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Designing a Fuzzy Logic-based Carbon Emission Cost-incorporated Inventory Model: A Comparative Analysis of Different Machine Learning Algorithms for Demand Forecasting with Memory Effects

Abstract. *Inventory control is a widely discussed topic in the real world, with organisations increasingly turning to Machine Learning models to manage stock levels based on specific product demand. This article presents an inventory model that addresses imperfect and deteriorating products within a fuzzy environment. It allows for shortages, which may be partially backlogged, and connects closely with concerns about carbon emissions and global warming. Traditional inventory models, often based on integer-order differential equations, typically overlook the system's memory aspect. Addressing inventory management is crucial in our efforts to combat global warming. This paper introduces a novel approach by integrating carbon emission costs within a fuzzy environment. To capture the memory effect of the system, Fractional Calculus is a powerful mathematical tool is employed. In the real world, entrepreneurs frequently face challenges in pinpointing exact parameter values. Therefore, this study considers uncertain factors such as ordering costs, deterioration rates, and demand rates, treating them as triangular fuzzy numbers. The objective is to determine the optimal ordering quantity and replenishment period to minimise average overall costs, including carbon emissions. The defuzzification process utilises the Graded mean integration method (GMIM), Centroid methods (CM), and Signed Distance method (SDM). Seasonal demand forecasting is approached using Machine Learning methodologies. Numerical results are analysed through the lens of memory concepts to validate the proposed mathematical model.*

Keywords: *inventory control, Machine Learning algorithms, Fractional Calculus Derivatives, carbon emission costs, seasonal demand forecasting.*

JEL Classification: 90B05, 90B60, 91B42.

1. Introduction

Seasonal and weather conditions exert a significant influence on global market demand, a pivotal factor in inventory management across all business sectors due to its inherent challenges. Seasonal demand, exacerbated by events like Christmas or Black Friday, drives heightened consumer spending, while weather conditions directly impact sales, such as increased garden furniture purchases in spring. Despite most studies relying on deterministic demand models, real-world demand fluctuates seasonally. Accurate demand forecasting enables companies to optimise inventory management, curb unnecessary costs, and elevate customer service levels. Machine

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learning (ML) stands out as an innovative tool to enhance the precision and reliability of demand forecasts. Its applications span diverse fields including retail, economics, military, and healthcare, with researchers actively developing algorithms like Decision Tree-based Approaches to bolster demand forecasting capabilities (de Almeida Neto and Castro, 2017). In our study, we implement the decision tree classifier algorithm for seasonal demand forecasting. The deterioration of physical products over time, such as flowers, vegetables, and medicines, poses substantial challenges during transit and storage. Deterioration, defined as damage that renders a product unfit for its intended use, contributes significantly to food waste, estimated at 20-40% between harvest and consumption (Sethi, 2006). Products are seldom perfect due to manufacturing defects, handling issues, and transit of economic phenomena, operations research, mathematical programming, game theory, marketing, statistical analysis methods and techniques, artificial intelligence, expert systems, neuronal networks, and software tools for modelling and analysis of economic phenomena.

Damage, complicating inventory management. While traditional models assume deterministic deterioration rates, real-world rates are uncertain and can be treated as fuzzy variables in advanced inventory models. Fuzzy inventory models, accommodating uncertain deterioration rates, have been extensively studied (Naserabadi, 2014). The unpredictability extends to defective product percentages in deliveries, also treated as fuzzy variables. Modern concerns about carbon emissions from industrial activities underscore the imperative to reduce the environmental impact in inventory management practices. To the best of our knowledge, the impact of demand forecasting on imperfect deteriorating products has rarely been addressed. With this in mind, two research problems emerge: (a) How can machine learning-based demand forecasting techniques enhance the accuracy and predictability of seasonal demand forecasts for deteriorating products? (b) What are the advantages of employing machine learning-based monthly predicted demand over fixed demand in inventory management, integrated with metaheuristic algorithms? To tackle these challenges, this article develops a machine learning-based fuzzy inventory model considering imperfect deteriorating items under carbon emissions.

2. Literature review

In contemporary business strategies, the primary objective of inventory management is increasingly focused on reducing carbon emissions (Singh and Mishra, 2021). Many organisations are committed to this goal as a means to address the challenge of global warming, which is exacerbated by carbon emissions. To encourage sustainable practices, carbon emission costs are now integrated into inventory models. Memory-based inventory systems, including fractional order Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models (Pakhira et al., 2023), have emerged as pivotal aspects of modern inventory management. Traditionally, the integer-order EOQ model has dominated the literature but has shown limitations in accurately capturing the intricate dynamics of real-world demand patterns. This limitation has spurred researchers to explore fractional order models, rooted in fractional calculus concepts introduced by mathematicians such as Riemann and Liouville. These models incorporate past input influences to determine future outputs, offering a more adaptable representation of memory-affected systems in inventory management. For retailers who navigate this terrain, it is crucial to model inventory systems that account for memory effects on consumer behaviour influenced by factors such as advertisements. Recent advances have particularly focused on scenarios involving deteriorating items with partial backlogging and quadratic demand rate dynamics, showcasing the effectiveness of fractional calculus in handling these complexities (Ghosh et al., 2022; Thirthar et al., 2023; Jana et al., 2024). In a broader context, fuzzification is utilised to manage uncertainties inherent in system components, providing a more realistic approach.

In the realm of inventory systems, various cost parameters exhibit time-dependent uncertainty, underscoring the necessity for models that operate within a fuzzy environment (Kumar et al., 2023). The globally recognised vehicle manufacturer Ford has significantly increased the use of green and renewable energy sources, bolstering the efficiency of its global production operations. Ruidas et al. (2022) explored an interval-valued green pro- duction inventory model that integrates controllable carbon emissions and green subsidies using particle swarm optimisation. Their study highlighted a direct correlation between product environmental sustainability and government subsidy intensity. It also underscored positive outcomes for both manufacturing firms investing in green innovation (GI) and emission reduction technology (ERT), as well as environmental benefits.

Recently, Paul et al. (2022) applied meta-heuristic algorithms to develop a production inventory model incorporating green investments and managed carbon emissions. Our research focuses on the critical challenge of reducing industrial carbon emissions through innovative manufacturing strategies and technologies. We emphasise the significant role of regulatory policies and emission reduction technologies (ERTs) in addressing environmental challenges on a global scale. Introducing the Graded Mean Integration Method (GMIM), our study employs this mathematical tool to represent fuzzy numbers in the context of environmental sustainability and industrial emission reduction. The GMIM is chosen for its balance of simplicity and accuracy, aligning seamlessly with our goal of providing clear and precise representations in the complex realms of environmental assessment and fuzzy logic, particularly concerning fuzzy environmental data associated with industrial sector carbon emissions. In summary, the deliberate use of GMIM in our research meets the demand for precise and manageable representations in the intricate fields of environmental sustainability and industrial emission reduction. These mathematical methods significantly enhance the clarity and accuracy of our findings, enabling us to navigate the complex landscape of fuzzy environmental data effectively.

The existing literature on the field of machine learning for demand forecasting is limited, and the use of forecasting tools in business strategy is an ongoing debate. Demand forecasting for business strategy is one of the most challenging tasks in modern business research. Generally, the global market demand depends on seasonal and weather conditions. The demand forecasting model helps to predict the overstocking and under-stocking situations and when demand rises or falls (Wright & Schultz, 2018). For example, on Christmas Day, Amazon sells more goods than on other days of the year. Therefore, instead of fixing demand, forecasted demand is necessary. Different researchers used different forecasting methods over time in inventory management. (Kirshners et al., 2010) studied the joint analysis of continuous and discrete data using inductive decision trees. Recently, (Zohdi et al., 2022) implemented various machine learning algorithms such as K-nearest neighbours, decision tree, and gradient boosting to forecast demand and examine its accuracy and performance compared to other approaches. But the forecasting approach in inventory modelling has not been addressed. Therefore, this study outlines the use of machine learning for demand forecast in inventory management systems. This paper conducts a thorough review of the current literature on pricing strategies and market analysis, with a focus on integrating machine learning models for personalised dynamic pricing and predicting market trends. Its goals include providing scalable pricing solutions, assisting retailers in making informed decisions swiftly, and establishing reliable pricing strategies. Methodologically, the study emphasises meticulous data preprocessing, feature engineering, exploratory data analysis, and developing innovative features to enhance machine learning model performance. Ensemble learning technique such as Decision Tree Classifier Singh (as see in Fig. 2) and Mishra (2024) is introduced for their speed and accuracy advantages with Fuzzy and Fuzzy learning both approaches.

2.1 Research Gap

In recent papers (Mishra et al.,2021; Jaggi et al., 2023; Rahaman et al., 2022; Santra et al., 2023), ordinary differential equations have been utilised to develop models incorporating fractional calculus in inventory systems or accounting for carbon emission costs. This research addresses this gap by formulating an inventory model in a fuzzy setting that considers memory-dependent factors along with associated carbon emission costs. Key gaps identified are:

1. We find that traditional inventory models often overlook the uncertain- ties in deterioration rates and defective percentages, which can impact the accuracy of demand forecasts and inventory management.
2. In existing models, we typically observe a reliance on fixed demand assumptions rather than incorporating dynamic, machine learning-based demand forecasting methods.
3. We note a lack of focus on integrating carbon emission costs into inventory models, which is crucial for addressing sustainability concerns.
4. We have incorporated memory-effect factors, which are rarely considered in inventory models, despite their potential to enhance understanding of how past actions influence current inventory systems.

2.2 Research contribution

- We utilised fractional calculus to introduce memory effects into the inventory model within a fuzzy environment, capturing the impact of past actions on the inventory system.
- We extended the inventory model by including carbon emission costs, transforming it into a green inventory model that reflects growing environmental sustainability concerns.
- We developed a machine learning-based fuzzy inventory model using a decision tree classifier to accurately forecast seasonal demand for deteriorating products.
- We demonstrated through numerical experiments that using seasonally forecasted demand significantly reduces overall costs compared to relying on fixed demand assumptions.
- We innovatively combined memory effects and carbon emission costs within a fuzzy and fuzzy learning environment, providing a multidimensional approach to inventory management.
- We showcased the potential for businesses to reduce their ecological footprint by optimising ordering quantities and replenishment periods to minimise total average costs, including carbon emission costs.

In summary, our study focuses on enhancing inventory management by introducing memory effects, accounting for carbon emissions, and exploring their interactions in a fuzzy environment. This multidimensional approach offers a comprehensive understanding of inventory systems and paves the way for more sustainable and efficient inventory management practices.

3. Model Formulation

3.1 Notations

In the subsection 3.1 the notations are outlined in Table 1 and rest of the notation as usual meaning. In the subsection 3.2, the assumptions of the model are discussed.

Table 1. Notations

Notations	
d_f	Forecasted Demand,
E_c	Carbon emission cost associated with item ordering,
O_c	Ordering cost/Setup cost,
O_{fc}	Ordering cost/Setup cost in fuzzy environment,
α_m, β_m	Memory effects parameters related to total average cost,
TC_f	Minimised total average cost with fuzzy parameters,
TC_{fl}	Minimised total average cost with fuzzy learning parameters,
$HC_{f(\alpha_m,\beta_m)}$	Total carrying cost in fractional-order model with fuzzy parameters,
$HC_{fl(\alpha_m,\beta_m)}$	Total carrying cost in fractional-order model with fuzzy learning parameters,

Notations	
$SC_{f(\alpha_m,\beta_m)}$	Total shortage cost for fractional order model with fuzzy parameters,
$SC_{fl(\alpha_m,\beta_m)}$	Total shortage cost for fractional order model with fuzzy learning parameters,
$LC_{f(\alpha_m,\beta_m)}$	Total Lost sales cost for fractional order model with fuzzy parameters,
$LC_{fl(\alpha_m,\beta_m)}$	Total Lost sales cost for fractional order model with fuzzy learning parameters,
$PC_{f(\alpha_m)}$	Total purchasing cost for fractional order model with fuzzy parameters,
$PC_{fl(\alpha_m)}$	Total purchasing cost for fractional order model with fuzzy learning parameters,
p_c	Purchasing cost for fractional order model,
p_{fc}	Purchasing cost for fractional order model with fuzzy parameters,
p_{flc}	Purchasing cost for fractional order model with fuzzy learning parameters,
s_c	Shortage cost for fractional order model,
s_{fc}	Shortage cost for fractional order model with fuzzy parameters,
s_{flc}	Shortage cost for fractional order model with fuzzy learning parameters,
L_c	Lost sale cost for fractional order model,
L_{fc}	Lost sale cost for fractional order model with fuzzy parameters,
L_{flc}	Lost sale cost for fractional order model with fuzzy learning parameters,
h_c	Holding cost for fractional order model,
h_{fc}	Holding cost for fractional order model with fuzzy parameters,
h_{flc}	Holding cost for fractional order model with fuzzy learning parameters,
h_l	Holding cost with fuzzy learning rate,
s_l	Shortage cost with fuzzy learning rate,
p_l	Purchasing cost with fuzzy learning rate,
L_l	Lost sale cost with fuzzy learning rate,
O_l	Ordering cost with fuzzy learning rate.

Source: Author's own creation.

3.2 Presumptions

- Lead time is negligible.
- An infinite time horizon is considered.
- The demand pattern of green products is based on forecasted demand.
- Shortages are allowed which are partially backlogged, and lost sales are also considered.
- The holding cost, ordering cost, shortage cost, and purchasing cost are represented as triangular fuzzy numbers.
- Carbon emission costs are factored in for acquisition, transportation, ordering, inventory holding, and shortages.
- The model focuses on a single type of deteriorating product with an infinite replenishment rate.

- Defective products arise from imperfect manufacturing and handling issues, with k represented as an interval trapezoidal fuzzy number and k is considered an interval trapezoidal fuzzy number.
- Retailers conduct a 100% screening process to identify defective products, which are removed after screening.
- Screening and demand rates occur simultaneously, with the screening rate exceeding the demand rate.

4. Model Formulation

To account for the impact of memory effects, we can express the following differential equations:

$$\frac{dI_1(t)}{dt} = -d_f\,, \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\delta_f d_f\,, \qquad t_1 \le t \le T_f \tag{2}$$

By incorporating kernel functions, the differential equations can be expressed as:

$$\frac{dI_1(t)}{dt} = \int_0^t k(t-t')\, d_f dt'\,, \qquad 0 \le t \le t_1$$

$$\frac{dI_2(t)}{dt} = \delta_f \int_0^t k(t-t')\, d_f dt'\,, \qquad t_1 \le t \le T_f$$

Incorporating the kernel function, denoted as k(t−t′), is crucial. This specific type of kernel, k(t−t′), often exhibits inherent scaling characteristics, making it a prevalent choice in modelling various natural phenomena. To create a fractional-order model, we define the kernel function as:

$$k(\mathrm{t}-\mathrm{t}') = \frac{1}{\Gamma(\alpha-1)}(t-t')^{\alpha-2}$$

where $0 < \alpha \le 1$ and $\Gamma(\alpha)$ represents the gamma function. By applying the definition of the Caputo fractional-order derivative (Ghosh et al.,2015; Pakhira et al., 2024), we can express equations (1)-(2) as fractional differential equations with fractional integration in the following form:

$$\frac{dI_1(t)}{dt} = -M_t^{1-\alpha} d_f\,, \qquad 0 \le t \le t_1 \tag{3}$$

$$\frac{dI_2(t)}{dt} = -M_t^{1-\alpha} \delta_f d_f\,, \qquad t_1 \le t \le T_f \tag{4}$$

Next, we apply the Caputo fractional derivative of order $(\alpha - 1)$ to both sides of equations (3) and (4). Utilising the inverse relationship between derivatives and integrals, we can retrieve the original fractional differential equations (3) and (4) for the model:

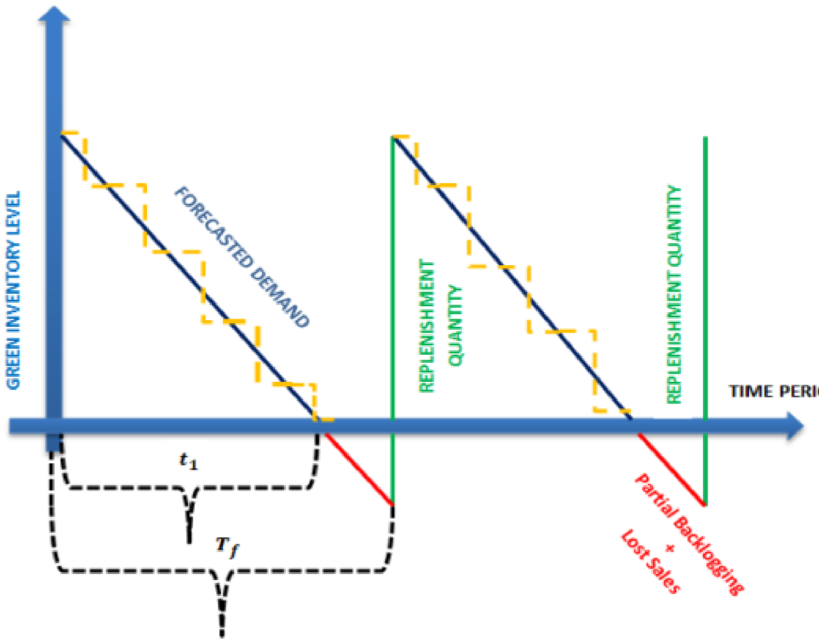

Figure 1. Graphical representation of the proposed inventory model
Source: Author's own creation.

$$\frac{d^\alpha I_1(t)}{dt} = -d_f, \qquad 0 \le t \le t_1 \tag{5}$$

$$\frac{d^\alpha I_2(t)}{dt} = -\delta_f d_f, \qquad t_1 \le t \le T_f \tag{6}$$

subject to boundary conditions: $I_1(t_1) = 0$ and $I_2(t_1) = 0$.

4.1 Analysis of the economic order quantity (EOQ) model with memory effects

The fractional-order inventory model described by equations (5)-(6) is operated upon by the fractional integral operator on both sides, with the initial conditions $I_1(t_1) = 0$ and $I_2(t_1) = 0$, as follows:

$$I_1(t) = \frac{d_f(t_1^\alpha - t^\alpha)}{\Gamma(\alpha - 1)} \tag{7}$$

$$I_2(t) = \frac{\delta_f d_f (t_1^{\alpha} - t^{\alpha})}{\Gamma(\alpha - 1)} \tag{8}$$

In this context, $I_1(t)$ represents the memory-dependent positive inventory level at time t, and $I_2(t)$ signifies the memory-dependent negative inventory level at time t. Within this framework, α denotes the order of the fractional derivative, which signifies the rate of change in the inventory level, commonly referred to as the differential memory index.

Given that the inventory level diminishes over time (t), we define the maximum positive inventory level I_{max} at $t = 0$ in the following manner:

$$I_{max} = I_1(0) = \frac{d_f (t_1^{\alpha})}{\Gamma(\alpha - 1)} \tag{9}$$

Here, the maximum backorder units during shortage time becomes,

$$S_{max} = -I_1(T_f) = \frac{\delta_f d_f (T_f^{\alpha} - t_1^{\alpha})}{\Gamma(\alpha - 1)} \tag{10}$$

Hence, the order size denoted as Q during the entire ordering interval [0, T_f] is the combination of the maximum positive inventory level and the maximum backorder units, given by

$$Q = I_{max} + S_{max} \tag{11}$$

Our main objective is to minimise the costs associated with the inventory system, which include holding costs and shortage costs. The total average cost is calculated as the average of holding costs, shortage costs, purchasing costs, and ordering costs over the ordering interval. The individual costs for this system affected by memory effects are evaluated as follows:

4.2 Some Associated Inventory Costs

Holding costs vary over time and are not constant throughout the entire cycle of the system. Therefore, we assume the inventory holding cost per unit as a time-dependent function in the form of $h_{fc} t^{\alpha}$ This leads to the calculation of the β_m^{th} order inventory holding cost, denoted as

$$HC_{f(\alpha_m, \beta_m)}(T_f) = h_{fc} M^{-\beta_m} (t^{\alpha_m} I_1(t)) \tag{12}$$

In this context, the symbol $M^{-\beta_m}$ for $0 \leq t \leq t_1$ signifies the fractional integration of order β_m which is employed in the Riemann-Liouville sense.

The parameter β_m indicates the integral memory index. The computation of the β_m^{th}order shortage cost with fractional effects, denoted as:

$$SC_{f(\alpha_m,\beta_m)} = s_{fc} M^{-\beta_m} I_2(t) \tag{13}$$

Here, for the time interval $t_1 \leq t \leq T_f$, $M^{-\beta_m}$ represents fractional integration of order β_m used in the Riemann-Liouville sense. Additionally, s_{fc} denotes the shortage cost per unit item.

The purchasing cost for the fractional-order model, indicated as:

$$PC_{f(\alpha_m)} = p_c * Q \tag{14}$$

The Lost sale cost for the fractional-order model, indicated as:

$$LC_{f(\alpha_m,\beta_m)} = L_{fc} M^{-\beta_m} (1\text{-}\delta_f)\, d_f \tag{15}$$

where, p_{fc} is considered as per unit cost, Q is the total order quantity. Hence, the total average cost for the fractional-order inventory model in a fuzzy environment can be expressed as:

$$TC_f = \frac{HC_{f(\alpha_m,\beta_m)} + SC_{f(\alpha_m,\beta_m)} + PC_{f(\alpha_m)} + LC_{f(\alpha_m,\beta_m)} + E_c + O_{fc}}{T_f}$$

Similarly, the total average cost for the fractional-order inventory model in fuzzy learning environment can be expressed as:

$$TC_{fl} = \frac{HC_{fl(\alpha_m,\beta_m)} + SC_{fl(\alpha_m,\beta_m)} + PC_{fl(\alpha_m)} + LC_{fl(\alpha_m,\beta_m)} + E_c + O_{fc}}{T_f}$$

Algorithm 1 Decision Tree Regression for Seasonal Demand Forecasting

Import Libraries
```
import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.tree import DecisionTreeRegressor
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
import matplotlib.pyplot as plt
from sklearn.tree import plot_tree
```
Create Dataset
Define `data` as a dictionary and create DataFrame `df`
Convert Categorical Data
Convert `'season'` column to numerical codes
Define Features and Target
```
X = df[['month ', 'season']]
y = df['demand ']
```
Split Data
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=1)
```
Train Model
Create a `DecisionTreeRegressor` object and fit it to `X_train` and `y_train`
Predict
Use the trained model to predict `y_pred` for `X_test`
Evaluate
Calculate and print Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and R-squared (R2)
Show Predictions
Create a DataFrame of actual vs. predicted values and print it
Monthly Predictions
Create a DataFrame `months` for each month with corresponding season, convert `'season'` to numerical codes, predict demand for each month, and print the results
Plot Decision Tree
Plot the Decision Tree using `plot_tree` with features `'month'` and `'season'`

Figure 2. Algorithmic approach of Decision tree classifier method
Source: Author's own creation.

where, the learning rate of holding cost is as below similarly for other shortage, lost sale, setup, and purchasing cost also.

$$h_{flc} = h_c + \frac{h_l}{l^{\lambda}}, 0 < \lambda < 1$$

4.3 Proposed Methodology for data forecasting of seasonal demand

Decision trees are highly effective methods widely used in fields such as machine learning, image processing, and pattern recognition (Charbuty & Abdulazeez, 2021). These models work by successively combining a series of basic tests in an efficient and cohesive manner, where the numeric features are compared. The construction of conceptual rules in decision trees is significantly simpler than developing the numerical weights used in neural networks (Priyanka & Kumar, 2020). Primarily utilised for classification, decision trees consist of nodes and branches; each node represents features within a category to be classified, and each subset defines a potential value for the node (Mahesh, 2020). Due to their simplicity and accuracy across various data types, decision trees have been widely implemented in numerous applications.

4.4 Resulting optimisation problem

For Optimisation applying four methods three with triangular fuzzy parameters i.e., Graded mean integration method (GMIM), Centroid methods (CM) and Signed Distance method (SDM) and other is applying fuzzy learning method. In the numerical section, a example is provided to validate the model's application. The author estimates the seasonal demand for a deteriorating product based on the observed data trends. Seasonal demand data is presented in Tables 2 and 3 and the dataset is then divided into training and testing sets. Three forecasting methods are employed: a Decision Tree-based classifier (Algorithm 1). Eighty percent of the data is used for training, and the remaining twenty percent is reserved for testing. The month-wise forecasted demand is then obtained by inputting the month value as a parameter.

The complete initial data for the numerical example is as follows:

Table 2. Empirical Parameters for solving numerical

Parameters	Units	Data
α_m, β_m	unit	0.3, 1.0
g	\$/unit/year	0.05
(h_c^1, h_c^2, h_c^3)	\$/unit/year	(2.5,2.6,2.7)
(s_c^1, s_c^2, s_c^3)	\$/unit/year	(1.2,1.3,1.4)
(O_c^1, O_c^2, O_c^3)	\$/unit/year	(25,26,27)
(p_c^1, p_c^2, p_c^3)	\$/unit/year	(20,22,27)
(L_c^1, L_c^2, L_c^3)	\$/unit/year	(2.2,2.3,2.4)

Source: Author's own creation.

Table 3. Season-wise product demand

Season name	Month	Product demand
Winter	12,1	1.0-1.2
Spring	2,3	1.2-1.3
Summer	4,5,6	1.3-1.4
Monsoon	7,8,9	1.4-1.6
Autumn	10,11	1.6-1.8

Source: Author's own creation.

After getting month-wise demand, the outcome of the optimal total cost for an individual month per unit time for Example 1 with both approaches i.e., Fuzzy and Fuzzy learning are depicted in Table 4a and 4b. Due to the highly non-linear cost function, we have used MAPLE and PYTHON software to determine optimal values.

Table 4a. Tabular Representation of Decision Tree-based Classifier Optimal Results by using Graded mean integration method (GMIM) and Centroid Method (CM)

Forecasted Demand (DTC)	Graded mean integration method (GMIM)				Centroid Method (CM)			
Values	t_1^G	T_f^G	TC_{α_m,β_m}^G	Q^G	t_1^C	T_f^C	TC_{α_m,β_m}^C	Q^C
1.15	0.46	*7.03*	*31.48*	*1.21*	*0.31*	*5.18*	*42.26*	*1.08*
1.45	*0.57*	*7.93*	*50.90*	*1.61*	*0.37*	*5.95*	*66.86*	*1.44*
1.47	*0.58*	*8.02*	*52.70*	*1.64*	*0.38*	*6.03*	*69.11*	*1.46*
1.32	*0.50*	*7.44*	*40.97*	*1.42*	*0.33*	*5.53*	*54.35*	*1.27*
1.35	*0.52*	*7.54*	*43.02*	*1.46*	*0.34*	*5.61*	*56.95*	*1.31*
1.37	*0.53*	*7.61*	*44.46*	*1.49*	*0.35*	*5.67*	*58.77*	*1.33*
1.42	*0.55*	*7.80*	*48.36*	*1.56*	*0.36*	*5.84*	*63.67*	*1.39*
1.49	*0.59*	*8.10*	*54.57*	*1.67*	*0.39*	*6.11*	*71.46*	*1.49*
1.60	*0.66*	*8.64*	*66.55*	*1.85*	*0.51*	*7.42*	*77.06*	*1.73*
1.62	*0.68*	*8.75*	*69.06*	*1.88*	*0.44*	*6.69*	*89.45*	*1.68*
1.10	*0.45*	*6.96*	*29.25*	*1.15*	*0.30*	*5.12*	*39.39*	*1.03*

Source: Author's own creation.

Table 4b. Tabular Representation of Decision Tree-based Classifier Optimal Results by using Signed Distance Method (SDM), and Fuzzy Learning (FL)

Forecasted Demand (DTC)	Signed Distance Method (SDM)				Fuzzy Learning (FL)			
Values	t_1^S	T_f^S	TC_{α_m,β_m}^S	Q^S	t_1^{fl}	T_f^{fl}	$TC_{\alpha_m,\beta_m}^{fl}$	Q^{fl}
1.15	*0.36*	*5.91*	*37.16*	*1.13*	3.06	12.78	22.30	1.94
1.45	*0.44*	*6.74*	*59.36*	*1.50*	3.54	12.62	33.69	2.52
1.47	*0.45*	*6.82*	*61.40*	*1.53*	3.59	12.61	34.75	2.57
1.32	*0.40*	*6.29*	*48.04*	*1.33*	3.25	12.69	27.88	2.25
1.35	*0.41*	*6.38*	*50.38*	*1.37*	3.31	12.67	29.08	2.31
1.37	*0.41*	*6.45*	*52,03*	*1.39*	3.35	12.66	29.92	2.35
1.42	*0.46*	*6.63*	*56.46*	*1.46*	3.47	12.64	32.20	2.46
1.49	*0.43*	*6.91*	*63.53*	*1.56*	3.65	12.60	35.85	2.61
1.60	*0.53*	*6.59*	*86.34*	*1.65*	3.99	12.57	42.97	2.86

1.62	*0.51*	*7.53*	*79.89*	*1.76*	4.06	12.56	44.49	2.91
1.10	*0.36*	*5.85*	*34.58*	*1.08*	3.06	12.79	20.96	1.85

Source: Author's own creation.

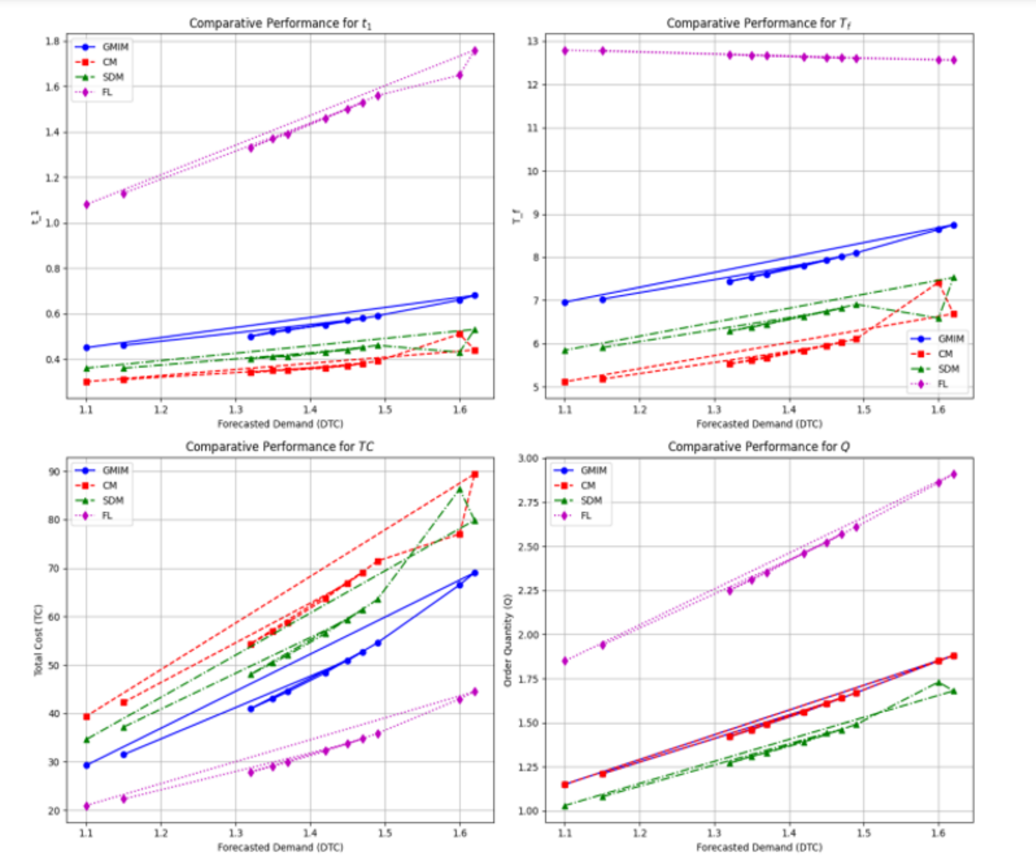

Figure 3. Comparative performance of Graded mean integration method (GMIM), Centroid Method (CM), Signed Distance Method (SDM), and Fuzzy Learning (FL) with respect to t_1 , T_f , TC_f, and Q

Source: Author's own creation.

4.5 Results and Discussions

Table 4 and Figure 3 presents the optimal results of a Decision Tree-based classifier using three fuzzy methods: Graded Mean Integration Method (GMIM), Centroid Methods (CM), and Signed Distance Method (SDM), along with Fuzzy Learning (FL). The parameters considered include the forecasted demand (DTC), replenishment cycle start time t_1 replenishment cycle finish time T_f total cost $TC^{fl}_{\alpha_m,\beta_m}$, and order quantity (Q).

Analysis of Methods

Graded Mean Integration Method (GMIM): GMIM generally shows moderate values for t_1 , T_f and Q across different forecasted demands. The total cost $TC^G_{\alpha_m,\beta_m}$ in GMIM ranges from 29.25 to 69.06, indicating relatively lower total costs compared to other methods, which suggests its efficiency in cost management. The order quantity (Q) is also stable, ranging from 1.15 to 1.88.

- CM has the lowest start times t_1 and finish times T_f indicating a quicker cycle, but this comes with higher total costs. The total cost $TC^C_{\alpha_m,\beta_m}$ ranges from 39.39 to 89.45, which is the highest among the methods, indicating that while CM may be faster, it is not cost-effective. The order quantities (Q) are slightly lower than GMIM, ranging from 1.03 to 1.73.
- SDM shows intermediate values for t_1 , T_f and Q. The total cost $TC^G_{\alpha_m,\beta_m}$ ranges from 34.58 to 79.89, higher than GMIM but lower than CM. The order quantity (Q) is relatively consistent with a range of 1.08 to 1.76.
- FL method results show a significant deviation in replenishment cycle start time $t_1$and finish time T_f with notably higher values compared to other methods. The total cost $TC^{fl}_{\alpha_m,\beta_m}$in FL ranges from 20.96 to 44.49, demonstrating that FL is effective in cost management, potentially the most efficient in terms of total cost. The order quantities (Q) for FL are higher, ranging from 1.85 to 2.91, indicating that FL suggests higher inventory levels.

Discussion

- GMIM and FL show better performance in terms of minimising total cost compared to CM and SDM. This is significant for businesses looking to optimise their inventory systems while keeping costs low. FL particularly stands out with the lowest total cost values, suggesting it may be the most effective method for cost-sensitive environments.
- CM provides the quickest replenishment cycles with the lowest t_1 and T_f making it suitable for environments where quick turnaround is crucial despite higher costs. GMIM and SDM offer a balanced approach with moderate cycle times and costs.
- The higher order quantities in FL suggest a more conservative approach, ensuring ample inventory. In contrast, CM recommends lower order quantities, potentially reducing holding costs but risking stockouts. GMIM and SDM provide middle ground solutions, balancing between inventory levels and holding costs.
- As the forecasted demand (DTC) increases, the total costs and order quantities (Q) also increase across all methods. This trend aligns with the expectation that a higher demand requires more frequent and larger replenishments.
- Businesses can choose the appropriate method based on their priorities. For cost efficiency, GMIM and FL are preferable, while for quicker replenishment cycles, CM is better suited. The choice of method should align with the company's inventory strategy, considering factors like cost sensitivity, demand variability, and service level requirements.

5. Conclusions

This study has comprehensively addressed the complexities of modern inventory management, integrating green practices, memory effects, and uncertain parameters modelled as fuzzy variables. By incorporating carbon emission costs and addressing backlogging scenarios during shortage periods, we have highlighted the nuanced trade-offs and strategies essential for optimising profitability and sustainability in inventory systems. Our findings underscore that the accounting for memory effects in inventory dynamics significantly impacts profitability. Specifically, scenarios with long or strong memory effects tend to yield higher profits, emphasising the need for adaptive inventory policies tailored to different environmental and operational conditions. Sensitivity analysis identified shortage costs and demand rates as critical factors influencing profitability, guiding effective decision-making in inventory management.

Furthermore, leveraging machine learning techniques, such as the Decision Tree-based Classifier, has enhanced the realism and efficiency of inventory models for imperfect deteriorating products. By treating deterioration rates and defective quantities as fuzzy variables and directly forecasting seasonal demand, our approach demonstrated substantial cost savings over traditional fixed-demand strategies. These results show that the Fuzzy Learning method provides the best performance in terms of minimising total costs, making it highly effective for cost-sensitive environments. The Graded Mean Integration Method also shows strong performance with low costs and stable order quantities, making it a reliable choice.

We expand the current model by integrating alternative demand forecasting approaches such as the Autoregressive Integrated Moving Average (ARIMA) and AdaBoost classifier methods. Additionally, conducting comparative studies among various forecasting techniques could enhance model robustness. Future extensions may incorporate type-2 fuzzy variables to handle uncertain parameters more effectively.

References

[1] Charbuty, B., Abdulazeez, A. (2021), *Classification based on decision tree algorithm for machine learning. Journal of Applied Science and Technology Trends*, 2(01), 20-28.

[2] De Almeida Neto, F.A., Castro, A. (2017), *A reference architecture for educational data mining*. In *2017 IEEE Frontiers in Education Conference*, 1-8, IEEE.

[3] Dinata, M.C., Suharjito, S. (2024), *Materials inventory optimization using various forecasting techniques and purchasing quantity in packaging industry. Journal of Industrial Engineering and Management*, 17(2), 321-343.

[4] Ghosh, U., Thirthar, A.A., Mondal, B., Majumdar, P. (2022), *Effect of fear, treatment, and hunting cooperation on an eco-epidemiological model: Memory effect in terms of fractional derivative. Iranian Journal of Science and Technology, Transactions A: Science*, 46(6), 1541-1554.

[5] Ghosh, U., Sengupta, S., Sarkar, S., Das, S. (2015). *Analytic solution of linear fractional differential equation with Jumarie derivative in term of Mittag-Leffler function,* arXiv preprint arXiv:1505.06514.

[6] Gurnani, P., Hariani, D., Kalani, K., Mirchandani, P., CS, L. (2022), *Inventory Optimization Using Machine Learning Algorithms.* In *Data Intelligence and Cognitive Informatics: Proceedings of ICDICI 2021*, 531-541. Singapore: Springer Nature Singapore.

[7] Jana, D.K., Das, A.K., Islam, S. (2024), *Effect of memory on an inventory model for deteriorating item: fractional calculus approach. OPSEARCH*, 1-19.

[8] Jaggi, C.K., Priyamvada, Kamna, K.M. (2023), *Sustainable production system with preservation strategy and renewable energy under different carbon tax policies. International Journal of Modelling and Simulation*, 43(4), 523-532.

[9] Keswani M., Khedlekar U. (2024), *Optimal inventory management strategies for sequentially convertible items with different distribution patterns: A comprehensive stochastic approach. Numerical Algebra, Control and Optimization*, 14(2).

[10] Keswani M., Khedlekar U. (2024), *Optimizing pricing and promotions for sustained profitability in declining markets: A Green-Centric inventory model. Data Science in Finance and Economics*, 4(1), 83-131.

[11] Kirshners, A.K., Parshutin, S.V., Borisov, A.N. (2010), *Combining clustering and a decision tree classifier in a forecasting task. Automatic Control and Computer Sciences*, 44(3), 124-132.

[12] Kumar, V., Kumar, A., Shekhar, C., Gupta, C.B. (2023), *A fuzzy inventory model with permissible delay in payments for non-instantaneous deteriorating items under inflation*. In *Modeling and Applications in Operations Research*, 139-154. CRC Press, England&Wales.

[13] Mishra, U., Wu, J.Z., Sarkar, B. (2021), *Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions. Journal of Cleaner Production*, 279, 123699.

[14] Naserabadi, B., Mirzazadeh, A., Nodoust, S. (2014), *A new mathematical inventory model with stochastic and fuzzy deterioration rate under inflation. Chinese Journal of Engineering*, 347857, 21-30.

[15] Ntakolia, C., Kokkotis, C., Karlsson, P., Moustakidis, S. (2021), *An explainable machine learning model for material backorder prediction in inventory management. Sensors*, 21(23), 7926.

[16] Pakhira, R., Mondal, B., Ghosh, U., Sarkar, S. (2024), *An EOQ model with fractional order rate of change of inventory level and time-varying holding cost. Soft Computing*, 28(5), 3859-3877.

[17] Pakhira, R., Mondal, B., Thirthar, A.A., Alqudah, M.A., Abdeljawad, T. (2024), *Developing a fuzzy logic-based carbon emission cost-incorporated inventory model with memory effects. Ain Shams Engineering Journal*, 15(6), 102746.

[18] Paul, A., Pervin, M., Roy, S.K., Maculan, N., Weber, G.W. (2022), *A green inventory model with the effect of carbon taxation. Annals of Operations Research*, 309(1), 233-248.

[19] Priyanka, Kumar, D. (2020), *Decision tree classifier: a detailed survey. International Journal of Information and Decision Sciences*, 12(3), 246-269.

[20] Rahaman, M., Mondal, S.P., Chatterjee, B., Alam, S., Shaikh, A.A. (2022), *Generalization of classical fuzzy economic order quantity model based on memory dependency via fuzzy fractional differential equation approach. Journal of Uncertain Systems*, 15(01), 2250003.

[21] Ruidas, S., Seikh, M.R., Nayak, P.K. (2022), *A production inventory model for green products with emission reduction technology investment and green subsidy. Process Integration and Optimization for Sustainability*, 6(4), 863-882.

[22] Santra, P.K., Mahapatra, G.S., Kumar, A. (2023), *Effect of reliability and memory on fractional inventory model incorporating promotional effort on demand. RAIRO-Operations Research*, 57(4), 1767-1784.

[23] Singh, R., Mishra, V.K. (2021), *An inventory model for non-instantaneous deteriorating items with substitution and carbon emission under triangular type demand. International Journal of Applied and Computational Mathematics*, 7(4), 127.

[24] Singh, R., Mishra, V.K. (2024), *Machine learning based fuzzy inventory model for imperfect deteriorating products with demand forecast and partial backlogging under green investment technology. Journal of the Operational Research Society*, 75(7), 1223-1238.

[25] Shah N.H., Keswani M., Khedlekar U.K., Prajapati N.M., (2023), *Non-instantaneous controlled deteriorating inventory model for stock-price-advertisement dependent probabilistic demand under trade credit financing. Opsearch*, 60, 1-39.

[26] Thirthar, A.A., Sk, N., Mondal, B., Alqudah, M.A., Abdeljawad, T. (2023), *Utilizing memory effects to enhance resilience in disease-driven prey-predator systems under the influence of global warming. Journal of Applied Mathematics and Computing*, 69(6), 4617- 4643.

[27] Wright, S.A., Schultz, A.E. (2018), *The rising tide of artificial intelligence and business automation: Developing an ethical framework. Business Horizons*, 61(6), 823-832.

[28] Zohdi, M., Rafiee, M., Kayvanfar, V., Salamiraad, A. (2022), *Demand forecasting based machine learning algorithms on customer information: an applied approach. International Journal of Information Technology*, 2022, 1-11.