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Using the Omega Ratio for Optimal Asset Allocation in Commodities

Abstract. *This paper calculates the Omega ratio of 16 commodities from four classes – agriculture, precious metals, industrial metals, and energy. The analysis considers two subsamples and five different threshold levels, taking into account individual commodities as well as optimised portfolios. Palladium has the best reward-to-risk ratio in the pre-crisis period, while all grains recorded high Omega in the crisis period, and soybean stands out as the best of them. Specific market circumstances caused these results in the pre-crisis and crisis periods. As for the optimised Omega portfolios, the precious metal portfolio proved to be the best in the pre-crisis period, while the agricultural commodity portfolio has the highest Omega in the crisis period.*

Keywords: *the Omega ratio, commodities, optimised portfolios.*

JEL classification: D81, G32, Q02.

1. Introduction

Commodity markets have become an important alternative investment in the past decade, according to Andreasson et al. (2016) and Olson et al. (2017), where physical delivery is no longer the main focus of commodity market participants. This process is significantly fuelled by the so called commodity market financialisation, where non-commercial traders enter commodity futures markets with the sole purpose to realise a profit (Wang et al., 2023). The analysis of global commodity prices, their gains and losses, has drawn much of attention by academics and practitioners after the global financial crisis, when all commodities experienced wild swings to a greater or lesser extent. Now this topic regains attractiveness due to the recent and ongoing global developments – the COVID-19 pandemic and the war in Ukraine, which have caused severe turbulences in global commodity markets. The Coronavirus pandemic has inflicted significant damage to global supply chains, increasing transportation costs and decreasing world economic activity, which

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substantially deflated the prices of the major global commodities. On the other hand, the war in Ukraine has increased uncertainty in many global commodity markets, especially in those where Russia and Ukraine are the key suppliers, such as energy and agricultural products. This caused significant price increase for global commodities at the beginning of the war. In conditions of increased uncertainty, every investor in commodities is keen to know which investment is the most profitable with the lowest risk.

However, this question is not straightforward in practice because investors usually face the problem how to properly evaluate the risk-return performance of their investments. The classic approach in this respect refers to the Sharpe ratio of Sharpe (1966), which is inspired by the Markowitz mean-variance theory. However, the serious drawback of the Sharpe ratio reflects in the fact that it is a valid measure only in the very strict case when an asset or portfolio follows a normal distribution or quadratic preferences (Zakamouline and Koekebakker, 2009), which is not realistic to assume in daily commodity time series. In other words, the Sharpe ratio takes into account only the first and second moment of the returns, which means that it is an adequate risk-reward measure only if risk can be gauged by standard deviation. When the return distribution exhibits fat tails and skewness, the Sharpe ratio can lead to misleading conclusions and counterintuitive performance (Kapsos et al., 2014a).

In order to address this issue of the Sharpe ratio, Keating and Shadwick (2002) proposed a new performance measure, which they called the Omega ratio. The key advantage of the Omega ratio over the Sharpe ratio is the use of an entire probability distribution of an asset or portfolio, which significantly alleviates the assumption of normality. In other words, Omega takes into account the influence of all the moments instead of each one individually, which is of practical significance because it is difficult to separate which moment is more important. In particular, the omega ratio defines a threshold value (τ) to distinguish gains from losses, which can be interpreted as the probability weighted ratio of gains to losses, relative to the given threshold. When the threshold level observes a particular rate of return, this indicates to what extent the obtained result exceeds the expectations of the investor, while returns below the threshold are seen as losses.

Yu et al. (2022a) lists advantages of the Omega ratio over conventional risk-return measures, such as Sharpe, Sortino, and Traynor ratio. First, Omega gives flexibility to the investor in selecting the threshold value according to the principle – no level is "better" than another, which reflects the particular risk preference of a decision maker. Second, the Omega ratio does not have to assume distribution of returns, meaning that it can be applied to any asymmetrical distribution. Third, the Omega ratio does not require covariance matrix in the portfolio calculation and can be linearised, which overcomes computational complexity in the case when the portfolio consists of a large number of assets. In addition, Keating and Shadwick (2002) argue that the simplicity of the Omega calculation gives an advantage over more sophisticated statistical measures, involving estimation of higher order moments, when time-series deal with non-normal distributions.

This paper hypothesises situation of an agent who prefers to invest in commodities, where the goal is to choose an asset with the best reward-risk characteristics. In this task, we use the Omega ratio as an execution tool, and the reason for that is twofold. First, the Omega ratio overcomes many deficiencies of traditional indicators, which makes it a superior and more reliable performance measure. The second and more important reason is the fact that no paper thus far has researched the Omega ratio from the aspect of global commodities as an investment vehicle, to the best of our knowledge. This gives us an opportunity to contribute to the literature, and this is where we find a motive for this research.

In particular, the paper considers 16 commodity futures from four different classes – energy, precious metals, industrial metals, and agriculture, covering the period of six and a half years. We examine futures prices rather than spot prices because futures markets are more liquid, which means that external information builds up in futures prices more quickly, making them more realistic. All the selected commodities are traded on the Chicago Mercantile Exchange, and they are – Brent oil, gas oil, heating oil, natural gas, gold, silver, platinum, palladium, aluminium, copper, lead, zinc, corn, wheat, soybean and oats. The wide range of selected assets gives diversity to the analysis, and also provides an opportunity to determine which class of commodities had the best performance of the Omega ratio, and which commodity stands out as the best one.

In addition to the analysis of individual commodities, the paper tries to determine which group of assets produces the highest Omega ratio when they are combined in a four-asset portfolio. This analysis determines the shares of assets in these optimal portfolios, which is useful for investors. It is important to note that both individual and portfolio analysis are conducted at different threshold levels, which shows how realised performance of the Omega ratios differs in various scenarios. According to Yu et al. (2022a), the floating threshold settings gives higher realised performance of the Omega calculations, modelling better the market dynamics.

Considering the specificity of the observed sample, where we have a relatively calm period before the pandemic and a the war in Ukraine, and very turbulent period afterwards, it is reasonable to assume that the calculated Omega ratios significantly differ between these two sub-periods. In this regard, both individual and portfolio analysis of the Omega ratio is conducted by separating the full sample into the two subsamples – pre-crisis and crisis. In this way, it can be assessed how different commodities perform in different market conditions, which can give market participants a clue as to how to behave in diametrically opposed market circumstances.

2. Literature review

From an asset performance perspective, relatively few papers have used the Omega ratio in inspecting the reward-risk characteristics of investments. Most of the studies checked the Omega performance on stock indices and mutual funds. For instance, Yu et al. (2022a) modelled the Omega portfolio with various settings of threshold as a mechanism to adapt market dynamics. They used a fixed threshold as the benchmark, and compared the realised performance of the modified Omega models with two floating thresholds – treasury security yields and CVaR. They also researched how the parameters governing the investor preference between loss and return affect the results. Using daily prices of exchange traded funds on the Morgan Stanley Capital International (MSCI) stock indices from 19 countries and composite stocks of the S&P 500 Index, they reported that setting floating return threshold in the Omega models realise higher performance. Taylor (2022) presented a new model for Value-at-Risk and expected shortfall, in which expected shortfall is modelled as the product of Value-at-Risk and a factor that is a simple function of a dynamic Omega ratio. The empirical analyses considered the 1%, 2.5% and 5% probability levels, and the research is conducted on the five U.S. stocks (Apple, Microsoft, Amazon, Berkshire Hathaway, and JP Morgan). The results showed that the dynamic Omega formulation for the expected shortfall produced slightly better forecast accuracy than previously proposed expected shortfall formulations.

Yu et al. (2022b) proposed a multiple objective programming model to generate time-varying return thresholds by maximising the threshold (τ) value and maximising the deviation between gain and loss, taking into account short sales and trading costs in portfolio rebalances. The investigation considered the composite stocks in the S&P 500 index over 13 years. Their results suggested that the Omega (Max τ) model realises higher performances than the widely applied portfolio models and the fixed-τ Omega models under different rebalancing frequencies. The paper of Sehgal and Mehra (2021) proposed robust portfolio optimisation models for several reward–risk ratios – Omega, semi-mean absolute deviation ratio, and weighted stable tail adjusted return ratio (STARR). The research evaluated the performance of the robust reward–risk ratio models on the listed stocks of FTSE100, Nikkei225, S&P500, and S&P BSE 500. They found that robust portfolio optimisation models outperform their conventional counterpart models in terms of statistics measured by the standard deviation, value at risk (VaR), conditional value at risk (CVaR), Sharpe ratio, and STARR ratio.

Sharma et al. (2017) redefined the Omega ratio for a loss averse investor by taking the stochastic threshold point as the conditional Value-at-Risk at α confidence level ($CVaR_\alpha$). Higher values of α indicate a higher loss averse attitude of an investor. They empirically investigate the performance of the Omega-$CVaR_\alpha$ portfolios and robust Omega- $CVaR_\alpha$ portfolios under a mixed uncertainty set for listed stocks of the S&P500 index. They asserted that the optimal portfolios resulting from the Omega-$CVaR_\alpha$ model exhibit a better performance compared to the classical $CVaR_\alpha$ model in the sense of higher expected returns, Sharpe ratios, modified Sharpe ratios, and lower losses in terms of VaR_α and $CVaR_\alpha$ values. Botha (2007) investigated the performance of 35 South African hedge funds. He researched the differences between the Omega and Sharpe ratios in their ranking of hedge funds, trying to determine which of the two is the more accurate and reliable. The data sample comprises monthly data over a period of 90 months. Four different types of hedge fund were selected for the survey: fixed interest (3), long–short equity (13), market neutral (11) and trading (8). He concluded that Omega emerges as the superior measure. Hentati et al. (2010) researched the relevance of four performance measures – the Sharpe Ratio, the returns on VaR and on CVaR, and the Omega ratio, when they are used to determine optimal portfolios including hedge funds. They asserted that both CVaR and Omega measures are more appropriate, especially when the Cornish-Fisher expansion is introduced to calculate the CVaR. Both static and dynamic optimisations are calculated. The results indicated that portfolio, which maximises the Omega measure, has more stable performances, when compared to the return-on-CVaR portfolio.

3. Research methodology

3.1 The Omega ratio

Global commodities are prone to large and abrupt movements, which was particularly the case from the onset of the pandemic. Hence, investors in commodities have a great interest to properly measure performances of risky assets. However, traditional measures (like the Sharpe, Sortino, or Traynor ratio), cannot capture all the information in empirical distribution because they approximate returns only with the mean and standard deviation. In this way, the normal distribution is implicitly assumed, which discards higher-moment effects, such as skewness or kurtosis. The Omega ratio of Keating and Shadwick (2002) overcomes deficiencies of the traditional ratios because it divides all the returns into the two parts – above and below a threshold. In Equation (1), F(x) is the cumulative probability distribution, τ is a threshold value selected by the investor, while a, b are the upper and lower investment intervals. It can be said that Omega is equal to the probability weighted gains divided by the probability weighted losses, taking into account the threshold (τ) that is determined by an investor.

$$\Omega(\tau) = \frac{\int_{\tau}^{a}(1-F(x))dx}{\int_{b}^{\tau}F(x)dx} \tag{1}$$

Omega ratio is a convenient tool for measuring asset performance because it does not require assumptions about risk preferences or utility function, but we just need to define a simple decision rule (Avouyi-Dovi et al., 2004). This significantly simplifies decision-making because more money is better than less money, which means that a commodity with a higher value of Omega is preferable to one with a lower value. Kane et al. (2009) state that Omega can be calculated directly from the historical data, which actually portrays the empirical returns distribution. This improves performance measuring based only on mean and variance.

An important aspect in Omega calculation is the proper setting of a threshold value. Vilkancas (2014, 2016) argues that Omega should be assessed within a range of τ values because floating thresholds may enhance portfolio performance, as they better reflect market dynamics than fixed thresholds. In this regard, we calculate the Omega ratio with the five different thresholds, which form the Omega function. We use the same τ values to calculate the Omega of the individual commodities and for portfolio optimisation. The lower threshold level is set to zero, which means that all returns below zero are seen as losses, while the upper zero returns are gains. When determining the upper level of a threshold in a portfolio, we took into account the limitation that is imposed in the portfolio optimisation procedure. This restriction implies that ex post portfolio returns can never be higher than the threshold level.

3.2 Linear model for the Omega ratio optimisation

In addition to calculating the omega ratio for each commodity, we also construct four four-asset portfolios with the highest Omega. This reveals which group of commodities gives the highest Omega ratio, regarding the different threshold values. Following Mausser et al. (2006) and Yu et al. (2022b), we use a non-parametric approach to construct the optimal Omega portfolios. Non-parametric method does not assume any return distribution, but use historical distributions and the sample measure associated to them. Calculating the Omega portfolio is different from the traditional mean-variance optimisation because it does not use mean and variance directly to optimise a portfolio, nor covariance matrix. However, the Omega ratio observes an entire distribution via returns above and below the threshold, which indirectly includes all four moments of the empirical distribution. In relation to the mean-variance portfolio, the Omega portfolio can have higher volatility, but it reduces the impact of tail-risk. This is particularly important for assets that have highly unstable price dynamics, which is an intrinsic feature of all commodities in greater or lesser extent.

According to Yu et al. (2022b), the Omega ratio optimisation can be translated into a linear model:

$$\text{Max } \Omega, \tag{2}$$

subject to:

$$\delta\left(\sum_{i=1}^{n} w_i r_i - \tau\right) - (1-\delta)\frac{1}{T}\sum_{t=1}^{T}\eta_t \geq \Omega, \tag{3}$$

$$\eta_t \geq \tau - \sum_{i=1}^{n} r_{it} w_i, \qquad t = 1,2,\ldots,T, \tag{4}$$

$$\eta_t \geq 0, \qquad t = 1,2,\ldots,T, \tag{5}$$

$$\sum_{i=1}^{n} w_i = 1, \tag{6}$$

$$\sum_{i=1}^{n} w_i r_i \geq \tau, \tag{7}$$

$$w_i \geq 0, \qquad i = 1,2,\ldots,n \tag{8}$$

The objective function is the maximum Omega ratio, which shows the deviation between portfolio return and loss. r_i denotes the average return of the selected commodities (i), while w_i is the weight of security (i) in the Omega portfolio. δ is the parameter determining return-risk weights. τ is the threshold of portfolio return. The first part of Equation (3), $(\sum_{i=1}^{n} w_i r_i - \tau)$, indicates the excess portfolio gain over the threshold, while the second part, $\frac{1}{T}\sum_{t=1}^{T}\eta_t$, stands for the portfolio loss. Equations (4) and (5) measure the periodical loss of the portfolio. The necessary condition in all portfolio optimisations is that sum of all asset weights is equal to one (Equation 6). The portfolio return cannot be less than the required return, and the required return is a threshold level (Equation 7). Equation 8 indicates that all weights are positive or equal to zero, which means that short selling is not allowed.

4. Dataset and descriptive statistics

This paper uses daily near-maturity futures prices of 16 globally-traded commodities: Brent oil, gas oil, heating oil, natural gas, gold, silver, platinum, palladium, aluminium, copper, lead, zinc, corn, wheat, soybean, and oats. All commodities are traded on the Chicago Mercantile Exchange, and they are all collected from the investing.com website. The sample comprises the period between January 2017 and September 2023. All futures prices are transformed into log-returns ($r_{i,t}$) according to the expression $r_{i,t} = 100 \times log(P_{i,t}/P_{i,t-1})$, where P_i is the price of a particular commodity. Before the omega ratio calculation, all assets from the same class are synchronised in order to have consistent results between Omegas of individual assets and the portfolios.

Table 1 shows descriptive statistics of the selected commodities, dividing the whole sample in the two sub-periods – pre-crisis and crisis. The breaking point is January 2020, which means that the crisis period covers the COVID-19 pandemic and the war in Ukraine. It can be seen that all mean values of the commodities are relatively low, while some assets have negative average returns. It is interesting to note that average daily returns of palladium are very high in the pre-crisis period, amounting to 0.056%. This happened due to the high global demand of palladium and the metal shortages on the spot market in the pre-COVID period, which was resulted in an all-time high price of USD 2,795 on February 8, 2020. After reaching this level, palladium plummeted by almost 45% in March 2020, primarily because the global automotive industry was devastated by the pandemic. This explains why the average returns of palladium entered very negative zone during the pandemic, reaching the mean of -0.022% in the crisis period.

Table 1. Descriptive statistics of the selected commodities in the two periods

	Pre-crisis period				Crisis period			
	Mean	**St. dev.**	**Skewness**	**Kurtosis**	**Mean**	**St. dev.**	**Skewness**	**Kurtosis**
Brent oil	0.008	0.781	-0.121	8.920	0.009	1.360	-1.488	19.798
Gas oil	0.011	0.650	0.154	5.786	0.010	1.463	-2.176	29.026
Heating oil	0.008	0.657	-0.010	6.375	0.008	1.379	-1.079	11.679
Natural gas	-0.030	1.178	-0.217	10.053	0.001	1.928	-0.104	4.450
Gold	0.015	0.293	0.120	4.983	0.011	0.467	-0.242	6.852
Silver	0.005	0.495	-0.168	5.265	0.013	0.967	-0.496	7.662
Platinum	-0.004	0.485	-0.184	4.339	0.002	0.924	-0.318	6.571
Palladium	0.056	0.671	-0.639	5.609	-0.022	1.254	-0.412	10.613
Aluminium	0.004	0.486	0.154	7.149	0.008	0.645	-0.072	4.719
Copper	0.001	0.504	-0.032	4.575	0.007	0.665	-0.201	4.382
Lead	-0.002	0.590	-0.108	4.221	0.004	0.634	0.064	4.653
Zinc	-0.005	0.618	-0.049	3.956	0.002	0.731	-0.138	4.191
Corn	0.004	0.578	0.049	5.601	0.015	0.821	-1.804	18.984
Wheat	0.003	0.389	1.280	15.031	0.012	0.719	1.115	18.286
Soybean	-0.001	0.460	0.057	4.799	0.019	0.626	-0.548	6.433
Oats	0.016	0.883	0.037	6.998	0.004	1.324	-2.006	34.976

Source: Authors' calculation.

Having an insight on the average returns of the commodities is useful from the aspect of determining dynamic threshold levels. As has been said, in the portfolio setting, the *ex post* portfolio returns can never be higher than the threshold level. This means that the maximum threshold level should be determined by the highest historical return of the commodity from the class of commodities that have the lowest average returns. In this way, a consistency can be achieved throughout all classes of commodities. At high level of threshold, the portfolio with low-return assets will only consist of the commodity with the highest returns, while all other low-return commodities will be excluded from the portfolio. According to Table 1, it can be seen that the class of industrial metals recorded the lowest average returns in the observed periods, where aluminium has the highest mean in the two sub-periods, 0.004% and 0.008%, respectively. Guided by these numbers, we set the upper threshold level at 0.008%, while the lowest level is set to zero. In other words, the range of five threshold levels changes by the increment of 0.002%. These threshold levels form the Omega function, which is strictly descending because when the return threshold increases, the possibility to achieve higher yields decreases. The mean of aluminium in the pre-crisis period is 0.004, which means that the Omega portfolio of industrial metals cannot be calculated at higher daily threshold levels, i.e., 0.006% and 0.008%, in the pre-crisis period.

5. Research results

5.1 The Omega ratio of the commodities

This section presents the Omega ratio results of the individual commodities, taking into account the two sub-periods. The Omega ratio captures the characteristics of entire distribution, including higher moments, which is especially valuable for non-normal investments, such as commodities. Investors should favour the assets with a higher Omega ratio, which provides a greater potential to achieve the desired level of return while minimising the likelihood of extreme losses. Table 2 contains the dynamic results of the Omega ratios, calculated with the five threshold levels, while Figure 1 shows a graphical illustration of the results. Changing threshold levels refer to the risk tolerance of investors, where one could conclude that the lower the threshold, the more risk tolerant the investor is. However, Avouyi-Dovi et al. (2004) state that the term “risk tolerant” can be confusing because smaller threshold does not mean that the investor is less risk averse, but rather that he fears only a smaller outcome.

According to Table 2, all Omega ratios move around one, whereas it should be said that Omega equals one when the threshold value is the average return. Only by this information, investor can conclude which commodities have achieved relatively high returns. In particular, in the first (non-crisis) subsample, palladium has by far the highest profit-to-loss ratio, while gold and gas oil follow. On the other hand, in the crisis period, soybean is the commodity with the highest Omega, while gold and corn follow. The explanation of these results lies in the global circumstances that caused them. As has been said, the high Omega of palladium is due to the increase of global demand for palladium and insufficient supply in the spot market in 2019 and early 2020. On the other hand, in the crisis period, all grains recorded high levels of Omega, particularly soybean, corn, and wheat. This outcome is caused by the war in Ukraine, which triggered the global food insecurity, causing food prices to rise.

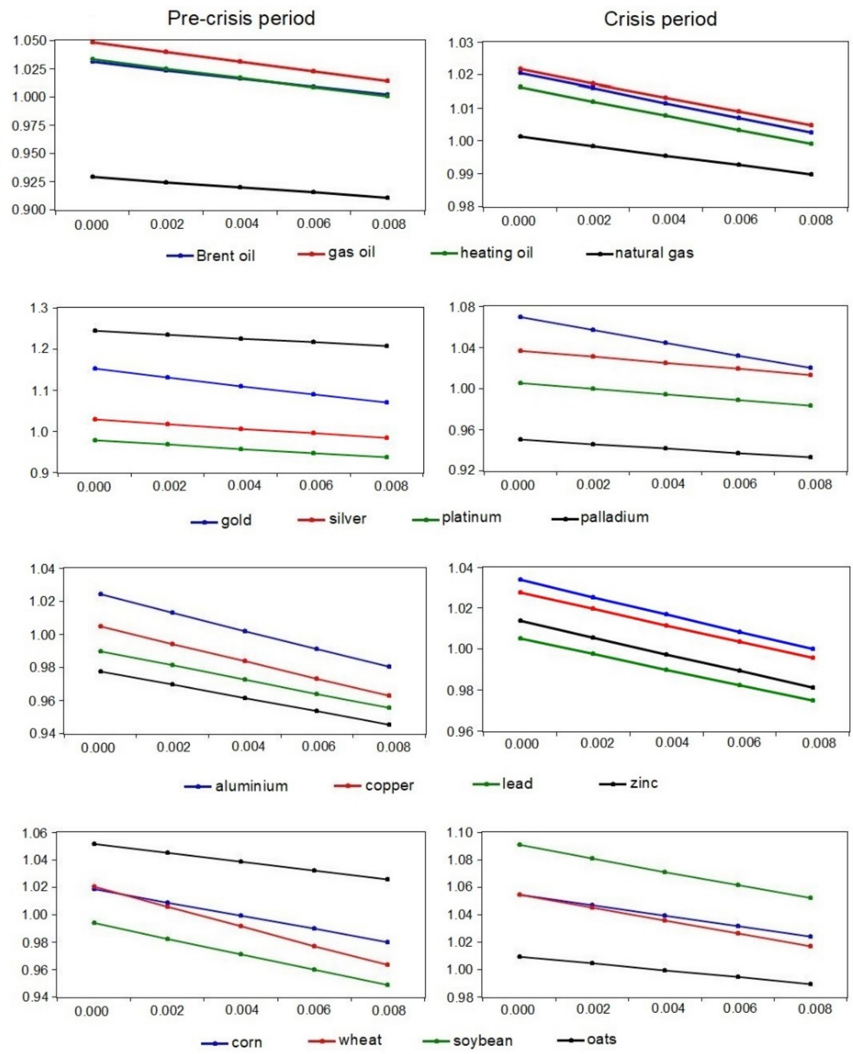
Table2. Omega ratios of the selected commodities in the two sub-periods

Panel A: Pre-crisis period								
	Brent	**Gas oil**	**H. oil**	**N. gas**	**Gold**	**Silver**	**Platinum**	**Palladium**
$\tau = 0.000$	1.031	1.048	1.033	0.929	1.151	1.028	0.978	1.245
$\tau = 0.002$	1.023	1.040	1.025	0.924	1.130	1.017	0.967	1.235
$\tau = 0.004$	1.016	1.031	1.017	0.920	1.109	1.005	0.957	1.225
$\tau = 0.006$	1.009	1.023	1.008	0.915	1.089	0.994	0.946	1.216
$\tau = 0.008$	1.001	1.014	1.000	0.911	1.069	0.983	0.936	1.206
	Aluminium	Copper	Lead	Zinc	Corn	Wheat	Soybean	Oats
$\tau = 0.000$	1.024	1.005	0.977	0.990	1.018	1.020	0.994	1.051
$\tau = 0.002$	1.013	0.994	0.969	0.981	1.009	1.006	0.982	1.045
$\tau = 0.004$	1.002	0.983	0.961	0.972	0.999	0.991	0.971	1.038
$\tau = 0.006$	0.991	0.973	0.953	0.964	0.989	0.977	0.960	1.032
$\tau = 0.008$	0.980	0.963	0.945	0.955	0.980	0.963	0.948	1.025
Panel B: Crisis period								
	Brent	Gas oil	H. oil	N. gas	Gold	Silver	Platinum	Palladium
$\tau = 0.000$	1.020	1.022	1.016	1.001	1.069	1.037	1.005	0.950
$\tau = 0.002$	1.016	1.017	1.012	0.998	1.057	1.031	1.000	0.946
$\tau = 0.004$	1.011	1.013	1.008	0.996	1.044	1.025	0.994	0.941
$\tau = 0.006$	1.007	1.009	1.003	0.993	1.032	1.019	0.989	0.937
$\tau = 0.008$	1.003	1.005	0.999	0.990	1.020	1.013	0.983	0.933
	Aluminium	Copper	Lead	Zinc	Corn	Wheat	Soybean	Oats
$\tau = 0.000$	1.034	1.028	1.005	1.014	1.054	1.055	1.091	1.009
$\tau = 0.002$	1.025	1.020	0.997	1.006	1.047	1.045	1.081	1.004
$\tau = 0.004$	1.017	1.011	0.990	0.997	1.039	1.035	1.071	0.999
$\tau = 0.006$	1.008	1.004	0.982	0.989	1.032	1.026	1.061	0.994
$\tau = 0.008$	1.000	0.996	0.975	0.981	1.024	1.017	1.052	0.989

Source: Authors' calculation.

It is interesting to note that gold is the second best commodity in both sub-periods, which means that value of gold maintains steady rise regardless of global circumstances, which favours gold as a good investment (see, e.g., Bonato, 2021), compared to all other commodities. On the other hand, industrial metals have recorded the poorest Omega results in both sub-periods, while lead and zinc are among the worst. This indicates that industrial metals are not a good investments to put money into, if an agent want to achieve a high profit-to-loss ratio.

Following Botha (2007), the Omega ratio holds two more useful information for investors. The first one says that the slower the Omega function tends to zero, the larger the potential for positive asset returns. On the other hand, the steeper the slope of the Omega function, the lower the risk. The inclination of the Omega function cannot be perceived by looking at Table 2, so we plot all Omega functions in Figure 1.



Note: X axis denotes threshold values, while Y axis stands for the Omega ratio.

Figure 1. Omega functions of the selected commodities in the two sub-periods

Source: Authors' calculation.

The Omega functions are aligned one on the top of the other, so it is relatively easy to spot which commodity has a higher slope, i.e. lower risk. Most omega functions are parallel to a greater or lesser extent, while only in the two cases a particular function has a steeper slope. These are gold in both periods and wheat in the pre-crisis period. This indicates that these commodities have lower risk compared to the risk of other assets from the same class. The Omega results suggest correctly because descriptive statistics in Table 1 clearly shows that gold has the lowest standard deviation (0.293) in the pre-crisis period and (0.467) in the crisis period, while this is the case with wheat (0.389) in the pre-crisis period. As for the highest

Omega ratios among the various classes of assets, gas oil is the best one of the energy commodities, gold is the best precious metal, while aluminium is the best industrial metal. These results apply to both subsamples. On the other hand, in the class of agricultural commodities, the best performing commodity is different between the sub-periods. Oats recorded the highest Omega in the pre-crisis period, while soybeans took the lead in the crisis period.

5.2 Optimisation of the Omega ratio portfolios

It is valuable for investors to know what the reward-risk performance of each commodity is. But now the question can be raised whether investors can do better if they choose to invest in a portfolio of commodities. This section tries to answer the question by constructing the four-asset portfolios with each commodity class, where the goal is to find the highest Omega ratio of the portfolios. Table 3 shows the calculated weights of the commodities in the four portfolios, while Table 4 presents the results of the Omega ratios of these portfolios. All portfolios are calculated at five different threshold levels in the two sub-periods, which generates a total amount of 40 portfolios. Half of all portfolios are the single-asset portfolio, 16 are the two-asset portfolio, and 2 of them are the three-asset portfolio. In the two cases, the portfolio cannot be created. It is obvious that the assets with the highest Omega dominate the portfolios, which is expected.

In particular, gas oil prevails in the energy portfolio in the pre-crisis period, while in the crisis period, it shares the portfolio with Brent oil, taking into account the threshold levels from 0.000 to 0.006. In the pre-crisis period, gas oil has the highest Omega in all threshold levels, which excludes all other assets in the portfolio. However, in the crisis period, Brent oil also has a relatively high Omega, but slightly lower than oil gas, which gives Brent a significant share in the portfolio, particularly at a lower threshold level. As the threshold increases, the level of Brent declines because its Omega decreases faster than the Omega of gas oil. At the highest threshold level, Brent vanishes from the portfolio.

As for the precious metals portfolio, gold actually has a higher weight (51%) than palladium (0.49%) at the zero threshold level, although palladium has a higher Omega at all threshold levels. The advantages of gold *vis-à-vis* palladium at the zero threshold level suggest that gold and palladium go well together, which produces a synergistic effect of these two commodities. At higher threshold levels, the share of gold decreases, amounting eventually 20% at the highest threshold level. These results can be linked with the paper of Živkov et al. (2022), who hedged energy commodities with precious metals in the mean-variance portfolio, and found that the highest share have gold and palladium. On the other hand, in the crisis period, gold is the only element in the portfolio, where it should be noted that gold simultaneously reports the highest Omega at all threshold levels, and also its Omega function has the steepest slope. This indicates very good safe haven properties of gold during the crisis period.

Table 3. Constructed maximum Omega portfolios in the two periods

	Pre-crisis period					Crisis period				
Threshold	0.000	0.002	0.004	0.006	0.008	0.000	0.002	0.004	0.006	0.008
	Energy commodity portfolio									
Brent	0%	0%	0%	0%	0%	47%	47%	42%	27%	0%
Gas oil	100%	100%	100%	100%	100%	53%	53%	58%	73%	100%
Heating oil	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Natural gas	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Precious metals portfolio									
Gold	51%	44%	35%	26%	20%	100%	100%	100%	100%	100%
Silver	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Platinum	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Palladium	49%	56%	65%	74%	80%	0%	0%	0%	0%	0%
	Industrial metals portfolio									
Aluminium	100%	100%	100%	NA	NA	58%	67%	77%	100%	100%
Copper	0%	0%	0%	NA	NA	29%	33%	23%	0%	0%
Lead	0%	0%	0%	NA	NA	13%	0%	0%	0%	0%
Zinc	0%	0%	0%	NA	NA	0%	0%	0%	0%	0%
	Agricultural commodity portfolio									
Corn	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Wheat	23%	0%	0%	0%	0%	22%	16%	12%	6%	0%
Soybeans	0%	0%	0%	0%	0%	78%	84%	88%	94%	100%
Oats	73%	100%	100%	100%	100%	0%	0%	0%	0%	0%

Note: NA indicates that portfolio cannot be optimised.

Source: Authors' calculation.

In the industrial metals portfolio, aluminium has 100% share in the pre-crisis period, and this applies only for the first three threshold levels. The last two portfolios cannot be optimised because average returns of aluminium are lower than the last two thresholds. The bad Omega performance of industrial metals in the pre-crisis period is also evident in Figure 1, since all metals, except aluminium, have average returns below one at almost all threshold levels. In the crisis period, the situation changes in a sense that aluminium is no longer the only element in the portfolio at the first three thresholds, but copper and even lead have the share in the portfolio. The price of copper rose in 2021 and 2022, which increases its Omega, giving a share to copper of 29%, 33%, and 23% at the first three threshold levels, respectively. However, it is surprising that lead has 13%, while zinc has 0%, although zinc has significantly higher Omega at the first threshold (see Figure 1). It seems that lead returns better fit to aluminium and coper returns in terms of maximum Omega, which explains why the optimisation lefts no room for zinc. As the threshold increases, only aluminium and copper are the parts of the portfolio, whereas aluminium gains dominance at the last two threshold levels.

Oats prices recorded the highest rise in the pre-crisis period, which explains why oats dominate in the agricultural portfolio. Wheat and corn are also part of the portfolio at the first threshold, while at all other levels, oats make a 100% share. Soybeans are not part of the portfolio at any threshold level in the pre-crisis period. However, in the crisis period, the situation changes diametrically. Now, soybeans and wheat make the whole portfolio, while oats and corn are excluded completely.

Soybean dominates because it has significantly higher Omegas compared to all other agricultural commodities (see Figure 1). Also, it is interesting to note in Figure 1 that corn has gentler slope, which means that corn has higher potential for positive returns than wheat, but still corn is excluded from the portfolio. It seems that soybeans and wheat are better aligned in terms of the maximum Omega ratio.

Table 4 shows the calculated Omega values of the portfolios in the two sub-periods. It can be noted that the Omega ratios of portfolios with two or three assets are higher than the Omega ratios of individual assets, which indicates that all optimisations are performed well. Comparing the different portfolios, portfolio with precious metals has the best gain-to-loss performance in the pre-crisis period, while the agricultural commodity portfolio has the highest Omega in the crisis sub-period.

Table 4. Calculated Omega values of the constructed portfolios in the two periods

Created portfolios	Number of assets in the portfolios	Threshold levels				
		0.000	0.002	0.004	0.006	0.008
		Panel A: Pre-crisis period				
ECP	1	1.048	1.040	1.032	1.023	1.014
PMP	2	1.264	1.247	1.233	1.220	1.208
IMP	1	1.024	1.013	1.002	NA	NA
ACP	3 and 1	1.052	1.045	1.038	1.032	1.025
		Panel B: Crisis period				
ECP	2	1.024	1.019	1.014	1.009	1.005
PMP	1	1.069	1.057	1.044	1.032	1.020
IMP	3, 2 and 1	1.034	1.025	1.015	1.008	1.000
ACP	2 and 1	1.092	1.082	1.071	1.061	1.052

Note: ECP, PMP, IMP and ACP denote energy commodities portfolio, precious metals portfolio, industrial metals portfolio, and agricultural commodities portfolio, respectively. The greyed values indicate the highest omega ratios.

Source: Authors' calculation.

6. Conclusions

This paper investigates which commodity has the best reward-to-risk characteristics, using the Omega ratio. Sixteen commodities are examined, and they are selected from four classes – agriculture, precious metals, industrial metals and energy. The Omega ratios are calculated from the aspect of five different threshold levels, taking into account individual commodities, as well as a four-asset portfolio. The observed sample is divided into the two subsamples – pre-crisis and crisis.

According to the results, palladium has the best reward-to-risk ratio in the pre-crisis period, while gold and gas oil follow. Specific market circumstances, such as strong global demand for palladium and a shortage of the metal in the spot market in the pre-COVID period, are responsible for this result. On the other hand, all grains recorded high Omega in the crisis period, while soybean recorded the best result. The reason for this scenario is the war in Ukraine, which triggered the global food insecurity, causing food prices to rise.

As for the optimised Omega portfolios, the precious metal portfolio proved to be the best in the pre-crisis period, while agricultural commodity portfolio has the highest Omega in the crisis period.

This research can be useful to commodity investors for a number of reasons. First, the Omega ratio is a much more practical reward-to-risk indicator than the classical Sharpe ratio because it takes into account all four moments, making the results less biased. The paper indicates which commodities produce the best investment opportunity in different market conditions. Besides, portfolio investors can learn how to optimise their commodity portfolios.

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