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Downside Risk and Risk-Adjusted Performances of Industrial Metals

Abstract. *This paper investigates the level of risk and the risk-adjusted returns of five industrial metals. In the research process, we use several sophisticated approaches – EGARCH-NIG model, parametric and semiparametric CVaR risk measures, and four return-to-risk ratios. Aluminium has the lowest parametric CVaR at all probabilities, whereas lead has the upper hand in semiparametric CVaR. This happens because lead has the highest positive skewness and the lowest kurtosis. However, the riskiest metal is tin because of the highest negative skewness and highest kurtosis. As for the calculated ratios, copper is the best metal in the three out of four cases (Sharpe, Sortino, and modified STARR), primarily because copper recorded the highest price rise in the observed period. Aluminium has the best Treynor ratio because of the relatively low beta and the relatively high mean, whereas lead can serve as a good auxiliary instrument in combination with the S&P500 index due to the lowest beta.*

Keywords: *parametric and semiparametric downside risk, GARCH-NIG, risk-adjusted ratios.*

JEL Classification: C51, G32, Q02.

1. Introduction

Industrial metals are crucial raw materials in numerous manufacturing industries, such as machinery manufacturing, building construction, aviation, automotive, and mining. Shen et al. (2021) stated that aluminium and copper are among the most traded commodities, in general. Zhu et al. (2021) asserted that the purchase cost of non-ferrous metals accounts for almost 60% of the total cost in the aforementioned production and processing sectors, which means that price stability of non-ferrous metals is essential for successful development and growth of global economies. However, prices of industrial metals are heavily influenced by a wide range of factors, inter alia, imbalances in global production and consumption, rapid urbanisation and industrialisation of emerging markets, fluctuations in exchange rates, import and export policies, as well as short- and long-term trading strategies

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of funds (Gil-Alana and Tripathy, 2014; Zhong et al., 2019). In addition, the world has been in a turbulent mode in the last three years due to the pandemic and the Ukrainian war, which exerted a high price rise of industrial metals (see Figure 1). This means that heavy price fluctuations are an intrinsic part of non-ferrous metals (Irfanullah and Iqbal, 2023), where several reasons entail an accurate risk modelling and assessment of risk in these commodities. Most obviously, the presence of high price risk in industrial metal markets causes great uncertainty for producers, consumers, importing and exporting countries, but also produce huge costs to market participants who are involved in portfolio selection, derivative pricing, and risk management, as Wang et al. (2021) contended.

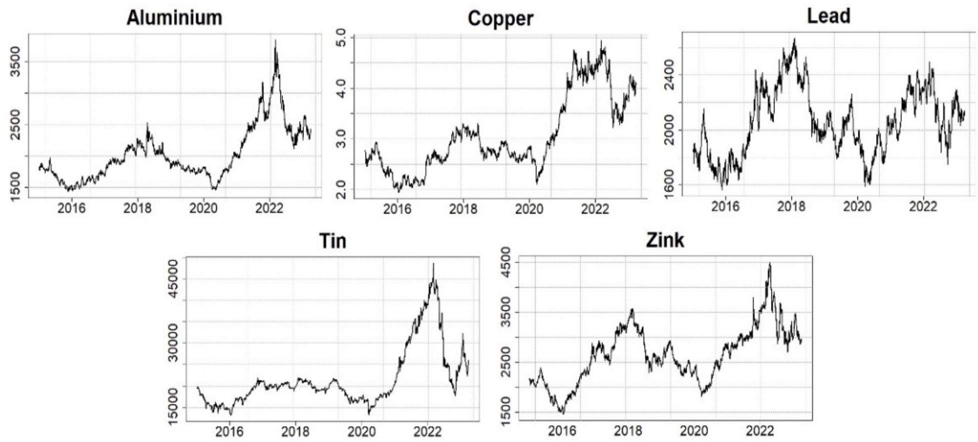

Figure 1. Empirical dynamics of industrial metal prices
Note: Price of copper is expressed in US cents per pound,
while all other commodities are expressed in USD per metric ton.
Source: Authors' calculation.

However, despite the significance of risk assessment in industrial metals markets from both empirical and theoretical perspectives, very few research papers have been done in this area, according to Wang et al. (2021). This leaves enough room for our contribution to the literature.

According to the above, this paper measures risk level as well as risk adjusted returns of the five industrial metals, aluminium, copper, lead, tin, and zinc. We take this dual approach because risk is only one side of the coin, while market agents are even more eager to know how much they can earn. Our motivation to do this research stems from the fact that very few papers investigated level of risk and risk-adjusted returns of industrial metals, while none of the papers used sophisticated methodological approaches in this process. Generally speaking, most papers measure risk *via* variance, which is not an efficient way of risk evaluation, because common variance considers equally positive and negative returns. This approach can yield biased conclusions, whereas the only risk that matters to the market participant is the risk of losses or the downside risk. The starting point in downside risk calculation is parametric Value-at-risk (VaR), but this approach is not an ideal risk measure because it is permeated with numerous drawbacks. A serious flaw of parametric VaR comes to the fore when the expected loss is greater than VaR at certain confidence levels, according to Aloui and Ben Hamida (2015). In this regard, instead of VAR, we consider the more conservative measure of losses – Conditional Value-at-Risk (CVaR) of Rockafellar and Uryasev (2002). In other words, VaR only observes a snapshot on the left tail of distribution at a particular probability, while CVaR indicates an average expected loss at the same level of probability. In this way, CVaR as a stricter measure of risk, gives higher estimates of losses compared to VaR.

Another shortcoming of a common VaR appears when the distribution does not follow a Gaussian function. This can result in a significant underestimation of losses that particular asset might endure, and this issue applies for both VAR and CVaR. Therefore, we follow the paper of Živkov et al. (2021), who researched agricultural commodities and utilised an improvement of common VaR, i.e., modified VaR (mVaR) of Favre and Galeano (2002). This particular method takes into account all the four moments of distribution, unlike parametric VaR, which considers only the first two moments. mVaR is based on the Cornish-Fisher (1938) expansion, and mVaR is better known as the semiparametric risk measure. Since CVaR is a more prudent risk measure than VaR, we calculate mCVaR instead of mVaR.

It is important to say that a reliable calculation of downside risk cannot be done if time-series are not independently and identically distributed (see e.g., Patra, 2021). In order to resolve this issue, we first create white noise error residuals with EGARCH model, which can efficiently deal with possible existence of autocorrelation and heteroscedasticity in empirical time-series. Besides, EGARCH is an asymmetric model that can recognise asymmetric response of volatility to positive and negative shocks. In addition, many authors, such as Kyrtsou et al. (2004), Iregui and Otero (2013), Dinică and Armeanu (2014), reported that non-ferrous metals are characterised by heavy tails and asymmetry. These findings give us enough reason to combine the EGARCH model with unconventional normal inverse Gaussian distribution (NIG) of Barndorff-Nielsen (1997). This elaborate function is appropriate because it has two parameters that can recognise the asymmetric and fat-tailed properties of empirical time series very well.

Having an information about the risk-level of an assets can indicate whether investment needs to be hedged, but this is only a half of a story, because every investor wants to know how much its investment is lucrative. Only from this aspect, an investor can grasp a whole picture and make proper decision whether to diversify investment, abandon it, or continue to invest. Therefore, besides downside risk calculation, we also calculate four different risk-adjusted ratios – Sharpe, Treynor, Sortino, and modified STARR (Stable Tail Adjusted Return Ratio). The first one is a standard return-to-risk indicator that observes the relation between risk-adjusted returns and a common standard deviation. However, the Sharpe ratio is biased because it takes into account both positive and negative returns, which can produce misleading estimates. To improve the ordinary Sharpe ratio, we calculate two additional indicators – Sortino and modified STARR. The former one uses standard deviation of only negative returns, which is known as downside deviation, i.e. it calculates the level of earnings per unit of average downside deviation. On the other hand, mSTARR is even stricter risk measure because it observes only specified set of negative returns under certain level of probability, which are placed at the left tail of the distribution. In other words, referring to Martin et al. (2003), we calculate a modified STARR by putting mCVaR in denominator instead of CVaR. At the end, the Treynor ratio combines empirical returns and a measure of systemic risk as denominator, which is beta (β). This ratio observes the level of earnings per unit of systemic risk.

Besides the introduction, the rest of the paper has the following structure. The second section presents a brief literature review. The third section explains used methodologies – GARCH-NIG model, risk measures and return-to-risk ratios. The fourth section contains the dataset and preliminary findings. The fifth section reveals results of calculated downside risk and four risk-adjusted ratios. The last section concludes.

2. Literature review

Risk evaluation of industrial metals was rarely a research subject in the existing literature. Papers are mostly focused on risk forecasting and risk spillover effect between non-ferrous markets and other commodity or financial markets, than on actual measurement of risk. For instance, Wang et al. (2021) used the dynamic model average (DMA) approach to forecast future realised volatility of four non-ferrous metals – aluminium, copper, lead, and zinc. They decomposed the realised volatility into continuous components and jump components and measured the leverage effect through either semi-variances or signed jumps. They concluded that volatility components of 1-day lag and short-term volatility transfer components are the most important predictors for future realise volatility of between metals. Mensi et al. (2021) investigated the dependence structure, risk spillovers and conditional diversification benefits between oil and six non-ferrous metals futures markets (aluminium, copper, lead, nickel, tin, and zinc), using a variety of copula functions and CVaR measure. They found significant lower tail dependence and upper tail independence between oil and non-ferrous metals markets. They asserted that lower temporal dependence is positive and heterogeneous between oil and non-ferrous markets, whereas for copper, lead, and tin markets, it intensified during the onset of the global financial crisis. The paper of Todorova et al. (2014) applied the HAR model to consider the volatility spillovers between the five of the most liquid non-ferrous metals contracts (aluminium, copper, lead, nickel, and zinc). They found that volatility series of other industrial metals appear to contain useful incremental information for future price volatility. However, they asserted that their own dynamics are often sufficient for describing most future daily and weekly volatility, with the most pronounced volatility spillovers identified in the longer term.

As for papers that studied risk measurement of non-ferrous metals, Brunetti and Gilbert (1995) analysed aluminium, copper, nickel, lead, tin and zinc in the period of 24 years, considering monthly data. They contended that volatility has shown no tendency to increase over this period, while only in the case of tin volatility levels were beneath their historic average levels over 1993-95, which is a period of increased speculative interest in the metal markets. They concluded that volatility of non-ferrous metals in this period was very volatile. The paper of McMillan and Speight (2001) researched six non-ferrous metals price volatility (aluminium, copper, nickel, lead, tin, and zinc) through GARCH component analysis. They decomposed volatility into long-run and short-run components, and concluded that non-ferrous metals prices are not only exposed to volatility persistence but also have some degree of long memory, which are ultimately stationary and mean-reverting. Gil-Alana and Tripathy (2014) investigated volatility persistence and the leverage effect across six non-ferrous metals, considering both spot and futures markets in India. They detected volatility persistence *via* the ARCH/GARCH class models. The leverage effect was tested using TGARCH and EGARCH models, and they found an asymmetric effect in seven out of twelve series using TGARCH model, while EGARCH captures the leverage effect in ten time-series. They also tested long memory features of the data and found that non-ferrous metal time-series are I(1), but the squared returns display long memory features.

3. Used methodologies

3.1 EGARCH model

In order to avoid biased estimates of downside risk, first task is to create white noise residuals that have no problem with autocorrelation and time-varying variance. Referring to Gil-Alana and Tripathy (2014), who found a leverage effect in industrial metals, we employ an asymmetric EGARCH model. However, unlike these authors, we also assume presence of fat tails and asymmetry in distribution of the selected time-series. Aiming to recognise these stylised facts, we combine unconventional NIG distribution with EGARCH model, since NIG has two parameters – skew (τ) and shape (ν), $\varepsilon \sim NIG(0, \sigma_t^2, \tau, \nu)$, which can successfully capture third and fourth moments of empirical distributions. In this way, the EGARCH-NIG model produces more accurate and reliable residuals, compared to the common GARCH model with traditional distributions. We overcome possible autocorrelation in the EGARCH model by using the first autoregressive term AR(1) in the mean equation, while the variance equation by default deals with heteroscedasticity. Following Moravcova (2018), specifications of the variance equations in EGARCH-NIG model are:

$$ln(h_{t,i}) = \alpha_i + \beta_i\, ln(h_{t-1,i}) + \gamma_i \left|\frac{\varepsilon_{t-1,i}}{\sqrt{h_{t-1,i}}}\right| + \delta_i \frac{\varepsilon_{t-1,i}}{\sqrt{h_{t-1,i}}}; \quad \varepsilon \sim NIG(0, \sigma_t^2, \tau, \nu) \qquad (1)$$

3.2 Downside risk measurement

In the process of risk measurement, we apply two downside risk methods – parametric and semiparametric CVaR. Both CVaR measures calculate an average loss of tail distribution assuming a certain probability. Following Yu et al. (2018), parametric CVaR is an integral of VaR, and can be calculated as in equation (2):

$$CVaR_{\alpha} = -\frac{1}{\alpha}\int_{0}^{\alpha} VaR(x)dx, \tag{2}$$

where $VaR(x)$ is Value-at-Risk of a particular industrial metal, while α denotes the left quantile of the standard normal distribution.

Although CVaR gives better risk assessment than VaR, both measures suffer from a common problem, i.e. they consider only the first two moments of distribution, while higher moments remain neglected. This issue can be especially pronounced in commodity time-series, which are characterised by negative skewness and heavy tails. In order to circumvent this problem, we additionally calculate semiparametric or modified CVaR, which takes into account all the four moments of distribution. More specifically, mCVaR penalises negative features of distribution, such as negative skewness and high kurtosis, which have an adverse effect on investors. On the other hand, mCVaR rewards favourable distributional characteristics, which are positive skewness and low kurtosis (see, e.g., Boo et al., 2017). This means that mCVaR risk might be lower than CVaR if positive characteristics of distribution prevail, but also these measures can be significantly higher than parametric risk measures if negative traits of distribution dominate. Equation (3) shows how mCVaR is calculated, and this measure is based on the Cornish–Fisher expansion presented in Equation (4):

$$mCVaR_{\alpha} = -\frac{1}{\alpha}\int_{0}^{\alpha} mVaR(x)dx, \tag{3}$$

where $Z_{CF,\alpha}$ is the non-normal-distribution percentile adjusted for skewness and kurtosis according to the Cornish–Fisher equation.

$$Z_{CF,\alpha} = Z_{\alpha} + \frac{1}{6}(Z_{\alpha}^{2} - 1)S + \frac{1}{24}(Z_{\alpha}^{3} - 3Z_{\alpha})K - \frac{1}{36}(2Z_{\alpha}^{3} - 5Z_{\alpha})S^{2}, \tag{4}$$

where S and K denote skewness and kurtosis of the particular non-ferrous metal.

3.3 Risk-adjusted return measures

Knowing how big the risk of losses might be is important, but investors are also very interested to know how much they can earn. To this end, this subsection presents the way in which four different return-to-risk ratios are calculated, where all the ratios take into account different risk-measures as denominator.

The first risk-adjusted measure is the well-known Sharpe ratio, which observes risk-adjusted returns *vis-à-vis* a common standard deviation (σ):

$$Sharpe\ ratio = \frac{R - R_f}{\sigma} \tag{5}$$

where R is an average log-return of a particular industrial metal, R_f is risk-free rate, and σ is standard deviation of a particular asset. Yields of 3M treasury bills denote a risk-free rate.

The Treynor ratio places risk-adjusted returns in relation to systemic risk, represented by beta (β).

$$Treynor\ ratio = \frac{R - R_f}{\beta}; \quad \beta = \frac{COV(R, R_M)}{\sigma_M^2} \tag{6}$$

where COV is covariance, while R_M represent the whole market, and the proxy for this variable is S&P500 index.

The Sortino ratio and mSTARR overcome the problem of using biased standard deviation in the denominator. Sortino ratio observes standard deviation calculated upon only negative portfolio returns (σ_D), which gives more realistic risk-adjusted returns.

$$Sortino\ ratio = \frac{R - R_f}{\sigma_D} \tag{7}$$

On the other hand, mSTARR is an even stricter indicator than Sortino ratio, in a sense that it uses the measure of downside risk calculated by the mCVaR metric as denominator.

$$mSTARR = \frac{R_p - R_f}{|mCVaR|} \tag{8}$$

4. Dataset and preliminary findings

This study uses daily data of short-maturity futures prices of the five industrial metals that are traded on the Chicago Mercantile Exchange – aluminium, copper, lead, tin, and zinc. We consider futures prices rather than spot prices, since futures prices process all available information faster than spot prices, making them more reliable. Our data-span covers the period between January 2015 and March 2023, and all the data is collected from investing.com website. All daily prices ($P_{i,t}$) are transformed into log-returns ($r_{i,t}$) according to the expression $r_{i,t} = 100 \times log(P_{i,t}/P_{i,t-1})$. Table 1 contains descriptive statistics of the selected assets, as well as Ljung-Box test results for level and squared returns. According to Table 1, only tin reports autocorrelation, while all the assets have problem with time-varying variance. ARMA-GARCH models can resolve these empirical issues, but time-series that enters GARCH must be stationary. In this regard, we present results of Dickey-Fuller generalised least square (DF-GLS) test, which refutes any doubt that selected time-series have a unit root. Regarding the first four moments, zinc has the highest standard deviation, while tin follows. This could indicate that these two metals have the highest CVaR risk, because the second moment is the key factor determining CVaR. High skewness is recorded only for tin, while all metals have relatively high kurtosis, which justifies the usage of NIG distribution that can recognise these stylised facts. Due to the high third and fourth moments, this might suggest that tin has the highest mCVaR. Also, high third and fourth moments indicate that none of the time-series follows a Gaussian function, which is confirmed by the Jarque-Bera test.

Table 1. Descriptive statistics of the selected industrial metals

	Mean	St. dev.	Skew.	Kurt.	JB	LB(Q)	LB(Q^2)	DF-GLS
Aluminium	0.006	0.555	0.026	5.503	544.9	0.580	0.000	-9.459
Copper	0.008	0.595	-0.081	4.629	238.638	0.325	0.000	-5.164
Lead	0.003	0.625	0.135	4.592	226.512	0.503	0.000	-6.403
Tin	0.006	0.654	-0.752	9.138	3469.255	0.000	0.000	-7.238
Zinc	0.006	0.685	0.039	4.685	247.285	0.980	0.000	-9.719

Notes: LB(Q) and LB(Q^2) tests indicate to p-values of Ljung-Box Q-statistics of level and squared residuals of 10 lags. 1% and 5% critical values for DF-GLS test with 5 lags, assuming only constant, are -2.566 and -1.941, respectively. JB refers to value of Jarque-Bera coefficients of normality.
Source: Authors' calculation.

In order to create white noise residuals that are required for downside risk calculation, we use the asymmetric ARMA(1,0)-EGARCH(1,1)-NIG model, and these results are presented in Table 2. According to Table 2, the ARCH effect is present in the two out of five cases, while a high persistence of conditional variance is recorded in all the five industrial metals. Asymmetric parameter (γ) is positive and significant for all the metals, which means that positive shocks have stronger effect on conditional variance than negative shocks. The estimated conditional variance parameters coincide with the findings of Gil-Alana and Tripathy (2014).

Table 2. Estimated EGARCH parameters

	Aluminium	Copper	Lead	Tin	Zinc
Panel A: EGARCH parameters					
α	-0.152^{***}	-0.021^{**}	-0.004	-0.020	-0.007
β	0.998^{***}	0.985^{***}	0.991^{***}	0.951^{***}	0.987^{***}
γ	0.271^{***}	0.064^{***}	0.053^{***}	0.284^{***}	0.073^{***}
Panel B: NIG distribution parameters					
τ	0.037	-0.030	0.052^{**}	-0.161^{***}	-0.052
ν	0.048^{***}	1.279^{***}	3.559^{***}	0.769^{***}	5.485^{***}
Panel C: Diagnostic tests					
LB(Q)	0.263	0.338	0.759	0.114	0.133
LB(Q^2)	0.954	0.218	0.441	0.174	0.767

Notes: LB-Q and LB-Q^2 test denote p-values of Ljung-Box Q-statistics for level and squared residuals of 10 lags. *, **, *** represent statistical significance at the 1%, 5% and 10% level, respectively.
Source: Authors' calculation.

τ parameter measures asymmetry in the NIG distribution, and it is only significant for lead and tin, i.e. the two metals that have the highest skewness. Shape parameter (ν) is significant for all the metals, because in all the cases relatively high kurtosis is recorded. Diagnostic tests indicate that the EGARCH model resolves autocorrelation and heteroscedasticity problems in the empirical time-series. This means that estimated residuals are white noise error terms (see Figure 2), which can be used in the downside risk calculation.

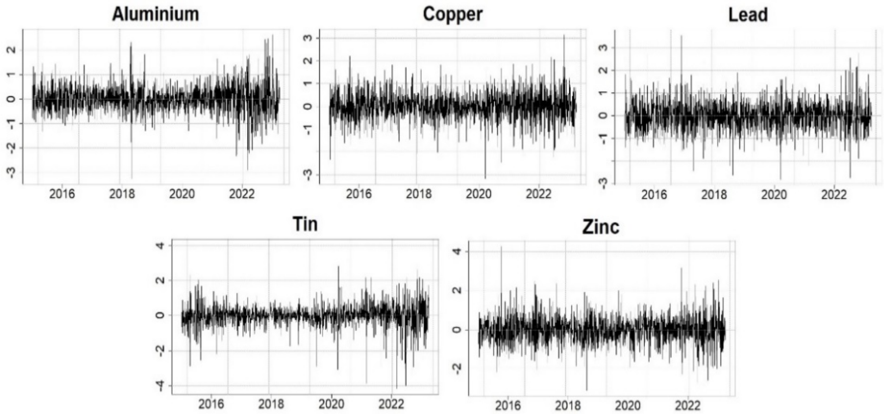

Figure 2. Estimated residuals of the five industrial metal
Source: Authors' calculation.

5. Empirical results

5.1 Parametric and semiparametric risk measures

This subsection presents the results of parametric and semiparametric downside risk measures, where Table 3 contains numerical estimates, while Figure 3 graphically illustrates these findings. Both CVaR and mCVaR are calculated at five different risk-levels, 96%, 97%, 98% 99%, and 99.5%, which portrays different aversion towards risk. We do not calculate downside risks under 96% because semiparametric risk estimates are accurate only above the level of 95.84%, according to Cavenaile and Lejeune (2012). This rule does not apply to parametric risk, but to make results comparable, we also calculate CVaR only above the 96% confidence level. Both CVaR and mCVaR are interpreted as an average loss at a certain level of probability. For instance, at 99.5% probability, CVaR of aluminium is -1.598, which means that potential loss of an investor might be 1.598% in a single day or worse.

According to Table 3, it is clear that mCVaR downside risk is higher than the CVaR counterpart at all the probability levels. This happens because CVaR uses only the first two moments in calculating the downside risk, while mCVaR considers all the four moments. Since none of the industrial metals follows a Gaussian distribution, it can be expected that mCVaR measures are higher than those of the CVaR peers.

Table 3. Results of parametric and semiparametric CVaR

	Probability	Aluminium	Copper	Lead	Tin	Zinc
CVaR	96%	-1.189	-1.275	-1.344	-1.403	-1.470
	97%	-1.252	-1.343	-1.415	-1.478	-1.548
	98%	-1.337	-1.434	-1.511	-1.578	-1.653
	99%	-1.472	-1.579	-1.663	-1.738	-1.820
	99.5%	-1.598	-1.714	-1.805	-1.886	-1.976
mCVaR	96%	-1.438	-1.485	-1.473	-2.345	-1.668
	97%	-1.576	-1.609	-1.591	-2.651	-1.807
	98%	-1.777	-1.788	-1.760	-3.107	-2.007
	99%	-2.140	-2.103	-2.058	-3.949	-2.360
	99.5%	-2.526	-2.432	-2.368	-4.865	-2.729

Note: Greyed values denote the lowest downside risk.
Source: Authors' calculation.

Looking at Figure 3, it is interesting to note that all the CVaR risk measures are neatly aligned one below the other without intersecting. This can be explained by the fact that the level of variance increases gradually with the rise of probability, and this transfers perfectly to the rise of CVaR because the second moment is the key in calculating CVaR. On the other hand, this kind of consistency does not exist in the mCVaR plot because some lines are overlapped. This happens because mCVaR risk is affected additionally by skewness and kurtosis, and these moments do not rise gradually as probability increases. This explains why tin has significantly higher mCVaR compared to the all other industrial metals, while on the CVaR results, tin is the second worst commodity. In other words, tin has the highest kurtosis (9.138) and the highest negative skewness (-0.752), which pushes tin to the last place in the mCVaR results. On the other hand, tin does not have the highest second moment (0.654), but zinc (0.685), which gives tin the second highest CVaR.

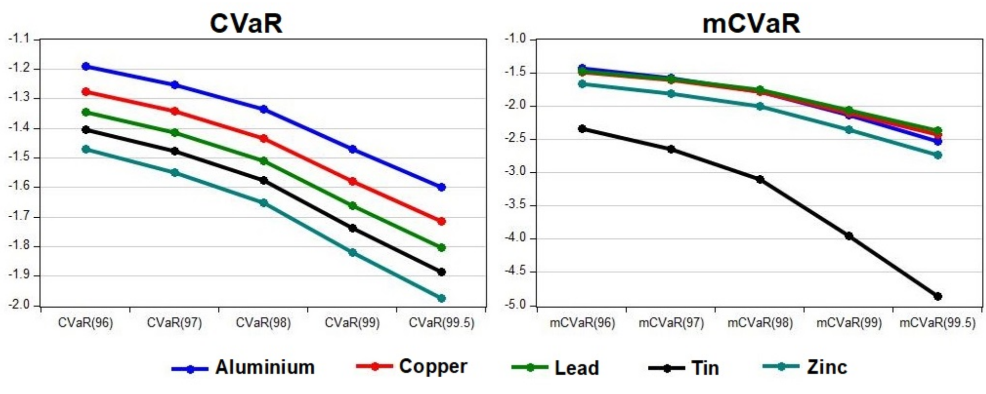

Figure 3. Graphical presentation of the calculated CVaR and mCVaR risks
Source: Authors' calculation.

As for the other industrial metals, aluminium has the lowest CVaR because it has the lowest second moment (0.555), while copper, lead, tin, and zinc follow, respectively. On the other hand, regarding the mCVaR metric, aluminium, copper, and lead have almost identical downside risk because the third and fourth moments come to the fore in this calculation. In other words, although aluminium has the lowest variance, it has the fourth largest kurtosis, which offsets its lowest second moment. On the other hand, lead has a relatively high second moment (0.625), but it has the lowest kurtosis (4.592), which decreases mCVaR of lead. Zinc has the second worst mCVaR at all probabilities, although it has positive skewness, but zinc has the highest second moment, which puts zinc behind aluminium, copper and lead. Overall results clearly indicate that CVaR risk is biased, and could lead to wrong conclusions.

5.2 Results of risk-adjusted returns

Knowing the level of potential losses is always relevant for investors, but market participants are even more eager to know the potential benefits of their investments. In this regard, this subsection presents the results of the four return-to-risk ratios that measure risk-adjusted returns of industrial metals, where all four ratios observe the level of risk from a different perspective. We calculate Sharpe ratio, Treynor ratio, Sortino ratio, and mSTARR, where the denominator is standard deviation, beta, downside deviation, and mCVaR, respectively. Table 4 contains these results, whereas Figure 4 graphically illustrates the findings, providing a visual comparative picture which metal has the best (worst) return-to-risk relation.

Table 4. Results of four return-to-risk ratios

	Sharpe ratio	Sortino ratio	Treynor ratio	mSTARR
Aluminium	0.0107	0.0165	0.0381	0.0024
Copper	0.0127	0.0188	0.0238	0.0031
Lead	0.0043	0.0069	0.0203	0.0011
Tin	0.0085	0.0101	0.0312	0.0011
Zink	0.0085	0.0130	0.0242	0.0021

Note: Greyed values denote the highest risk-adjusted ratio.
Source: Authors' calculation.

According to the results, copper has the best result in three out of four ratios, while aluminium follows. In the case of the Treynor ratio, copper and aluminium change their places, and now aluminium is the best, while copper follows. Regarding the Sharpe ratio, copper has an advantage, although it does not have the lowest second moment (0.595), but aluminium (0.555). However, copper has the highest mean (0.008), which gives upper hand to copper vis-à-vis all the other metals. Looking at Figure 1, all the metals recorded a steep rise during the pandemic, but copper had the smallest decline in 2022, which means that the price of copper rose the most on average. This gives a very good result of copper taking into account all the four ratios, regardless of the relative risk level of copper. On the other hand, lead has by far the worst result primarily because lead has the lowest mean (0.003), that is, the price of lead rose the least in the observed period.

Table 5. Beta value and downside standard deviation of the industrial metals

	Aluminium	Copper	Lead	Tin	Zinc
σ_D	0.360	0.402	0.389	0.549	0.446
Beta	0.156	0.317	0.132	0.177	0.240

Source: Authors' calculation.

As for the Sortino ratio, copper also has the best result, but not as good as the previous indicator. This happens because the downside standard deviation of copper is the third one (0.402), but the best Sortino result comes from the highest mean value (see Table 1). Aluminium has the lowest downside deviation, which puts it in a very good second place. Zinc has better results than tin, compared to the Sharpe ratio, because zinc has significantly lower downside deviation (0.446) than tin (0.549). Lead has the second lowest σ_D (0.389), but lea has the lowest mean, putting it in the last place.

The Treynor ratio gauges the relation between risk-adjusted returns and the level of systemic risk, which is beta. Beta measures the synchronisation of industrial metals with the benchmark, which is a whole market; in our case, it is S&P500 index. According to this indicator, aluminium has by far the best result, because the beta of aluminium is the second lowest, amounting 0.156, but aluminium has relatively high mean, which gives it the highest Treynor ratio. Lead has the lowest beta (0.132), which means that lead has the least synchronous movements with the whole market, which is good for diversification in combination with the S&P500 index. Tin has the second best Treynor ratio because tin has a relatively low beta (0.177), and a relatively high mean (0.006). Once again, due to the low mean, the Treynor ratio of lead is the worst one, while copper has higher Treyor ratio than lead, although copper has 2.4 times higher beta than lead, but copper has the highest mean.

mSTARR can be regarded as the strictest ratio, but also the most realistic one because it puts the most accurate measure of downside risk in the denominator, which is mCVaR. In this case, mCVaR is observed at 99.5% of probability. Positions of mSTARR values slightly alters in respect to Sortino ratio, according to Figure 4, because levels of mCVaR changes. In particular, lead is no longer the single worst metal, but it shares the last position with tin. Tin drops because it has by far the highest mCVaR (-4.865). On the other hand, lead has the lowest mCVaR (-2.368), but it also has the lowest mean, which equals mSTARR of lead and tin. Zinc increases the advantage with respect to tin, although these two metals have equable average returns because tin has a far worse level of downside risk (-4.865) in the form of mCVaR than zinc (-2.729).

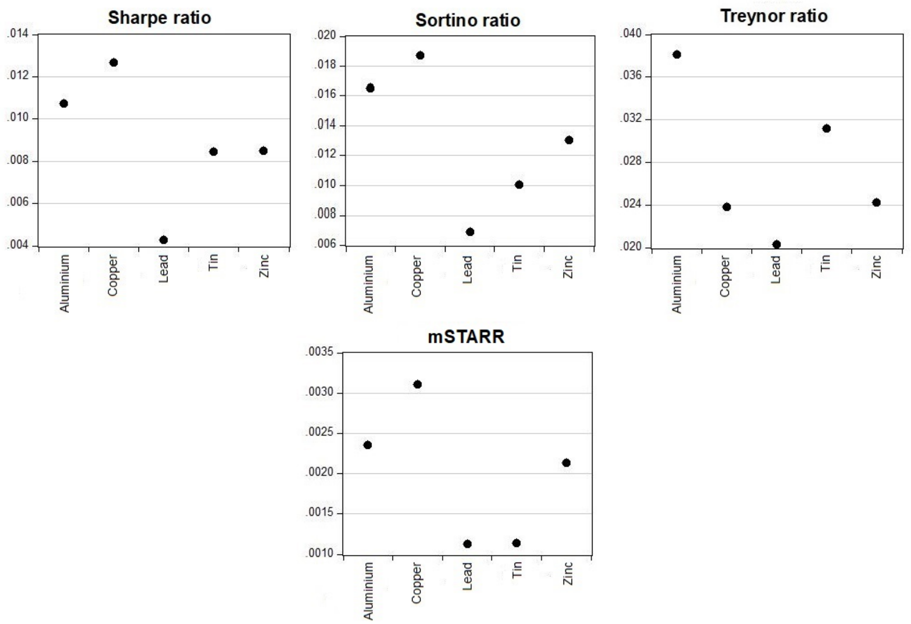

Figure 4. Graphical presentation of four return-to-risk ratios
Source: Authors' calculation.

6. Conclusions

This paper analyses five industrial metals from the aspects of risk and risk-adjusted returns. For the calculation purposes, we use several non-conventional and elaborate methodological approaches, in order to calculate accurate and reliable estimates. In particular, all the risk measurements are done using white noise residuals obtained from the EGARCH-NIG model. Instead of common variance, we use parametric and semiparametric CVaR measures. At the end, the risk-adjusted returns are gauged *via* four different ratios, which take into account four different risk measures in the denominator, providing a comprehensive picture about the selected metals.

Based on the results, several noteworthy findings can be reported. First, estimates of semiparametric downside risk are more accurate than parametric counterparts, because all the industrial metals have non-Gaussian properties. Due to these reasons, an order of calculated risk levels is different between parametric and semiparametric estimates, which justifies the usage of mCVaR. Second, aluminium has the lowest CVaR at all probabilities, while lead takes the upper hand in mCVaR when higher probabilities are observed. This happens because lead has favourable properties of the third and fourth moments, i.e. the highest positive skewness and the lowest kurtosis. On the other hand, the riskiest metal is tin because it has the highest negative skewness and the highest kurtosis.

As for the calculated ratios, copper is the best metal in the three out of four cases (Sharpe, Sortino, and mSTARR), primarily because copper has the highest mean. Aluminium has the best Treynor ratio because of its relatively low beta and relatively high mean.

Based on the results, it can be concluded that economic agents who work with tin need to hedge their positions because tin is prone to significant losses. On the other hand, the least risky metals are lead and aluminium. Copper is the best metal in terms of risk-adjusted results, while lead can be used as a good secondary instrument in combination with S&P500 index in a portfolio because it has the lowest beta.

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