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Valuation of Intangible Assets: The Value of Voluntary Certifications

Abstract. *Among the intangible assets acquired by companies, voluntary certifications have gained increasing importance. The most notable of these is the certification of quality management systems, but demands by citizens and social groups have brought about the introduction of other voluntary certifications in diverse areas such as the environment or corporate social responsibility. In this paper, we present a tool to facilitate their valuation. The methodology proposed is based on discounted cash flows in fuzzy environments. However, given its voluntary nature, one of the features of certification is that every organisation can choose to discontinue if they wish to do so. This option to discontinue certification will be valued using fuzzy real options. In order to calibrate the real possibility of a company opting to discontinue, expert opinion will be included and aggregated by using Ordered Weighted Averaging (OWA). Finally, the proposed valuation model will be applied to a case study.*

Keywords: *intangible assets, fuzzy logic, business valuation, voluntary certifications, case study*.

JEL Classification: C02, M10, M49.

1. Introduction

One of the main problems involved in valuing a production unit is quantifying the value of its intangible assets. However, identifying an intangible asset can also be problematic at times. In line with Itami (1987) and from a perspective of human, structural, and relational capital, Capcha (2003) defines intangible resources as assets that lack physical substance because they are based on information and knowledge, which makes it difficult to identify and quantify them.

Belkaoui (1992) identifies two basic types of intangible assets: those that are easily identifiable, such as patents, and those that are difficult to identify. Among the latter, an item that has recently gained greater relevance is recognition from different certifications that companies decide to adopt voluntarily. The most notable of these is the certification of quality management systems according to the ISO 9001 standard. In fact, since its inception in 1987, the ISO 9000 series has spread profusely, and despite its voluntary nature, it is assuredly the most widely known

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and practiced set of standards worldwide. Nevertheless, demands by citizens and social groups have brought about the introduction of other voluntary certifications in diverse areas such as the environment, corporate social responsibility, or occupational risks.

This paper therefore provides an innovative methodology that is able to approximate the value that voluntary certifications generates for companies. On the one hand, it will be possible to obtain an objective and verifiable value of these intangible elements, providing a judgement element to establish their profitability. On the other hand, however, certification not only represents a portion of business value, but its voluntary nature means it can be discontinued if a company considers it necessary, which per se contributes additional value.

The quintessential valuation model is one based on a company's expected discounted cash flows. However, despite its wide applicability, authors like Mascareñas et al. (2004) point out that this methodology is insufficient when valuing contingent projects that have operational flexibility and high volatility, so for these situations incorporating the use of real options is particularly recommended. A real option gives the right, but not the obligation, to undertake a defined action at a fixed price and over a determined period.

However, using real options does not solve the uncertainty and subjectivity problem that underlies any valuation process. The estimation of cash flows, in particular, is quite imprecise, so fuzzy mathematics is very useful, and it is one of the key contributions of our model. Fuzzy logic is an especially useful tool when working in situations that are highly subjective (Zadeh, 1975; 1995). It is particularly suited because it is able to model natural language through a set of rules (Mamdani & Assilian, 1975). In this respect, Yao, Chen and Lin (2005) extended the discounted cash flow model by developing a fuzzy logic system that takes into account both uncertain cash flows and imprecise discount rate with high uncertainty. In addition, Tsao (2012) presents a set of algorithms to calculate current net value in situations where estimations are imprecise.

In this sense, Collan et al. (2009a; 2009b) introduced the fuzzy payoff method to evaluate real options using triangular fuzzy numbers (TFNs). This method is based on the valuation of options by approximation as applied to a Boeing Corporation project (Datar & Mathews, 2007). Although the initial valuation of the options was made using stochastic procedures, some authors argue that when management's actions can affect value, this is not a simple random variable (Kinnunen &Valuing, 2010). Real options valued by fuzzy approximation using the binomial method was introduced by Liao & Ho (2010). Collan & Heikkila (2011) value a patent with this methodology based on the pay-off method; Wang (2011) proposes the binomial method for valuing options in a fuzzy environment, and You et al. (2012) use real options to value an Enterprise Resource Planning Investment. The binomial method for valuing real options is the most used because it is the most intuitive and easiest for managers to understand (Macareñas et al.,2004).

Finally, the methodology proposed would be incomplete if it was not able to formally incorporate the fact that although certification is voluntary, external factors can condition the possibility of discontinuation. So as to calibrate this possibility, the proposed model facilitates obtaining a discontinuation coefficient. To do so, expertise techniques and Ordered Weighted Averaging (OWA) operators, specifically Power OWA, are used. In this way, it will be possible to combine the subjective opinions, whereby the analysis performed to have greater consistency and objectivity. Expertise theory, resulting from the studies by Kaufman (1987), has been used by authors such as Sirbiladze et al. (2014) or Brotons & Sansalvador (2018), among others. The OWA operator was introduced by Yager (1988) and its extensions have been used in a variety of applications (see examples in the following: Yager, 1996; Liu, 2008; Merigó & Casanovas, 2011; Mendoza-Carbajal et al. 2023; Flores-Sosa et al., M. 2022).

Taking the above-mentioned studies as our starting point, we will develop an innovative methodology that enables the valuation of certifications voluntarily adopted by a company, thereby improving and completing the business valuation processes that a company may be immersed in. Subsequently, this tool will be applied to a case study.

2. Proposed methodology

2.1 Introduction to the methodology: general expression to be used.

The company is valued through the discounted free cash flows method. First, net business turnover must be estimated for the coming financial years, and to do so, growth rates must be estimated for the projected timeline, where having voluntary certification or not must be considered, as well as for the residual period. This will allow one to project profit and loss for the coming financial years and obtain free cash flows. The discount rate to be applied is determined and business value is obtained from the following expression:

$$V = \sum_{t=1}^{T} CF_1 \cdot \left(1+f\right)^{t-1} \left(1+i\right)^{-t} + CF_T \left(1+g\right)\left(1+i\right)^{-T} \left(i-g\right)^{-1} \tag{1}$$

Where T is the time line, t the year, i the discount rate used, CF estimated cash flow for the first year, f estimated growth rate of cash flow in the timeline and g growth rate of cash flow in the residual period.

2.2 Estimation of growth rates

a. Growth rate in the timeline

We are aware of the uncertainty and subjectivity inherent in the valuation process of intangible resources such as the value that voluntary certification generates for companies. In this respect, fuzzy logic is especially suitable since it permits the processing of the information provided, not in specific terms, but by incorporating existing ambiguity and uncertainty into the model. Consequently, the growth rate for the timeline is estimated by using triangular fuzzy numbers (TFNs), which include both business information and that of the corresponding sector.

A triangular fuzzy number can be defined by three values $\tilde{A}=(b,a,c)$, where a is the central value, and *b* and *c* are the left and right extremes, respectively. The belonging function $\mu(x)$ is defined as $\mu(x)=(x-b)/(a-b)$ si $b<x<a$; 1, if $x=a$; $(c-x)/(c-a)$ if $a<x<c$ and 0 otherwise. The use of TFNs not only contemplates extreme cases (the best and worst situation) and the intermediate situation, but also the whole range of values that they encompass.

At this point, two different scenarios must be considered: on the one hand, the real situation where a company has voluntary certification; and on the other hand, the hypothetical situation that would have occurred if the company had not adopted such a certification.

a1. A company's growth rate considering they have voluntary certification.

The average growth rates of the certified companies in the sector for the last few years are ordered from the highest to the lowest, and the TFN $\tilde{C}$ (where lower and upper extremes are the minimum and maximum growth rate of these companies and median is the central value) is also created. The range of this TFN is usually excessively large, so the minimum and maximum of α-cut 0.25 is taken. Generally, α-cut of a TFN is defined as $C_\alpha=\{x\in X\ /\ \mu(x)\geq\alpha\}$.

Based on the information about the company and the sector (through α-cut 0.25), it is possible to construct the TFN growth rate of the certified company in the timeline ($\tilde{f}$):

$$\tilde{f}=\left(min\left(x\in C_{0,25}\right),a,max\left(x\in C_{0,25}\right)\right) \tag{2}$$

Where expected business growth rate for the central value of the TFN (*a*) is taken as being that of the last few years; the lower extreme of the TFN, $min\left(x\in C_{0,25}\right)$, corresponds to the lower extreme of the growth rate for the sector that has obtained certification, adopting a pessimistic position and assuming that behaviour is similar to the worst certified companies in the sector; and the upper extreme of the TFN, $max\left(x\in C_{0,25}\right)$, corresponds to the higher growth rate of the sector that has obtained certification; that is to say, an optimistic scenario is assumed, whereby the company will behave in a similar way to the best certified companies in the sector.

a2. Growth rate of the company considering they do not have voluntary certification

As before, the average growth rates of the companies in the sector for the years prior to the company obtaining certification are ordered from highest to lowest, but this time considering those who have chosen not to be certified. Likewise, α-cut 0.25 is used.

However, we are aware that the economic situation could have changed since the company obtained the certificate. This circumstance can be measured by the growth rate of the companies in the sector, particularly through the median of non-certified companies. Hence, we will call h the difference between the mean growth rate of the non-certified sector on the valuation date and the mean growth rate of the non-certified sector in the year prior to the company obtaining it. To construct the TFN growth rate of the non-certified company in the timeline, the following three items are taken:

i) central value: $a+h$, that is, the say business growth rate in the years before obtaining certification plus the difference h;

ii) lower extreme: $min\left(x\in C_{N\ 0,25}\right)+h$, that is to say the lower extreme of the non-certified sector's growth rate, plus the difference h,

iii) the upper extreme: $max\left(x\in C_{N\ 0,25}\right)+h$, that is to say the upper extreme of the growth rate of the sector without certification plus the difference h. In summary, growth rate of the non-certified company in the timeline is:

$$\tilde{f}_N=\left(min\left(x\in C_{N\ 0,25}\right)+h,a+h,max\left(x\in C_{N\ 0,25}\right)+h\right) \tag{3}$$

b. Growth rate in the residual period

This period includes the rest of the company's life, which is assumed to be perpetual. The growth rate considered in the residual period g is 50% of the GDP growth rate, under the assumption of having and not having certification.

2.3 Estimation of the costs of obtaining and maintaining certification.

Obtaining and maintaining certification will foreseeably require some investments in human and material resources, which, at least in the short term, will lead to an increase in costs. In determining the costs incurred by certification (cc), it is possible to distinguish two components. i) The cost invoiced by the certifying company. Although the costs are set out in the contract and are well known, the amount can vary from one year to the next according to whether an audit is required for the renewal of the certificate or not. ii) Internal costs. These consist of the costs of internal staff involved in obtaining and maintaining the certificate, as well as the costs incurred from equipment and/or materials used in the process.

2.4 Profit and loss projections

To make a profit and loss projection, it is necessary to take into account that a company that decides not to adopt certification will save on the costs of obtaining and maintaining it (cc), so they must be deducted from the projections made.

In this way, if CF_1 is the free cash flows for the first year, estimated under the hypothesis of certification, $CF_1^* = CF_1 - (-cc)$ will be the estimates assuming the company had not obtained this certification, and consequently would not have incurred costs from obtaining and maintaining certification (cc).

2.5 Estimation of the value of certification

An approximation of the value of certification is obtained as the difference between the value of the company having and not having certification. In the following expression, the terms Y / N refer to the value of the corresponding variables according to whether they have (Y) or do not have (N) certification

$$\tilde{V}_{Y/N} = \sum_{t=1}^{T} CF_1^{Y/N} \cdot \left(1+\tilde{f}_{Y/N}\right)^{t-1} \left(1+i\right)^{-t} + CF_T^{S/N} \left(1+g\right)\left(1+i\right)^{-T} \left(i-g\right)^{-1} \quad (4)$$

However, the results obtained according to the above expressions are fuzzy numbers. Terceño et al. (1995a; 1995b; 1997) and Perrone and La Diega (1999) demonstrated that the expressions of current value of a set of fuzzy equities can be correctly approximated with a TFN. Nevertheless, if we want to work with exact values, they will have to be defuzzified, which can be obtained from expression (5) where $Sop\left(\tilde{V}\right)$ is the support of the cited fuzzy number, that is to say, the set of values with positive belonging function:

$$V = \int_{Sop(\tilde{V})} x \cdot \mu_{\tilde{V}}\left(x\right)dx \Big/ \int_{Sop(\tilde{V})} \mu_{\tilde{V}}\left(x\right)dx \quad (5)$$

For this reason, the value that voluntary certification (V_C) generates for the company is the difference between the business value obtained by assuming that it has obtained certification, V_Y , and the value under the assumption that it has not V_N . Evidently, we assume that voluntary certification is the only differential factor between both values. In the event of considering other possible differential factors, management should weigh up the importance of certification against the other factors, which would increase the uncertainty and subjectivity of the process and would therefore justify the use of fuzzy mathematics even more so. Finally, $c = 1 - V_N / V_Y$ is the profit that certification generates for the company expressed per unit $c = 1 - V_N / V_Y$ c, logically, without yet considering the discontinuation option.

2.6 Estimation of the the value of the discontinuation option

In Section 2.5, a first approximation of the value of certification was obtained. However, the value obtained in this way does not take into account the operational flexibility that allows management to discontinue certification if it considers that the benefits it generates are lower than the costs. To value opting to discontinue certification, we propose the following methodology:

a) Estimation of the volatility of the business value obtained.

The methodology proposed by Haahtela (2007) will be followed to estimate volatility. This author defines the variable z as the logarithm of a division where the numerator is the sum of business value in a defined period plus free cash flow generated in this period, and the denominator is business value in the period before. Volatility is the standard deviation of this variable.

b) The diffusion process of the value of a certified company

Given that TFNs are used, one binominal tree is constructed for the lower extreme, another for the upper extreme, and a third one for the central value. In all three cases, the rest of the columns are obtained from the initial valuation (V) multiplied by the ascending ($u = e^{\sigma\sqrt{t/n}}$) and descending coefficients ($d = e^{-\sigma\sqrt{t/n}}$); that is to say, the coefficients by which the business value must be multiplied in one period to obtain the company's value for the following period, where σ is the previously defined volatility, t is the number of years until expiry and n is the number of periods that t is subdivided into. In this way, different business values are obtained for the different periods according to the ascending and descending coefficients. Since it is a TFN, these values must be obtained for the lower, middle, and upper extremes of business value.

c) Value of opting to discontinue certification

When the time comes to renew certification, the company must choose between: discontinuing certification, thereby saving on costs (cc) but losing $c \cdot 100\%$ of its value; or continuing, whereby it will have to meet these costs. In this way, the value of opting to discontinue in the case of two consecutive ascents is obtained as the maximum between business value minus certification costs and business value if it did not have certification $C_{uu} = max\left(V_{uu} - cc; V_{uu}\left(1-c\right)\right)$ (similarly C_{ud} and C_{dd} will be obtained). The value of the option at the end of the first period results from the one obtained at the end of the second period multiplied by the implicit probabilities of ascent (p) and of descent ($1-p$) for the business value of a certified company, where p is the implicit probability of ascent, $p = \left(1 + r_f - d\right) / \left(u - d\right)$, r_f being the type of risk-free interest, $C_u = p \cdot C_{uu} + \left(1-p\right) \cdot C_{ud}$, y $C_d = p \cdot C_{ud} + \left(1-p\right) \cdot C_{dd}$. Finally, the value of the discontinuation option is obtained from the values of the option towards the end of the first period multiplied by the implicit probabilities of ascent and descent, $C = p \cdot C_u + \left(1-p\right) \cdot C_d$

Therefore, the lower extreme of the discontinuation option (C_b) is obtained if based on the minimum values of the TFN business value ($\tilde{V}$), a central value (C_a) is obtained if based on the central value of a certified company, and the upper extreme

(C_c) is obtained if based on the upper extreme of the value of a certified company. The value obtained in this way can be considered a TFN: $\tilde{C} = \left(C_b, C_a, C_c\right)$. This TFN can be defuzzified according to (5) so as to obtain the Crisp or defuzzified value of the discontinuation option (C_{oa}), which indicates the added value that the option to discontinue the corresponding certification generates if the situation is adverse for the company.

2.7 Final value of the certificate

As can be seen, the value of certification increases as a consequence of the discontinuation option inherent in the voluntary nature of certification, this being a component that is commonly ignored. Therefore, the final value of the certificate (V_{FC}), or in other words, the final contribution of the certificate to the business valuation is really the sum of two components: the value that this certification generates at the beginning (V_C), and the value generated by being able to freely opt to maintain certification or not (C_{oa}), $V_{FC} = V_C + C_{oa}$

However, not all companies can make use of the discontinuation option in the same way. On occasions, the regulations for carrying out certain activities or submitting certain tenders forces them to adopt this certification. In other cases, although this certification is not obligatory, in practice, companies that do not have it will be less competitive in the market. Hence, a group of experts are asked to value the visibility of this discontinuation option so as to estimate a discontinuation coefficient that will take values between 0 and 1. The aggregator Ordered Weighted Averaging (OWA) is proposed to summarise the values given by the experts, thereby increasing the objectivity of the results. Among the different OWAs, Power OWAs (POWA) are recommended, as they combine the flexibility of OWAs with the possibility of considering the measure of confidence each expert merits.

An OWA operator (Yager, 1988) of dimension n is an application $F_{OWA} : \mathbb{R}^n \to \mathbb{R}$ that has a corresponding weighting vector $W = \left[\omega_1, \omega_2, ..., \omega_n\right]$ such that $\omega_i \in \left[0,1\right]$ y $\sum_{i=1}^{n} \omega_i = 1$ defined as:

$$F_{OWA}\left(a_1, a_2, ..., a_n\right) = \sum_{i=1}^{n} \omega_i a_{\sigma(i)} \tag{6}$$

Where $a_{\sigma(i)}$ is the value of argument a_i in non-decreasing order, being $a_{\sigma(i)} \geq a_{\sigma(i+1)}$. The essence of OWAs (Yager, 1988) is the reordering of the elements a_i, which associates the arguments with the weightings. The OWA operator is a nonlinear function of elements, since it involves an ordering process. It has commutative, monotonic, bounded, and idempotent properties.

A POWA operator is a function $F_{POWA}: \mathbb{R}^n \to \mathbb{R}$ of dimension n if it has an associated weighting vector W, with $\sum_{j=1}^{n} \omega_j = 1$ y $\omega_j \in [0,1]$ and a probability vector V, with $\sum_{i=1}^{n} \upsilon_i = 1$ y $\upsilon_i \in [0,1]$, such that:

$$POWA(a_1, a_2, ..., a_n) = \beta \sum_{j=1}^{n} \omega_j a_{\sigma(i)} + (1-\beta) \sum_{i=1}^{n} \upsilon_i a_i \quad (7)$$

where $a_{\sigma(i)}$ is the value of argument a_i in non-decreasing order, being $a_{\sigma(i)} \geq a_{\sigma(i+1)}$ y $\beta \in [0,1]$

In conclusion, to obtain the final value of the certificate (V_{FC}), the value that this certificate generates at first (V_C) will be added to the value of the discontinuation option (C_{oa}) multiplied by the discontinuation coefficient $POWA(a_1, a_2, ..., a_n)$, which depends on the values provided by the experts $a_1, a_2, ..., a_n$: $V_{FC} = V_C + POWA(a_1, a_2, ..., a_n) \cdot C_{oa}$

3. Case study

This research is completed by applying the proposed methodology to a case study. To do so, a SME, dedicated to manufacturing, mounting, and commercialising machines for the chemical industry, analysis centres, and all types of laboratories, was selected. This company had implemented and certified its quality management system in accordance with the ISO 9001 standard in 2012.

In this context, the aim is to estimate the value generated in 2020 by having voluntary certification, in this case, the quality certificate ISO 900, taking into account the value of the discontinuation option among other valuation components. To be exact, a small company was chosen to validate the proposed methodology because, as pointed out by other authors like Mears & Voehl (1995), Feng et al. (2008) or Climent (2005), small companies frequently obtain worse results for ISO 9001 certification than larger companies due to an inadequate implementation of the standard, whose content in many cases does not become duly incorporated by most companies. Table 1 shows the main financial magnitudes for the target company from 2008 to 2019.

Table 1. Financial data of the company from 2008 to 2019 (in thousands of euros)

Year	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008
Income	1,400	1,188	1,113	1,328	1,585	1,397	1,768	1,721	1,325	1,174	1,468	956
Materials	579	494	467	507	661	585	795	813	555	390	644	392
Staff	285	271	350	267	335	387	353	342	339	522	551	520
Amortization	15	27	18	10	9	5	12	8	8	3	3	3

Year	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008
Financial exp.	4	4	3	1	0	0	2	1	2	28	1	1
Other exp.	432	436	375	537	455	351	458	419	280	94	95	-52
Results	87	-44	-101	6	126	69	148	139	141	138	173	92
Cash Flow	25	46	6	34	65	95	57	71	38	69	21	23

Source: Own source.

Table 2 shows the different growth rates for the timeline and the residual period. The TFNs company's estimated sales for 2020 to 2025 have been constructed considering the company as certified and non-certified. It also shows the upper and lower extremes and the central value of these TFNs.

Table 2. Growth rates for sales in the timeline (TL) and those of cash flow in the residual period (RP), and TFN estimation of the company's net sales with and without certification (in thousands of euros) for 2020 to 2025

Year	Growth rate sales TL (%)	Growth rate CF RP (%)	2020	2021	2022	2023	2024	2025
With certification								
Upper extreme	4.92	1.39	1,469	1,542	1,617	1,697	1,781	1,804
Central value	-4.71	1.44	1,334	1,272	1,212	1,155	1,100	1,115
Lower extreme	-12.40	1.52	1,227	1,075	941	825	722	732
Without certification								
Upper extreme	-1.69	1.42	1,377	1,354	1,331	1,308	1,286	1,303
Central value	-10.13	1.49	1,258	1,131	1,016	913	821	832
Lower extreme	-17.03	1.60	1,162	964	800	663	550	558

Source: Own source.

The cash flows for each of the abovementioned extremes are obtained from the information in Table 2 under the assumptions with and without certification. Table 3 outlines the profit and loss projection in its central value assuming the company has the ISO 9001 certificate. The summary of cash flows obtained for each of the assumptions established is shown in Table 4.

Table 3. Profit and loss projection assuming the company has certification (thousands of euros)

	2020	2021	2022	2023	2024	2025
Net Sales	1,334	1,272	1,212	1,155	1,100	1,115
Materiales	569	543	518	494	471	477
Staff expenses	432	413	395	377	360	365
Allowance fixed assets depreciation	37	35	34	32	31	31
Other expenses	157	148	140	133	126	128
Financial result	-5	-5	-5	-5	-5	-5

	2020	2021	2022	2023	2024	2025
Result before tax	135	128	121	114	108	109
Corporate tax	34	32	30	29	27	27
Financial year profits	101	96	91	86	81	82
Cash Flow	138	131	124	118	112	113

Source: Own source.

Table 4. TFN cash Flow estimation for company with and without certification (thousands of euros)

Year	2020	2021	2022	2023	2024	2025
With certification						
Upper extreme	153	161	170	179	188	190
Central value	138	131	124	118	112	113
Lower extreme	126	109	94	81	70	71
Without certification						
Upper extreme	143	140	138	135	133	134
Central value	129	115	102	91	81	82
Lower extreme	119	97	78	63	50	51

Source: Own source.

The business value TFNs (in euros) are for the company with a quality certificate (650,507; 1,241,478; 2,173,664) and without a certificate (519,794; 955,823; 1,612,539). By defuzzifying these TFNs, the business value obtained with certification is 1,298,347 € and 992,604 € under the assumption of not having this certificate. The difference between both amounts gives a first approximation of the value that the voluntary certificate generates for the company in principle (V_C), in this case, 305,743 €. However, this value does not consider the value generated from being able to choose to maintain certification or not (C_{oa}).

Before valuing the discontinuation option, the following have been considered. On the one hand, the increase in costs incurred (correctly updated) from certification, obtaining the TFN certification costs (60,371; 65,184; 71,731) whose defuzzified value is 65,473 €. On the other hand, if the company were to have to discontinue certification, it would lose 23.55%.

From this information, the binomial tree is obtained for the current value of the certified company, (current net value of the company plus savings in costs), assuming the discontinuation option arises after a year. To generate this tree, a volatility of 0.66 has been obtained, whereby we obtain ascending and descending coefficients of 1.200 and 0.827 respectively, having considered monthly increases. This process is carried out for each of the extremes of the TFN current business value.

To obtain the binomial tree of the certified company's current net value with the discontinuation option, the last column of Table 5 is obtained by considering the values in Table 2. The rest of the columns are obtained by applying implicit ascending and descending probabilities.

Table 5. Table 5 of the certified company's current net value with the certification discontiuing option in thousands of euros (upper extreme) (thousands of euros)

2.3	2.7	3.3	4.0	4.9	5.9	7.1	8.6	10.4	12.5	15.1	18.2
	1.9	2.3	2.7	3.3	4.0	4.8	5.8	7.1	8.5	10.3	12.4
		1.5	1.8	2.2	2.7	3.3	4.0	4.8	5..	7.0	8.5
			1.2	1.5	1.8	2.2	2.7	3.3	3.9	4.8	5.8
				1.0	1.2	1.5	1.8	2.2	2.7	3.2	3.9
					0.8	1.0	1.2	1.5	1.8	2.2	2.6
						0.7	0.8	1.0	1.2	1.5	1.8
							0.5	0.7	0.8	1.0	1.2
								0.4	0.5	0.6	0.8
									0.3	0.4	0.5
										0.3	0.3
											0.2

Source: Own source.

If we subtract the values of the certified company (2,245,395 €) from the current value of the company with the discontinuation option (2,267,238 €) and add the cost of certification (71,731 €), the value of the discontinuation option in the upper extreme is obtained (93,574 €). If we repeat this process for the lower extreme and for the central value, we can obtain the TFN in euros (35,164; 57,574; 93,574), which can approximate the value of the discontinuation option, whose defuzzified value is 59,839 €.

Finally, we asked for the opinion of eight experts to determine the discontinuation coefficient. Table 6 shows the values given by the experts, the level of confidence they merit, the weights considered for this confidence, and the weights considered for the OWA. This leads to the discontinuation coefficient (0.558), resulting from the aggregation of the probability component, 0.470, and OWA component, 0.645, considering a β coefficient of 0.500.

Therefore, the final value of certification is estimated at 339,115 €, as the sum of the initial value of the certificate 305,743 € plus the value of the discontinuation option 59,839 € multiplied by the discontinuation coefficient (0.5577).

Table 6. Valuation, level of confidence in experts and weights used for OWAs

Expert	Discontinuation coefficient	Confidence	Confidence weights	OWA weights
1	0.6	0.70	0.11	0.23
2	0.4	1.00	0.16	0.2
3	0.5	0.70	0.11	0.17
4	0.3	1.00	0.16	0.14
5	0.8	0.50	0.08	0.11
6	0.9	0.50	0.08	0.08
7	0.2	1.00	0.16	0.05
8	0.5	0.70	0.11	0.02

Source: Own source.

4. Conclusions

The main aim of this paper is to provide a model that facilitates the valuation of certifications that companies adopt voluntarily. Therefore, this will improve obtaining information that is necessary in any valuation process that companies may find themselves immersed in.

The first step in this methodology is to establish an approximation of the certification value by obtaining the difference between the value of a company that has voluntary certification and the hypothetical case of not having it.

The valuation method proposed is discounted cash flow in fuzzy environments. Discounted cash flows is a widely used method, which, however, presents high levels of subjectivity, so it is useful to use fuzzy mathematics.

To estimate future flows, it is usual to differentiate a timeline from a residual period. Triangular fuzzy numbers (TFNs) are used to determine the growth rate in the timeline. On the one hand, TFNs can ascertain the growth of the company to be valued in the las few years, and on the other hand, they can determine the growth of the companies in the sector. To make the projection of profits and losses in the timeline as realistic possible, different premises have been used to construct TFNs according to whether or not they have voluntary certification or not. In addition, when a company is being valued under the hypothesis of not having certification, the costs incurred from obtaining and maintaining this certification must be deducted from the expenditure items. These costs will be considered again in the subsequent valuation of the discontinuation option.

The value of the company obtained in each case is another TFN, whose minimum and maximum values indicate a company's range of values, while the central value presents the greater likelihood of occurrence. Consequently, it is necessary to defuzzify it, before obtaining, for differences, the first approximation of the value that a voluntary certificate generates for the company. However, the value obtained in this way does not take into account the operational flexibility, which allows management to discontinue certification if it considers that the benefits that it contributes are lower than the costs incurred.

To value this discontinuation option, fuzzy real options are used through the binomial method, estimating the company's volatility through the methodology by Haahtela (2007). The final value of the certificate is determined by the sum of the first approximation and that of the values of the discontinuation option weighted by a discontinuation coefficient. This coefficient allows us to calibrate the possibility of exercising the discontinuation option, since even though it is a voluntary certification, depending on its situation the company will have greater or less freedom to renounce certification without being penalised. We would like to stress the importance of considering the value that the discontinuation option contributes to the company. Otherwise, in any valuation process which voluntarily certified companies participate in, they will be discriminated against with respect to other countries.

Finally, we decided to apply the methodology to a real case. The use of a case study as a research method is fully justified when the aim is to illustrate or represent a theoretical model, as in this case. Nevertheless, in future research, we aim to broaden the application of the proposed methodology to a large sample.

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