Hyungwook SHIM, PhD (corresponding author)
shw@kisti.re.kr
National Supercomputer Center
Korea Institute of Science and Technology Information, Korea

Nash Equilibrium Analysis of the Public Supercomputing Duopoly Ecosystem

Abstract. *The supercomputing industry has been leveraging individual supercomputers owned by many countries and organisations in communal systems in response to the rapid influx of AI computing demand. Using Korea as a case study, we conducted a Nash equilibrium analysis to develop an effective promotion strategy for the industry, which has transitioned from a single national centre system to a dual system that includes specialised centres. The analysis resulted in the derivation of the demand function for the duopoly market and the calculation of the Nash equilibrium points of the response curve for each centre. In the duopoly market, it was discovered that for effective operation of each centre, the total demand, usage fees, new inflow demand, and demand-sharing ratio for each centre must be taken into consideration.*

Keywords*: supercomputer, Nash equilibrium, demand function, logit model, regression model.*

JEL Classification: C02, C13, C35, C52, C54.

1. Introduction

Maintaining a large number of supercomputing resources in the public sector nationally is challenging, as it requires operational technology and experts, as well as the economic burden of building infrastructure. In most countries, including Korea, the available resources for general users, not private companies, to access supercomputers are the national centre's single resource. Recently, however, with the advancement of artificial intelligence (AI) technology and the explosive increase in demand for convergence technology development, many countries are transitioning from a single-resource model to a duopoly ecosystem, incorporating specialised centres for each field. In the future, the demand for supercomputing will be divided into new centres, including national and specialised centres, and users will choose a resource that reflects their individual preferences. Therefore, in the case of an existing single resource, if a government-led operation plan has been created to maximise its usage by as many users as possible, it is important to consider the user's perspective and encourage appropriate sharing of usage demands. Various demand management methods are applicable to supercomputing resources, including infrastructure performance, service types and levels, and usage fees. Currently, the government is promoting a management policy that incorporates usage fees to improve industry sustainability, and national and specialised centres have received

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approval for operating plans that consider annual usage fees. In this context, this study derives policy implications through a Nash equilibrium analysis of game theory to establish an effective operational strategy for the duopoly public-sector supercomputing ecosystem. The Nash equilibrium point was determined by estimating the future usage demand model and sharing ratio, a method that has not yet been attempted in Korea, and deriving the response curve.

The remainder of this paper is organised as follows: Section 2 explains the background of this study and its academic value, including its progressiveness and differentiation, compared to existing research. Section 3 explains the theoretical background, and Section 4 presents the research procedures, methodology, and case study results of the Nash equilibrium analysis. Finally, Section 5 summarises the results and explains their implications and limitations.

2. Literature Review

In this chapter, the purpose, method, and main results of the research are analysed based on previous research, and the background and academic value are derived. Nezarat (2015) proposed an auction-based method that determines the auction winner by applying a game theory mechanism and holding a repetitive game with incomplete information in a noncooperative environment. In this method, users calculate suitable price bids with their objective function during several rounds and repetitions and send them to the auctioneer, who chooses the winning player based on the suggested utility function. Waichman (2014) investigated the impact of communication on outcomes in Cournot duopoly and triopoly experiments. Communication was implemented by two different devices: a “standardised communication” and a “free communication” device. Using both students and managers as subjects, it was found that managers behaved similarly when using both communication devices, while students colluded slightly better when using the free communication device compared to when using the standardised communication device. Hasnas (2014) analysed the case of substitutability in an OI setup within a Cournot duopoly, where knowledge spillovers were endogenously determined through the R&D process. The game yielded multiple steady states, allowing for an asymmetric solution in which a firm could trade R&D investment against information absorption by its rival. Technical analysis and numerical simulations indicate that a firm that commits to a higher level of OI absorption produces smaller output but enjoys higher profits than its rivals. Nagurney (2014) developed a game theory model for a service-oriented Internet, where profit-maximising service providers offered substitutable services, competing with the quantities of services in a Cournot–Nash manner, whereas network transport providers, which transport the services to the users in the demand markets and are also profit-maximisers, competed with prices in a Bertrand fashion and on quality. Consumers responded to the composition of services and network provision through demand price functions, which were both quantity- and quality-dependent. The model derived the governing equilibrium conditions of an integrated game and demonstrated that it satisfied the variational inequality problem. Luong (2018) reviewed the economic and pricing approaches proposed to address resource management issues in 5G wireless networks, including user association, spectrum allocation, interference, and power management. Furthermore, they presented applications of economic and pricing models for wireless caching and mobile data offloading. Finally, they highlighted important challenges, open issues, and future research directions for applying economic and pricing models to 5G wireless networks. Using the Cournot and Nash equilibria based on game theory as tools, meaningful research results have been published in various fields, such as the interpretation of competitive situations, best selection alternatives, and implementation strategies. However, few empirical studies have been conducted on game theory that targets supercomputing resources. This is because supercomputer resources are still being built with a focus on suppliers; therefore, the need for demand management is not felt in terms of policy. Consequently, it is judged that not much empirical research has been conducted due to the limited number of estimated cases of microeconomic models, which are essential for establishing the economic structure and promotion strategy of the industry.

3. Theoretical Background

Von Neumann (1947) first established a theory that applied game theory to economics in the 1940s in his book “The Theory of Games and Economic Behaviour”. The theory’s background suggests that various aspects, such as economic competition or armed confrontation, which are not typically considered as games, can be analysed as games. This theory later became known through a representative example of game theory called the prisoner's dilemma, as presented in the work of Tucker (1950). Game theory has gradually become an essential analytical tool for making rational choices in various problem-solving scenarios, and it has been widely expanded and applied to diverse fields, including economics, political science, and social science. Game theory studies the strategic choices made by interacting rational individuals and is called interactive decision theory. The primary objective of this analysis is to determine the most effective strategy for an individual in response to the strategies chosen by others. Here, following the precedent of neoclassical economics, the best response is defined as a strategy that gives one player the maximum benefit, given the strategies that other players have chosen or are expected to choose. Cournot developed this concept to understand the principles of price determination in duplexes. This model assumes that each company determines the amount of production to bring to the market and that the price is determined by the total supply. The key factors in this model are the response curve and the Cournot equilibrium. First, in the Cournot model, the reaction curve represents one’s optimal response to another party’s actions. This refers to a curve that connects the corresponding points where one party’s profit is maximised under the assumption that the other party's output is given. The Cournot equilibrium, also called the Nash equilibrium, refers to the intersection of two reaction curves (Bimpikis, 2019).

The Cournot model can be summarised mathematically as follows: First, the inverse demand or cost function for the entire market can be expressed as shown in Equation (1). P refers to the cost, and Q_n refers to the production volume of company n.

$$P = f(Q_1 + Q_2) \tag{1}$$

This can be expressed as the demand function of an individual company, as shown in Equations (2) and (3).

$$P_1 = f_1(Q_1 + Q_2) \tag{2}$$

$$P_2 = f_2(Q_1 + Q_2) \tag{3}$$

Next, the product cost can be expressed as a function of Equations (4) and (5).

$$C_1 = g_1(Q_1) \tag{4}$$

$$C_2 = g_2(Q_2) \tag{5}$$

Using the demand and cost functions, the profit function for each company is defined in Equations (6) and (7) as the difference between the two functions.

$$\Pi_1 = P_1 Q_1 - C_1 \tag{6}$$

$$\Pi_2 = P_2 Q_2 - C_2 \tag{7}$$

Next, the company pursues profit maximisation; thus, Equation (8) is the partial differentiation of Equations (6) and (7) with respect to Q_n. This applies to the conditions of Equation (9) that are generally accepted in economics. MR_n is the marginal revenue, and MC_n is the marginal cost.

$$\partial g_n / \partial Q_n = P_n - Q \cdot \partial f_n / \partial Q_n \tag{8}$$

$$MC_n = MR_n \tag{9}$$

Summarising Equation (8) for Company 1 with respect to Q, it becomes Equation (10):

$$\mathrm{Q} = \{-P_1 + (\partial g_1 / \partial Q_1)\} / (\partial f_1 / \partial Q_1) = (-P_1 + MC_1) / (\partial f_1 / \partial Q_1) \tag{10}$$

By differentiating the above equation with respect to Q_2 and integrating it as a function of Q_1, the company's response function, Equation (11), can be obtained.

$$Q_1 = h_1(Q_2) \tag{11}$$

The response function for Company 2 can be obtained using the same method. Finally, by using the two response functions, a Nash equilibrium is said to exist at point A, which is the intersection of the two curves in Figure 1 and can be considered stable.

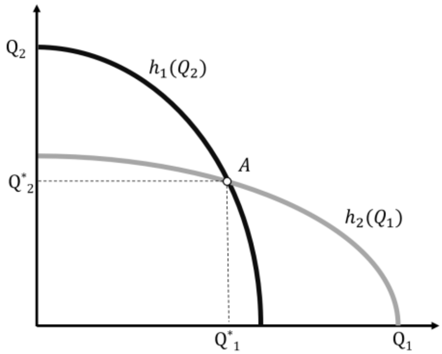

Figure 1. Nash equilibrium point (A)
Source: Illustration by authors.

4. Nash Equilibrium Analysis of The Public Supercomputing Ecosystem

Currently, the supercomputer industry is divided into national and specialised centres. The national centre is designated as the Korea Institute of Science and Technology Information (KISTI), providing resources of 25.3 PF. To date, the national centre has handled all demands from the public sector. The specialised centre is the second public sector supercomputer resource to be operated in 2023. It provides resources and professional services in response to the computing needs of the 10 fields listed in Table 1.

Table 1. Ten fields of the specialised centre

Material /Nano	**Bio/Health**	**ICT**	**Meteorology/ Climate /Environment**	**Autonomous driving**
Space	Nuclear fusion/Accelerator	Manufacturing technology	Disaster	Defense/Security

Source: Organized by the author.

As the existing single national centre system evolves into a duopoly, it is necessary to predict the various impacts of their interactions and operate a stable infrastructure in advance. This situation is similar to the Nash equilibrium in game theory. In other words, assuming that each competitor knows the other competitor's equilibrium strategy in a noncooperative game between two competitors, this is a situation in which neither competitor changes its strategy. Therefore, this study attempts to derive meaningful results for government strategy preparation using a Nash equilibrium analysis. In addition, an empirical study was conducted by estimating the demand model in the duopoly ecosystem and applying each centre's sharing ratio to the new incoming demand (Choi, 2016; Shim, 2023). The analysis procedure and main methodology for each step are shown in Figure. 2. First, data on the preferred doctors for each centre were collected through a survey, and the sharing ratio was estimated using a logit model. Through regression analysis, the demand model for each centre was estimated, and the Nash equilibrium considering the sharing ratio was calculated.

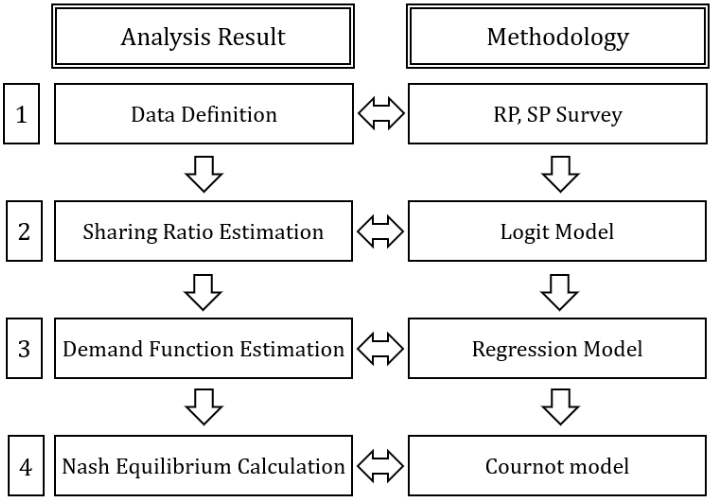

Figure 2. Research procedure
Source: Illustrated by authors.

The sharing ratio is essential for calculating the demand for each new centre entering the duopoly ecosystem. In many previous studies, a logit model based on utility theory is applied to the sharing ratio, and when there are two alternatives, a binomial logit model is applied. The logit model is an individual behaviour model that can be applied when multiple alternatives exist and follows the principle of maximising utility, in that the alternative with the highest level of utility is selected from all alternatives available for an individual to choose from. The alternative selection probability is given by Equation (12).

$$P_a = \exp(V_a)/\sum_{n=1}^{J} \exp(V_J) \quad (12)$$

The observable utility V_j can be expressed as Equation (13) when assumed to be in the form of a linear function, considering the alternative characteristics. α is the alternative characteristic constant, β is the coefficient, x is the alternative general variable, and x_j is the alternative characteristic variable (kim, 2014; Lee, 1999; Werden, 1996).

$$V_j = \alpha + \beta x + \beta_j x_j \quad (13)$$

The utility function in Equation (14) was applied based on the findings of Shim (2023). $time_{NC}$ is a general variable for national centre time (alternative general variable), and $cost_{NC}$ is a general variable for national centre cost (alternative general variable). This utility function is derived from the findings of the most recent demand estimation research targeting the Korean supercomputer ecosystem (national and specialised centres).

$$2.71 + 0.013 \cdot time_{NC} - 0.877 \cdot cost_{NC} \tag{14}$$

The proportion of new computational demand was 40.5% for national centres and 59.5% for specialised centres (Shim, 2023).

The demand function was estimated using a linear regression analysis. The independent variable was defined as "fee", the dependent variable was defined as "usage demand," and the estimate was calculated based on the change in usage demand in response to the change in fee. The data for each variable were obtained through surveys. Respondents were limited to existing supercomputer users. The fees in the options were based on the national centre's fee system and were differentiated from the current fee level of $3 by up to ±30%. The results are summarised in Table 2.

Table 2. Survey table

	Price (Unit CPU/day)	**Alternatives**
1	$2	0. Use 1. Non-use
2	$3 (Reference Value)	
3	$4	

Source: Defined by author.

As a result of the survey, 55 people responded, constituting a sample with a confidence level of 90% and a margin of error of 10%. Table 3 summarises the survey results.

Table 3. Survey results

	Price (Unit CPU/day)	**Utilisation Rate (%)**
1	$1	121.0
2	$3 (Reference Value)	100.0
3	$5	88.6

Source: Calculated by author.

Next, a regression analysis was performed using the survey results. As shown in Table 4, R^2, which indicates the explanatory power of the entire model, is 0.972, and the significance probability p value is significant within 5%; therefore, the model can be considered to be at an appropriate level.

Table 4. Model test

	$\mathbf{R^2}$	**p**
Model	0.972	0.01

Source: Calculated by author.

Looking at the parameter estimate results in Table 5, the constant term was derived as 15, and the coefficient of usage demand V was derived as -0.01. For the constant term, according to the t-test, the p value was found to be 0.08 for usage fees P and 0.1 for V.

Table 5. t-test results

	β	t	p
Constant	15.0	7.20	0.08
V	-0.01	-5.84	0.10

Source: Calculated by author.

As a result of the regression analysis, the demand function can be expressed as Equation (15).

$$P = -0.01V + 15 \quad (15)$$

The first step in estimating the Nash equilibrium is to obtain the individual profit functions of national and specialised centres. Therefore, the marginal cost was calculated based on the existing fee system, and the sum of the demands of each centre was applied to the demand currently covered by the national centre and the newly incoming demand. In the case of new incoming demand, the workload can be divided between the national centre and the specialised centre. Therefore, using the sharing ratio α, the annual increase in demand for the use of existing national centres was assumed to be new inflow demand and added to improve the accuracy of the results. To estimate the Nash equilibrium, we can re-express the demand function as shown in Equation (16). The marginal cost m is three for national centres and six for specialised centres, depending on the usage fee system.

$$P = -0.01V + 15 \quad (16)$$

Total demand can be defined by Equation (17).

$$V_N + V_S + v_N + v_s = V \quad (17)$$

Here, the inflow demand can be expressed as Equation (18). α is a coefficient that indicates the rate of increase or decrease compared to existing demand.

$$v_N + v_S = \alpha \cdot (V_N + V_S) = \varepsilon \quad (18)$$

Using Equations (17) and (18) to create a profit function and perform partial differentiation for V_N and V_S, the response curves of the national and specialised centres can be derived as Equations (19) and (20).

$$V_N = -1/2\, V_S + (a - m)/2b(1 + \alpha) \quad (19)$$

$$V_S = -1/2\, V_N + (a - m)/2b(1 + \alpha) \quad (20)$$

Since α is a value that can have various ranges, the response curve must be estimated according to the range. Therefore, the range can be defined as listed in Table 6.

Table 6. Scope of α

	α	ε
1	< 1	$< V_N + V_S$
2	$= 1$	$= V_N + V_S$
3	> 1	$> V_N + V_S$

Source: Organized by the author.

Therefore, in the third range, there are cases where the incoming demand exceeds the existing demand. However, even in existing research cases, the new inflow demand appears to be smaller than the existing demand; therefore, a correction is necessary (Novshek, 1985). Consequently, this study adopts a new approach and improves the accuracy of the results by introducing additional assumptions in Equations (21) and (22): ρ represents the total amount of existing demand, and α represents the coefficient indicating the ratio of new inflow demand. Considering the value of the existing research cases, α did not exceed half of the existing demand, and all cases were analysed in 10% increments.

$$V_N + V_S = \rho \tag{21}$$

$$v_N + v_s \leq \alpha\rho \tag{22}$$

If we use Equation (21) again and apply the marginal cost to each centre, it becomes equivalent to Equations (23), (24), and (25).

$$V_N = -1/2\, V_S + 600/(1+\alpha) \tag{23}$$

$$V_S = -1/2\, V_N + 450/(1+\alpha) \tag{24}$$

$$0 < \alpha < 0.4 \tag{25}$$

The demand function is schematised as shown in Figures 3 and 4.

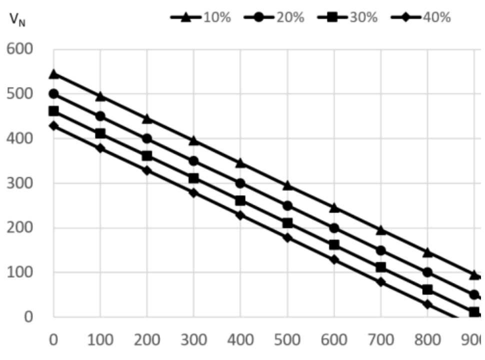

Figure 3. National centre demand function

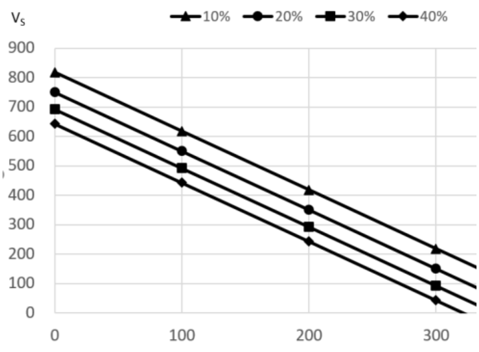

Figure 4. Specialised centre demand function

Source: Illustrated by authors.

The diagrams of all demand functions for deriving each Nash equilibrium point for α are shown in Figures. 5, 6, 7, and 8.

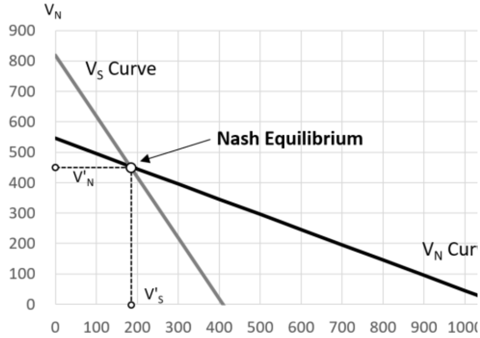

Figure 5. Nash equilibrium point for α = 10%

Source: Illustrated by authors.

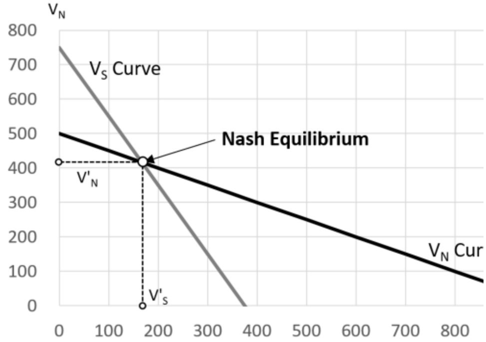

Figure 6. Nash equilibrium point for α = 20%

Source: Illustrated by authors.

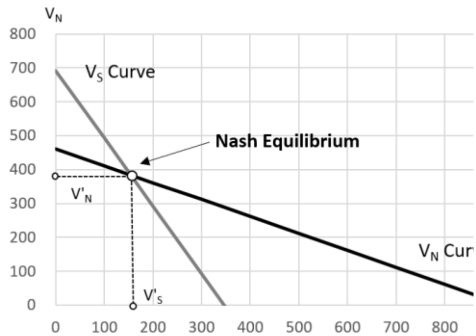

Figure 7. Nash equilibrium point for α = 30%

Source: Illustrated by authors.

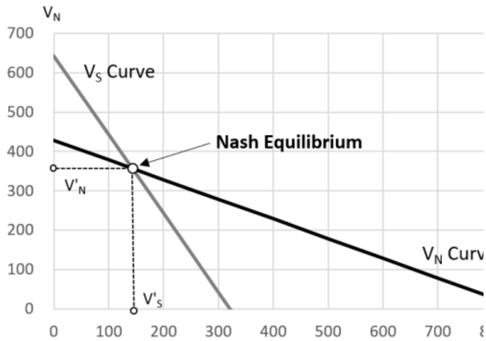

Figure 8. Nash equilibrium point for α = 40%

Source: Illustrated by authors.

The demand for national and specialised centres at the Nash equilibrium point is summarised in Table 7.

Table 7. Equilibrium point

α	Equilibrium Point	
	V_N	V_S
10%	455	182
20%	417	167
30%	385	154
40%	357	143

Source: Calculated by authors.

The Nash equilibrium point is schematised as shown in Figure 9, and it is evident that as the sharing ratio increases, the Nash equilibrium point gradually decreases.

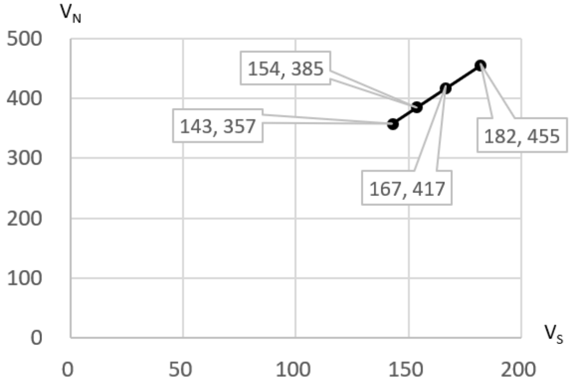

Figure. 9. Nash equilibrium
Source: Illustrated by authors.

In addition, the new inflow demand is calculated to derive policy implications. Table 8 presents the new inflow demand for each centre by applying the sharing ratio θ from Equation (26).

$$v_N + v_S = v_N + \theta \cdot v_S \quad (26)$$

The inflow demand calculation results are listed in Table 8. The total demand was calculated to be 700.

Table 8. Demand for each centre

k	Existing demand		Inflow demand		Total
	V_N	V_S	v_N	v_s	
10%	455	182	26	38	700
20%	417	167	47	70	700
30%	385	154	65	97	700
40%	357	143	80	120	700

Source: Calculated by authors.

5. Conclusions

When public supercomputer resources expand internationally, various economic analyses of duopoly and oligopoly industrial structures are required. This study calculates the demand model and Nash equilibrium point in the supercomputer industry, which comprises national and specialised centres. In the existing demand model, we applied an improved demand model that additionally considered the induced demand due to duplexing and calculated the change in the Nash equilibrium point according to the scale of the induced demand. If the government wants to pursue policies for a duopoly supercomputer industry in the future, four factors must be considered: First, the total demand; second, the usage fees for each centre; third, the new inflow demand; and fourth, the demand-sharing ratio for each centre. This can be expressed mathematically as Equation (27), and it is believed that effective decision-making based on quantitative evidence will be possible in future policy decisions.

$$f(x) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 \quad (27)$$

This study interprets the international duopoly phenomenon in the public supercomputer industry economically and derives policy implications for the establishment of effective promotional strategies. In the new duopoly ecosystem, usage demand and fee systems were modelled, and quantitative analysis results were derived. Based on the results of the analysis, the key decision-making factors for the government's strategy are presented. This is the first study in Korea to address the duopoly ecosystem and can serve as a reference for various studies, including future usage fee design and demand management. However, this study may contain some of the most basic survey errors and has limitations, such as errors in assessing the overall scale of demand for specialised centres. Therefore, in the future, we plan to conduct follow-up research based on actual operational data from a specialised centre.

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