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An Integrated Multi-Objective SORM Cross-Efficiency Model: An Application in Portfolio Selection

Abstract. *This paper presents a novel multi-objective framework for portfolio selection model integrated with the cross-efficiency approach of the Semi-oriented radial measure (SORM). This article suggests a two-stage method for choosing a portfolio. Although the SORM model was created as a workaround for the traditional DEA model's inability to handle negative data, in Stage 1, we extended it to peer evaluation as opposed to just the self-evaluation model. In the second stage, we enhance its application as a portfolio selection and optimisation tool by incorporating it into the multi-objective framework. We introduce the maverick index and diversity index as an innovative risk/return measure, that proves to be a valuable gauge of sensitivity to environmental volatility and diversification in the context of portfolio selection. The case study demonstrates the model's ease of application, its capacity to distinguish between stocks, and its effectiveness in portfolio selection.*

Keywords: *Data Envelopment Analysis, Semi-oriented radial measure, negative data, portfolio selection, cross-efficiency.*

JEL Classification : C02, C44, C60, C61, C67.

1. Introduction

Markowitz (1952) introduced the mean–variance (MV) model, which has significantly advanced the theory of modern portfolio selection. Many researchers have investigated and expanded the MV approach since the MV model was first introduced. Guo et al. (2019) examined how investors' decisions were affected by background risk using the MV, mean-VaR, and mean-CVaR frameworks. Xu et al. (2016) applied constraints on the weights in the standard CVaR-based portfolio selection model. Cui et al. (2019) introduced a strategy for the multiperiod mean–CVaR portfolio selection model that is both time-consistent and self-coordinating.

Most of the literature discussed above focusses on offering different models for portfolio selection. However, portfolio performance assessment is also crucial in the

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financial market. To assess the relative performance of a particular group of decision-making units (DMUs), the DEA technology (CCR model), first introduced by Charnes et al. (1978) and later expanded by Banker et al. (1984), (BCC model) can manage multiple inputs and outputs simultaneously. As a result, it is widely utilised for performance assessment and portfolio selection. Chen et al. (2021) explored various DEA-based fuzzy portfolio evaluation models employing different risk measures. Gupta et al. (2020) formulated a comprehensive approach to streamlined portfolio evaluation, incorporating a dynamic rebalancing strategy. This empowers investors to construct optimal portfolios within a credibilistic framework. Zhou et al. (2018) introduced a DEA frontier improvement approach, offering investors a strategy for portfolio rebalancing. Traditional DEA models have a limitation—they only work with non-negative input and output values. However, in many cases, some inputs and/or outputs can be negative. Certain DEA models capable of handling negative data include those introduced by Cooper et al. (1999) known as Range Adjustment measure (RAM), Portela et al. (2004), known as Range Directional Measure (RDM), by Sharp et al. (2007) referred to as Modified Slack Based Measure (MSBM), and by Emrouznejad et al. (2010a). known as the SORM model. The radial DEA (BCC) model, under the VRS (variable return to scale) condition, offers an efficiency assessment for each DMU and accommodates negative values in inputs or outputs. However, it becomes inapplicable when negative values are present in both inputs and outputs.

To overcome the challenge of the weak discriminative power of DEA, a commonly used method is cross-efficiency evaluation, an extension to DEA introduced by Sexton (1986) which involves the utilisation of the weights chosen by each DMU to calculate its efficiency as a common set of weights for determining the efficiency of all other DMUs. This approach assesses the efficiency of each decision-making Unit (DMU) in two stages: self-evaluation and peer evaluation. Cross-efficiency assessment is commonly employed in the evaluation of production technologies exhibiting constant returns to scale, as negative efficiencies are not observed in such instances. In the case of variable returns to scale, de Mello et al. (2013) introduced the concept of constraining multiplier values to ensure only positive efficiencies across all DMUs. Wu et al. (2016) and Lim and Zhu (2015) represent notable methodologies for determining cross-efficiencies in the context of VRS production technology. Addressing limitations in prior research, Kao and Liu (2020) have introduced a slacks-based measure model for assessing cross-efficiency in DMUs, ensuring the absence of negative efficiencies. This model offers reasonable efficiencies for DMUs that exhibit only weak efficiency.

The literature ignores a cross-efficiency evaluation's inherent flaw. Negative values in inputs and outputs are not handled by the traditional cross-efficiency evaluation because it is built on the CCR model. Nonetheless, in many circumstances, inputs or outputs have negative values. There are not many studies in the literature that discuss cross-efficiency when dealing with negative data. Lim et al. (2014) presented the cross-efficiency form of the RAM model capable of handling negative data. Lin (2020) presented a DDF-based cross-efficiency evaluation method under the VRS technology framework that is based on the RDM model, presented by Portela et al. (2004) and the duality theory. Wu et al. (2009) present a BCC VRS cross-efficiency model that can handle the problem of negative cross-efficiency by adding a non-negativity restriction.

Due to the continuous advancements in the evaluation of cross-efficiency using DEA and its exceptional characteristics, this approach has found widespread application across various fields. Researchers have explored the utilisation of DEA cross-efficiency methods in the realm of portfolio selection. Lim et al. (2014), for instance, introduced a DEA mean-variance cross-efficiency model to enhance the effectiveness of portfolio selection. Wu (2016) applied DEA cross-efficiency within the context of asset/stock organisation and portfolio optimisation, while Lim et al. (2014) proposed an innovative method for portfolio selection on the Korean stock exchange using DEA cross-efficiency. Amin and Hajjami (2021) demonstrated the impact of alternative optimal solutions in constructing a cross-efficiency matrix and verified the improved performance of the mean-variance portfolio selection method. Deng et al. (2019) integrated DEA prospect cross-efficiency evaluation into the novel Mean-Variance-Maverick (MVM) framework for fuzzy portfolio selection. Chen et al. (2021) delved into a portfolio selection problem that incorporates fuzzy Data Envelopment Analysis cross-efficiency evaluation, considering both undesirable fuzzy inputs and outputs.

Emrouznejad et al. (2010a) employed a partitioning approach to model negative data, leading to the introduction of the SORM model for evaluating the performance of observed production units. The SORM model is designed to address situations where some DMUs exhibit positive values, while others display negative values for a variable. It is also applicable when DMUs have both negative inputs and outputs simultaneously. A distinctive feature setting the SORM model apart from other approaches like RDM and MSBM is its conversion of each input-output variable into the sum of two components – one representing its negative value and the other its positive value. This allows for a positive format without requiring changes of origin that might otherwise be necessary to obtain positive values.

The above-mentioned studies on cross-efficiency evaluation in DEA evaluate the DMU's performance through a comprehensive perspective derived from its peers (Wu 2016). The average column values of the cross-efficiency matrix (CEM) are commonly used to evaluate the relative performance of a given DMU using a two-stage DEA model. Additionally, it is believed that the row averages in the cross-efficiency matrix can also be utilised to measure the diversity of performance within the same evaluation system for the unit (Talluri et al. (2013).

This paper makes four theoretical and novel contributions.: (i) We proposed a novel cross-efficiency technique that can handle negative data based on an input-oriented SORM model. (ii) Specifically, we extended the SORM cross-efficiency analysis in DEA that uses row and column means for optimal portfolio selection. (iii) We aim to develop a novel return indicator in the portfolio area and present a multi-objective model for portfolio selection by considering the diversity index as a return and maverick as a risk. (iv) Furthermore, we covered the detailed sensitivity analysis by changing several realistic constraints such as cardinality and buy-in thresholds.

In this study, we broaden the SORM model into its cross-efficiency counterpart to gauge efficiency while accommodating variables that exhibit positive values for some DMUs and negative values for others. Here, we concurrently evaluate the performance diversity score (derived from the CEM row average) and the input-oriented SORM cross-efficiency score (based on the CEM column average). This approach provides a comprehensive method for identifying DMUs that excel not only in comparison to their peers, but also possess unique capabilities that differentiate them.

The remaining sections of the paper are organised as follows: In Section 2, we delve into the traditional input-oriented SORM cross-efficiency model. Section 3 introduces the proposed SORM cross-efficiency model, marking the initial stage of the proposed methodology. Section 4 outlines the second stage of the work, presenting a multi-objective portfolio selection model. The practical application of the proposed approach is illustrated in Section 5 to underscore its validity and effectiveness. Finally, Section 6 serves as the conclusion, where we analyse the results and suggest potential avenues for future research.

2. The input-oriented SORM model to deal with negative data

The traditional DEA models cannot be used directly to evaluate the efficiency of DMUs where some inputs and/or outputs may assume negative data. The semi-oriented radial measure (SORM) model was introduced by Emrouznejad et al. (2010a) to evaluate such DMUs. Without requiring any transformation, their method can be applied to negative data and produce an efficiency metric that is comparable to the radial measures in conventional DEA. We consider the input-oriented SORM model in dual form as described in Emrouznejad et al. (2010b). The fundamental concept of this approach involves replacing an input/output variable, which may have positive or negative data for different units, with the difference of two non-negative variables. The first part of this replacement involves exclusively positive values, while the second part incorporates the absolute values of the negative segment.

Let us assume there are n DMUs ($DMU_j, j = 1,2, \dots, n$) that are associated with m inputs x_{ij} ($i = 1,2 \dots m$) and s outputs $y_{rj}(r = 1,2..s)$. Also, let

$I = \{i \in (1,2 \dots m): x_{ij} \geq 0 \ \forall \ j = 1,2 \dots n \}$

$L = \{l \in (1,2 \dots m): j \in (1,2..,n) \ such \ that \ x_{lj} < 0 \}$ and

$R = \{r \in (1,2 \dots s): y_{rj} \geq 0 \ \forall \ j = 1,2 \dots n \}$

$K = \{k \in (1,2 \dots m): j \in (1,2..n) \ such \ that \ y_{kj} < 0 \}$

Emrouznejad et al. (2010a) define two output variables y_{kj}^1 and y_{kj}^2 such that

$$y_{kj}^1 = \begin{cases} y_{kj} & if \ \ y_{kj} \geq 0 \\ 0 & otherwise \end{cases} \qquad y_{kj}^2 = \begin{cases} -y_{kj} & if \ \ y_{kj} < 0 \\ 0 & otherwise \end{cases}$$

Similarly, two input variable x_{kj}^1 and x_{kj}^2 are defined as

$$x_{kj}^1 = \begin{cases} x_{kj} & if\ \ x_{kj} \geq 0 \\ 0 & otherwise \end{cases} \qquad x_{kj}^2 = \begin{cases} -x_{kj} & if\ \ x_{kj} < 0 \\ 0 & otherwise \end{cases}$$

The standard input-oriented SORM model is presented as follows:

Model I

$$e_{dd} = \max \sum_{r \in R} y_{rd} u_{rd} + \sum_{k \in K} y_{kd}^1 u_{kd}^1 - \sum_{k \in K} y_{kd}^2 u_{kd}^2 + u_0$$

$$s.t.\ \ -\sum_{i \in I} x_{ij} v_{id} - \sum_{l \in L} x_{lj}^1 v_{ld}^1 + \sum_{l \in L} x_{lj}^2 v_{ld}^2 + \sum_{r \in R} y_{rj} u_{rd} + \sum_{k \in K} y_{kj}^1\, u_{kd}^1 - \sum_{k \in K} y_{kj}^2 u_{kd}^2 + u_0 \leq 0$$

$$\sum_{i \in I} x_{id} v_{id} + \sum_{l \in L} x_{ld}^1 v_{ld}^1 - \sum_{l \in L} x_{ld}^2 v_{ld}^2 = 1$$

$$u_{rd} \geq 0, \forall\, r \in R,\ \ v_{id} \geq 0, \forall\, i \in I,\ \ u_{kd}^1, u_{kd}^2 \geq 0, \forall\, k \in K\ ,\ \ v_{ld}^1, v_{ld}^2 \geq 0, \forall\, l \in L$$

In Model I, the objective is to find a set of input-output weights that maximise the efficiency value of DMU_d. The optimal weights are referred as $u_{rd}^*, u_{kd}^{1*}, u_{kd}^{2*}, v_{id}^*, v_{ld}^{1*}, v_{ld}^{2*}\ and\ u_0^*$ and they determine the relative efficiency (self-efficiency) of DMU_d denoted as e_{dd}. The efficiency measure of the above model will only indicate the radial contraction of absolute input values. In the model I, the efficiency of DMU_j might become negative when assessed with DMU_d multipliers due to the free variable u_0^*. However, these negative efficiencies typically do not manifest in the results unless cross-evaluation models are utilised. Hence, we proposed a modified SORM model for cross-evaluation in the next section.

One of the best methods for choosing a portfolio is the DEA cross-efficiency method, since it examines stocks based on efficiency rather than absolute output and more fully rationalises stocks. The construction of the proposed method consists of two stages. Stage-I focuses with the evaluation of the efficiency of the stocks based on input-oriented SORM cross-efficiency model. Subsequently, the multi-objective portfolio selection model is formulated in Stage-II.

3. Stage 1-Proposed input-oriented SORM cross-efficiency model

As explained previously, the SORM model proves to be an efficient approach to address negative data in the input-output scenario. However, Model I is unsuitable for cross-efficiency due to its input orientation, which may result in negative cross-efficiencies, as noted by Lim and Zhou (2013). To overcome this limitation, we introduce an extension of the SORM model, referred to as Model II, specifically designed for cross-efficiency evaluation. This enhanced model effectively handles the issue of negative cross-efficiency as well as negative input output.

Model II

$$e_{\tilde{dd}} = \max \sum_{r\in R} y_{rd}u_{rd} + \sum_{k\in K} y^1_{kd}u^1_{kd} - \sum_{k\in K} y^2_{kd}u^2_{kd} + u_0$$

$$s.t. \ -\sum_{i\in I} x_{ij}v_{id} - \sum_{l\in L} x^1_{lj}v^1_{ld} + \sum_{l\in L} x^2_{lj}v^2_{ld} + \sum_{r\in R} y_{rj}u_{rd} + \sum_{k\in K} y^1_{kj}\, u^1_{kd} - \sum_{k\in K} y^2_{kj}u^2_{kd} + u_0 \leq 0$$

$$\sum_{r\in R} y_{rd}u_{rd} + \sum_{k\in K} y^1_{kd}u^1_{kd} - \sum_{k\in K} y^2_{kd}u^2_{kd} + u_0 \geq 0$$

$$\sum_{i\in I} x_{id}v_{id} + \sum_{l\in L} x^1_{ld}v^1_{ld} - \sum_{l\in L} x^2_{ld}v^2_{ld} = 1$$

$$u_{rd} \geq 0, \forall\, r \in R,\ \ v_{id} \geq 0, \forall\, i \in I,\ \ u^1_{kd}, u^2_{kd} \geq 0, \forall\, k \in K\ ,\ \ v^1_{ld}, v^2_{ld} \geq 0, \forall\, l \in L$$

The 2nd constraint of model II will ensure the non-negativity of the cross-efficiency values. The optimal weights of Model II are referred as $u^*_{rd}, u^{1*}_{kd}, u^{2*}_{kd}, v^*_{id}, v^{1*}_{ld}, v^{2*}_{ld}$ *and* u^*_0 and they determine the relative efficiency (self-efficiency) of DMU_d denoted as $e_{\tilde{dd}}$. In the above context of cross-efficiency evaluation, model II needs to be computed n times for each evaluated DMU_d to obtain the cross-efficiency values for all DMUs. Consequently, each DMU has both a self-efficiency value and $n-1$ peer-efficiency value. Let $e_{\tilde{dj}}$ is the cross-efficiency of DMUj obtained with the weights of DMU_d and defined as

$$e_{\tilde{dj}} = \frac{\sum_{r\in R} y_{rj}\, u^*_{rd} + \sum_{k\in K} y^1_{kj}u^{1*}_{kd} - \sum_{k\in K} y^2_{kj}u^{2*}_{kd} + u^*_0}{\sum_{i\in I} x_{ij}\, v^*_{id} + \sum_{l\in L} x^1_{lj}v^{1*}_{ld} - \sum_{l\in L} x^2_{lj}v^{2*}_{ld}} \quad (1)$$

A cross-efficiency matrix (CEM) has been obtained as $E = \left(e_{\tilde{dj}}\right), d, j = 1,2\ldots n$. Thus, the final cross-efficiency value of DMU_j (The column average of the CEM) is defined as $e^C_j = \frac{1}{n}\sum^n_{d=1} e_{\tilde{dj}}$. While the column average scores (averaged appraisal by peers) from the CEM offer valuable insights, our attention is also directed towards the row mean scores, representing the averaged appraisals of peers. Examining these rows mean scores enables us to understand how a DMU excels in comparison to other DMUs when utilising its optimal weights. The row

average of CEM is as follows: $e^r_j = \frac{1}{n}\sum^n_{d=1} e_{\tilde{jd}}$. The row averages in the CEM offer insights into how a specific DMU evaluates its peers. A lower row average in the CEM suggests that the DMU possesses distinctive capabilities, contributing to increased performance diversity within the group of DMUs.

Maverick Index and Diversity index

The results obtained from the CEM can be employed for assessing the maverick index, which contrasts the column average cross-efficiency score with the selfevaluated efficiency. The calculation of the maverick index is given as $M_j = \frac{e_j^{\sim} - e_j^c}{e_j^c}$. DMUs characterised by low Maverick index scores (high mean cross-efficiency scores) are recognised as effective performers overall.

The calculation of the diversity index follows a similar methodology to that of the maverick index. To determine the performance of a test DMU (stock), its self-efficiency score is contrasted with the row average cross-efficiency score of all other DMUs, using the weights specific to the test DMU. The diversity index is computed as follows: $\mathrm{D}_j = \frac{e_j^{\sim} - e_j^r}{e_j^r}$. The diversity index brings to the table a set of capabilities that sets it apart from its counterparts. A DMU that exhibits high self-efficiency and a robust diversity index is regarded as having unique resources, positioning it to outshine other DMUs and achieve superior performance.

4. Stage 2-A multi-objective framework of portfolio selection based on SORM cross-efficiency evaluation

The Lim et al. (2014) demonstrates that although the straightforward application of cross-efficiency evaluation in portfolio selection successfully considers the risk of changing the weights of specific DMUs chosen within a portfolio, it does not consider the risk for the portfolio. While inter-DMU risk is not considered, the straightforward application of cross-efficiency evaluation successfully lowers individual DMU risk. Thus, the SORM cross-efficiency approach alone can generate a moderately risk-consistent portfolio, particularly for individual risks. To solve this problem, we create the following multi-objective framework for portfolio selection based on cross-efficiency evaluation.

Let a portfolio Ω consist of n stocks with w_j $(j = 1,2 \dots n)$ proportion vector. For a given j^{th} stock the return and risk degree measure are defined as diversity index D_j and maverick index M_j respectively defined in section 3.

We maximise the weighted diversity index P_Ω (return) while minimise the maverick index (risk) I_Ω and define as $P_\Omega = \sum_{j=1}^{n} D_j w_j$ and $I_\Omega = \sum_{j=1}^{n} M_j w_j$ respectively. The optimal portfolio Ω^* cn be obtained using the following multi-objective portfolio selection model:

Model III

$$\begin{cases} Min\ I_{\Omega} = \sum_{j=1}^{n} M_j w_j \\ Max\ P_{\Omega} = \sum_{j=1}^{n} D_j w_j \\ \sum_{j=1}^{n} z_j \leq h \\ l_j z_j \leq w_j \leq \varepsilon_j z_j \\ \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0 \end{cases}$$

The terms l_j and ε_j correspond to the lower and upper bounds, respectively, representing a share of the capital budget available that can be invested in the j^{th} asset. The first constraints of Model III represent a cardinality constraint, imposing a limit on the number of assets (h) included in the portfolio. Investors can have varying opinions regarding the values of h, l_j and ε_j in this context.

In summary, the proposed method comprises two stages for portfolio selection. In the first stage, we apply model II and equation 1 to calculate the cross-efficiency (both row and column efficiency). In the second stage, we choose the portfolio by considering the maverick index for risk and the diversity index as return. A preferable portfolio exhibits a low maverick index for risk and a high diversity index.

5. An application to stock portfolio selection

To showcase the efficacy of the proposed two-stage approach in handling negative input-output scenarios, we have applied it to analyse authentic data sourced from the CSI 300 index. The evaluation of eleven specifically chosen stocks is conducted based on two inputs and seven outputs. The relevant financial input/output parameters are detailed in Table 1 for the selected nine parameters, and their standardised input output data are provided in Table 2.

Table 1. The Input-output parameters

Type	Parameter	Classification
Input	Solvency ratio-I (x1)	Total liability divided by total assets
	Solvency ratio-II (x2)	Total liability divided by shareholder's equity
Output	Return on equity (y1)	Net income divided by shareholders equity
	Return on assets (y2)	Net income divided by the total assets
	Net profit margin (y3)	Net income divided by revenue for the period
	Basic earnings per share (basic EPS) (y4)	Net income minus dividends and preferred stock dividend divided by common shares
	Revenue growth rate (y5)	Current period revenue divided by the previous period revenue minus one
	Net profit growth rate (y6)	Current period net income divided by the previous period net income minus one

Type	Parameter	Classification
	Basic earnings per share growth rate (y7)	Current period basic EPS divided by the previous period basic EPS minus one

Source: Sourced from Chen et al. (2021).

Table 2. The standardised input-output data with efficiency values

Stocks	x1	x2	y1	y2	y3	y4	y5	y6	y7
S_1	0.8135	0.3036	0.0855	0.0545	0.0189	0.0129	0.0031	0.0098	0.0290
S_2	0.4392	0.0936	0.6122	0.6862	0.1880	0.3365	0.0017	-0.0001	-0.0004
S_3	0.4655	0.105	0.3457	0.3495	0.2065	0.1054	0.0046	-0.0006	-0.0044
S_4	0.4481	0.0911	0.6088	0.6475	0.4377	0.3959	0.0133	0.0016	-0.0015
S_5	0.1235	0.0184	0.0384	0.0338	0.4418	0.0180	0.0063	0.0016	-0.0132
S_6	0.4384	0.0888	0.2430	0.2498	0.4978	0.2108	0.0061	0.0011	-0.0062
S_7	0.5197	0.1302	0.1559	0.1476	0.1948	0.0823	0.0193	-0.0026	-0.0059
S_8	0.8124	0.3278	0.0481	0.0292	0.0428	0.0090	0.0028	-0.0068	-0.0189
S_9	0.8275	0.3446	0.2248	0.1218	0.1296	0.0977	0.0051	-0.0005	-0.0014
S_{10}	0.3764	0.0758	0.4310	0.4924	0.2181	0.3332	0.0166	0.0034	-0.0058
S_{11}	0.4224	0.0835	0.5124	0.5424	0.4117	0.5116	0.0229	0.0097	-0.0012

Source: Sourced from Chen et al. (2021).

5.1 Results and discussion

Initially, we address model II to determine the self-efficiency of the chosen stocks. Subsequently, the optimal weights obtained from model II and equation (1) are utilised to assess the CEM, as presented in Table 3. Additionally, the column and row average efficiencies derived from the CEM are also outlined in Table 3. This marks the completion of the first stage, resulting in three types of efficiencies: self-efficiency, column average efficiency, and row average efficiency.

Table 3. Different efficiency values

Stocks	**SORM self-efficiency**	**Column average efficiency**	**Row Average efficiency**	**Stocks**	**SORM self-efficiency**	**Column average efficiency**	**Row Average efficiency**
S_1	1	0.2643	0.3084	S_7	0.7622	0.4096	0.6371
S_2	1	0.6254	0.4862	S_8	0.325	0.0825	0.6973
S_3	0.8387	0.5133	0.7436	S_9	0.4873	0.1630	0.7118
S_4	1	0.7637	0.5001	S_{10}	1	0.7796	0.5319
S_5	1	0.6588	0.2217	S_{11}	1	0.9023	0.3596
S_6	1	0.5994	0.5685				

Source: Sourced from MATLAB on solving Model II.

Table 4. The input-oriented SORM cross-efficiency matrix

Stocks	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
S_1	1.00	0.00	0.00	0.15	0.75	0.11	0.00	0.00	0.00	0.386	1.00
S_2	0.08	1.00	0.53	0.98	0.22	0.40	0.22	0.04	0.20	0.82	0.87
S_3	0.43	0.87	0.84	0.94	1.00	0.85	0.72	0.20	0.33	1.00	1.00
S_4	0.06	0.99	0.48	1.00	0.22	0.44	0.21	0.04	0.18	0.88	1.00
S_5	0.01	0.12	0.12	0.28	1.00	0.32	0.11	0.02	0.04	0.16	0.27
S_6	0.01	0.46	0.55	0.95	1.00	1.00	0.48	0.04	0.00	0.77	1.00
S_7	0.23	0.50	0.62	0.79	1.00	0.68	0.76	0.24	0.18	1.00	1.00
S_8	0.31	0.70	0.81	0.84	1.00	0.86	0.72	0.32	0.21	1.00	0.89
S_9	0.54	0.99	0.75	0.93	1.00	0.70	0.60	0.00	0.49	0.84	1.00
S_{10}	0.24	0.62	0.77	0.84	0.00	0.86	0.57	0.00	0.07	1.00	0.89
S_{11}	0.00	0.63	0.17	0.73	0.06	0.39	0.12	0.00	0.09	0.72	1.00

Source: Sourced from MATLAB on solving Model II and equation 1.

In stage 2, we solve the proposed multi-objective portfolio selection model (model III). The stock selection process through the proposed multi-objective model possesses distinctive resources and strengths, enabling it to differentiate or align with the performance of other assets. As a result, the assets chosen through this approach contribute to the formation of a portfolio with enduring stability in performance over the long term.

The solution of a multi-objective portfolio selection model is addressed by employing a genetic algorithm with default parameters in MATLAB 2022a. The implementation of the genetic algorithm resulted in a set of Pareto optimal solutions for model III. In each run of the GA, a collection of optimal Pareto solutions is generated, allowing the decision maker to choose a preferred portfolio based on their individual preferences. For example, in the model proposed herein, investors can evaluate the portfolios produced, weighing the trade-offs between

portfolio return rate and risk. They can then opt for the portfolio that offers the highest return rate (diversity index) while maintaining an acceptable level of risk. In this study, the portfolio that exhibits the highest return rate and the lowest risk is specifically chosen. Decision-makers can then choose a portfolio based on their individual preferences from this diverse set of portfolios.

To solve model III, we set the cardinality constraint h=5 while vary the values of l_j and ε_j.Within the set of Pareto solutions obtained, we choose the portfolio with the highest diversity index and the lowest maverick index. The results of both scenarios are presented in Table 5.

Table 5. The weights of the stocks in the selected portfolio for h=5

Stock	$\boldsymbol{l_j = 0.05,}$ $\boldsymbol{\varepsilon_j = 0.5}$		$\boldsymbol{l_j = 0.1}$, $\boldsymbol{\varepsilon_j = 0.4}$		$\boldsymbol{l_j = 0.06}$, $\boldsymbol{\varepsilon_j = 0.3}$	
S_1	0.35	0	0.3	0	0.3	0
S_2	0.05	0.05	0	0.1	0.1023	0.06
S_3	0	0	0	0.1	0	0
S_4	0.05	0.35	0.1	0.3	0.2377	0.28
S_5	0.5	0.05	0.4	0.1	0.3	0.3

Stock	$\boldsymbol{l_j = 0.05}$, $\boldsymbol{\varepsilon_j = 0.5}$		$\boldsymbol{l_j = 0.1}$, $\boldsymbol{\varepsilon_j = 0.4}$		$\boldsymbol{l_j = 0.06}$, $\boldsymbol{\varepsilon_j = 0.3}$	
S_6	0.05	0.05	0.1	0	0.06	0.06
S_7	0	0	0	0	0	0
S_8	0	0	0	0	0	0
S_9	0	0	0	0	0	0
S_{10}	0	0.5	0.1	0.4	0	0.3
S_{11}	0	0	0	0	0	0
Return	2.68(max)	1.056	2.3(max)	1.1215	2.2(max)	1.706
Risk	1.318	0.34(min)	1.1681	0.41(min)	1.1652	0.40(min)

Source: Sourced from MATLAB on solving model III.

Table 6. The statistics from obtained pareto solutions for h=5

Criteria		$l_j = 0.05$, $\varepsilon_j = 0.5$	$l_j = 0.1$, $\varepsilon_j = 0.4$	$l_j = 0.06$, $\varepsilon_j = 0.3$
return	Max	2.6813	2.3411	2.1174
	Min	2.0744	1.121	1.7063
	Average	1.0563	2.083	1.9203
Risk	Max	1.3118	1.1681	1.1652
	Min	0.3389	0.4124	0.4028
	Average	0.6761	0.7909	0.7905

Source: Sourced from MATLAB on solving model III.

Additionally, the statistics of the obtained pareto solutions in each case are presented in Table 6. Moreover, by adjusting the cardinality constraints, one can obtain more diversified portfolios.

Comparative study: In the literature, few studies address cross-evaluation in the presence of negative input-output scenarios**.** The approach proposed by Lin (2020) is non-oriented and offers self-evaluations and evaluations by peers for DMUs, simultaneously measuring inefficiencies in both inputs and outputs. While the proposed SORM model provides the radial efficiency, and cross-efficiency, which reflect radial contraction only of absolute input values, hence the direct comparison of the results obtained by these two methods is not possible.

The input-oriented BCC VRS cross-efficiency model by Wu et al. (2009) can also be utilised when negative values are present exclusively in either inputs or outputs, but it is not suitable for situations where negative values occur in both inputs and outputs. But our proposed method can be applied in such cases effectively. The detailed analysis presented in Table 7 further compares the proposed work with the existing literature available.

Table 7. Analysis of proposed work and existing work in the literature

Characteristic	**Cross-evaluation**	**orientation**	**Limitation**
Cooper et al. (1999) (RAM)	Yes	non-oriented	Resulting in the most distant targets on the frontier for

Characteristic	Cross-evaluation	orientation	Limitation
		(measure inefficiency)	inefficient DMUs and failing to offer a genuine measure of efficiency.
Wu et al. (2009)	Yes	Input/output oriented (Measure efficiency)	Not valid when both inputs and outputs exhibit negative values.
Lin (2020) (DDF based RDM)	Yes	non-oriented (measures inefficiency)	The efficiency score fails to accurately represent the actual performance of the DMU because it cannot encompass all the sources of inefficiency, as pointed out in Portela et al. (2004).
Sharp et al. (2007) (MSBM)	No cross-evaluation exist	Non-oriented	Applied in case of natural negative input output cases only
Proposed work	Yes	Input-oriented/ measure efficiency	Applicable when there is at least one input with a positive value, with no conditions imposed on any of the outputs. (Emrouznejad et al. (2010b).)

Source: The table is compiled from an extensive survey of the literature.

To the best of our knowledge and following an extensive examination of the literature, there is currently no available method which takes the diversity index as a return indicator while selecting the optimal portfolio, so direct comparison of the proposed method with existing methods is not possible.

6. Conclusions

This study introduces two significant novel contributions within the proposed method. Firstly, the SORM model introduced can be viewed as enhancements and extensions to the DEA cross-efficiency evaluation. Additionally, this study marks the application of the SORM cross-efficiency approach to portfolio management, positioning it as a promising tool for the evaluation of financial assets. This study takes a unique approach to the problem, introducing an integrated multi-objective framework that surpasses the conventional mean-variance method. It introduces an additional layer of sophistication by melding a risk measure derived from the maverick index, computed through column average cross-efficiency, with a return component based on the diversity index, determined through row average cross-efficiency. Moreover, the proposed portfolio selection model adeptly addresses real-world constraints such as maximum and minimum capital fractions, as well as cardinal constraints, offering substantial advantages in portfolio management. The numerical findings indicate that the proposed two-stage method offers a mechanism for decision-makers to achieve a more balanced and rationalised portfolio diversification.

The proposed work has the following limitations and possible future directions. First off, this study only looked at the input-oriented SORM cross-efficiency. Including the output-oriented SORM cross-efficiency would be beneficial to increase the significance of efficiency comparisons. The impact of fuzzy input-output data could be considered in future research by expanding the modelling scope to include ordinal and interval data values.

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