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Optimal Inventory Policies for Fresh Perishable Products with Preservation Operations and Greening Efforts under Inflationary Environment

Abstract. *The business of perishable products depends on their shelf life and greening level. To maintain the freshness level and extend the shelf life of the product by reducing the deterioration rate, the product needs proper preservation operations. Inflation is a key economic factor that needs to be considered for inventory management. Inflation is typically correlated with the rapidly rising selling price of products. Demand is depends on selling price and greening efforts. The objective of this study is to ascertain the optimum selling price, the optimum time for replenishment, and the optimum cost of green initiatives simultaneously while considering the maximisation of retailers' total profit under inflation and the controllable deterioration rate for fresh perishable products. To represent real-world scenarios, a mathematical formulation of the inventory system has been developed and verified using numerical analysis. Optimum value of decision variables determined using the classical optimisation method. The concavity of the objective function was verified theoretically, numerically and graphically. Results derived from sensitivity analysis and the model validated through sensitivity analysis. Our results show that the higher inflation rate reduces the total profit but investment in greening efforts and preservation operations helpful to extend product shelf life and hence increases the average profit. After presenting several important managerial perspectives, the article discusses the future direction of relevant research in its conclusion.*

Keywords: *inventory, deterioration, perishable product, greening efforts, inflation, price dependent demand, preservation.*

JEL Classification: C05.

1. Introduction

Perishable goods such as fruits, vegetables, meat, dairy products, and some beverages products are currently purchased based not only on their freshness but also on their cost and greening level. The good's quality diminishes with time as a

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result of deterioration. Manufacturers and retailers who are steadfast in their commitment to maintaining consistent product freshness have made several efforts. Emerging technologies that can be utilised to regulate and assess deterioration rate effectively have been made possible by recent technical advancements. Adequate storage and handling strategies can regulate perishable food deterioration.

Greening efforts are the steps taken to guarantee environmentally friendly goods and reduce the effect that trade has on the environment. Producing nutrient-dense and health-conscious products is a small but important step towards going green. Aside from the environmental benefits, having fresh products on hand draws clients to a business by increasing their selections for high-quality goods and encouraging purchases.

Inflation and the temporal worth of money are quantified. Inflation, even in modest amounts, can have a favourable economic impact. Although not always the case, inflation is often perceived as undesirable. The impact of inflation varies among individuals.

An inventory model for perishable goods is developed with all these factors in mind. In this model, the demand rate is determined by the selling price and the amount of greening done under inflation.

1.1 Significance of study

Perishable products deteriorate at considerable and varying rates as they go through their storage processes or operational channels. The rate of deterioration continues to decline through the use of FPE, investments in green endeavours, and the creation of environmentally friendly products that improve customer well-being and sustainability in an inflationary environment is the new idea of this study. Remember this; the following is a summary of our study's major contributions:

- A freshness quality procedure is used to control the rate at which perishable goods deteriorate.
- Determined the optimum greening efforts of perishable products for sustainability.
- Identified the impact of inflation rate and other parameters on decision results.
- Under the inflation, controlled the rate of deterioration and investment in greening level to maximise the total profit.

The objective of the proposed research is to obtain the optimum cycle time, selling price, and cost of greening efforts to maximise the retailer's total revenue under inflation. Numerical example and sensitivity analysis authenticated this model.

The article is formatted as follows for the remaining portion. The literature study and research gap are given in Section 2. The notations and assumptions are mentioned in Section 3. In Section 4, Mathematical formulations for the proposed model with theorems for convexity of the profit function and solution procedure are mentioned. Numerical example, sufficient condition validations, graphical representation of objective function is carried out in Section 5. Section 6, the sensitivity analysis, and managerial perspectives is derived and finally, draws conclusions and indicates future research directions.

2. Literature review

A review of the literature that is pertinent to our suggested research is presented in this section.

2.1 Deterioration

The deterioration rate is one important factor that accurately depicts the qualities of perishable commodities. (Ghare & Schrader, 1963) initiated the inventory modelling with a constant rate of deterioration. Thereafter, (Covert & Philip, 1973) extend the research of (Ghare & Schrader, 1963) by using weibull distribution deterioration. An overview of current inventory management strategies for deteriorating inventory systems is provided by many authors like(Raafat, 1991), (Shah & Shah, 2000),(Li et al., 2010),(Goyal & Giri, 2001),(Bakker et al., 2012), (Janssen et al., 2016), etc.

2.2 Freshness Preservation Efforts

(Hsu et al., 2010) provide deteriorating inventory approaches with optimal freshness preservation technology costs. (Dye & Hsieh, 2012)examine the effects of using preservation technology to reduce the rate of deterioration by assuming that the cost of preservation technology is a function of the duration of the replenishing cycle. According to (Cai et al., 2010), the freshness index influences product demand. Subsequently,(Yang et al., 2020) created the relationship between deterioration and freshness index and determined the ideal deterioration rate in terms of product quality. By including trade credit payment policies and shortages, (Jani et al., 2023) expand on the research of (Yang et al., 2020). Recently (Katariyaet al., 2024)developed an inventory model with deterioration controlled by freshness quality technology with greening efforts.

2.3 Greening level and selling price-based demand

Retailers and manufacturers have recently launched several initiatives to show their unwavering commitment to being green. (Srivastava, 2007) gave a comprehensive overview of the literature on eco-friendly supply chains. Channel coordination in a supply chain with financial considerations for greening efforts was proposed by (Swami & Shah, 2013). (Ghosh & Shah, 2015) examined cost-sharing agreements in a supply chain involving environmentally conscious customers.Inventory models for collaborative pricing and greening effort decisions with discounts were developed by (Raza & Faisal, 2018). (Meemken & Qaim, 2018) released an annual report on environmental challenges, food security, and organic agriculture. From 2000 to 2015, they looked into the effects of the environment, ecosystems, and soil quality on sustainable farming. In addition to greening initiatives, (Shah et al., 2022) developed an inventory model for perishable goods for price and stock-dependent demand rate. (Shah et al., 2022) determined how a greengrocer's price selections were influenced by advertising and greening initiatives. (Katariya and Shukla, 2022) developed an EPQ model with green investment and price sensitive demand.

2.4 Inflation

Different effects of inflation are felt by an economy. An increase in the purchasing power per unit of money is indicated by high inflation. Under the influence of inflation, (Buzacott, 1975) and (Misra, 1975) worked on the inventory model concurrently. An economic production quantity (EPQ) model for time-declining demand with an imperfect production process was examined by (Shah & Shah, 2014) for dependability and inflationary situations. In their investigation, (Yang et al., 2001) also considered inflation. The two-warehouse inventory model for deteriorating items with shortages under inflation and time-value of money was developed by (Singh et al., 2009). An imperfect production process with an exponential demand rate and Weibull deterioration under inflation was devised by (Singh & Singh, 2011) and (Singh & Rana, 2020) studied the optimal refilling policy under inflation.

Research gap and our contribution

The previous literature emphasises several types of methodologies that have been published in the area of green inventory management. Perishable products freshness is maintained by applying freshness preservation efforts (FPE) as well optimising the greening level, cycle time, and selling price under inflation. The demand for products depends on the sale price and greening efforts, which are a unique study compared to previous published research. By considering every aspect, the present study seeks to maximise the retailer's average profit. The cited literature is used to construct a comparison table that displays the literature that is most relevant to our planned study as well as the research gap in **Table 1**.

Table 1. Most relevant literatures to this study and gap of research

Author(s)	Deterioration	Greening efforts	Price dependent demand	Product preservation during holding operations	Inflation
(Shah & Shah, 2000)	Yes	No	No	No	No
(Ghosh & Shah, 2015)	No	Yes	No	No	No
(B. Li et al., 2016)	No	Yes	Yes	No	No

Author(s)	Deterioration	Greening efforts	Price dependent demand	Product preservation during holding operations	Inflation
(Raza & Faisal, 2018)	No	Yes	Yes	No	No
(Jamali & Rasti-Barzoki, 2018)	No	Yes	Yes	No	No
(Shah & Vaghela, 2018)	Yes	No	No	No	Yes
(Yang et al., 2020)	Yes	No	No	Yes	No
(Singh & Rana, 2020)	Yes	No	Yes	No	Yes
(N. Shah et al., 2022)	Yes	Yes	Yes	No	No
(Paul et al., 2023)	Yes	Yes	Yes	No	No
(Jani et al., 2023)	Yes	No	No	Yes	No
(Shah et al., 2023)	Yes	No	Yes	No	Yes
(Katariya et al., 2024)	Yes	Yes	Yes	Yes	No
Proposed article	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**

Source: Authors' processing.

3. Notations and Assumptions

3.1 Notations

Table 2. Notations for proposed study

Parameters	
K	Fresh product's ordering cost (in \$/order).
C	Procurement cost (constant) (\$/unit).
h	Fixed holding cost (in \$/unit/unit time).
h_m	The maximum cost of holding including preservation operations (in \$/unit/unit time).
h_{cm}	Controllable marginal holding cost with preservation operations (in \$/unit/unit time) .
I_f	Freshness-preservation efforts (FPE) indicator, $\mathrm{I}_f = h_{cm}/h_m$, $0 < \mathrm{I}_f < 1$
ϖ	Deterioration rate and FPE factor, $\varpi > 0$.
d_c	Deteriorationcost (in \$/unit) .
r	Rate of inflation per unit time, where $0 \le r < 1$.
λ	Greening efforts effectiveness parameter ($\lambda > 0$) .
θ	Rate of deterioration of product ($0 < \theta < 1$).

Parameters	
Decision variables	
T	Replenishment cycle time; (in year).
p	Selling Price (in \$/unit), $S_p > C$.
g_e	Cost of greening efforts (constant), (in \$/unit).
Objective function	
$AP(T, g_e, p)$	Retailer's average profit per cycle (in \$).
Expressions and functions	
$R\left(p, g_e\right)$	Demand function.
$S(t)$	Inventory level at time t during the time interval $[0, T]$.
Q	Ordering quantity (in units).

Source: Authors' processing.

3.2 Assumptions

- The inventory system only applies to a single type of fresh perishable product.
- Fresh perishable product affect by the deterioration at a constant rate $\theta (0 < \theta < 1)$ of the existing inventory during the cycle time.
- Freshness-preservation efforts (FPE) applied to control the rate of deterioration. FPE denoted $\mathrm{I}_f = h_{cm} / h_m$, where h_m the maximum cost of holding including preservation operations. h_{cm} is controllable marginal holding cost with preservation operations and $0 < h_{cm} < h_m$.(Yang et al., 2020),(Jani et al., 2023),(Katariya, et al., 2024).
- The higher value of I_f , increase the freshness of product. The relation between I_f and rate of deterioration θ is defined as $\theta = 1 - (\mathrm{I}_f)^{\frac{1}{\varpi}}$, where $\varpi > 0$ is the relationship coefficient between deterioration rate and effort to maintain quality, the relation stated that the higher value of I_f , the less the deterioration rate of perishable food, generation fresh items for extended time horizons. (Cai et al., 2010)
- The relation between I_f and θ also denotes as $\mathrm{I}_f = (1-\theta)^{\varpi}$, in other words $h_{cm} = h_m (1-\theta)^{\varpi}$, i.e. if $\theta \to 0$ then $\mathrm{I}_f \to 0$ and $h_{cm} \to h_m$. Likewise, $\theta \to 1$ then $\mathrm{I}_f \to 0$ and $h_{cm} \to 0$.(Yang et al., 2020)
- Deteriorated product cannot be repaired or replaced and products have no salvage value. The waste of perishable products is ignored.

- The firm's/retailer's greening effort requires a capital investment in greening efforts over a certain time frame rather than raising the unit price of the product. The firm/retailer uses green investments to produced or maintain organic/ green products.
- Total greening investments per unit time t is $\int_0^t \int_0^{g_e} \lambda \cdot g_e \, dg_e dt = \frac{\lambda \cdot g_e^2 \cdot t}{2}$.

So, the total investment for the time duration T is $\frac{\lambda \cdot g_e^2 \cdot T}{2}$. (Swami & Shah, 2013),(Raza & Faisal, 2018).

- The demand rate is based on the selling price of the product, as well as greening efforts. Demand rate function defined as, $R(p, g_e) = \alpha - \beta p + \gamma g_e, 0 \le t \le T$, where $\alpha > 0$ represent the constant market demand , $\beta > 0$ is the price elasticity factor, $\gamma > 0$ is greening investments effectiveness scale.
- The lead time is negligible, the replenishment rate infinite, and shortages are not permissible.
- Inflation is typically associated with rapidly increasing prices that reduce the purchasing power of money, which varies greatly depending on the period.

4. Mathematical modellingof the inventory system

At the initial time $t = 0$, in inventory system have a stock Q units of fresh items and during the span $0 < t < T$ level of inventory depletes due to the combined effect of market demand and effect of deterioration and stock level approached to zero at time $t = T$ .The following differential equations demonstrated the level of inventory at time t over the period $[0, T]$ is,

$$\frac{dS(t)}{dt} + \theta S(t) = -(\alpha + \beta p + \gamma g_e), 0 \le t \le T \tag{1}$$

At condition $I(T) = 0$, the solution of Eq.(1) is,

$$S(t) = \frac{(\alpha + \beta p + \gamma g_e)}{\theta}(e^{(T-t)\theta} - 1) \tag{2}$$

Therefore, the order quantity of fresh products per scheduled time T is as under:

$$Q = \frac{(\alpha + \beta p + \gamma g_e)}{\theta}(e^{T\theta} - 1) \tag{3}$$

The following cost components included with inventory system,

Ordering costs are expenses incurred when a fresh product is ordered.
Ordering cost of fresh product per order is, $C_o = K$ (4)

We take into account the two components of the holding costs for fresh green perishable inventory: the first is the constant holding cost h, which includes the cost of physical storage, and the additional component is the cost of necessary preservation management, i.e., freshness preservation cost h_{cm}.
The holding cost with preservation operations under inflation per unit time is,

$$C_h = [h + h_m(1-\theta)^{\varpi}]\int_0^T \frac{(\alpha+\beta p+\gamma g_e)}{\theta}(e^{(T-t)\theta}-1)e^{-rt}dt \tag{5}$$

$$= [h + h_m(1-\theta)^{\varpi}]\frac{(\alpha+\beta p+\gamma g_e)}{\theta}\left[\frac{1}{\theta+r}\left(e^{-rt}-e^{-\theta T}\right)+\frac{1}{r}\left(e^{-rt}-1\right)\right] \tag{6}$$

The procurement cost of fresh product,

$$C_p = CQ$$
$$= C\left[(\alpha+\beta p+\gamma g_e)(e^{T\theta}-1)\right].$$

The retailer spends some cost due to product deterioration characteristic, so the deteriorating cost is,

$$C_d = d_c\left[Q-\int_0^T\left(\alpha+\beta p+\gamma g_e\right)e^{-rt}dt\right] \tag{7}$$

$$= d_c(\alpha+\beta p+\gamma g_e)\left[\frac{(e^{T\theta}-1)}{\theta}-\frac{(1-e^{-rT})}{r}\right]$$

Depending on the product's level of greening, retailers may have to incur extra expenses to increase the product's quality. Greening efforts investment is,

$$G_{EI} = \int_0^T\int_0^{g_e}\lambda\cdot g_e\,dg_e dt = \frac{\lambda\cdot g_e^2\cdot T}{2} \tag{8}$$

The revenue generated by each sales cycle under inflation is,

$$R_s = p\int_0^T(\alpha+\beta p+\gamma g_e)e^{-rt}dt \tag{9}$$

$$= \left(\frac{p(\alpha+\beta p+\gamma g_e)}{r}(e^{-rT}-1)\right)$$

Retailer's average profit per cycle time T is,

$$AP(T,g_e,p)=\frac{1}{T}\left(R_s-C_o-C_p-C_h-C_d-G_{EI}\right) \tag{10}$$

$$=\frac{1}{T}\left[\begin{array}{l}\left(\dfrac{p(\alpha+\beta p+\gamma g_e)}{r}(e^{-rT}-1)\right)-K-C\left[\dfrac{(\alpha+\beta p+\gamma g_e)}{\theta}(e^{T\theta}-1)\right]\\ -[h+h_m(1-\theta)^{\varpi}]\dfrac{(\alpha+\beta p+\gamma g_e)}{\theta}\left[\dfrac{1}{\theta+r}\left(e^{-rt}-e^{-\theta T}\right)+\dfrac{1}{r}\left(e^{-rt}-1\right)\right]\\ -d_c(\alpha+\beta p+\gamma g_e)\left[\dfrac{(e^{T\theta}-1)}{\theta}-\dfrac{(1-e^{-rT})}{r}\right]-\dfrac{\lambda\cdot g_e^2\cdot T}{2}\end{array}\right]$$

For the simplicity of profit function, approximate the exponential function in Eq. (10) by the first three terms of the Taylor series.

$$AP=\frac{1}{T}\left[\begin{array}{l}\left(\dfrac{p(\alpha+\beta p+\gamma g_e)}{r}(-rT+\dfrac{r^2T^2}{2})\right)-K-C\left[\dfrac{(\alpha+\beta p+\gamma g_e)}{\theta}(T\theta+\dfrac{T^2\theta^2}{2})\right]\\ -[h+h_m(1-\theta)^{\varpi}]\dfrac{(\alpha+\beta p+\gamma g_e)}{\theta}\left[\dfrac{1}{\theta+r}\left(-rt+\dfrac{r^2t^2}{2}+T\theta-\dfrac{T^2\theta^2}{2}\right)+\left(-T+\dfrac{rT^2}{2}\right)\right]\\ -d_c(\alpha+\beta p+\gamma g_e)\left[\dfrac{T^2}{2}(\theta+r)\right]-\dfrac{\lambda\cdot g_e^2\cdot T}{2}\end{array}\right] \tag{11}$$

Noticed that the Eq. (11) is a nonlinear form, we adopted the following solution methodology for obtaining optimum value of decisions variable and corresponding results.

4.1 Theoretical results and optimal solution

In this section, we derive the optimal value of decision variables T, g_e, p which maximise the total profit per unit time of the inventory system. To prove this result, the optimal solution must satisfy the necessary condition:

Taking the first order partial derivative of Eq. (11), with respect to T and equating to zero, we have,

$$\frac{\partial AP}{\partial T}=\left[\begin{array}{l}\left(\dfrac{p(\alpha+\beta p+\gamma g_e)}{r}(\dfrac{r^2}{2})\right)-C\left[\dfrac{(\alpha+\beta p+\gamma g_e)}{\theta}(\dfrac{\theta^2}{2})\right]\\ -[h+h_m(1-\theta)^{\varpi}]\dfrac{(\alpha+\beta p+\gamma g_e)}{\theta}\left[\dfrac{1}{\theta+r}\left(\dfrac{-1}{T^2}\left(-rt+\dfrac{r^2t^2}{2}\right)-\dfrac{\theta^2}{2}\right)+\left(\dfrac{r}{2}\right)\right]\\ -d_c(\alpha+\beta p+\gamma g_e)\left[\dfrac{1}{2}(\theta+r)\right]\end{array}\right]=0 \tag{12}$$

Taking the first order partial derivative of Eq. (12), with respect to g_e and equating to zero,

$$\frac{\partial AP}{\partial g_e}=\frac{1}{T}\left[\begin{array}{l}\left(\frac{p(\gamma)}{r}(-rT+\frac{r^2T^2}{2})\right)-C\left[\gamma(T+\frac{T^2\theta}{2})\right]\\ -[h+h_m(1-\theta)^{\varpi}]\frac{(\gamma)}{\theta}\left[\frac{1}{\theta+r}\left(-rt+\frac{r^2t^2}{2}+T\theta-\frac{T^2\theta^2}{2}\right)+\left(-T+\frac{rT^2}{2}\right)\right]\\ -(\gamma)d_c\left[\frac{T^2}{2}(\theta+r)\right]-\lambda\cdot g_e\cdot T\end{array}\right]=0 \tag{13}$$

Final, to find the first order partial derivative of Eq. (12), with respect to p and equating to zero,

$$\frac{\partial AP}{\partial p}=\frac{1}{T}\left[\begin{array}{l}\left(\frac{(\alpha+2\beta p+\gamma g_e)}{r}(-rT+\frac{r^2T^2}{2})\right)-C\left[\frac{(\beta)}{\theta}(T\theta+\frac{T^2\theta^2}{2})\right]\\ -[h+h_m(1-\theta)^{\varpi}]\frac{(\beta)}{\theta}\left[\frac{1}{\theta+r}\left(-rt+\frac{r^2t^2}{2}+T\theta-\frac{T^2\theta^2}{2}\right)+\left(-T+\frac{rT^2}{2}\right)\right]\\ -(\beta)d_c\left[\frac{T^2}{2}(\theta+r)\right]\end{array}\right]=0 \tag{14}$$

Theorem 1. Given positive value of $p\ (p>C)$ and g_e,if the average profit is a strictly concave function of T, then

1. The eq. (13) has one and only one solution.
2. The solution in (1) fulfilled the second order condition for the maxima.

Proof: See Appendix A1.

Theorem 2. Given positive value of $p\ (p>C)$ and cycle timeif the average profit is a strictly concave function of g_e, then

1. The eq. (14) has one and only one solution.
2. The solution in (1) fulfilled the second order condition for the maxima.

Proof: See Appendix A2.

Theorem 3. Given any positive selling price $p\ (p>C)$, average profit function fulfilled the Hessian matrix conditions for concavity at the optimal value of T^* and g_e^*.

Proof: See Appendix A3.

Theorem 4. At the optimum value of T^* and g_e^*,if the average profit function is a strictly concave function of p, then

1. There exists unique optimal p^* that satisfies Eq. (15).

2. The solution of (1) fulfilled the second order condition for the maxima.
Proof: See Appendix A4.

4.2 Solution procedure

Step 1: First assigned the value of inventory parameters in their proper unit in Eq.(11) except decisions variables.
Step 2: Choose any positive fix value of p ($p > C$).
Step 3: Solving the simultaneous equations stated in Eq. (12) and Eq. (13) using the mathematical software Maple XVIII or Matlab or Mathematica to find T^* and g^*.
Step 4: Verify the sufficient conditions stated as Eq. (A1.2), Eq. (A2.1) and Eq. (A3.2) at T^* and g_e^* , if not satisfied go to step 2 and choose other value of p and other value of parameters in step 1, repeat process till Eq. (A1.2), Eq. (A2.1) and Eq. (A3.2) satisfied.
Step 5: Using Maple XVIII orMatlab, to find p^* at T^* and g_e^*,solve Eq.(14) and verify Eq. (A4.2), otherwise go to step 1 choose other parametric value.
Step 6: Find profit $AP(T^*, g_e^*, p^*)$ from Eq.(11).
Step 7: Evaluate Q^* value at (T^*, g_e^*, p^*) from Eq. (3).

5. Real example with numerical validations

5.1 Real life examples

The retailer of fresh perishable goods and the requirement of ongoing product preservation are the subjects of the current investigation. Perishable goods, such as fruits, meat, vegetables, etc., must be preserved continuously to increase their shelf life. Retailers must invest in preservation technology to carry a product with a fixed holding cost. Using green technologies, perishable goods can be made healthier. Other perishable products,products, such as dairy products are processed using green technology and green packaging is employed for sustainability. This paradigm can be used for perishable health-conscious goods such as nutrient-dense foods, green dairy products, green beverages, and organic agricultural products.

5.2 Numerical Analysis

In order to illustrate the model being presented and the method of solving it, a numerical example is shown in this section.

Example 1: This section's parameters were inferred from several cited literature and were determined to be appropriate for the model that was suggested.

$\alpha = 900$ units, $\beta = 5.5$, $\gamma = 0.5$, K =\$500/order, d_c =\$2/unit, C =\$12/unit, h =\$2/unit, h_m=\$3/unit, $\varpi = 0.5$, $\lambda = 4$, $r = 0.05$, $\theta = 0.05$. The optimal results found as under **Table 3** as per section 4.2 solution procedures.

Table 3. Optimal results

T^* (in year)	g_e^* (in \$)	p^* (in \$)	$AP(T^*, g_e^*, p^*)$ (in \$)	Q^* (Units)
0.4881	**9.32**	**89.02**	**29741.51**	**205**

Source: Authors' processing.

5.3 Numerical validation of sufficient conditions

From the eq. (A1.1), (A2.1), (A3.2), observed that the sufficient conditions at T^*, g_e^*, p^* are $\frac{\partial^2 AP}{\partial T^2} = -8575.90 < 0, \frac{\partial^2 AP}{\partial g_e^2} = -4 < 0, \frac{\partial^2 AP}{\partial p^2} = -10.87 < 0$ and from eq.(A3.2), $|H| = 34293.20 > 0$.

5.4 Graphical validation of concavity of the objective function

The graphical demonstration of concavity of the profit function as mentioned in Figure 1, Figure 2 and Figure 3.

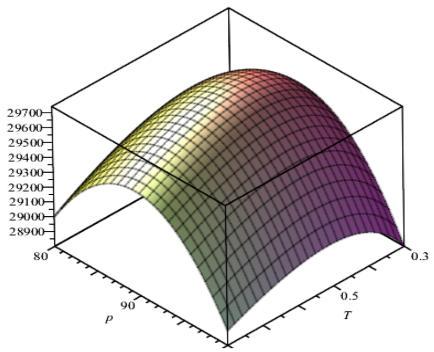

Figure 1. Concavity of total profit with respect to p^* and T^*

Source: Authors'own creation.

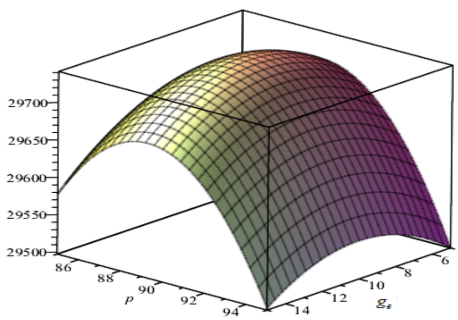

Figure 2. Concavity of total profit with respect to p^* and g_e^*

Source: Authors'own creation.

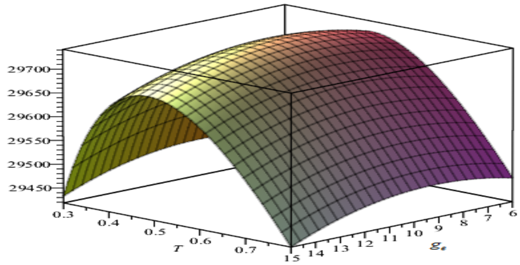

Figure 3. Concavity of total profit with respect to T^* and g_e^*.

Source: Authors'own creation.

6. Sensitivity investigations, discussion and results

The sensitivity analysis is performed by changing key parameters from -20% to +20% independently while keeping the other parameters stable to examine the effects of various parameters on the optimum outcome of the proposed inventory model.

Table 4. Effect of inflation rate r on optimum results

r	T^*	g_e^*	p^*	AP^*	Q^*
0.04	0.5105	9.33	89.04	29833.53	215
0.045	0.4989	9.33	89.03	29786.98	210
0.055	0.4779	9.31	89.01	29697.05	201
0.06	0.4684	9.30	89.01	29653.55	197

Source: Authors' processing.

Table 4 shows that the ordering quantity, total profit, and decision variables are all negatively correlated with the inflation rate (r).

Table 5. Effect of FPE factor ϖ on optimum results

ϖ	T^*	g_e^*	p^*	AP^*	Q^*
0.4	0.4877	9.32	89.02	29739.99	205
0.45	0.4879	9.32	89.02	29740.75	205
0.55	0.4883	9.32	89.02	29742.27	205
0.6	0.4884	9.32	89.02	29743.02	205

Source: Authors' processing.

Noticed from **Table 5**, the higher value of freshness preservation efforts factor ϖ reduces the impact of deterioration with reduces the cycle time. Increases the freshness of product resulted to increases the shelf life of product and hence increases the profit.

Table 6. Effect of maximum holding cost h_m on optimum results

h_m	T^*	g_e^*	p^*	AP^*	Q^*
2.4	0.5028	9.321	88.97	29801.64	211
2.7	0.4953	9.320	89.00	29771.35	208
3.3	0.4812	9.317	89.05	29712.11	202
3.6	0.4746	9.316	89.07	29683.12	199

Source: Authors' processing.

Table 6 shows that the increases -20% to 20% in h_m resulted to cycle time, greening cost, profit and number of ordering quantity decreases and on other side selling price increases slightly.

Table 7. Effect of Greening efforts effectiveness parameter λ on optimum results

λ	T^*	g_e^*	p^*	AP^*	Q^*
3.2	0.4876	11.66	89.13	29784.99	205
3.6	0.4879	10.36	89.07	29760.82	205
4.4	0.4883	8.47	88.98	29725.73	205
4.8	0.4884	7.76	88.95	29712.59	205

Source: Authors' processing.

According to **Table 7**, the increases in λ in -20% to 20% then greening cost highly decrease and profit also decreases. Selling price slightly decreases and cycle time moderately increases with increases in λ .

Parameters variation impact on retailer's total profit demonstrated in the Figure 4, Figure 5 and Figure 6.

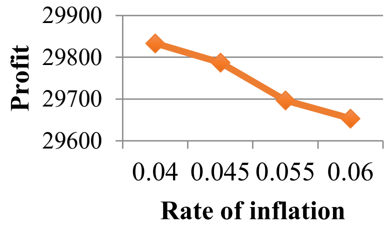

Figure 4. Rate of inflation effect on profit
Source: Authors'own creation.

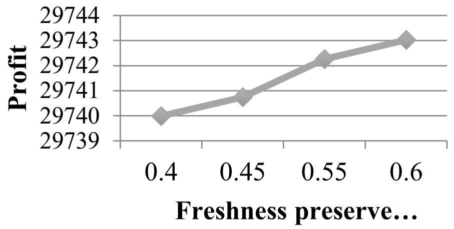

Figure 5. Freshness preserve effort factor effect onprofit
Source: Authors'own creation.

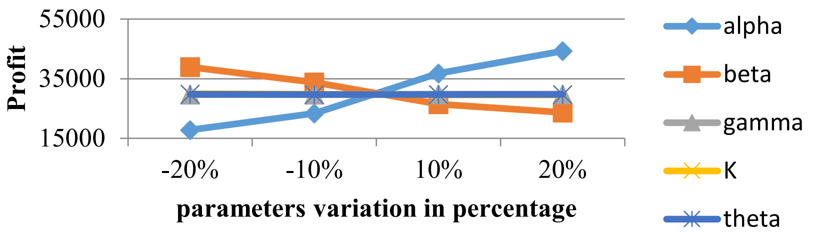

Figure 6. Effect of parameters on profit
Source: Authors'own creation.

Figure 4 indicates that a higher inflation rate decreases the total profit,but a higher freshness of the product as per **Figure 5** increases the total profit. From Figure 6, see that constant demand α and greening effectiveness parameter γ increases then total profit increase but price elasticity β, cost parameters K, C, h and deterioration rate θ increases then total profit decreases.

Parameters variation impact on ordering quantity is demonstrated in the **Figure 7** as below.

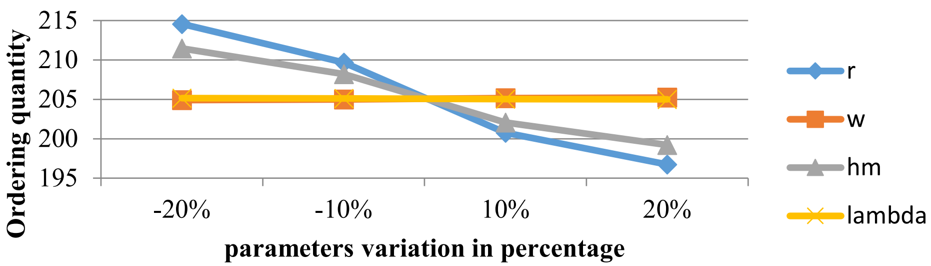

Figure 7. Effect of parameters on ordering quantity
Source: Authors'own creation.

Figure 7 indicated that the ordering quantity decreases due to decrease in r and h_m but ordering quantity remain stable for ϖ , λ and other parameters.

6.1 Managerial perspectives as the results of analysis

The following perspectives on management are distilled from the sensitivities, findings, and mathematical evaluation mentioned above.

- Higher freshness index increases the product shelf life and hence increased total profit.
- Total profit reduces due to increases in inflation rate.
- The investment in greening efforts resulted to increases the sustainability and product greening level.

- Product deterioration reduces through preservation operations during the holding the inventory. Low deterioration rate increases the total profit.
- The retailer will try to reduce the ordering cost and deterioration cost and other cost to increase the profit.
- Retailers should preserve the freshness and greenness while maximising overall profit by optimisingsales prices, cycle times, and greening initiatives.

7. Conclusions

This article presents an integrated structure for joint optimum pricing, cycle time, and greening effort (investment) tactics for a fresh perishable product to maximise retailer profits. The demand of fresh perishable green products depends on selling price and greening efforts dependent. Product freshness maintaining through the investment in preservation technology and optimising greening level with investment in greening efforts under inflationary environment is the novel idea of this study. We conclude with pertinent managerial implications and apply sensitivity analysis to validate the analytical results. According to the findings, a greater investment in greening and a longer shelf life for goods with a short deterioration period increase overall profit. This study's results can guide decisions about managing perishable inventories, accounting for product deterioration, freshness, and environmental initiatives.

This model could be extended to consider various payment schemes and concepts of carbon emissions, such as carbon taxation, cap and trade, and carbon limit policies. Additionally, by permitting shortages, the suggested model can be expanded.

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Appendices

Appendix A1. Proof of theorem 1

Suppose take any positive fix value of selling price p and greening cost g_e, using the second order partial derivative of Eq. (13) with respect to T, we have

$$\frac{\partial^2 AP}{\partial T^2} = \left[-[h + h_m(1-\theta)^{\varpi}] \frac{(\alpha + \beta p + \gamma g_e)}{\theta} \left[\frac{1}{\theta + r} \left(\frac{2}{T^3} \left(\frac{r^2 t^2}{2} - rt \right) \right) \right] \right] \quad \text{(A1.1)}$$

Observed from, above equation (A1.1) , $\frac{r^2t^2}{2} > rt \Rightarrow \frac{r^2t^2}{2} - rt > 0, 0 < r < 1$

$$h + h_m(1-\theta)^{\varpi} > 0, \varpi > 0, 0 < \theta < 1 \Rightarrow \frac{\partial^2 TP}{\partial T^2} < 0, p > 0, p > C, g_e > 0 \quad \text{(A1.2)}$$

For the fixed positive value of p and g_e, we obtained the fixed positive value of T^*,hence the Eq. (13) has an one and only one solution and satisfies the second order condition for the maximum as per (A1.2). Resulted, given positive particular value of p and g_e the solution T^* which maximised (13), not only exists but is also unique.

Appendix A2. Proof of theorem 2

For any positive particular value of p and T, the second order partial derivative of Eq. (14) with respect to g_e, we have

$$\frac{\partial^2 AP}{\partial g_e^2} = -\lambda < 0 \text{ at } (T^*, g_e^*) \quad \text{(A2.1)}$$

Therefore, there exist one and only one value of g_e which maximised Eq. (13).

Appendix A3. Proof of theorem 3

Taking second-order derivative in Equation (14) with respect to g_e and simplifying terms is given as follows:

$$\frac{\partial^2 AP}{\partial T \partial g_e} = \begin{bmatrix} \frac{p\gamma r}{2} - \frac{C\theta\gamma}{2} - \gamma d_c \left[\frac{1}{2}(\theta + r) \right] \\ -[h + h_m(1-\theta)^{\varpi}] \frac{(\gamma)}{\theta} \left[\frac{1}{\theta + r} \left(\frac{-1}{T^2} \left(-rt + \frac{r^2 t^2}{2} \right) - \frac{\theta^2}{2} \right) + \left(\frac{r}{2} \right) \right] \end{bmatrix} \quad \text{(A3.1)}$$

The optimum value T^* and g_e^* obtained from Eq.(12) and Eq. (13), the conditions below are fulfilled at T^* and g_e^*.

$$\left|\left[\frac{\partial^2 AP}{\partial T^2}\right]_{(T^*,g_e^*)}\right| \geq \left|\left[\frac{\partial^2 AP}{\partial T \partial g_e}\right]_{(T^*,g_e^*)}\right|, \left|\left[\frac{\partial^2 AP}{\partial {g_e}^2}\right]_{(T^*,g_e^*)}\right| \geq \left|\left[\frac{\partial^2 AP}{\partial T \partial g_e}\right]_{(T^*,g_e^*)}\right| \Rightarrow |H| = \left[\left[\frac{\partial^2 AP}{\partial T^2}\right]\left[\frac{\partial^2 AP}{\partial {g_e}^2}\right] - \left[\frac{\partial^2 AC}{\partial T \partial g_e}\right]^2\right]_{(T^*,g_e^*)} > 0 \quad (A3.2)$$

the hessian matrix associated with $AP(T, g_e, p)$ is negative definite. Therefore, $AP(T, g_e, p)$ in equation (13) is concave in T and g_e for the fix positive value of p $(p > C)$.

Appendix A4. Proof of theorem 4

The second order partial derivative of the Eq. (14) with respect to p $(p > C)$, with given value of T^* ***and*** g_e^*,

$$\frac{\partial^2 AP}{\partial p^2} = -2\beta(1 - \frac{rT}{2}) \quad (A4.1)$$

Since, $1 - \frac{rT}{2} > 0, 0 < r < 1$, for $T^* > 0, g_e^* > 0$ and hence, $\frac{\partial^2 TP}{\partial p^2} < 0$ at p^*. (A4.2)

Hence the equation (14) has one and only one solution and fulfilled the second order condition for the maxima. Hence, for given positive fix value of T^* and g_e^* the solution p^* which maximise (14) not only exists but is also unique. The conditions Eq. (A3.2) and Eq. (A4.1) also proved by numerically in section 5. Proof of theorem 4 completed.