperative games. However, these works did not discuss RE and storage devices.

In this paper, we focus on the complex smart grid where there are multiple sellers and multiple users, and each user is equipped with RE and storage devices. We propose a distributed real-time pricing algorithm for utility companies to institute electricity buying-back schemes based on social welfare maximisation, in which we consider coupled constraints that couple the energy demand of all users and over all time slots. In the proposed model, profit function of RE is introduced to encourage people to use more renewable energy. In addition, RE is assumed to be kept for the user's own use or to be sold to the smart grid and the depreciation of the storage capacity is considered. By applying the dual decomposition method, we divide the primal problem into a set of single-buyer and single-seller-single-time-slot subproblems, through which we derive the relationship between prices of electricity and Lagrangian multipliers and further design a distributed algorithm to obtain its optimal solution. In addition, we demonstrate the strong duality of the original problem. The distributed approach can help reduce the difficulty to exchange private information of users and utility companies. At the same time, it ensures the scalability of the system. The contributions of this paper are summarised as follows.

1) A multi-time slots distributed real-time pricing algorithm is designed for multiple-seller smart grid to institute electricity buying-back schemes based on social welfare maximisation.

2) Dual decomposition is introduced to separate the primal problem into a set of single-buyer and single-seller-single-time-slot subproblems, through which the relationship between prices of electricity and Lagrangian multipliers is derived.

3) The strong duality of the original problem is demonstrated, and a distributed algorithm is further designed to determine appropriate prices for energy sales and buying-back.

The rest of the paper is organised as follows. The system model is proposed in Section 2. In Section 3, we formulate RTP framework based on social welfare maximisation and further divide the primal optimisation problem into subproblems by Lagrangian dual decomposition. Besides, we demonstrate the Lagrangian duality of the original problem. In Section 4, a distributed algorithm is presented. In Section 5, simulation results and analysis are reported. Section 6 concludes this paper.

2. System model

Our grid system includes multiple utility companies and multiple users, which are denoted by $\mathsf{M} = \{1,2,\cdots,M\}$ and $\mathsf{N} = \{1,2,\cdots,\mathrm{N}\}$ respectively. Let $\mathsf{T} = \{1,2,\cdots,T\}$ denote the set of time slots. A day is divided into T time slots. Each user is equipped with an energy storage device and distributed renewable resources such as wind turbines and solar photovoltaics. Storage devices charge or discharge electricity with certain depreciation. Each user installs a smart metre to control energy consumption and determine the amount of RE sold back to the grid. The users and the utility companies exchange information related to selling/buying-back pricing, energy requirements, and RE sale schedules through a communication infrastructure such as a local area network.

2.1 Users

Let x_{ij}^k denote the amount of energy consumed by user i supplied by utility company j in time slot k. Considering both the baseline and semi-inelastic demand requirement, we have the following users' demand constraints:

$$\sum_{j\in\mathbf{M}} x_{ij}^k \geq b_i^k, \forall i \in \mathsf{N}, \forall k \in \mathsf{T}, \tag{1}$$

$$\sum_{k\in\mathsf{T}} \sum_{j\in\mathbf{M}} x_{ij}^k \geq \sum_{k\in\mathsf{T}} b_i^k + e_i, \forall i \in \mathsf{N}, \forall k \in \mathsf{T}, \tag{2}$$

where b_i^k represents the baseline demand of user i in time slot k, e_i denotes the total energy consumption of semi-inelastic load of user i.

Each user has RE, i.e., dispatchable resources and nondispatchable renewable energy sources. Nondispatchable RE has fixed cost, and is generated at its maximum available power, therefore, there is no strategy regarding energy production. Denote the output of the nondispatchable RE of microgrid i in time slot k as $g_{i,1}^{k}$, which has fixed value. Let $g_{ij,1}^{k}$ denote the amount of nondispatchable renewable energy sold back to utility company j or kept for its own use with utility company j by user i in time slot k. It satisfies

$$\sum_{j\in \mathbf{M}} g_{ij,1}^{k} = g_{i,1}^{k}. \tag{3}$$

The output and cost of the dispatchable resources are variable; therefore, the microgrids are interested in optimising their dispatchable resources production strategies. Let $g_{i,2}^{k}$ denote the dispatchable resources of user i in time slot k, $g_{ij,2}^{k}$ denote the amount of dipatchable RE sold back to utility company j or kept for its own use with utility company j by user i in time slot k. It satisfies

$$\sum_{j\in \mathbf{M}} g_{ij,2}^{k} = g_{i,2}^{k}, \tag{4}$$

$$0 \le \sum_{j\in \mathbf{M}} g_{ij,2}^{k} \le g_{i,2}^{k,\max}, \tag{5}$$

where $g_{i,2}^{k,\max}$ is the maximum energy production capability for microgrid i in time slot k.

Suppose that each user is equipped with a storage device, energy is stored with depreciation rate γ, and the energy stored by user i at the beginning slot is S_i^0. Denote B_i as the maximum storage capacity. The maximum charging and discharging rates of the battery are identical and denoted by g_b. Define r_{ij}^{k} as the amount of energy charged or discharged from by user i in time slot k, we have $r_i^k = \sum_{j\in \mathrm{N}} r_{ij}^{k}, -g_b \le r_i^k \le g_b$. Since the stored energy cannot exceed the storage capacity, we have

$$0 \le (1-\gamma)^{k-1} S_i^0 + \sum_{t=1}^{k} (1-\gamma)^{k-t} r_i^t \le B_i, \forall k \in \mathrm{T}. \tag{6}$$

Let l_{ij}^{k} denote the energy load of user i from utility company j in time slot k. l_{ij}^{k} satisfies

$$l_{ij}^{k} = x_{ij}^{k} + r_{ij}^{k} - g_{ij,1}^{k} - g_{ij,2}^{k}. \tag{7}$$

when $l_{ij}^{k} > 0$, user i needs to load electricity from utility company j. Otherwise, user i sells back the surplus RE or dispatchable resources.

1) Utility function

The utility function represents the level of satisfaction obtained by the user as a function of its power consumption. It is increasing and concave(Samadi et al., 2010).

The quadratic utility function is usually considered, corresponding to decreasing marginal benefit. For example, for user i in time slot k, the utility derived by energy demand is denoted as

$$U(x_{ij}^k,\omega_{ij}^k)=\begin{cases}\omega_{ij}^k x_{ij}^k-\dfrac{\alpha_{ij}^k}{2}(x_{ij}^k)^2, 0\le x_{ij}^k\le\dfrac{\omega_{ij}^k}{\alpha_{ij}^k},\\ \dfrac{(\omega_{ij}^k)^2}{2\alpha_{ij}^k}, x_{ij}^k\ge\dfrac{\omega_{ij}^k}{\alpha_{ij}^k},\end{cases} \tag{8}$$

where $\omega_{ij}^k,\alpha_{ij}^k$ are parameters, ω_{ij}^k represents user's preference.

2) Energy storage cost function

When an energy storage device charges and discharges electricity from the electric company, it causes operating cost, which depends on how much/quickly it charges and discharges electricity from the electric company. The same as Chiu et al. (2017), we consider the cost of operating energy storage to be a convex function of r_{ij}^k as follows:

$$D_s(r_{ij}^k)=\delta_{ij}^k*(r_{ij}^k)^2+\beta_{ij}^k, \tag{9}$$

where $\delta_{ij}^k>0,\beta_{ij}^k$ are parameters.

3) Cost function of dispatchable RE

Let $CW(g_{i,2}^k)$ denote the variable production cost for generating $g_{i,2}^k$ dispatchable RE in time slot k, which is convex and increasing, with $CW(0)=0$. The same as Atzeni et al. (2013), $CW(g_{i,2}^k)$ is defined as

$$CW(g_{i,2}^k)=\delta_{i,2}^k*(g_{i,2}^k)^2+\sigma_{i,2}^k g_{i,2}^k, \tag{10}$$

where $\delta_{i,2}^k>0$, $\sigma_{i,2}^k$ are given parameters.

4) Profit function of distributed energy

When user i generates g_i^k RE (including nondispatchable RE and dispatchable resources) in time slot k, the user can translate carbon footprints reduction into substantial profits through carbon emission trading. The same as Chiu et al. (2017), the profit that user i corresponding to the amount of RE can obtain is denoted as follows:

$$H(g_i^k)=-m(g_i^k)^2+ng_i^k, \tag{11}$$

where $m>0$, n are given parameters.

User i charges/discharges r_{ij}^k from utility company j, and generates $g_{i,1}^k$ nondispatchable renewable energy and $g_{i,2}^k$ dispatchable renewable energy in time slot k. Electricity from utility company j is charged at price P_j^k. Consequently, the welfare for each user can be denoted as

$$W(x_{ij}^k,\omega_{ij}^k)=\sum_{k\in T}(\sum_{j\in M}(U(x_{ij}^k,\omega_i^k)-D_S(r_{ij}^k)-CW(g_{ij,2}^k)-P_j^k*(x_{ij}^k+r_{ij}^k-g_{ij,1}^k-g_{ij,2}^k))$$
$$+H(g_{i,1}^k)+H(g_{i,2}^k)), \tag{12}$$

where $U(.)$ denotes the user's utility function, $H(.)$ is the profit from carbon emissions trading, $D_S(r_{ij}^k)+CW(\mathrm{g}_{ij,2}^k)+P_j^k(x_{ij}^k+r_{ij}^k-g_{ij,1}^k-g_{ij,2}^k)$ is the user's cost. When $l_{ij}^k=x_{ij}^k+r_{ij}^k-g_{ij,1}^k-g_{ij,2}^k>0$, user i needs to load electricity from the utility company j, which means that the user needs to pay the utility company for loading electricity. Otherwise, the user obtains the pay from the utility company.

2.2 Utility sellers

There are M utility sellers in the supply side. Utility sellers not only determine how much electricity should be generated but also consider which user and how much electricity they should sell. Let L_j^k denote the total electricity supplied by utility company j in time slot k. Due that the amount of electricity of utility company j in time slot k cannot exceed the maximum supply capacity of utility company j, L_j^k satisfies the following constraint:

$$0\le L_j^k\le L_j^{k,\max}, \tag{13}$$

where $L_j^{k,\max}$ the maximum supply capacity in time slot k

For utility sellers, when they generate electricity, the production cost arises. Suppose $C_{kj}(L_j^k)$ indicates the production cost of the utility company j to generate L_j^k electricity in time slot k. From the perspective of microeconomics, the production cost is increasing and convex. We model it as Samadi et al. (2010):

$$C_{kj}(L_j^k)=a_{kj}(L_j^k)^2+b_{kj}L_j^k+c_{kj}, \tag{14}$$

where $a_{kj}>0; b_{kj},c_{kj}\ge 0$ are predetermined parameters.

In time slot k, utility seller generates L_j^k electricity and sells it to users at price P_j^k. Hence, the net profit of utility company j is represented as:

$$\pi(L_j^k)=P_j^kL_j^k-C_{kj}(L_j^k), \tag{15}$$

where $C_{kj}(.)$ represents the cost of generating electricity, $P_j^kL_j^k$ is the profit from selling energy to end users.

3. Problem formulation and transformation

In this section, we formulate the RTP scheme for the above smart grid based on social welfare maximisation. In addition, to solve the problem in a distributed fashion efficiently, we introduce dual decomposition and divide the primary problem into a set of single-user and single-seller-single-time-slot subproblems, which can be individually solved by users and utility companies.

3.1 Problem formulation

Considering the interconnection of different time slots and from the perspective of social fairness, we formulate an RTP scheme for the multiseller-multibuyer smart grid integrated with RE and storage devices into a convex optimisation problem as follows:

(P1)

$$\max_{z}\left(\sum_{k}\sum_{i}\Big(\sum_{j}\big(U(x_{ij}^{k})-CW(g_{ij,2}^{k})-D_{S}(r_{ij}^{k})\big)+H(g_{i,1}^{k})+H(g_{i,2}^{k})\Big)-\sum_{k}\sum_{j}C_{kj}(L_{j}^{k})\right) \quad (16)$$

$$\text{s.t.}\quad (1)-(7),(13),$$

$$\sum_{i\in \mathsf{N}}(x_{ij}^{k}-g_{ij,1}^{k}-g_{ij,2}^{k}+r_{ij}^{k})\le L_{j}^{k},\forall j\in \mathsf{M},\forall k\in \mathsf{T}, \quad (17)$$

here $z=\left\{x_{ij}^{k},g_{ij,1}^{k},r_{ij}^{k},g_{ij,2}^{k},L_{j}^{k}\,\middle|\, i\in \mathsf{N},j\in \mathsf{M},k\in \mathsf{T}\right\}$. $U(x_{ij}^{k})$, $D_S(r_{ij}^{k})$ and $CW(g_{ij,2}^{k})$ are defined in (8), (9) and (10) respectively. The equalities or inequalities (1)-(7) are constraints about the baseline and semi-inelastic home appliances, RE and storage devices. Some of them couple all time slots together, such as (2) and (6). $C_{kj}(L_j^k)$ are specified in (14), which represents cost of utility company j in time slot k. The inequality (17) represents the net requirement of all users from utility company j cannot exceed the electricity that utility company j generates in time slot k.

For simplicity, we denote the opposite of the objective function of Problem (P1) as $f(z)$, $g_{kj}(z)=\sum_{i\in \mathsf{N}}(x_{ij}^{k}-g_{ij,1}^{k}-g_{ij,2}^{k}+r_{ij}^{k})-L_{j}^{k}$, $j\in \mathsf{M}$, $k\in \mathsf{T}$. Then (P1) can be further denoted as the following optimal problem:

(P2)

$$\min f(z) \quad (18)$$

$$\text{s.t.}\quad (1)-(7),(13),$$

$$g_{kj}(z)\le 0,\ j\in \mathsf{M},k\in \mathsf{T}, \quad (19)$$

Because $U(x_{ij}^{k})$, $D_S(r_{ij}^{k})$, $CW(g_{ij,2}^{k})$, $H(.)$ are convex, the objective function in (18) is convex. In addition, the constraints (1)-(7), (13) and (19) are linear functions, so the feasible set is a convex set. Thus, optimisation problem (P2) is convex and we can solve it in a central manner in which we need to know the exact information of users and utility companies. In practice, due to that the information of users and utility companies are private, we may not have sufficient information to solve the problem (P2).

3.2 Lagrangian dual method

Constraint (19) couple all users and utility companies. Such constraints make Problem (P2) difficult to solve in a distributed way. Due to that Problem （P2） is convex, we use Lagrangian multiplier to relax the constraints and transform the primal optimisation problem into the dual argument. That is, Problem （P2） can be solved through minimising its Lagrangian and maximising the corresponding dual function. The Lagrangian function corresponding to (P2) is as follows:

$$
\begin{aligned}
&L(z,\lambda)\\
&= f(z)+\sum_{k\in\mathrm{T}}\sum_{j\in\mathrm{M}}\lambda_{kj}g_{kj}(z)\\
&=-\Bigg[\sum_{k\in\mathrm{T}}\sum_{i\in\mathrm{N}}\Big(\sum_{j\in\mathrm{M}}(U(x_{ij}^k)-CW(g_{ij,2}^k)-D_S(r_{ij}^k))+H(g_{i,1}^k)+H(g_{i,2}^k)\Big)-\sum_{k\in\mathrm{T}}\sum_{j\in\mathrm{M}}C(L_j^k)\\
&\qquad -\sum_{k\in\mathrm{T}}\sum_{j\in\mathrm{M}}\lambda_{kj}\Big(\sum_{i\in\mathrm{N}}(x_{ij}^k-y_{ij}^k-z_{ij}^k+r_{ij}^k)-L_j^k\Big)\Bigg]\\
&=-\Bigg[\sum_{k\in\mathrm{T}}\sum_{i\in\mathrm{N}}\Big(\sum_{j\in\mathrm{M}}(U(x_{ij}^k)-CW(g_{ij,2}^k)-D_S(r_{ij}^k))+H(g_{i,1}^k)+H(g_{\mathrm{i},2}^k)\Big)\\
&\qquad -\sum_{k\in\mathrm{T}}\sum_{j\in\mathrm{M}}\lambda_{kj}\sum_{i\in\mathrm{N}}(x_{ij}^k-g_{ij,1}^k-g_{ij,2}^k+r_{ij}^k)+\sum_{k\in\mathrm{T}}\sum_{j\in\mathrm{M}}(\lambda_{kj}L_j^k-C(L_j^k))\Bigg]\\
&=-\Bigg(\sum_{i\in\mathrm{N}}\sum_{k\in\mathrm{T}}\Big(\sum_{j\in\mathrm{M}}(U(x_{ij}^k)-CW(g_{ij,2}^k)-D(r_{ij}^k))+H(g_{i,1}^k)+H(g_{i,2}^k)\\
&\qquad -\sum_{j\in\mathbf{M}}\lambda_{kj}(x_{ij}^k+r_{ij}^k-g_{ij,1}^k-g_{ij,2}^k)\Big)+\sum_{j\in\mathrm{M}}\sum_{k\in\mathrm{T}}[\lambda_{kj}L_j^k-C(L_j^k)]\Bigg),
\end{aligned}
\tag{20}
$$

where $\lambda=\left\{\lambda_{kj}>0 \,\middle|\, k\in\mathrm{T},\, j\in\mathrm{M}\right\}$ are Lagrangian multipliers. The corresponding dual function is as follows:

$$
\begin{aligned}
g(\lambda)&=\min_z L(z,\lambda)\\
&=-\Big(\sum_{i\in\mathrm{N}}\phi_i(\lambda)+\sum_j\psi_j(\lambda)\Big),
\end{aligned}
\tag{21}
$$

where

$$
\begin{aligned}
\phi_i(\lambda)=\max_{x_{ij}^k,g_{ij,1}^k,g_{ij,2}^k,r_{ij}^k}\sum_{k\in\mathrm{T}}\Big(\sum_{j\in\mathrm{M}}&(U(x_{ij}^k,\omega_i^k)-D_S(r_{ij}^k)-CW(g_{ij,2}^k)-\lambda_{kj}*(x_{ij}^k+r_{ij}^k-g_{ij,1}^k-g_{ij,2}^k))\\
&+H(g_{i,1}^k)+H(g_{i,2}^k)\Big)
\end{aligned}
\tag{22}
$$

$$\text{s.t.}\quad (1)-(7),$$

and

$$
\begin{aligned}
\psi_j(\lambda)=\max_{L_{ij}^k,Q_{ij}^k,L_j^k,Q_j^k}&\sum_{k\in\mathrm{T}}[\lambda_{kj}L_j^k-C(L_j^k)]\\
&\text{s.t.}\quad (13).
\end{aligned}
\tag{23}
$$

Now we have separated the Lagrange dual function into a set of single-user subproblems (22) and single-company subproblems (23). If we set $P_j^k = \lambda_{kj}$, subproblems (22) and (23) are, respectively, equal to the maximisation of the user welfare function and the company profit function. In addition, because the objective function and the constraints in (23) are separable in each time slot, the subproblem (23) can be further transformed into a set of single-company-single-time-slot optimisation problems as follows:

$$\psi_{jk}(\lambda) = \max_{L_j^k}(\lambda_{kj} L_j^k - C(L_j^k)), \quad \text{s.t.} \quad (13). \tag{24}$$

The dual problem of the primal optimisation problem (P2) is

$$\max_{\lambda} g(\lambda) \tag{25}$$

Theorem 1. Problem (P2) satisfies strong duality condition and there is zero duality gap between (P2) and its dual problem.

Proof: According to Section 3.1, (P2) is convex. Besides, because the inequality and equality constraint functions (1)-(7), (13) and (19) are affine, (P2) satisfies Slater conditions. Therefore, the extreme value point z^* is the KKT point, i.e., there is (z^*, λ^*) that satisfies the following KKT conditions:

$$\nabla f(z^*) + \sum_{k \in \mathrm{T}} \sum_{j \in \mathrm{M}} \lambda_{kj}^* \nabla g_{kj}(z^*) = 0, \tag{26}$$

$$\lambda_{kj}^* \geq 0, g_{kj}(z^*) \leq 0, \lambda_{kj}^* g_{kj}(z^*) = 0, \; j \in \mathrm{M}, k \in \mathrm{T}, \tag{27}$$

Because $f(z)$, $g_{kj}(z)$, $j \in \mathrm{M}$, $k \in \mathrm{T}$. are all convex, the condition (26) is equivalent to $L(z^*, \lambda^*) = \min_z L(z, \lambda^*)$, i.e. $L(z^*, \lambda^*) \leq L(z, \lambda^*)$. On the other hand, $L(z^*, \lambda^*) = f(z^*) \geq L(z^*, \lambda, \mu)$. Therefore, (z^*, λ^*) is a saddle point. Consequently, z^* and $\{\lambda^*, \mu^*\}$ are the solutions of the primal problem (P2) and the dual problem (25) respectively. At the same time, we have $f(z^*) = L(z^*, \lambda^*) = \max_{\lambda} L(z^*, \lambda)$, thus there is no duality gap. This completes the proof.

According to Theorem 1, we can solve the dual problem instead of the primal problem. Furthermore, we can solve the dual problem in a distributed fashion, because $g(\lambda)$ has been decomposed into independent subproblems in the form of (22) and (24), respectively, representing the user and company sides. Meanwhile, we can obtain the price of electricity by Lagrangian multipliers, because the selling price satisfies $P_j^k = \lambda_{kj}$.

4. Distributed RTP algorithm

In this section, we design distributed algorithms to solve the dual problem and obtain the price of electricity. We use the gradient projection method to obtain the optimal solution of the dual problem in an iterative manner as follows:

$$\lambda_{kj}^{t+1} = \left(\lambda_{kj}^{t} + \rho_t \left. \frac{\partial g(\lambda)}{\partial \lambda_{kj}} \right|_{\lambda_{kj}=\lambda_{kj}^{t}} \right)^{+}$$
$$= \left(\lambda_{kj}^{t} + \rho_t (\sum_{i \in \mathsf{N}} (x_{ij}^{k,*}(\lambda_{kj}^{t}) + r_{ij}^{k,*}(\lambda_{kj}^{t}) - g_{ij,1}^{k,*}(\lambda_{kj}^{t}) - g_{ij,2}^{k,*}(\lambda_{kj}^{t})) - L_j^{k,*}(\lambda_{kj}^{t})) \right)^{+}, \quad (28)$$

where $t \in \mathsf{T}_1$, T_1 denotes the set of iteration instances, ρ_t is the step size of the sub-gradient method. $\left\{ (x_{ij}^{k,*}(\lambda_{kj}^{t}), g_{ij,1}^{k,*}(\lambda_{kj}^{t}), g_{ij}^{k,*}(\lambda_{kj}^{t}), r_{ij}^{k,*}(\lambda_{kj}^{t})) \mid k \in \mathsf{T} \right\}$ is the optimiser of subproblem (22) for user i, and $\left\{ L_j^{k,*}(\lambda_{kj}^{t}) \mid k \in \mathsf{T} \right\}$ is the optimiser of subproblem (24) for a given $\left\{ (\lambda_{kj}^{t}) \mid k \in \mathsf{T} \right\}$.

The interactions between users and utility sellers are shown in Figure 1. In the whole RTP scheme, each utility seller first sets selling prices $\lambda_{kj}^{0}, k = 1, 2, \cdots T$, and sends them to users. Each user obtains the optimal solution by solving (24) for the given prices. Based on this feedback, the company calculates new prices to maximise profits, and then continues to update and resend the new pricing information to all users. This loop is guaranteed to eventually converge to an optimal solution if ρ_t follows two rules: $\sum_{i=k}^{\infty} \rho_i^2 < \infty$ and $\sum_{i=k}^{\infty} \rho_i = \infty$. When this loop converges to the optimal solution, we obtain the electricity price $\{ \lambda_{kj}^{t,*} \mid k \in \mathsf{T}, j \in \mathsf{M} \}$.

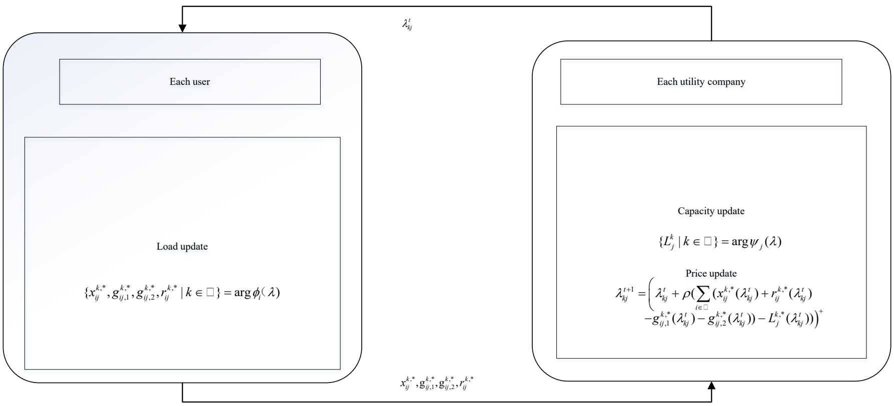

Figure 1. Interactions between users and utility companies

Source: Figure 1 is drawn by authors based on Microsoft word.

The distributed algorithms of each user and each utility seller are summarised in Algorithms 1 and 2, respectively. In Algorithm 1, in Step 0, each user starts with its initial condition, which is assumed to be random. In Steps 1 to 5, each user receives pricing information, obtains the optimal values of $x_{ij}^{k,*}(\lambda_{kj}^t), g_{ij,1}^{k,*}(\lambda_{kj}^t), g_{ij,2}^{k,*}(\lambda_{kj}^t)$ and $r_{ij}^{k,*}(\lambda_{kj}^t)$ by solving subproblem (22) and sends the updated values of $x_{ij}^{k,*}(\lambda_{kj}^t), g_{ij,1}^{k,*}(\lambda_{kj}^t), g_{ij,2}^{k,*}(\lambda_{kj}^t)$ and $r_{ij}^{k,*}(\lambda_{kj}^t)$ to the utility companies. The loop in Steps 1 to 5 continues during the operational cycle of the system. In Algorithm 2, each utility company starts with random initial conditions in Step 0. The loop in Steps 1 to 5 continues during the operational cycle of the system. Within this loop, each utility seller calculates $L_j^{k,*}(\lambda_{kj}^t)$ by solving the subproblem (24), updates λ_{kj}^t in each instance $t \in T$, and further sends the new pricing information to users.

Algorithm 1	Performed by each user
Step 0:	Initialisation.
Step 1:	for each $t \in \mathrm{T}_1$
Step2:	Obtain the optimal solutions $x_{ij}^{k,*}(\lambda_{kj}^t), g_{ij,1}^{k,*}(\lambda_{kj}^t), g_{ij,2}^{k,*}(\lambda_{kj}^t)$ and $r_{ij}^{k,*}(\lambda_{kj}^t)$ by solving subproblem (22) for the given pricing information λ_{kj}^t.
Step 3:	Send $x_{ij}^{k,*}(\lambda_{kj}^t), g_{ij,1}^{k,*}(\lambda_{kj}^t), g_{ij,2}^{k,*}(\lambda_{kj}^t)$ and $r_{ij}^{k,*}(\lambda_{kj}^t)$ to utility companies.
Step 4:	end for

5. Numerical tests

In this section, we perform a simulation to illustrate the effectiveness of the proposed RTP scheme.

Suppose that there are 2 electric sellers and 10 residential users. The whole day is divided into 24 time slots. Each utility company has cost as $ax^2 + bx + c$, where parameter a is randomly chosen from uniform distribution [0.01, 0.02], b=0, c=0. Each user has utility function as (6), where w is randomly selected from the interval [0.5, 4.5]. The parameter α of the utility function is set as 0.1. Each user has a storage device with a depreciation rate of 0.1 and the maximum charging and discharging rates of 2 kw. We assume that each user deploys solar PV following a normal distribution with an expectation of 3 and a variance of 1 in their homes. The generation cost parameters are supposed to be 0.1. The parameters of the profit function of carbon emission trading are set as $m = 0.001$ and $n = 4$. The total load of semi-inelastic appliances of each user follows a uniform distribution [14, 18]. To demonstrate the effectiveness of the proposed algorithm, we study the convergence of prices. In the meantime, to show the load balance of the whole system, we use gird variance to evaluate the smoothness of the grid load curve over a day. The maximum grid loading is also an important consideration for smart grids to prevent power outages. Thus, the peak-to-average ratio (PAR), a widely adopted metric used in previous studies of smart grid operations, is used to demonstrate that our pricing scheme can efficiently reduce peak-time loading without incurring power outages. Besides, we compare the social welfare of the proposed algorithm with that of the other two methods.

By running the above algorithm, we obtain the optimal RTP of electricity and RE to be announced by the providers. Meanwhile, the optimal power consumption, storage, and RE sold back to grid planning for each user and the optimal power production and assignment scheduling for the utility companies can be derived.

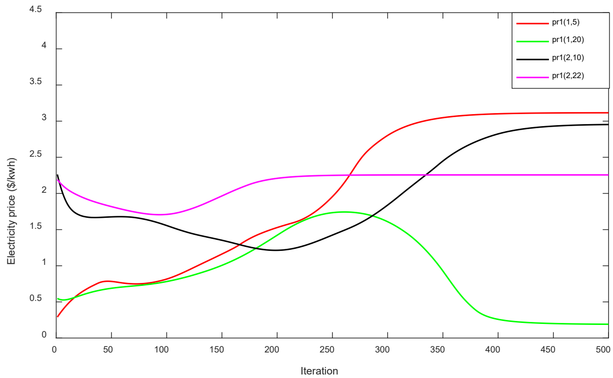

Figure 2. Convergence of electricity price

Source: Figure 2 is obtained by authors based on Matlab 2021b.

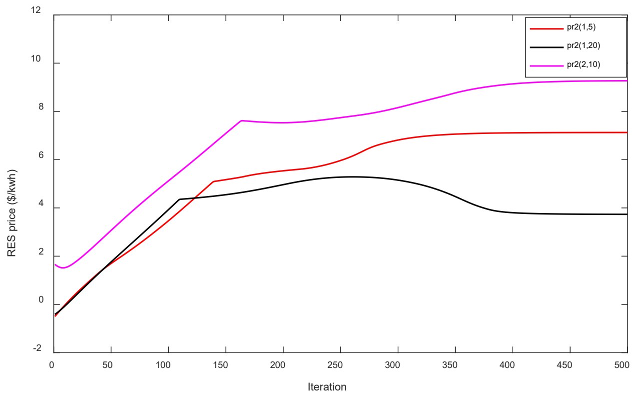

Figure 3. Convergence of RES price

Source: Figure 3 is obtained by authors based on Matlab 2021b.

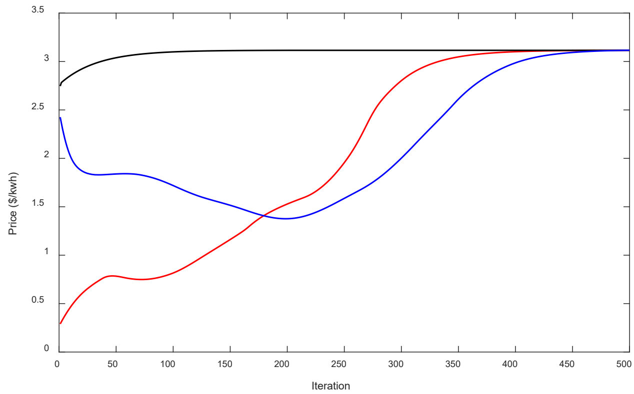

Figure 4. Impact of initial value on convergence
Source: Figure 4 is obtained by authors based on Matlab 2021b.

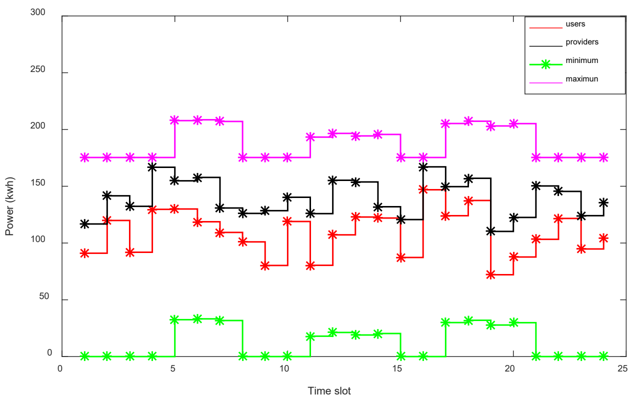

Figure 5. Analysis of demand and supply
Source: Figure 5 is obtained by authors based on Matlab 2021b.

Figure 2 and Figure 3 indicate that the real-time prices of electricity and RE will converge to the global optimum solutions with the growth of the number of iterations.

In Figure 4, we change the initial value of the prices of electricity and RE. We find that the prices finally converge to the same equilibrium regardless of any initial value. In Figure 5， the numerical results show that the optimal solution with our proposed algorithm satisfies each user's demand constraint, each utility company's supply constraint and the demand-supply balance constraint.

In Figure 6, we compare the social welfare in three different pricing strategies, i.e. flat selling with flat buyback pricing (FSFB), dynamic selling with flat buyback pricing (DSFB), and dynamic selling with dynamic buyback pricing (DSDB). As shown in Fig.6, the social welfare of our proposed algorithm (DSDB) is bigger than that of the other two traditional pricing strategies. That indicates that our approach is more suitable and attractive for utility companies.

Furthermore, we further compare the variance of grid loading to demonstrate the performance of our approach. Figure 7 shows that the PAR in our pricing strategy is lower than in the other two pricing strategies. Thus, our approach not only increases the social welfare of the whole grid but also guarantees that both users and utility companies' benefit, providing a win-win outcome for smart grids.

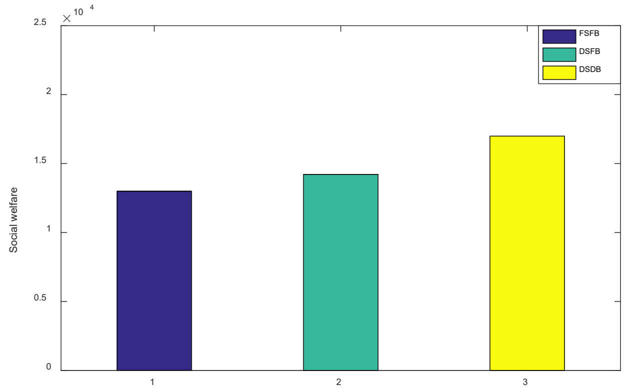

Figure 6. Comparison of social welfare
Source: Figure 6 is obtained by authors based on Matlab 2021b.

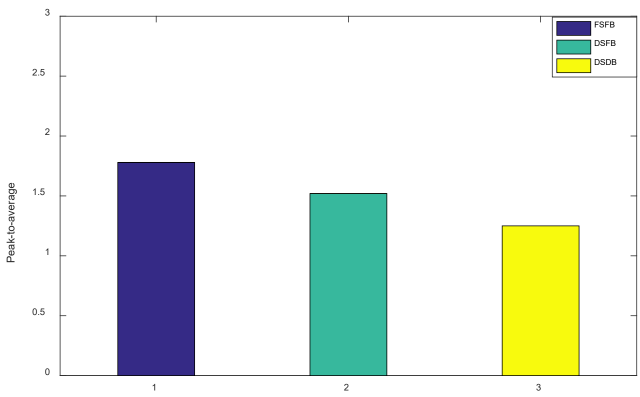

Figure 7. Comparison of PAR
Source: Figure 6 is obtained by authors based on Matlab 2021b.

6. Conclusions

In this paper, we conducted an extension of the existing studies and proposed a distributed RTP algorithm for the multiseller–multibuyer smart grid integrated with RE and energy storage devices based on social welfare maximisation. In the proposed model, profit function was introduced to encourage people to use more renewable energy. In addition, RE was assumed to be kept for the user's own use or to be sold to the smart grid and the depreciation of the storage capacity was taken into account, which are widespread in reality. The situation considered in this paper is of generality. The other situations can be regarded as its special ones. By Lagrangian dual decomposition, we divided the primal optimisation into a set of single-user and single-company-single-time-slot subproblems. With this method, each utility company can individually set clearing price to match supply and demand, and each user can individually decide from which utility company to buy electricity and how much to buy, which is beneficial to protecting privacies of any entity, lowering the complexity of operation and keeping the system scalability. Simulation results have shown that the algorithm can increase benefits of both users and utility companies while reducing the peak load by shifting the load demand to off-peak periods and balancing supply and demand. Therefore, the proposed method can be employed to effectively balance the energy allocation in the future smart grid. In the future work, we may focus on the impact of some other factors on RTP, such as uncertainties of the RE, some special load pattern adopted by users, etc.

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