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Analysis on Evolutionary Game of Risk Supervision over Public Pension Investment under Minimum Benefit Guarantee System

Abstract. *Modern pension systems are faced with severe challenges from longevity risks and fund shrinkage, while public pension safety not only concerns vital interests of retirees, or even social stability, but also needs to realise appreciation under the premise of ensuring safety. The earning rate of pension fund entrusted for investment should be greater than that of direct investment, but the principal-agent relation between the principal and investment organisations, as well as the obvious risk mismatches, motivate the investment organisations to expose the investment to more risks. This paper introduces a minimum benefit guarantee system to study the supervision mechanism of public pension investment risks on the basis of Evolutionary Game Theory, and draws conclusions as follows: the effect of returns on investment on avoiding investment risks is related to the earning rate of risk assets; greater minimum benefit guarantee levels and punishment intensity will reduce investment risks; greater returns on investment in risk assets will make investment behaviours more risky; investment environment will also influence practical effect of risk regulation policies.*

Keywords: *minimum benefit guarantee, public pension, investment risk, evolutionary game.*

JEL Classification: H55.

1. Introduction and Literature Review

The pension fund is an important component of the social security safety net, which offers important safeguard for economic security to individuals. At present, modern pension systems in many countries are faced with severe challenges from longevity risks and fund shrinkage (Popa et al., 2022). Past experience showed that the earning rates of entrusted investment of pension fund should be greater than that of direct investment (Nieuwerburgh and Veldkamp, 2010). For pension funds trusted for investment, government organs of fund management have authority over whether to invest the funds in a market-based manner and, in extreme cases, takes ultimate responsibility for pensions, playing the role of agent; while investment organisations

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enjoy the right of direct or entrusted investment, but for investment risks and losses, they mainly take hedging liability of minimal risk reserves, playing the role of agent. The agent and the investment organisations form a typical principal-agent relationship. When information is asymmetric, moral hazard and adverse selection will occur (Hamedani et al., 2021; Ciurea et al., 2022). Government organs of fund management aim at raising income levels of pension funds, realising maintenance and appreciation of fund values, and promoting healthy and sustainable development of old-age insurance system, while investment organisations seek to maximise the expected utility of their own profits (Yin et al., 2019). The obvious risk mismatches motivate investment organisations to make greater risk investment. However, pension safety not only concerns vital interests of retirees, or even social stability, but also needs to realise appreciation under the premise of ensuring safety (Tang and Hu, 2014). The Modern Portfolio Theory emphasises diversification to improve returns and reduce risks (Kim, 2021; Zhang et al., 2021; Anghelache et al., 2021). Scholars have studied the risk supervision mechanism in pension investment from various angles. Mao et al. (2021) proposed a novel prediction framework utilising technological innovation text mining data and ensemble learning to predict credit risk. Lai (2023) introduced a processing method (link processing method) for imbalance data based on the traditional early warning model and proved that the model was superior to XGBoost, NGBoost, Ada Boost, and GBDT in the prediction of default risk. Jang and Chae (2021) presented the funding ratio to divide the investment proportions in risky assets. Hosseininesaz and Jasemi (2022) used the lower partial moment measure to minimise investment portfolio risk of pension fund and employed a CVaR constraint to avoid loss. Ma et al. (2022) investigated the optimal strategies for a defined contribution pension plan with SAHARA utility under the DEV model. Wang et al. (2023) confirmed that both premium return provisions and information losses can make fund managers more cautious about venture investments. Dong et al. (2023) believed that under the non-concave utility framework, risk management based on finite expected loss constraints is not more beneficial than risk management based on VAR constraints. Yin et al. (2019) took into full consideration of entrusted pension investment, introduced minimum benefit guarantee, and analysed optimal asset allocation of pension funds both from the perspectives of the insured and investment organisations.

Most of the existing researches focus on how to allocate pension assets under minimum benefit guarantee system, and how to maximise expected utility of terminal wealth for pension funds from the perspectives of pension fund investment organisations. Boulier et al. (2001) studied optimal asset allocation issues of defined contribution pension funds under stochastic minimum benefit guarantee on the basis of the Vasicek Interest Rate Model (Vasicek, 1997) and fixed turn-over wealth value. On the research basis of Boulier et al. and the assumption that interests obey CIR Model, Wang et al. (2003) discussed optimal management issues of pension funds with internal benefit guarantee. Later, Deelstra et al. (2004) further studied optimal asset allocation of defined contribution pension funds on the basis of constraint on maximising the utility of the insured, and found that profit-sharing rule could transfer investment risks from agent to investment organisations. Liu et al. (2007) used the Black-Scholes Option Pricing Model to study relations between minimum rate of benefit guarantee and rate of administrative expense. Romaniuk (2007) categorised the minimum benefit guarantee into the internal benefit guarantee and the external benefit guarantee, and found that investment behaviours with an external benefit guarantee would be riskier. Liu et al. (2008) on the basis of Romaniuk's research, studied the influence of external benefit guarantee and profit sharing ratio over proportion of risk assets in pension funds investment, and found that the higher minimum benefit guarantees, the riskier investment behaviours. Guan and Liang (2014) studied optimal management strategy for defined contribution pension fund under internal benefit guarantee. MacKay and Ocejo (2022) studied a portfolio optimisation problem involving the loss averse policyholder of a variable annuity with a guaranteed minimum maturity benefit.

To sum up, fruitful results have been made on asset allocation of pension funds under minimum benefit guarantee system, but the principal-agent issue of pension fund investment has been overlooked. What investment organisations pursue in reality is not to maximise the expected utility of terminal wealth for pension fund, but to maximise the expected utility of their own gains (taking administrative expenses from the pension funds). Chen et al. (2017) realised this problem; however, only studied from the perspective of the insured, rather than the issue of entrusted pension investment. However, public pension investments are often made in a pooled investment. Pooled investment means that pension funds are collected and pooled by government organs, which act as principals to entrust the institutions to make direct or authorised investment. Principal-agent relations occur between government organs of fund management and investment organisations, which conforms more to reality of investment operation of public pension.

This paper takes into full consideration of the principal-agent relation between the principal and investment organisations in public pension investment, introduces minimum benefit guarantee system, and on the basis of Evolutionary Game Theory, analyses occurrence mechanism of public pension investment risks, and explores the supervision mechanism of public pension investment risks.

2. Fundamental Assumption

This paper only considers the system with one principal and one investment organisation. Being rational persons, none of the two parties in the initial phase of the evolutionary game has made optimal decisions; the participants adjusted their own decisions through constant learning and acquainting with strategies of their counterpart, until the two parties reached the equilibrium state (Sofikitis and Makris, 2022; Varga and Kiss, 2021; Clempner and Trejo, 2021). Policy space of the principal is (supervision, non-supervision), while policy space of the investment organisation is (investment as stipulated, non-investment as stipulated). The probability of "supervision" by the principal isx, probability of "non-supervision" is

$1 - x$; the probability of “investment as stipulated” by investment organisation is y, and the probability of “non-investment as stipulated” is $1 - y$.

As state organs, the principals are nominal owners of pension fund accounts, are responsible for collecting and pooling pension fund surplus, sign entrusted investment contract with investment organisations, supervise their investment behaviours, and take the ultimate minimum payment liability of pensions. Supervision by the principal will incur management and communication costs C. The gains and benefits of the investment in pension funds will alleviate the ultimate minimum payment pressure of the principal and bring about social benefits. Promissory pension investment will incur social benefit S.

Investment organisations are specialised in specific investment operations of pension assets. Investment organisations have two options for investment on the financial market: risk-free assets and risk assets. The amount of risk-free assets that investment organisation invests in is P_0, earning rate is r_0; the amount of risk assets that investment organisation invests in is P_1, earning rate is r_1, where $r_1 > r_0$. When pension fund investments get benefits, the investment organisation take certain percentage of investment income ρ as return on the investment. Investment organisations take a liability to minimum benefit guarantee when investing in risk assets. When pension fund investment incurs losses, and total assets are less that minimum benefit guarantee, investment organisation will make up the part below minimum benefit guarantee M, the principal will get minimum benefit guarantee W. The probability for risk assets to get benefits is α, the probability of incurring losses is $1 - \alpha$; the probability that total assets are higher than the minimum benefit guarantee is β, the probability that total assets are lower than the minimum benefit guarantee is $1 - \beta$.

The investment organisation may expand investment in risk assets against the rule in order to increase benefits ΔP, the principal imposes a fine on investment organisation B, policy executive strength is φ.

where, $x, y, r_0, r_1, \alpha, \beta, \rho, \varphi$ data range is $[0,1]$, $B, C, M, S, W, P_0, P_1, \Delta P$ data range is $[0, +\infty)$.

Game payoffs of principal and investment organisations are indicated as Table 1:

Table 1. Game Payoff Matrix

		Investment organisation	
		Promissory investment(y)	Non-promissory investment $(1 - y)$
Principal	Supervision (x)	$P_0 r_0 + \alpha P_1 r_1 + (1 - \alpha)(1 - \beta)W - C + S$, $\rho P_0 r_0 + \alpha\rho P_1 r_1 - (1 - \alpha)(1 - \beta)M$	$(P_0 - \Delta P)r_0 + \alpha(P_1 + \Delta P)r_1 + (1 - \alpha)(1 - \beta)W - C + \varphi B$, $\rho(P_0 - \Delta P)r_0 + \alpha\rho(P_1 + \Delta P)r_1 - \varphi B - (1 - \alpha)(1 - \beta)M$

	Investment organisation	
	Promissory investment(y)	Non-promissory investment $(1-y)$
Non-supervision $(1-x)$	$P_0r_0+\alpha P_1r_1+S$, $\rho P_0r_0+\alpha\rho P_1r_1$	$(P_0-\Delta P)r_0+\alpha(P_1+\Delta P)r_1$, $\rho(P_0-\Delta P)r_0+\alpha\rho(P_1+\Delta P)r_1$

Source: The content of table was created by the authors based on assumptions.

3. Analysis of Nash Equilibrium between Principal and Investment Organisation

3.1 Expected return

The principal "supervises", "not supervises" and average expected benefit are E_{11},E_{12} and E_1, respectively:

$$E_{11}=y(P_0r_0+\alpha P_1r_1+(1-\alpha)(1-\beta)W-C+S)+(1-y)((P_0-\Delta P)r_0+\alpha(P_1+\Delta P)r_1+(1-\alpha)(1-\beta)W-C+\varphi B) \tag{1}$$

$$E_{12}=y(P_0r_0+\alpha P_1r_1+S)+(1-y)((P_0-\Delta P)r_0+\alpha(P_1+\Delta P)r_1) \tag{2}$$

$$E_1=x(y(P_0r_0+\alpha P_1r_1+(1-\alpha)(1-\beta)W-C+S)+(1-y)((P_0-\Delta P)r_0+\alpha(P_1+\Delta P)r_1+(1-\alpha)(1-\beta)W-C+\varphi B))+(1-x)(y(P_0r_0+\alpha P_1r_1+S)+(1-y)((P_0-\Delta P)r_0+\alpha(P_1+\Delta P)r_1)) \tag{3}$$

Investment organisation "invests as agreed", "not invests as agreed" and average expected benefit are E_{21},E_{22} and E_2, respectively:

$$E_{21}=x(\rho P_0r_0+\alpha\rho P_1r_1-(1-\alpha)(1-\beta)M)+(1-x)(\rho P_0r_0+\alpha\rho P_1r_1) \tag{4}$$

$$E_{22}=x(\rho(P_0-\Delta P)r_0+\alpha\rho(P_1+\Delta P)r_1-\varphi B-(1-\alpha)(1-\beta)M)+(1-x)(\rho(P_0-\Delta P)r_0+\alpha\rho(P_1+\Delta P)r_1) \tag{5}$$

$$E_2=y(x(\rho P_0r_0+\alpha\rho P_1r_1-(1-\alpha)(1-\beta)M)+(1-x)(\rho P_0r_0+\alpha\rho P_1r_1))+(1-y)(x(\rho(P_0-\Delta P)r_0+\alpha\rho(P_1+\Delta P)r_1-\varphi B-(1-\alpha)(1-\beta)M)+(1-x)(\rho(P_0-\Delta P)r_0+\alpha\rho(P_1+\Delta P)r_1)) \tag{6}$$

3.2 Replicator Dynamics Equation

Replicator dynamics equation of principal is:

$$F(x)=\frac{dx}{dt}=x(1-x)(W(-1+\alpha)(-1+\beta)-C-B(-1+y)\varphi) \tag{7}$$

The replicator dynamics equation of investment organisation is:

$$F(y)=\frac{dy}{dt}=y(1-y)(Bx\varphi+\Delta P\rho(r_0-\alpha r_1)) \tag{8}$$

Therefore, the replicator dynamics equation of game system is:

$$\begin{cases} F(x) = x(1-x)(W(-1+\alpha)(-1+\beta) - C - B(-1+y)\varphi) \\ F(y) = y(1-y)(Bx\varphi + \Delta P\rho(r_0 - \alpha r_1)) \end{cases} \tag{9}$$

3.3 Analysis of evolutionary stable strategy of game-agent

(1) Jacobian matrix of game system

$$J = \begin{Bmatrix} \frac{dF(x)}{dx} & \frac{dF(x)}{dy} \\ \frac{dF(y)}{dx} & \frac{dF(y)}{dy} \end{Bmatrix} \tag{10}$$

where:

$$\frac{dF(x)}{dx} = (1-2x)(W(-1+\alpha)(-1+\beta) - C - B(-1+y)\varphi) \tag{11}$$

$$\frac{dF(x)}{dy} = -x(1-x)B\varphi \tag{12}$$

$$\frac{dF(y)}{dx} = y(1-y)B\varphi \tag{13}$$

$$\frac{dF(y)}{dy} = (1-2y)(Bx\varphi + \Delta P\rho(r_0 - \alpha r_1)) \tag{14}$$

In the dynamical system consisting of game-agent of two sides, make $F(x) = 0, F(y) = 0$, 5 pure strategy Nash equilibrium points of the system can be obtained: O(0, 0), A(0, 1), B(1, 0), C(1, 1), D(x_D,y_D).

where:

$$x_D = \frac{-\Delta P\rho r_0 + \alpha\Delta P\rho r_1}{B\varphi} \tag{15}$$

$$y_D = \frac{W - W\alpha - W\beta + W\alpha\beta + B\varphi - C}{B\varphi} \tag{16}$$

(2) Stable analysis of game system

Put the above 5 Nash equilibrium points into Jacobian matrix of the system respectively, determinant values and trace values can be obtained, as indicated in Table 2:

Table 2. Determinant and Trace of Jacobian

Equilibrium points	Determinant values and Trace values	
$O(0,0)$	detJ	$\Delta P\rho(W(-1+\alpha)(-1+\beta) + B\varphi - C)(r_0 - \alpha r_1)$
	trJ	$W(-1+\alpha)(-1+\beta) + B\varphi - C + \Delta P\rho(r_0 - \alpha r_1)$
$A(0,1)$	detJ	$-\Delta P\rho(W(-1+\alpha)(-1+\beta) - C)(r_0 - \alpha r_1)$
	trJ	$W(-1+\alpha)(-1+\beta) - C - \Delta P\rho(r_0 - \alpha r_1)$
$B(1,0)$	detJ	$-(W(-1+\alpha)(-1+\beta) + B\varphi - C)(B\varphi + \Delta P\rho(r_0 - \alpha r_1))$
	trJ	$-(W(-1+\alpha)(-1+\beta) - C) + \Delta P\rho(r_0 - \alpha r_1)$

Equilibrium points	**Determinant values and Trace values**	
$C(1,1)$	$\det J$	$(W(-1+\alpha)(-1+\beta)-C)(B\varphi+\Delta P\rho(r_0-\alpha r_1))$
	trJ	$-(W(-1+\alpha)(-1+\beta)+B\varphi-C)-\Delta P\rho(r_0-\alpha r_1)$
$D(x_D,y_D)$	$\det J$	$\emptyset = TN \neq 0$
	trJ	0

Source: The content of the table is calculated by substituting the Nash equilibrium point into the Jacobian matrix.

where:

$$T=\frac{(W(-1+\alpha)(-1+\beta)-C)(W(-1+\alpha)(-1+\beta)+B\varphi-C)}{B\varphi} \tag{17}$$

$$N=\frac{\Delta P\rho(-r_0+\alpha r_1)(-B\varphi-\Delta P\rho r_0+\alpha\Delta P\rho r_1)}{B\varphi} \tag{18}$$

The system stability is discussed as follows.

Case 1: when $W(1-\alpha)(1-\beta)+B\varphi<C$, and $\Delta P\rho r_0<\alpha\Delta P\rho r_1$, the system has the only evolutionary stable point O(0, 0), corresponding to strategy (non-supervision, non-promissory investment).

Case 2: When $W(1-\alpha)(1-\beta)+B\varphi<C$, and $\Delta P\rho r_0>\alpha\Delta P\rho r_1$, the system has the only evolutionary stable point A(0, 1), corresponding to strategy (non-supervision, promissory investment).

Case 3: When $W(1-\alpha)(1-\beta)>C$, and $\Delta P\rho r_0<\alpha\Delta P\rho r_1-B\varphi$, the system has the only evolutionary stable point B(1, 0), corresponding to strategy (supervision, non-promissory investment).

Case 4: When $W(1-\alpha)(1-\beta)>C$, and $\Delta P\rho r_0>\alpha\Delta P\rho r_1-B\varphi$, the system has the only evolutionary stable point C(1, 1), corresponding to strategy (supervision, promissory investment).

Case 5: When $C<W(1-\alpha)(1-\beta)<C+B\varphi$, and $\Delta P\rho r_0-\alpha\Delta P\rho r_1<-B\varphi$, the system has the evolutionary stable points A(0, 1) and B(1, 0), corresponding to strategy (non-supervision, promissory investment) and (supervision, non-promissory investment), respectively.

This paper chooses stability states of the system, as case 2, case 3 and case 5, to discuss the local stability as indicated in Table 3:

Table 3. Analysis of local stability

Equilibrium points	Case 2			Case 3		
	detJ	trJ	Stability	detJ	trJ	Stability
$O(0,0)$	-	uncertain	saddle point	-	uncertain	saddle point
$A(0,1)$	+	-	ESS	+	+	instability
$B(1,0)$	+	+	instability	+	-	ESS
$C(1,1)$	-	uncertain	saddle point	-	uncertain	saddle point
$D(x_D,y_D)$	$\emptyset$	0	saddle point	$\emptyset$	0	saddle point

(Continuation)

Equilibrium points	Case5		
	detJ	trJ	Stability
$O(0,0)$	-	uncertain	saddle point
$A(0,1)$	+	-	ESS
$B(1,0)$	+	-	ESS
$C(1,1)$	-	uncertain	saddle point
$D(x_D,y_D)$	∅	0	saddle point

Source: The content of table comes from the author's judgment on the stability of case 2, case 3 and case 5.

(3) Effects of various parameters on system evolution results in Case 5

Based on the above assumptions, the evolutionary stable strategy phase diagram of the two game participants in the system can be obtained, as shown in Figure 1.

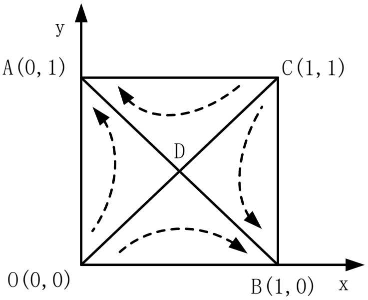

Figure 1. Phase diagram in Case 5

Source: Created by the authors.

Calculate the area of CDOA (S_A) and the area of CDOB (S_B) in Case5. When S_A increases， the evolutionary stable point is $A(0,1)$, When S_B decreases， the evolutionary stable point is $B(1,0)$.

$$S_A = \frac{1}{2}(\frac{-\Delta P\rho r_0 + \alpha\Delta P\rho r_1}{B\varphi} + \frac{-W + W\alpha + W\beta - W\alpha\beta + C}{B\varphi} \tag{19}$$

Proposition 1: Increasing returns on investment ρ will increase probability of “promissory investment” of the investment organisation.

Demonstrate: Taking the partial derivative of S_A with respect to ρ, with other parameters constant, get:

$$\frac{\partial S_A}{\partial \rho} = \frac{-\Delta P r_0 + \alpha\Delta P r_1}{2B\varphi} > 0 \tag{20}$$

With other parameters constant, S_A is an increasing function of ρ. The higher the rate of returns on investment, the larger the area of the quadrilateral CDOA, the greater the probability of “promissory investment” of the investment organisation.

Proposition 2: Reducing the minimum benefit guarantee level will promote the probability of “supervision” by the principal.

Demonstrate: Taking the partial derivative of S_A with respect to W, with other parameters constant, get:

$$\frac{\partial S_A}{\partial W} = \frac{-1 + \alpha + \beta - \alpha\beta}{2B\varphi} < 0 \tag{21}$$

With other parameters constant, S_A is a reducing function of W. The lower the minimum benefit guarantee level, the larger the area of the quadrilateral CDOA, the greater the probability of “supervision” of the principal.

Proposition 3: When the benefits of risk asset investment are more than risk-free asset investment, although the punishment is increased, investment organisation will still to break rules to expand investment in risk assets. The system stabilises at point $B(1,0)$.

Demonstrate: Taking the partial derivative of S_A with respect to φ, with other parameters constant, get:

$$\frac{\partial S_A}{\partial \varphi} = \frac{1}{2}\left(-\frac{-W + W\alpha + W\beta - W\alpha\beta + C}{B\varphi^2} - \frac{-\Delta P\rho r_0 + \alpha\Delta P\rho r_1}{B\varphi^2}\right) < 0 \tag{22}$$

With other parameters constant, S_A is an reducing function of φ. Greater punishment intensity will lower the area of the quadrilateral CDOA, increase probability of “promissory investment” of the investment organisation, and reduce probability of “supervision” by the principal. The system stabilises at point $B(1,0)$.

Proposition 4: When $\alpha \leq 0$, increasing return on risk asset investment will increase probability of “promissory investment” of the investment organisation, and reduce the probability of “supervision” of the principal. The system stabilises at point $A(0,1)$.

Demonstrate: Taking the partial derivative of S_A with respect to α, with other parameters constant, get:

$$\frac{\partial S_A}{\partial \alpha} = \frac{1}{2}\left(\frac{W - W\beta}{B\varphi} + \frac{\Delta P\rho r_1}{B\varphi}\right) > 0 \tag{23}$$

With other parameters constant, $S_A$is an increasing function of α. Higher return on risk asset investment, will larger the area of the quadrilateral CDOA, increase probability of “promissory investment” of the investment organisation, and decrease probability of “supervision” by the principal. The system stabilises at point $A(0,1)$.

4. Simulated analysis

This paper uses Matlab software to perform numerical simulation for the two intermediate stability states of the system, in order to verify the effectiveness of above-mentioned stability analysis and effects of various parameters on system evolution results. In view of constraints on reality and literature data, these paper set parameters with reference to behaviour characteristics of game-agent in the system, and assigned values to interactions of parameters in Case 2 and Case 3. Array 1: $\alpha = 0.5$, $\beta = 0.5$, $\Delta P = 150$, $r_0 = 3\%$, $r_1 = 5\%$, $B = 30$, $\varphi = 0.4$, $W = 100$, $C = 38$, $\rho = 0.2$; Array 2: $\alpha = 0.6$, $\beta = 0.2$, $C = 30$, $\Delta P = 100$, $r_1 = 6\%$, $B = 0.2$, and other parameters are the same as Array 1.

4.1 Original state

In order to test the effectiveness of the system evolution stability analysis, put Array 1 and Array 2 into the model for simulation, respectively, make probabilities of initial strategy for principal and investment organisation (0.5, 0.5), and computational results are indicated as Figure 2 and Figure 3:

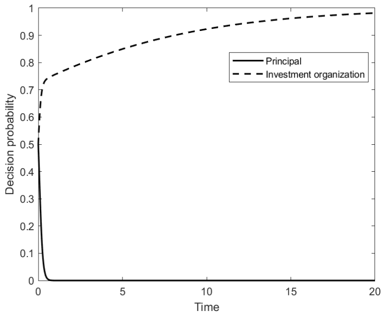

Figure 2. Initial state in Case 2
Source: Created by the authors.

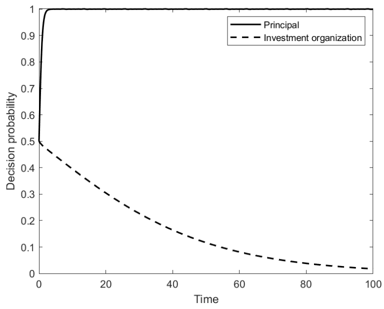

Figure 3. Initial state in Case 3
Source: Created by the authors.

In Array 1, when gains for investment in risk-free assets are greater than that in risk assets, investment organisation will invest in risk-free assets; supervision benefit of the principal mainly comes from minimum benefit guarantee and fines, when the probability of policy choice for investment organisation to make "promissory investment", the supervision benefits for principal gradually declines till to 0, and the evolutionary stable point of the two gaming parties is A(0, 1), corresponding to (non-supervision, promissory investment) strategy. In Array 2, when gains for investment in risk assets, even after being fined, are still greater than that in risk-free assets, investment organisation will break rules to expand investment in risk assets; at the moment, the principal by supervision gains minimum benefit guarantee and fines, when minimum benefit guarantee is far greater that supervision costs, the principal will choose "supervision" strategy. The evolutionary stable point of the two game parties B (1, 0), corresponding to strategy (supervision, non-promissory investment).

As indicated in Figure 2 and Figure 3, under Array 1 and Array 2, the system will evolve and stabilise at point A(0, 1) and point B(1, 0), respectively, conforming to the conclusions of system evolution stability analyses in Case 2 and Case 3. This verifies that the model is effective and has realistic guiding significance in clarifying principal-agent relationship, reducing investment risk, and realising the maintenance and appreciation of values for pension funds.

4.2 Return on investment ρ

With other parameters set, make ρ take 0.2, 0.5 and 0.8, the computational results are indicated as Figure 4 and Figure 5:

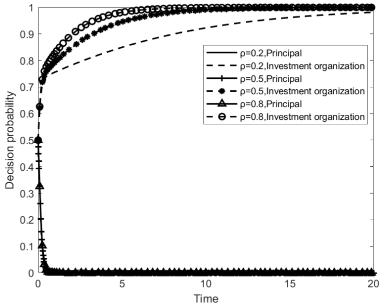

Figure 4. Influence of ρ in Case 2

Source: Created by the authors.

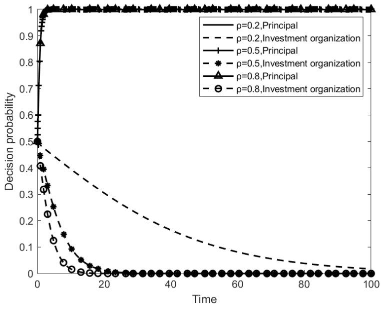

Figure 5. Influence of ρ in Case 3

Source: Created by the authors.

In Case 2, the optimal strategy choice for the two gaming parties is (non-supervision, promissory investment). Although investment in risk assets may have a higher earning rate, the probability is only $\alpha = 0.5$, that is, the benefit is αr_1; while investment in risk free assets has a lower earning rate, but $r_0 > \alpha r_1$, at the moment, investment organisation will invest in risk-free assets. The principal's strategy choice is free from the influence of ρ. As indicated in Figure 4, as the earning rate increasesρ, the system will stabilise at point A(0, 1) at a faster speed, but will not change the strategy selections of the two gaming parties.

In Case 3, the optimal strategy choice for the two gaming parties is (supervision, non-promissory investment). At the moment, the net benefit for the investment organisation to break the rules to expand the investment in risk assets is$\alpha \Delta P \rho r_1 - B\varphi$, greater than that of investment in risk-free assets $\Delta P \rho r_0$, investment organisation will choose not to invest as agreed. In this case, increasing the earning rate ρ, will make the investment organisation more inclined to choose investment in risk assets, and the probability of strategy selection for "non-promissory investment" will increase at a faster speed. The return on investmentρwill not influence the strategy selection of the principal. As indicated in Figure 5, increasing returns on investmentρ, will accelerate the system to stabilise at point B(1, 0), but will not change the strategy selection of the two gaming parties.

4.3 Minimum benefit guarantee W

With other parameters set, make W in Case 2 take100, 130 and 160, in Case 3 take 80, 100 and 120, the computational results are indicated in Figure 6 and Figure 7:

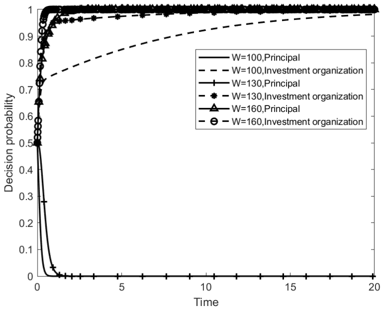

Figure 6. Influence of W in Case 2
Source: Created by the authors.

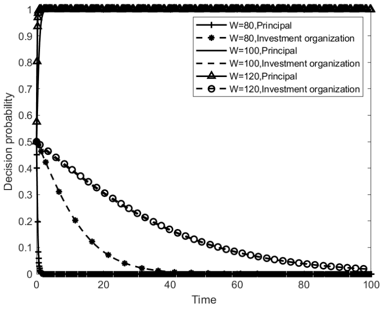

Figure 7. Influence of W in Case 3
Source: Created by the authors.

As known from Figure 6, in Case 2, minimum benefit guarantee W has a critical value γ_1 between 130 and 160. When $W < \gamma_1$, as Wincreases, the minimum benefit guarantee for principal to implement supervision gradually increases, but still less than supervision costs, the principal choose strategy of "non-supervision"; when $W > \gamma_1$, the minimum benefit guarantee for principal to implement supervision surpasses supervision costs, the principal choose strategy of "supervision". At the same time, as W increases, the investment organisation needs to make up a greater guarantee gap, the costs to break rules and expand investment in risk assets increase, the investment organisation is more inclined to choose "promissory investment". Therefore, in Case 2, increasing the minimum benefit guarantee level will cause the stable point A(0, 1) to evolve to point C(1, 1).

As known from Figure 7, in Case 3, minimum benefit guarantee W has a critical value γ_2 between 80 and 100. When $W > \gamma_2$, the minimum benefit guarantee for principal to implement supervision is greater than supervision costs, the principal choose strategy of "supervision"; when $W < \gamma_2$, the minimum benefit guarantee for the principal to implement supervision is less than supervision costs, the principal choose strategy of "non-supervision". At the same time, as W decreases, the costs for investment organisation to chooses not to invest as agreed decrease, the probability of "choosing not to invest as agreed" increases. Therefore, in Case 3, reducing the minimum benefit guarantee level will cause stable point B(1, 0) to evolve to point O(0, 0).

4.4 Punishment intensity φ

With other parameters set, make φ take 0.4, 0.6 and 0.8, the computational results are indicated as Figure 8 and Figure 9:

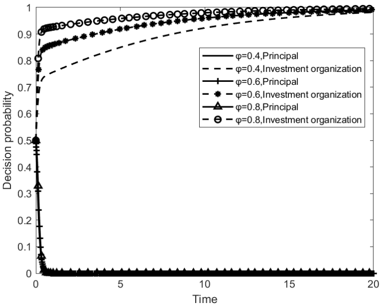

Figure 8. Influence of φ in Case 2
Source: Created by the authors.

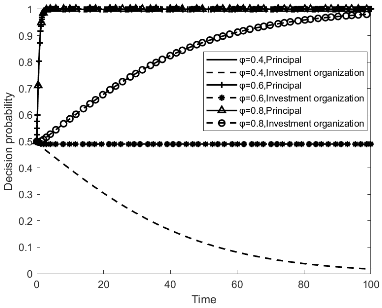

Figure 9. Influence of φ in Case 3
Source: Created by the authors.

As known from Figure 8, in Case 2, as punishment intensity increases, the costs for investment organisation to break rules and expand investment in risk assets increase, so the investment organisation will choose strategy of "promissory investment", at the moment, the gains through the minimum benefit guarantee and fines for principal to implement supervision is less that costs, so the principal will choose the strategy of "non-supervision". The system stabilises at point A(0, 1).

As known form Figure 9, in Case 3, the optimal strategy selection for the principal is "supervision", while investment organisation will adjust strategy selection according to the changes in punishment intensity. Punishment intensity φ has critical values γ_3 and γ_4 in 0.4-0.6 and 0.6-0.8, respectively. When $\varphi < \gamma_3$, the amercement outlays for the investment organisation breaking the rules and expanding the investment in risk assets are relatively small, its net benefits are greater than that of the promissory investment, the investment organisation will choose strategy of "not to invest as agreed", at the moment, the system stabilises at point B(1, 0); when $\gamma_3 < \varphi < \gamma_4$, the amercement outlays for investment organisation not performing investment as agreed gradually increase, whenφapproaches 0.6, the investment organisation will take a neutral attitude, and the probability of strategy selection gradually stabilises at 0.5. When $\gamma_4 < \varphi$, the amercement outlays become greater and greater, the net benefit for the investment organisation to break rules to expand investment in risk assets decreases, and less than that of promissory investment, the investment organisation will choose "promissory investment". At the moment, the system stabilises at point C(1, 1).

4.5 Return on risk asset investment α

With other parameters set, makeαtake 0.2, 0.5 and 0.8 in Case 2, and 0.2, 0.6 and 0.8 in Case 3, the computational results are indicated as Figure 10 and Figure 11:

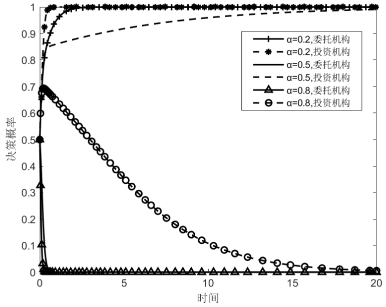

Figure 10. Influence of α in Case 2
Source: Created by the authors.

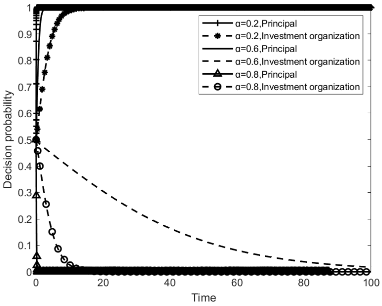

Figure 11. Influence of α in Case 3
Source: Created by the authors.

As indicated in Figure 10, in Case 2, when α = 0.5, the system stabilised at point A(0, 1). When α approaches 0.2, the probability of loss occurrence for investment organisation to invest in risk assets increases, the benefits of risk asset investment are less than that of risk-free asset investment, and the investment organisation is more inclined to make promissory investment in risk-free assets; the minimum benefit guarantee and fines that the principal obtains by implementing supervision increases, until surpassing supervision costs so as to choose "supervision" strategy, and the system stabilises at point C(1, 1). When α approaches 0.8, the probability of receiving benefits from the investment organisation to invest in risk assets increases, the benefits of the investment of risk assets are greater than those of the investment of risk-free assets, and the investment organisation is more inclined to break rules to expand the investment of risk asset investment; the minimum benefit guarantee obtained by the principal decreases as the probability of loss occurrence for the implementation of supervision is reduced, the minimum benefit guarantee decreases, the principal chooses the 'non-supervision' strategy and the system stabilises at point O(0, 0).Therefore, in case 2, as the return on investment increases, the system evolves from point C(1, 1) to point A(0, 1), then to point O(0, 0).

As indicated in Figure 11, in Case 3, when α = 0.6, the system stabilised at point A(1, 0). When α approaches 0.2, the probability that investment organisation breaks rules to expand risk asset investment and incurs losses will increase, and the investment organisation is more inclined to choose the strategy of "promissory investment"; the minimum benefit guarantee for principal decreases, but still greater that the supervision costs, and the principal chooses "supervision" strategy, and the system stabilises at point C(1, 1). When α approaches 0.8, the probability that investment organisation breaks rules to expand risk asset investment in order to get benefits increases, and the investment organisation is more inclined to choose strategy of “non-promissory investment”; the minimum benefit guarantee that the principal gets due to supervision decreases, but still less that supervision costs, and the principal chooses “non-supervision” strategy. Therefore, in case 3, as the return on investment increases, the system evolves from point C(1, 1) to point B(1, 0), then to point O(0, 0).

5. Conclusions

This paper introduces the minimum benefit guarantee system to the public pension investment, uses Evolutionary Game Theory to conduct a research on risk supervision mechanism of public pension investment, and through numerical simulation for Case 2 and Case 3, draws conclusions as follows:

- The influence of return on investment over strategy selection by investment organisation depends on relative size of return on risk-free asset investment the expected earning rate of risk assets. When earning rate of risk-free assets is relatively greater, the investment organisation will not break rules to expand risk assets investment, but to select “promissory investment” strategy more quickly; when expected earning rate of risk assets is relatively greater, the investment organisation will select strategy of “non-promissory investment”, and the investment behaviours will be riskier.
- Raising minimum benefit guarantee level will increase the principal’s minimum benefit guarantee, promote probability of strategy selection of “supervision” by the principal; the investment organisation is expected to make up greater guarantee gap, and part of investment risks are transferred from principal to investment organisation, such as to increase probability of “promissory investment” by the investment organisation, which is one of the routes to reduce pension fund investment risks.
- Greater punishment intensity will increase costs for investment organisations to expand risk asset investment against the rule, and promote probability of “promissory investment” by the investment organisation, which is another route to reduce pension fund investment risks.
- The greater return on risk asset investment will bring about greater gains for investment organisation to break rules to expand risk asset investment, and the investment behaviours will be riskier.
- The investment environment affects policy effects. Take the return rate of an investment organisation, for example, in Case 2, as return rate increases, the investment organisation is more inclined to select “promissory investment” strategy, and investment risk is reduced; in Case 3, as return rate increases, the investment organisation, however, is more inclined to select the “non-promissory investment” strategy, and the investment risks increase. Therefore, appropriate risk aversion policies should be selected on the basis of an in-depth analysis of the pension fund investment environment.

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