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Asymmetric Volatility Connectedness Among G7 Stock Markets

Abstract. *This paper investigates the asymmetries in volatility connectedness among the G7 stock markets. We provide ample evidence for asymmetric volatility connectedness based on daily realized semi-volatility indices obtained from intra-day data. We find that the impact of bad volatility strictly dominates that of good volatility in generating connectedness across financial markets. The global financial crisis, the European debt crisis, and the COVID-19 pandemic have witnessed the most influential episodes of volatility connectedness. We also discuss that the effect of the US stock market on other countries has been caused primarily by bad volatility.*

Keywords: *asymmetric connectedness, realized semi-volatility, G7 stock markets.*

JEL Classification: C32, G15.

1. Introduction

A long tradition in finance holds that stock prices tend to fall simultaneously but rise independently. A large body of finance literature has also discussed that volatility tends to rise (or fall) in response to "bad" (or "good") news. This empirical phenomenon is often referred to as asymmetric or leverage volatility.[1] Because asymmetric patterns in the financial market transmission mechanism are relevant for portfolio diversification and risk management strategies, the presence of asymmetric spillover may pose a challenge to investors. Hence, there have been many attempts to capture asymmetric connectedness across financial markets.

The GARCH (generalized autoregressive conditional heteroskedasticity) models have been widely used as a formal econometric approach to empirical analysis when measuring volatilities in financial markets. For asymmetric volatility correlation, the EGARCH (exponential GARCH) model of Nelson (1991) and the GJR (Glosten-Jagannathan-Runkle) specification of Glosten et al. (1993) have long been adopted in the financial literature. As discussed in Wu (2001), the presence of asymmetric volatility is most evident during stock market crashes when significant increases in market volatility are often led by a big drop in stock prices. Cappiello et al. (2006) proposed the asymmetric generalized dynamic conditional correlation

[1] Black (1976) is considered as the seminal work on this issue.

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model to explain asymmetric conditional correlations and variances for a multivariate framework.

Although the GARCH-based model is often used to estimate the volatility of financial data, it cannot capture the spillover dynamics in a multivariate framework. Recently, Diebold and Yilmaz (2014) developed the connectedness methodology, a unified framework for conceptualizing and empirically measuring network connectedness at various levels. Consequently, many authors have employed the methodology in investigating connectedness across various markets and countries. For instance, Tsai (2014) discussed the connectedness among stock markets, and Antonakakis (2012) applied this approach to the Forex markets. Claeys and Vašícˇek (2014) considered the bond market, and Lee and Lee (2019) examined the housing markets. Furthermore, the connectedness across different asset-class markets is also discussed in Diebold and Yilmaz (2015) and Lee and Lee (2020), among others. These studies suggested that connectedness in return or volatility is time-varying and crisis sensitive.

Given that recent decades have experienced large perturbations in the financial markets, such as the global financial crisis and the European debt crisis, a significant body of literature has emerged on connectedness dynamics across financial markets. Although the presence of asymmetric volatility in financial data has been well recognized in the literature since the seminal work of Black (1976), asymmetries in volatility connectedness remain in an early stage. As discussed by Garcia and Tsafack (2011), the proper quantification of such asymmetries is highly relevant to portfolio selection and risk management strategies.

Furthermore, the availability of high-frequency data has opened new avenues for volatility analysis of financial markets. For instance, Andersen and Bollerslev (1998) introduced a robust measure for the actual market volatility, called realized volatility. Barndorff-Nielsen et al. (2010) proposed RS (realized semi-variance) that decomposes the volatility measures into good and bad volatilities caused by positive and negative returns. Segal et al. (2015) defined bad (good) uncertainty as the volatility associated with negative (positive) innovations to quantities such as in output and return.

Several studies investigated the asymmetry in volatility connectedness across financial markets based on the Diebold-Yilmaz connectedness approach. Baruník et al. (2016) examined asymmetries in volatility spillovers that emerge from good and bad volatilities. Based on the spillover asymmetry measures, Baruník et al. (2017) presented evidence for dominating asymmetries of bad volatility over good news in spillovers across the major Forex markets. Caloia et al. (2018) examined five European Economic and Monetary Union (EMU) stock markets, and Mensi et al. (2021) investigated international stock markets. Wang and Wu (2018) and Wang and Li (2021) discussed the asymmetric relationship between oil and stock markets.

This paper investigates asymmetric volatility connectedness among the G7 stock markets. We assess the magnitude of asymmetric connectedness measures and the dynamic patterns of their transmission mechanisms. This study is related to

BenSaïda (2019) and Mensi et al. (2021), which examined the asymmetric connectedness among the international stock markets.

However, this study is distinguishable in several aspects. First, we estimate the stock market's spillover effects using the asymmetric connectedness approach proposed by Baruník et al. (2016) and Baruník et al. (2017). Hence, we can quantitatively analyse and compare the asymmetric characteristics of both approaches, unlike the previous studies that only employed the methodology of Baruník et al. (2016). Second, we can also identify the contribution of each volatility to total connectedness indices, by using a further decomposition into asymmetric connectedness indices from the combined VAR. Third, our sample includes the 2020-2021 COVID 19 pandemic episode, which can further analyse the risk transmission mechanism in stock markets during the recent financial turmoil. Forth, we use high-frequency realized measures, whereas BenSaïda (2019) inferred the volatility measure from the GJR-GARCH model. High-frequency data might improve the estimation of dynamic volatilities, and the availability of realized measures can provide more accurate forecasts (Hansen and Lunde, 2011). The realized variance measures are a more accurate estimator for current latent volatilities than those derived from the GARCH-based model, as discussed in Andersen et al. (2003).

The remainder of the paper is organized as follows. Section 2 discusses the methodology, and Section 3 describes the characteristics of the data. Section 4 presents the empirical results and discusses their implications. A summary and concluding remarks are provided in Section 5.

2. Empirical methodology

This section briefly discusses the asymmetric connectedness methodology. First, we introduce the concept of RV and RS measures. We then explain the connectedness indices and describe how to estimate asymmetries in volatility connectedness.

2.1 Realized variance (RV) and semi-variance

Consistent with Andersen and Bollerslev (1998), the RV can be defined as the sum of intraday squared returns, which can be derived as follows:

$$RV = \sum_{i=1}^{n} {r_i}^2, \tag{1}$$

where r_i is the intraday returns at five-minute intervals.

Barndorff-Nielsen et al. (2010) introduced the measure of RS, which can separate positive and negative movements in a financial time series to analyze the asymmetric effects of volatility. The positive and negative RSs (RS^+ and RS^-) are defined as follows:

$$RS+= \sum_{i=1}^{n} \mathrm{I}(r_i \geq 0){r_i}^2 \tag{2}$$

$$RS–= \sum_{i=1}^{n} \mathrm{I}(r_i < 0){r_i}^2, \tag{3}$$

where $I(\cdot)$ is the indicator function. The positive and negative RSs provide information on the upside opportunity and downside risk of the underlying variable.

The sum of positive and negative RSs is always equal to the RV (i.e., $RV_t = RS_t^+ + RS_t^-$). We can use the RSs to estimate the volatility connectedness measures because of good or bad volatilities and then quantify asymmetries in volatility connectedness across different financial markets.

2.2 Connectedness approach

We measure connectedness using the generalized variance decomposition approach discussed in Diebold and Yilmaz (2014). The primary advantage of the generalized method is to obtain connectedness indices robust to variable ordering. For a covariance stationary m-variable VAR (p) process:

$$RV_t = \sum_{i=1}^{p} \Phi_i RV_{t-i} + \varepsilon_t \text{ with } \varepsilon_t \sim (0, \Omega),$$

we have a moving average representation:

$$RV_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i},$$

where $m \times m$ coefficient matrices A_i are derived as: $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \cdots + \Phi_p A_{i-p}$ with $A_0 = I_m$ and $A_i = 0$ for $i < 0$.

The h-step-ahead forecast error variance decompositions are computed as:

$$\theta_{ij} = \frac{\omega_{jj}^{-1} \sum_{k=0}^{h-1} (e_i' A_k \Omega e_j)^2}{\sum_{k=0}^{h-1} (e_i' A_k \Omega A_k' e_i)} \quad (4)$$

where $\mathbf{\Omega}$ is the variance matrix for the error vector ε_t, ω_{jj} is the variance of ε_{jt}, and e_i is the selection vector with ith element unity and zero otherwise. Because $\sum_{j=1}^{m} \theta_{ij} \neq 1$, we normalize each entry using the row sum: [2]

$$\tilde{\theta}_{ij} = \frac{\theta_{ij}}{\sum_{j=1}^{m} \theta_{ij}}. \quad (5)$$

By construction, it holds that $\sum_{j=1}^{m} \tilde{\theta}_{ij} = 1$. Equation (5) represents a pairwise directional connectedness $\tilde{\theta}_{ij}$, from market j to market i (at horizon h), from which we can derive various connectedness measures. By denoting $\tilde{\theta}_{ij}$ as $C_{i \leftarrow j}$, we can explicitly indicate the direction of connectedness. We are also interested in the net pairwise directional connectedness, defined as:

$$C_{ij} = C_{i \leftarrow j} - C_{j \leftarrow i} \quad (6)$$

Next, the total directional connectedness has two measures: 'from' and 'to', which can be obtained as the off-diagonal row sum and column sum, respectively. The total directional connectedness received from others to i can be defined as:

[2] Although this row normalization scheme may lead to inaccurate measures of the net connectedness, as discussed in Caloia et al. (2018), it is most often used for interpretative purposes.

$$C_{i\leftarrow\bullet} = \sum_{\substack{i,j=1 \\ j\neq i}}^{m} \tilde{\theta}_{ij} \ . \tag{7}$$

Similarly, the total directional connectedness to others from i can be computed as:

$$C_{\bullet\leftarrow \mathrm{i}} = \sum_{\substack{i,j=1 \\ j\neq i}}^{m} \tilde{\theta}_{ji} \ . \tag{8}$$

Sometimes, we are also interested in net total directional connectedness, defined as the difference between the 'to' and 'from' others:

$$C_{\mathrm{i}} = C_{\bullet\leftarrow \mathrm{i}} - C_{i\leftarrow\bullet}. \tag{9}$$

The total connectedness is the ratio of the sum of the off-diagonal elements of the variance decomposition matrix to the sum of all its elements.

$$C_m = \frac{\sum_{\substack{i,j=1 \\ i\neq j}}^{m} \tilde{\theta}_{ij}}{\sum_{i,j=1}^{m} \tilde{\theta}_{ij}} = \frac{\sum_{\substack{i,j=1 \\ i\neq j}}^{m} \tilde{\theta}_{ij}}{m} \tag{10}$$

2.3 Asymmetric volatility connectedness

We estimate the asymmetric connectedness measures caused by good and bad volatilities using the decomposed RV_t indices: positive and negative semi-variances (RS_t^+ and RS_t^-). In this case, the asymmetric connectedness can be obtained using two approaches. First, we can estimate two separate VAR (vector autoregressive) models for positive and negative semi-variances, as examined in Baruník et al. (2016). Second, we can use a single VAR system that combines both positive and negative semi-variances, as discussed by Baruník et al. (2017).

(1) Connectedness asymmetry measures (CAM)

We capture the degree of asymmetries for individual market i using the directional connectedness asymmetry measure (CAM) as discussed by Baruník et al. (2017). The directional CAM for an individual market can be defined as the difference in responses to good and bad volatility shocks from market (or country) i to other markets.

$$CAM_{\bullet\leftarrow \mathrm{i}} = C^{+}_{\bullet\leftarrow \mathrm{i}} - C^{-}_{\bullet\leftarrow \mathrm{i}} \qquad (\text{for i} = 1,\cdots,m), \tag{11}$$

where $C^{+}_{\bullet\leftarrow \mathrm{i}}$ and $C^{-}_{\bullet\leftarrow \mathrm{i}}$ are the total directional connectedness to others from i for good and bad volatilities, respectively. This measure can be used to examine the asymmetries in volatility connectedness for a given market.

We can also quantify asymmetries in volatility connectedness for the entire system using the total directional CAM, defined as the difference between volatility connectedness measures caused by positive and negative returns from all markets (or countries) in the VAR system:

$$CAM_m = \sum_{i=1}^{m} (C^{+}_{\bullet\leftarrow \mathrm{i}} - C^{-}_{\bullet\leftarrow \mathrm{i}}). \tag{12}$$

The total directional CAM_mcharacterises the asymmetric pattern in volatility connectedness for the entire market system under investigation. For instance,

$CAM_m = 0$ indicates that RS^+ and RS^- have the same degrees of connectedness with no asymmetric effects. Otherwise, there are connectedness asymmetries. A negative CAM_m indicates that the volatility connectedness from bad news is higher than that from good news.

(2) Total connectedness for semi-variance

When we use a single VAR system, by stacking both positive and negative semi-variances, we must adjust the total connectedness measure in Equation (10), as proposed in Baruník et al. (2017). Besides the main diagonal elements $(i = j)$, we must exclude the cases for $|i - j| = m$, which denote own market connectedness between good and bad volatilities.

$$C_{2m} = \frac{\sum_{\substack{i,j=1, i\neq j \\ |i-j|\neq m}}^{2m} \tilde{\theta}_{ij}}{2m} \quad (13)$$

This measure of total connectedness represents the degree of connectedness across different financial markets when the RV is decomposed into RS^+ and RS^- series.

3. Data and descriptive statistics

We use daily observations on RSs obtained from intra-day returns for the G7 stock markets: S&P 500 (U.S.), S&P/TSX (Canada), FTSE 100 (UK), CAC 40 (France), DAX (Germany), FTSE MIB (Italy), and Nikkei 225 (Japan). The data span from May 2, 2002, to August 31, 2021, with 5,029 daily observations available from the Oxford-Man Institute's Quantitative Finance Realized Library. The Realized Library provides five-minute sampled RVs and RSs. When the market indices are unavailable on holidays, the previous-day indices are used.

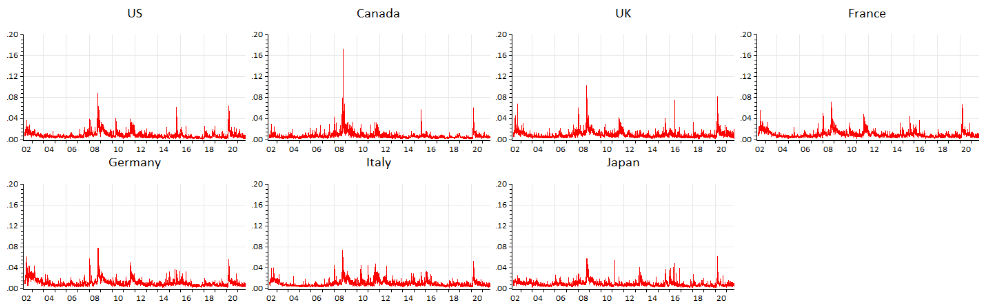

Figure 1. Time series plot of daily realized volatility

Note: This figure displays the time variations for G7 stock market volatilities from May 2, 2002 to August 31, 2021.

Source: https://realized.oxford-man.ox.ac.uk.

Figure 1 displays the time series plots of the realized volatility for the G7 stock markets. Highly persistent patterns in volatility dynamics are observed, with huge jumps in all volatilities around the global financial crisis and the recent COVID-19 pandemic.

Table 1. Descriptive statistics for log RV, log RS^+, and log RS^-

	US	Canada	UK	France	Germany	Italy	Japan
Panel A: $log\ \boldsymbol{RV}$							
Mean	-10.001	-10.319	-9.718	-9.562	-9.475	-9.605	-9.803
Max	-4.860	-3.527	-4.547	-5.274	-5.136	-5.245	-5.560
Min	-13.618	-13.403	-13.529	-12.352	-12.394	-14.083	-13.091
Std. dev	1.172	1.105	1.037	1.008	1.040	0.985	0.935
Skewness	0.491	0.904	0.634	0.480	0.540	0.446	0.465
Kurtosis	3.474	4.511	3.636	3.278	3.389	3.155	3.802
Panel B: $\frac{1}{2}log\ \boldsymbol{RS}^{+}$							
Mean	-5.364	-5.535	-5.229	-5.141	-5.100	-5.171	-5.276
Max	-2.734	-1.787	-2.975	-2.927	-2.756	-3.064	-3.220
Min	-7.107	-7.081	-7.198	-6.555	-6.612	-7.246	-7.076
Std. dev	0.589	0.560	0.527	0.505	0.519	0.494	0.480
Skewness	0.520	0.874	0.632	0.489	0.555	0.459	0.422
Kurtosis	3.557	4.465	3.566	3.358	3.527	3.245	3.705
Panel C: $\frac{1}{2}log\ \boldsymbol{RS}^{-}$							
Mean	-5.405	-5.582	-5.275	-5.148	-5.113	-5.163	-5.283
Max	-2.824	-2.682	-2.360	-2.883	-2.802	-2.778	-2.954
Min	-7.656	-7.225	-7.037	-6.827	-6.617	-7.588	-6.916
Std. dev	0.636	0.608	0.572	0.534	0.558	0.526	0.511
Skewness	0.432	0.801	0.635	0.419	0.473	0.346	0.417
Kurtosis	3.321	4.171	3.551	3.119	3.184	3.070	3.686

Source: authors' calculation.

Consistent with Andersen et al. (2003), we used the log transformation to obtain approximate Gaussian measures. Table 1 presents the descriptive statistics for the log-realized volatility and positive and negative log semi-volatilities. Most of the volatility series seem to follow approximate Gaussian processes. As expected, the standard deviations of negative semi-variances are higher than those of positive semi-variances for all the G7 stock markets.

4. Empirical results

In this section, we estimate the usual symmetric connectedness measures using the RV_t indices of the G7 stock markets. We then extended the framework to examine asymmetries in volatility connectedness across different financial markets. Finally, we assess the time-varying aspects of asymmetric connectedness using a rolling-sample estimation with 250-day windows.

4.1 Full-sample analysis

(1) Symmetric volatility connectedness analysis

Table 2 presents the estimation result on the full-sample (symmetric) volatility connectedness among the G7 stock markets. The results are based on the VAR (5) model, selected by the Schwarz information criterion, and ten-day ahead forecast error variance decompositions.

Table 2. Symmetric volatility connectedness table

	US	**Canada**	**UK**	**France**	**Germany**	**Italy**	**Japan**	**From**
US	39.29	15.99	9.82	12.68	11.56	8.75	1.92	60.71
Canada	20.86	45.49	8.67	9.22	7.56	6.70	1.50	54.51
UK	16.18	10.26	30.50	16.69	13.78	10.63	1.96	69.50
France	13.30	7.91	11.84	27.65	20.08	17.71	1.51	72.35
Germany	12.58	6.95	10.13	21.42	31.30	15.76	1.86	68.70
Italy	11.22	7.10	8.24	20.60	16.51	35.18	1.14	64.82
Japan	12.39	8.24	6.47	7.29	7.12	4.92	53.58	46.42
To	86.53	56.45	55.17	87.89	76.61	64.48	9.88	437.01
Net	25.82	1.95	-14.32	15.54	7.91	-0.34	-36.54	**62.43%**

Source: authors' calculation.

For the diagonal elements of the connectedness matrix in Table 2, the Japanese stock market has the highest own variance share (53.58%), followed by the Canadian market (45.49%). Japan has the lowest 'to' and 'from' connectedness (9.88% and 46.42%). These results suggest that the Japanese market contributes minimally to generating connectedness among the G7 stock markets.

European countries such as France, Germany, and Italy have relatively high pairwise connectedness with each other in terms of off-diagonal elements. The highest pairwise connectedness measures are observed between France and Germany (21.42% and 20.08%), similar to those of Diebold and Yilmaz (2015, Table 4.5). These results indicate a relatively strong tie between the two neighbouring stock markets. The French stock market has the highest 'to' connectedness (87.89%), followed by the US market (86.53%). The 'from' connectedness of France is also the highest (72.35%), followed by the UK (49.50%).

The net total directional connectedness measures ("to" and "from" others) vary substantially across countries. The US market has the highest net connectedness (25.82%), followed by the French market (15.54%), indicating that these countries are net transmitters of stock market volatilities. In contrast, the UK and Japan exhibit negative net connectedness measures (-14.42% and -36.54%, respectively). The total connectedness is 62.43%, indicating that 37.57% of the variations are caused by idiosyncratic shocks. The connectedness among the G7 markets seems relatively high. Similar observations were discussed in Diebold and Yilmaz (2015), although they examined the connectedness measures of financial markets from different combinations of countries and asset classes.

(2) Semi-volatility connectedness analysis

We analyzed the asymmetric connectedness by estimating two separate VAR models for the decomposed RV indices (i.e., positive and negative semi-variances: RS^+ and RS^-). Table 3 presents the estimation results on semi-volatility connectedness measures for the G7 stock markets. The results are based on VAR (5) model for the forecast error variance decompositions with a ten-day horizon.

The difference between the total connectedness measures of good and bad volatilities is noticeable, indicating asymmetries in volatility connectedness. This difference might be called the total CAM (connectedness asymmetric measure). The negative value of CAM_m = -10.20% (55.58–65.38%) indicates that cross-market linkages tend to strengthen when stock markets are under downside risk, compared with when stock markets exhibit upside variation. This result cannot be observed when we use only the realized volatility indices. Given the (symmetric) total connectedness measure (62.43%) in Table 2, we can conjecture that the total connectedness for positive volatility (58.58%) tends to be overestimated, whereas that for negative volatility (65.38%) it is underestimated when the potential asymmetries are not adequately considered.

Table 3. Semi-volatility connectedness table

	US	Canada	UK	France	Germany	Italy	Japan	From
Panel A : RS^+								
US	41.30	12.76	6.25	15.14	12.29	10.16	2.10	58.70
Canada	20.67	45.90	6.15	10.21	7.68	7.33	2.06	54.10
UK	15.70	7.67	32.92	17.77	13.35	10.57	2.04	67.08
France	12.60	4.90	8.43	32.33	20.96	19.79	0.99	67.67
Germany	11.59	4.19	7.13	23.53	35.23	17.10	1.24	64.77
Italy	9.88	4.08	5.26	23.00	16.82	40.41	0.55	59.59
Japan	10.85	5.32	3.18	7.26	6.76	4.77	61.86	38.14
To	81.28	38.92	36.40	96.91	77.86	69.71	8.97	410.05
Net	22.57	-15.18	-30.68	29.24	13.09	10.13	-29.17	**58.58%**
Panel B : RS^-								
US	36.10	16.91	11.69	12.76	11.76	9.04	1.73	63.90
Canada	20.14	40.16	10.87	10.47	8.74	8.17	1.44	59.84
UK	14.97	11.26	28.70	17.18	14.00	11.40	2.48	71.30
France	12.99	9.22	14.76	26.15	19.17	16.24	1.47	73.85
Germany	12.79	8.53	13.14	20.63	28.45	14.77	1.70	71.55
Italy	11.49	8.93	11.69	19.26	16.02	31.58	1.03	68.42
Japan	12.02	9.41	9.24	6.87	6.71	4.56	51.20	48.80
To	84.39	64.25	71.40	87.17	76.40	64.19	9.86	457.65
Net	20.50	4.41	0.10	13.32	4.85	-4.23	-38.95	**65.38%**

Source: authors' calculation.

(3) Asymmetric volatility connectedness analysis

The results of semi-volatility connectedness in Table 3 help examine the asymmetries in volatility connectedness. However, the analysis does not consider asymmetries in cross-market connectedness (i.e., between good and bad volatility). We address this issue by estimating asymmetric volatility connectedness by combining positive and negative semi-variances in a single VAR system, as discussed by Baruník et al. (2017).

Table 4. Asymmetric volatility connectedness table

		RS^+							RS^-							
		US	CAN	UK	FRA	GER	ITA	JP	US	CAN	UK	FRA	GER	ITA	JP	From
RS^+	US	**19.46**	4.71	1.64	4.23	3.11	2.47	0.22	**21.54**	10.80	8.48	8.27	7.78	5.76	1.54	59.00
	Canada	9.22	**29.07**	2.24	3.08	2.22	1.85	0.32	12.12	**18.94**	5.86	5.47	4.40	4.11	1.10	51.99
	UK	4.44	2.16	**19.79**	6.37	4.59	3.02	0.38	10.84	7.60	**12.97**	10.52	8.60	7.33	1.39	67.24
	France	3.19	1.05	2.48	**15.05**	7.89	7.58	0.08	9.61	6.44	9.46	**14.57**	11.55	9.80	1.25	70.38
	Germany	2.77	0.91	2.12	9.28	**16.49**	6.23	0.11	9.33	6.02	8.41	12.44	**15.45**	8.92	1.52	68.06
	Italy	2.13	0.75	1.16	8.67	5.62	**18.95**	0.02	8.40	6.11	7.59	12.28	10.09	**17.20**	1.03	63.85
	Japan	2.30	0.96	0.63	1.93	1.70	1.27	**36.43**	8.86	6.35	6.03	4.80	4.55	2.83	**21.36**	42.21
RS^-	US	**6.37**	1.35	0.42	2.68	1.88	1.50	0.05	**31.69**	14.31	10.12	10.53	9.97	7.57	1.57	61.94
	Canada	3.05	**3.88**	0.54	2.16	1.17	1.47	0.03	17.85	**35.54**	9.40	8.81	7.95	6.95	1.22	60.59
	UK	2.11	0.65	**1.89**	5.07	3.32	3.44	0.03	12.64	9.05	**24.44**	14.01	11.77	9.34	2.25	73.67
	France	1.90	0.52	1.07	**7.44**	4.52	4.78	0.05	10.44	7.19	11.57	**21.02**	15.44	12.76	1.32	71.54
	Germany	1.49	0.26	0.62	5.46	**6.63**	3.88	0.04	10.38	7.00	10.63	16.60	**23.31**	12.13	1.55	70.06
	Italy	1.22	0.33	0.48	4.80	2.81	**9.15**	0.02	9.55	7.19	9.32	15.37	13.42	**25.33**	1.01	65.51
	Japan	1.75	0.64	0.12	1.87	1.31	1.35	**9.17**	10.34	7.51	7.84	5.83	5.88	3.73	**42.66**	48.17
To		35.57	14.28	13.52	55.59	40.13	38.83	1.36	130.37	95.58	104.68	124.93	111.39	91.23	16.75	874.19
Net		-23.43	-37.71	-53.72	-14.79	-27.93	-25.02	-40.85	68.43	34.99	31.01	53.39	41.33	25.72	-31.42	**62.44%**

Source: authors' calculation.

Based on the asymmetric volatility connectedness in Table 4, we can distinguish how good and bad volatilities of individual markets propagate across other markets. The total asymmetric connectedness measure is 62.44%, similar to the total symmetric connectedness measure (62.43%) in Table 2. Besides the main diagonal elements, the cases for $|i-j|=m$ are also excluded because they indicate their own market connections between good and bad volatilities. All excluded numbers are highlighted in bold, and we sum (2m–2) numbers for every column.

The "to" connectedness reveals that the effects of bad volatilities are much larger than those of positive volatilities for all the G7 stock markets. The net connectedness measures of all good volatilities are minus, whereas those of bad volatilities are plus, except for Japan.[3] Table 4 indicates a limited role of the Japanese stock market among the G7 countries, consistent with the symmetric connectedness analysis in Table 2. The US bad volatility has the highest net connectedness measure (68.43%). These results can serve as further evidence that bad volatility dominates good volatility in most financial markets.

[3] Note here that the Japanese own market connectedness between good and bad volatilities displays dominating bad-to-good directional connectedness (21.36%) over good-to-bad connectedness (9.17%) in Table 4. However, the net connectedness of bad volatility for Japan is negative (-31.42%), because Japan is the largest net recipient of stock market volatility among the G7 countries, as presented in Table 2.

4.2 Dynamic analysis

The full-sample analysis in the previous subsection provides an 'average' aspect of connectedness for the entire sample period. However, the connectedness may vary over time depending on the economic conditions. The advantage of dynamic analysis is to monitor how the degrees of connectedness fluctuate, as evidenced by the GFC and the COVID-19 pandemic, which propagated across international financial markets. In this subsection, we provide a dynamic analysis by estimating 250-day rolling sample windows with a ten-day forecast horizon.

(1) Total connectedness

Figure 2 presents the time-varying pattern of the total volatility connectedness obtained from 250-day rolling-window samples. The symmetric and asymmetric total connectedness measures are based on the approaches in Tables 2 and 4. The two connectedness measures move very close to each other with almost the same trends.

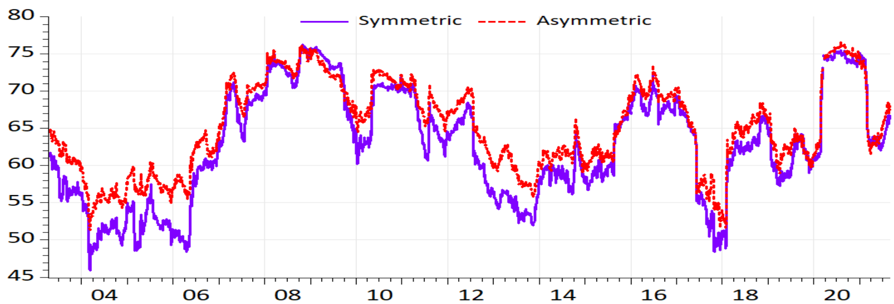

Figure 2. Dynamic symmetric and asymmetric total connectedness
Source: authors' calculation.

As expected, both total connectedness indices rapidly increased around the Lehman Brothers bankruptcy in September 2008. They decreased gradually until 2015, although they exhibited several peaks during several major economic events: the European debt crisis, which evolved from the bailouts of Greece in May 2010, the US credit rating downgrade in August 2011, and the Bernanke shock in May 2013. The total connectedness measures exhibited slight increases during 2015 and 2016, with increasing uncertainties concerning increases in the US federal funds rate and the Chinese stock market crash. The two indices also experienced a significant jump around the outbreak of the COVID-19 pandemic in March 2020.

Next, we decompose the asymmetric total connectedness into good-to-good, good-to-bad, bad-to-good, and bad-to-bad connectedness measures. In this analysis, we can decompose the 14×14 matrix in Table 4 into four 7×7 submatrices so that the sum of four components is equal to the asymmetric total connectedness. This decomposition helps identify the contribution of each volatility (i.e., good and bad) to the overall asymmetric connectedness.

First, the upper-left 7×7 submatrix in Table 4 can be viewed as the good-to-good connectedness ($C_{G-to-G} = \frac{1}{14}\sum_{i,j=1,i\neq j}^{7}\tilde{\theta}_{ij}$). Second, the lower-left 7×7 submatrix is concerned with good-to-bad connectedness ($C_{G-to-B} = \frac{1}{14}\sum_{i=8}^{14}\sum_{\substack{j=1,\\ |i-j|\neq 7}}^{7}\tilde{\theta}_{ij}$). Similarly, the bad-to-good and bad-to-bad connectedness measures are calculated as: $C_{B-to-G} = \frac{1}{14}\sum_{i=1}^{7}\sum_{\substack{j=8,\\ |i-j|\neq 7}}^{14}\tilde{\theta}_{ij}$, $C_{B-to-B} = \frac{1}{14}\sum_{i,j=8,i\neq j}^{14}\tilde{\theta}_{ij}$.

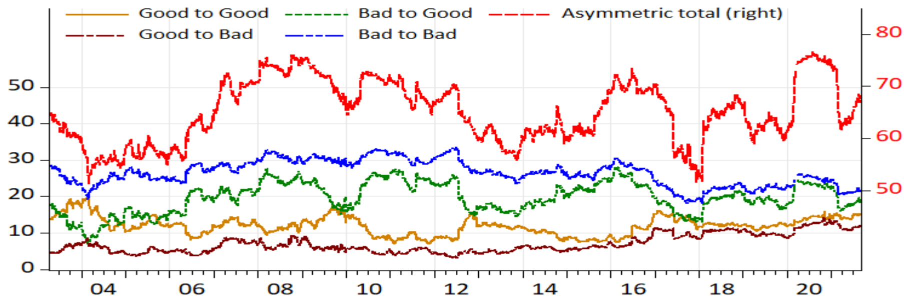

Figure 3. Decomposition of asymmetric total connectedness
Note: Good-to-good, bad-to-good, good-to-bad, and bad-to-bad measures, scaled on left axis, are decomposed from asymmetric total connectedness in Table 4. Asymmetric total connectedness is on the right axis.
Source: authors' calculation.

Figure 3 shows the decomposed asymmetric total connectedness indices. The connectedness measure for bad-to-bad volatility is the highest, whereas the good-to-good volatility connectedness is the lowest. This observation suggests that bad volatility contributes much more to total connectedness than good volatility.

(2) Connectedness asymmetric measure (CAM)

We now examine the asymmetric features of connectedness using the CAM introduced in Section 2.3. We can first define the CAM as the difference in the total connectedness between good and bad volatilities obtained from separate VARs in Table 3. Figure 4 displays the time series plot of the estimated CAM together with the total good and bad connectedness graphs. The total bad connectedness measures are relatively higher than the total good connectedness measures.

Consequently, most of the estimates for the CAM are negative, except from mid-2008 to mid-2009. This observation suggests that bad volatility strictly dominates good volatility in generating connectedness among the G7 stock markets. These results seem to differ from those discussed in BenSaïda (2019), which found negative values for the CAM only during the GFC and the European debt crisis (from 2007 to spring 2012). In this paper, we find much stronger evidence for the asymmetric effects of bad volatility over good volatility.

Figure 4 is based on two separate VAR models for positive and negative semi-variances. Because the above approach does not consider the interaction between good and bad volatilities, the result cannot fully capture the asymmetrical effects. Hence, we next investigate the degree of asymmetries in volatility connectedness using a single VAR system for both positive and negative semi-variances.

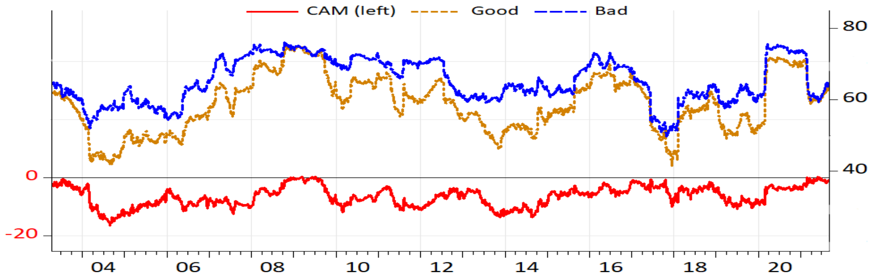

Figure 4. CAM obtained from separate VAR

Note: Total good and bad connectedness lines, scaled on right axis, are obtained from semi-volatility connectedness analysis as in Table 3. Estimated CAM is scaled on the left axis.

Source: authors' calculation.

Figure 5 displays the time-varying pattern of the CAM series obtained from the combined VAR, as presented in Table 4. Large negative values of the CAM are evident during most periods, suggesting that bad volatility strictly dominates good volatility. However, unlike Figure 4, the CAM reached its lowest values during the GFC and the EDC periods around 2008 and 2012. In this case, because the graphs in Figure 5 are obtained from a single VAR containing additional good-to-bad and bad-to-good elements, they are not directly comparable with those in Figure 4. For a fair comparison, we recalculated the CAM estimates using the difference between the sums of good-to-good (upper-left submatrix) and bad-to-bad (lower-right submatrix) elements in Table 4.

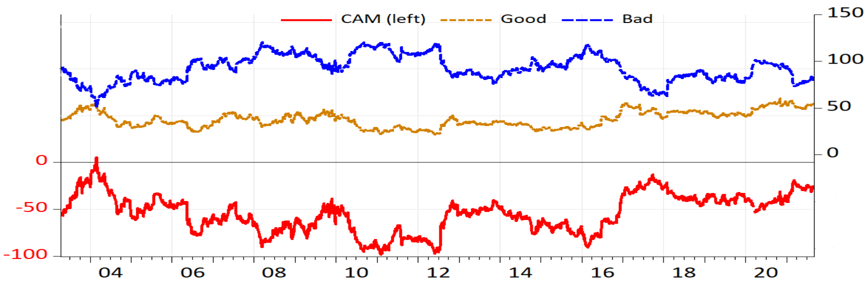

Figure 5. CAM obtained from the combined VAR

Note: Total good and bad connectedness lines obtained from asymmetric volatility connectedness analysis as in Table 4. See note in Figure 4.

Source: authors' calculation.

Figure 6 presents the decomposed CAM indices obtained from good-to-good and bad-to-bad semi-variances, as depicted in Figure 4. The decomposed CAM has almost the same trend as the CAM indices in Figure 5, although its scale is reduced by about half. The decomposed CAM in Figure 6 illustrates distinct different time-varying patterns with a larger amplitude than the CAM series in Figure 4. In this case, we expect that the asymmetrical effects are captured more accurately by a single combined VAR than by separate VAR models. Wang and We (2018) discussed that the CAM is useful in examining whether the markets are in an optimistic or pessimistic mood. The GFC and EDC episodes confirm significant negative values of the CAM, dominating the pessimistic mood.

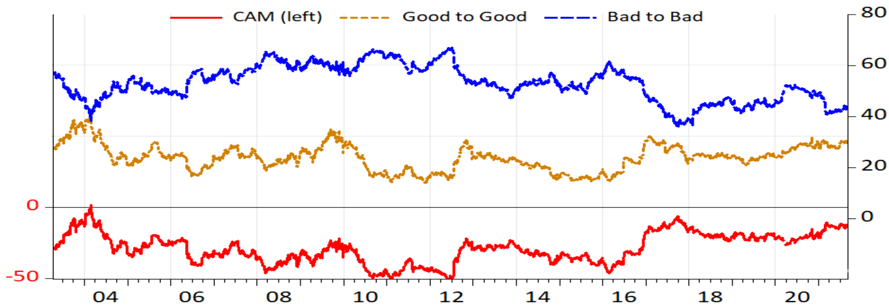

Figure 6. CAMs from good-good vs. bad-bad decompositions

Note: See note in Figure 5.

Source: authors' calculation.

(3) Directional connectedness asymmetric measures for individual markets

We can also investigate the dynamics of the directional CAMs for individual markets, as presented in Figure 7. The graphs are derived from a single VAR model, as presented in Table 4. Similarly to Figure 6, each country exhibits negative values for the directional CAM for most periods, indicating that the connectedness measures for bad volatility strictly dominate those for good volatility during the sample period. The magnitude of the directional CAM of the US is the highest among the G7 stock markets, whereas Japan has a much smaller directional CAM than other G7 stock markets.

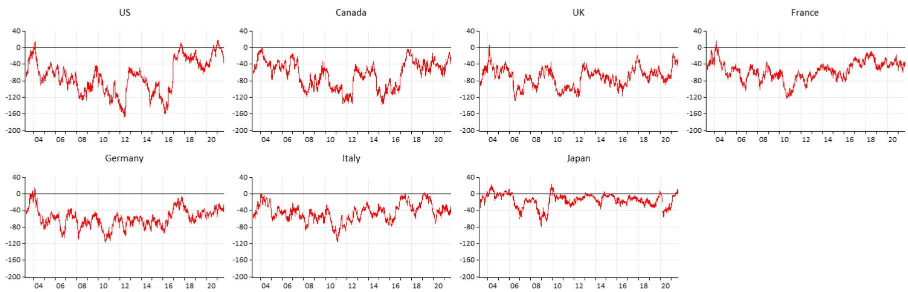

Figure 7. Directional CAM for individual markets

Source: authors' calculation.

(4) Net connectedness of US

Given that the US market has the most significant impact on the world financial market, this subsection examines the net connectedness of the US market. We focus on how good and bad volatilities of the US market propagate to other countries. Figure 8 presents the net directional connectedness measure of the US The net good and bad connectedness measures are calculated from the combined VAR model as $(\sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{j1} - \sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{1j})$ and $(\sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{j8} - \sum_{j=2,j\neq 8}^{14}\tilde{\theta}_{8j})$. The sum of net good and bad connectedness is the net total directional connectedness. The net bad connectedness accounts for most of the net total connectedness, whereas the net good connectedness is negative for most periods.

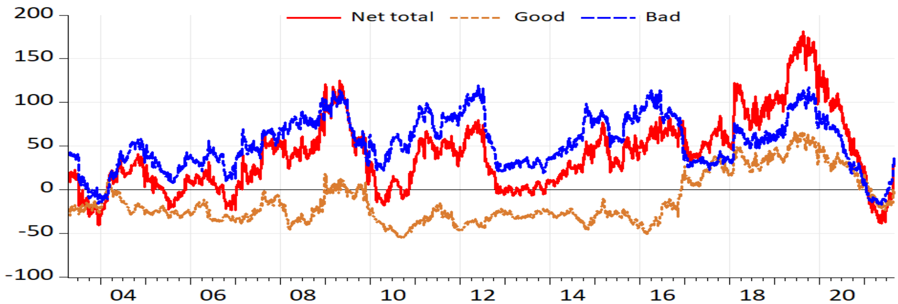

Figure 8. Net directional connectedness measures of the US stock market

Source: authors' calculation.

This result suggests that the spillover effects propagate from bad volatility rather than good volatility. US bad volatility is the predominant source of volatility connectedness among the international financial markets.

5. Conclusions

This paper examines asymmetric volatility connectedness among the G7 stock markets. We investigate the magnitude of asymmetries in volatility connectedness and their transmission mechanisms. The primary findings can be summarized as follows. First, we confirm that the effects of bad volatility strictly dominate those of good volatility in generating connectedness across financial markets. Second, the results of the full-sample symmetric volatility connectedness suggest that the US stock market is the dominant net transmitter of volatility shocks to other stock markets. Third, the asymmetric connectedness analysis, based on the semi-variances, emphasises the dominant role of the US in the world financial markets. We also present evidence that the influence of US shocks on other countries is caused primarily by bad volatility rather than good volatility.

The dynamic analysis suggests that both the symmetric and asymmetric connectedness measures fluctuate substantially over time. The observation that the connectedness measures displayed sharp peaks around the GFC, EDC, and COVID-19 pandemic periods, for example, also indicates that the connectedness across financial markets is time-varying and crisis-sensitive. By decomposing the total connectedness measures, we present further evidence for bad volatility contributing more to the total connectedness than good volatility, which is consistent with earlier results in this area that negative shocks lead to more significant impacts on other markets than positive ones.

The results from the CAM also provide evidence that bad volatility dominates good volatility in generating volatility connectedness across the G7 financial markets. The impact of the US volatility shocks on other countries is triggered primarily by bad volatility rather than good volatility. These findings suggest that bad volatility is the primary factor behind the global systemic risk transmission mechanism. Because asymmetries in the financial market transmission mechanism may pose a challenge for investors, the results in this paper raise important implications on risk management strategies for portfolio diversification.

While several interesting results are demonstrated in this paper concerning asymmetric connectedness among the G7 stock markets, much work remains to be done. For instance, given the importance of the risk spillover across different markets, it would be interesting to examine the risk reduction strategies of the stock-commodity portfolio by putting stock markets and other commodity markets together. It is also worth investigating the realized higher-order moments (i.e., realized skewness and kurtosis) among stock markets to better understand the risk transmission of global equity markets.

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