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A NOVEL SIMILARITY MEASURE AND SCORE FUNCTION OF PYTHAGOREAN FUZZY SETS AND THEIR APPLICATION IN ASSIGNMENT PROBLEM

Abstract. *In real-life problems, things are imprecise because of imprecision/inaccuracy, and the exact value of the measured quantities is impossible to get. Sometimes, due to time pressure/ incomplete knowledge, it is difficult for the decision-makers to provide their opinion. To describe the imprecision, the information in terms of the fuzzy is provided to allow the decision-makers to express their inputs freely. There is a valuable role of the fuzzy set (FS) and intuitionistic fuzzy set (IFS) to describe uncertainty under the uncertain situations. In the literature, various models are available for assignment problems under fuzzy sets and intuitionistic fuzzy sets. The Pythagorean fuzzy set (PFS) has a larger domain space than the intuitionistic fuzzy set to describe the membership grade. To handle the uncertainty in practical applications of assignment problems (AP), we have proposed a method to solve the Pythagorean fuzzy assignment problem (PFAP) using the proposed similarity measure and a score function. Numerical examples are given to explain the methodology.*

***Keywords**: Pythagorean Fuzzy Set, Similarity Measure, Score Function, Assignment Problem*

JEL Classification: C02, C44, C45, C60, C61, C62

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1. Introduction

An assignment problem is a linear programming problem (LPP) that deals with allocation and scheduling. The problem of assignment arises because available resources have varying degrees of efficiency for performing different activities. In classical assignment problems, it is assumed that the decision-maker is sure about the precise value of the cost of the assignment problem but, practically, these factors are imprecise.

Zadeh (1965) introduced the fuzzy set (FS) to deal with uncertainty in real-life problems. Gurukumaresan et al. (2020) used the centroid method for the solution of the fuzzy assignment problem. Tsai et al. (1999) worked on the multiobjective fuzzy deployment of manpower. Chanas et al. (1984) took the demand and supply as the fuzzy numbers in the transportation problem and used parametric programming for solving the problem. Verma & Merigó (2019) worked on generalised similarity measures for Pythagorean fuzzy sets and their applications to multiple attribute decision-making. Kumar and Gupta (2011) solved the fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions. Yuen and Ting (2012) performed the textbook selection using the fuzzy PROMETHEE II method. Thakre et al. (2018) worked on the placement of staff in LIC using the fuzzy assignment problems.

To consider the vague and imprecise information in the practical problem, the different extensions of the fuzzy set have been introduced by some authors. The intuitionistic fuzzy set (IFS) proposed by Atanassov (1984) is an extension of the fuzzy set. He considered the membership and nonmembership of the element. Roseline and Amirtharaj (2015) solved the intuitionistic fuzzy assignment problem by using the ranking of intuitionistic fuzzy numbers (IFN). Boran et al. (2012) used the TOPSIS method of the intuitionistic fuzzy set for solving the renewable energy problem. Mukherjee and Basu (2012) solved the assignment problem under IFS by using similarity measure and score function. Kumar and Bajaj [2014] introduced the problem of an interval-valued intuitionistic fuzzy assignment problem and solved it with similarity measure and score function.

Yager (2013) introduced a Pythagorean fuzzy set (PFS). Yager overcomes the situation when membership degree (τ) + nonmembership degree (ζ) >1 in IFS. PFS is an extension of IFS with the condition that the square sum of the membership degree and the nonmembership degree is less than or equal to 1 ($\tau_A(u)^2+\zeta_A(u)^2 \leq 1$). The concept of Pythagorean fuzzy sets (PFS) gives the larger preference domain for decision makers (DM). DMs can define their support and against the degree of membership as $\tau(x)$ = 4/5, $\zeta(x)$= 2/5. In this case, 4/5+2/5>1 is not valid in IFS but squaring $(4/5)^2 + (2/5)^2 < 1$ implies the Pythagorean fuzzy set is more suitable than the intuitionistic fuzzy set. Paul Augustine Ejegwa (2019) worked on the Pythagorean fuzzy set and its application in career placement using max-min composition. Fei and Deng [18] solved the problem of the Pythagorean fuzzy multi-criteria problem. Shahzadi et al. (2018)

proposed the solution of the decision-making approach under the Pythagorean fuzzy Yager weighted operators. Peng and Yang (2015) defined some results for Pythagorean fuzzy sets.

Over the period, score function and similarity measure of Pythagorean were introduced by many authors. Agheli et al. (2022) defined the new similarity measure for Pythagorean fuzzy sets and application on multiple-criterion decision- making. Zhang and Xu (2014) worked on TOPSIS for multi-criteria decision- making with PFS. Peng & Yang (2016) defined the score function and distance measure for the interval-valued Pythagorean fuzzy number (IVPFN) to analyse the problem. After that Garg [25] proposed the score function for PFN and IVPFN to overcome some limitations of the score function defined by Peng & Yang (2016).

In this work, we have developed a methodology to solve the assignment problem with Pythagorean fuzzy values. The score function defined by Garg (2017) has some limitations. To overcome these limitations, we have proposed a new score function. Additionally, we have defined the new similarity measure to validate our result. So far, there is no literature regarding Pythagorean fuzzy assignment problems using similarity measure and score function.

The paper is organised as follows: Some basic knowledge of FS, IFS, PFS, and arithmetic operations on Pythagorean fuzzy numbers are discussed in section 2. In section 3, we have proposed a novel similarity measure and score function. Also, the limitations of previously defined score functions have been pointed out. The methodology to solve PFAP using similarity measure and score function is given in section 4. Illustrative examples are also given in this section. Section 5, presents the comparative study and concluding remarks.

2. Preliminaries

In this section, we have discussed some basic definitions and arithmetic operations that are required for our work.

Definition 2.1 (1965) A fuzzy set (FS) $\tilde{A}$ is defined on universal set U as

$$\tilde{A} = \{\langle u, \tau_A(u) | u \in U \rangle\},$$

characterized by the membership function

$$\tau_A(u): U \to [0,1].$$

Here $\tau_A(u)$ is the membership degree of the element u to the set $\tilde{A}$.

Definition 2.2 (1984) An intuitionistic fuzzy set $\tilde{A}$ on U is defined as a set of ordered pair given by

$$\tilde{A} = \{\langle u, \tau_{\tilde{A}}(u), \zeta_{\tilde{A}}(u) \rangle | u \in U\},$$

where $\tau_{\tilde{A}}(u), \zeta_{\tilde{A}}(u): U \to [0,1]$ are the degree of membership and degree of non-membership of the element $u \in U$, with the condition $(\tau_{\tilde{A}}(u)) + (\zeta_{\tilde{A}}(u)) \leq 1$, the degree of indeterminacy is given by $\xi_{\tilde{A}}(u) = 1 - \tau_{\tilde{A}}(u) - \zeta_{\tilde{A}}(u)$.

Definition 2.3 (2013) A Pythagorean fuzzy set $\tilde{A}$ on U is defined as given by

$$\tilde{A} = \{< u, \tau_A(u), \zeta_A(u) > | u \in U\},$$

where $\tau_A(u), \zeta_A(u): U \to [0,1]$ are the degree of membership and degree of non-membership of the element $u \in U$, with the condition $(\tau_A(u))^2 + (\zeta_A(u))^2 \leq 1$, the degree of indeterminacy is given by $\xi_A(u) = \sqrt{1 - (\tau_A^2 + \zeta_A^2)}$.

The domain of a Pythagorean fuzzy set is larger than intuitionistic fuzzy sets. While working in the space of PFS, one may have much more choice of assigning value to member and nonmembership from [0, 1].

Definition 2.4 (2015) The addition, multiplication, and scalar multiplication on two PFNs $\tilde{A}_1 =< \tau_{A_1}(u), \zeta_{A_1}(u) >$ and $\tilde{A}_2 =< \tau_{A_2}(u), \zeta_{A_2}(u) >$ are defined as follows:

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = \left\langle \sqrt{\tau_{A_1}^2 + \tau_{A_2}^2 - \tau_{A_1}^2 \tau_{A_2}^2}, \zeta_{A_1}\zeta_{A_2} \right\rangle,$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = \left\langle \tau_{A_1}\tau_{A_2}, \sqrt{\zeta_{A_1}^2 + \zeta_{A_2}^2 - \zeta_{A_1}^2 \zeta_{A_2}^2} \right\rangle,$

(III) $k\tilde{A}_1 = \left\langle \sqrt{1 - (1 - \tau_{A_1}^2)^k}, \zeta_{A_1}^k \right\rangle, k > 0.$

3. Similarity Measure and Score Function of Pythagorean Fuzzy Set

In this section, we have defined the novel similarity measure and score function of Pythagorean fuzzy sets.

Definition 3.1: Suppose $\tilde{A}$ and $\tilde{B}$ be two PFSs. The similarity measure SM: $\tilde{A} \times \tilde{B} \to [0, 1]$ is defined as follows

$$S(\tilde{A}, \tilde{B}) = \frac{\sum_{j=1}^{m} \tau_A^2(u_j).\tau_B^2(u_j) + \zeta_A^2(u_j).\zeta_B^2(u_j)}{\sum_{j=1}^{m} \left[\left(\tau_A^4(u_j) \vee \tau_B^4(u_j)\right) + \left(\zeta_A^4(u_j) \vee \zeta_B^4(u_j)\right)\right]}$$

Theorem 3.1: Similarity measure (SM) between two PFS $\tilde{A}$ and $\tilde{B}$, then the following are true.

(S1) $0 \leq S(\tilde{A}, \tilde{B}) \leq 1$

(S2) $S(\tilde{A}, \tilde{B}) = 1$ iff A=B

(S3) $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$

(S4) $S(\tilde{A},\tilde{C}) \le S(\tilde{A},\tilde{B})$ and $S(\tilde{A},\tilde{C}) \le S(\tilde{B},\tilde{C})$ for all $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ such that $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$.

Proof: **(S1)** Since for all u_j, 1 ≤ j ≤ m, we have $\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) \le \tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)$ and $\zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j) \le \zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)$.

Therefore, for each u_j, we have

$$\left[\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) + \zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j)\right] \le \left[\left\{\tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)\right\} + \left\{\zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)\right\}\right]$$

Therefore, for all u_j, 1 ≤ j ≤ m, we have

$$\sum_{j=1}^{m}\left[\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) + \zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j)\right] \le$$

$$\sum_{j=1}^{m}\left[\left\{\tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)\right\} + \left\{\zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)\right\}\right]$$

$$0 \le S^s(\tilde{A},\tilde{B}) \le 1.$$

(S2). Suppose $S(\tilde{A},\tilde{B}) = 1$, $\dfrac{\sum_{j=1}^{m}\left[\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) + \zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j)\right]}{\sum_{j=1}^{m}\left[\left\{\tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)\right\} + \left\{\zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)\right\}\right]} = 1$

$$\sum_{j=1}^{m}\left[\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) + \zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j)\right] = \sum_{j=1}^{m}\left[\left\{\tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)\right\} + \left\{\zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)\right\}\right]$$

Now, we claim that $\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) = \tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)$ and $\zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j) = \zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)$.

Suppose $\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) \ne \tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)$, since $\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) \le \tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)$, there exists $k_1 > 0$ such that $\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) + k_1 = \tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)$.

Similarly, there exists $k_2 > 0$ such that $\zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j) + k_2 = \zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)$.

By hypothesis it follows that $k_1 + k_2 = 0$. This implies $k_1 = -(k_2)$, which is not possible. This implies that

$\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) = \tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)$ and $\zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j) = \zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)$. This implies that $\tau^2_{\tilde{A}}(u_j) = \tau^2_{\tilde{B}}(u_j)$ and $\zeta^2_{\tilde{A}}(u_j) = \zeta^2_{\tilde{B}}(u_j)$.

Hence $\tilde{A} = \tilde{B}$.

(S3) $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$ is trivial.

(S4) For three PFSs $\tilde{A}, \tilde{B}$ and $\tilde{C}$ in U. The similarity measures between $\tilde{A}, \tilde{B}$ and $\tilde{A}, \tilde{C}$ are given as

$$S(\tilde{A}, \tilde{B}) = \frac{\sum_{j=1}^{m}\left[\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j) + \zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j)\right]}{\sum_{j=1}^{m}\left[\left\{\tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{B}}(u_j)\right\} + \left\{\zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{B}}(u_j)\right\}\right]}$$

$$S(\tilde{A}, \tilde{C}) = \frac{\sum_{j=1}^{m}\left[\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{C}}(u_j) + \zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{C}}(u_j)\right]}{\sum_{j=1}^{m}\left[\left\{\tau^4_{\tilde{A}}(u_j) \vee \tau^4_{\tilde{C}}(u_j)\right\} + \left\{\zeta^4_{\tilde{A}}(u_j) \vee \zeta^4_{\tilde{C}}(u_j)\right\}\right]}.$$

Let $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$ for all $u_j \in \tilde{U}$, we have $\tau^2_{\tilde{A}}(u_j) \le \tau^2_{\tilde{B}}(u_j) \le \tau^2_{\tilde{C}}(u_j)$, $\zeta^2_{\tilde{A}}(u_j) \ge \zeta^2_{\tilde{B}}(u_j) \ge \zeta^2_{\tilde{C}}(u_j)$. This implies $\tau^4_{\tilde{A}}(u_j) \le \tau^4_{\tilde{B}}(u_j) \le \tau^4_{\tilde{C}}(u_j)$, $\zeta^4_{\tilde{A}}(u_j) \ge \zeta^4_{\tilde{B}}(u_j) \ge \zeta^4_{\tilde{C}}(u_j)$.

We claim that for all $u_j \in \tilde{U}$, we have

$$\frac{\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{B}}(u_j)}{\tau^4_{\tilde{B}}(u_j) + \zeta^4_{\tilde{A}}(u_j)} \le \frac{\tau^2_{\tilde{A}}(u_j).\tau^2_{\tilde{C}}(u_j)}{\tau^4_{\tilde{C}}(u_j) + \zeta^4_{\tilde{A}}(u_j)}$$

Similarly, we have

$$\frac{\zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{B}}(u_j)}{\tau^4_{\tilde{B}}(u_j) + \zeta^4_{\tilde{A}}(u_j)} \le \frac{\zeta^2_{\tilde{A}}(u_j).\zeta^2_{\tilde{C}}(u_j)}{\tau^4_{\tilde{C}}(u_j) + \zeta^4_{\tilde{A}}(u_j)}$$

by adding all above equations, we have

$$\frac{\sum_{j=1}^{m}\left[\tau_{\tilde{A}}^{2}(u_j).\tau_{\tilde{C}}^{2}(u_j)+\zeta_{\tilde{A}}^{2}(u_j).\zeta_{\tilde{C}}^{2}(u_j)\right]}{\sum_{j=1}^{m}\left[\left\{\tau_{\tilde{A}}^{4}(u_j)\vee\tau_{\tilde{C}}^{4}(u_j)\right\}+\left\{\zeta_{\tilde{A}}^{4}(u_j)\vee\zeta_{\tilde{C}}^{4}(u_j)\right\}\right]}\leq \frac{\sum_{j=1}^{m}\left[\tau_{\tilde{A}}^{2}(u_j).\tau_{\tilde{B}}^{2}(u_j)+\zeta_{\tilde{A}}^{2}(u_j).\zeta_{\tilde{B}}^{2}(u_j)\right]}{\sum_{j=1}^{m}\left[\left\{\tau_{\tilde{A}}^{4}(u_j)\vee\tau_{\tilde{B}}^{4}(u_j)\right\}+\left\{\zeta_{\tilde{A}}^{4}(u_j)\vee\zeta_{\tilde{B}}^{4}(u_j)\right\}\right]}.$$

Therefore $S(\tilde{A},\tilde{C})\leq S^{s}(\tilde{A},\tilde{B})$. Similarly $S^{s}(\tilde{A},\tilde{C})\leq S(\tilde{B},\tilde{C})$.

Next, we discuss the limitations of the previously defined score function and to overcome the limitations a new score function propose in this section.
Peng & Yang [7] defined score function and accuracy function for interval-valued Pythagorean fuzzy number (IVPFN):

Consider IVPFN $\tilde{W}=\langle[\alpha,\beta],[\gamma,\delta]\rangle$, the score function $E_1(\tilde{W})$ and accuracy function $F_1(\tilde{W})$ are defined as follows

$$E_1(\tilde{W})=\frac{\alpha^2+\beta^2-\gamma^2-\delta^2}{2}$$

$$F_1(\tilde{W})=\frac{\alpha^2+\beta^2+\gamma^2+\delta^2}{2}$$

Suppose that if we consider two IVPFN $\tilde{W}_1=\langle[0.1,0.2],[0.4,0.5]\rangle$ and $\tilde{W}_2=\langle[0.1,0.2],[0.5,0.5]\rangle$, the score and accuracy value calculated by Peng & Yang (2016) are $E_1(\tilde{W}_1)$=-0.1800, $E_1(\tilde{W}_2)$=-0.1800 and $F_1(\tilde{W}_1)$=0.2300, $F_1(\tilde{W}_2)$=0.2300.

According to Peng & Yang (2016) $\tilde{W}_1\sim\tilde{W}_2$, but we have seen that $\tilde{W}_1\neq\tilde{W}_2$. To overcome this limitation Garg (2017) defined the new score function $E_2(\tilde{W})$ and defined as

$$E_2(\tilde{W})=\frac{(\alpha^2-\gamma^2)(1+\sqrt{1-\alpha^2-\gamma^2})+(\beta^2-\delta^2)(1+\sqrt{1-\beta^2-\delta^2})}{2}\in[-1,1]$$

By this score function, the score value of above example are $E_2(\tilde{W}_1)=-0.3368$ and $E_2(\tilde{W}_2)=-0.3233$. Here $E_2(\tilde{W}_2)>E_2(\tilde{W}_1)$, hence $\tilde{W}_2>\tilde{W}_1$.

For IVPFN $\tilde{W}=\langle[0.81,0.87],[0.11,0.25]\rangle$, the score value $E_2(\tilde{W})$=1.0022, this is invalid because score value is greater than 1 i.e. $E_2(\tilde{W})=1.0022\notin[-1,1]$.

Proposed Score Function: We have seen from the above example that the score function defined by Garg (2017) is not giving the appropriate result. To improve this, we have proposed a novel score function as follows.

Definition 3.2 Let $\widetilde{W} = \langle[\alpha,\beta],[\gamma,\delta]\rangle$ be an interval valued Pythagorean fuzzy number (IVPFN). The score function for IVPFN is

$$E(\widetilde{W})=\frac{(\sqrt{3+\alpha-3\gamma})+(\sqrt{3+\beta-3\delta})}{4} \in [0,1].$$

In this if $\alpha = \beta = \tau$ and $\gamma = \delta = \zeta$, then the score function of Interval-valued Pythagorean fuzzy set will become score function of Pythagorean fuzzy set. So, the proposed score function of the Pythagorean fuzzy set is as follows:

$$E(\widetilde{W})=\frac{\sqrt{3+\tau-3\zeta}}{2} \in [0,1].$$

For any two IVPFN/PFN $\widetilde{W}_1$ and $\widetilde{W}_2$,

1. if $E(\widetilde{W}_1) > E(\widetilde{W}_2)$, then $\widetilde{W}_1 > \widetilde{W}_2$
2. if $E(\widetilde{W}_1) < E(\widetilde{W}_2)$, then $\widetilde{W}_1 < \widetilde{W}_2$
3. if $E(\widetilde{W}_1) = E(\widetilde{W}_2)$, then $\widetilde{W}_1 = \widetilde{W}_2$.

4. Application of the Pythagorean Fuzzy Assignment Problem

In this section, we introduce the assignment problem with Pythagorean fuzzy number (PFN) and give two methodologies to solve such problems. One is based on similarity measure and the other is based on a score function.

Pythagorean Fuzzy Assignment Problem (PFAP)

Min $\tilde{Y}=\sum_i^n \sum_j^n \tilde{c}_{ij}^{PFN} x_{ij}$

Subject to

$\sum_j^n x_{ij} = 1, i = 1,2,\ldots n$

$\sum_i^n x_{ij} = 1, j = 1,2,\ldots n$

$$x_{ij} \in \{0,1\}$$

In assignment problems, the cost is usually deterministic in nature. But in real-life problem, this is very difficult to judge the precise value of the cost. In this unstable condition, we calculate the preference value. Based on preference value, we get the preference for the j^{th} work to the i^{th} person in the form of a composite relative degree of similarity with an ideal solution, Thus we replace c_{ij} by composite relative degree.

4.1. Methodology for Pythagorean Fuzzy Assignment Problem using similarity measure

Step 1 First consider the Pythagorean fuzzy assignment problem decision matrix G={(L $_{ij}$)} $_{m\times n}$

(L $_{ij}$)=$< \tau_{ij}(x), \zeta_{ij}(x) >$, *i*=*1, 2....m, j= 1,.....n* are Pythagorean fuzzy numbers.

Step 2 Examine whether the problem is balanced or not. If it is not balanced, then add dummy variables so that the problem is converted into a balanced assignment problem.

Step 3 Calculate the similarity measure of each cost value from Pythagorean positive ideal solution (PPIS) $L^{+} = \langle 1,0 \rangle$ and Pythagorean negative ideal solution (PNIS) $L^{-} = \langle 0,1 \rangle$

$$S(L, L^{+}) = \frac{\sum_{j=1}^{m} \tau_L^2(x_j).\tau_{L^+}^2(x_j) + \zeta_L^2(x_j).\zeta_{L^+}^2(x_j)}{\sum_{j=1}^{m}\left[\left(\tau_L^4(x_j) \vee \tau_{L^+}^4(x_j)\right) + \left(\zeta_L^4(x_j) \vee \zeta_{L^+}^4(x_j)\right)\right]}$$

$$S(L, L^{-}) = \frac{\sum_{j=1}^{m} \tau_L^2(x_j).\tau_{L^-}^2(x_j) + \zeta_L^2(x_j).\zeta_{L^-}^2(x_j)}{\sum_{j=1}^{m}\left[\left(\tau_L^4(x_j) \vee \tau_{L^-}^4(x_j)\right) + \left(\zeta_L^4(x_j) \vee \zeta_{L^-}^4(x_j)\right)\right]}$$

Relative similarity matrix calculated column-wise

$$Q = \frac{S(L,L^{+})}{S(L,L^{+}) + S(L,L^{-})}$$

Similarly, relative similarity matrix calculated row-wise

$$R = \frac{S(L,L^{+})}{S(L,L^{+}) + S(L,L^{-})}.$$

Step 4 The composite matrix $[T]_{n\times n}$ is evaluated as $T = Q \times R = q_{ij} \times r_{ij}$, the resultant matrix T represents the preference that j th job is chosen by i th person.

4.2 Methodology for Pythagorean fuzzy assignment problem using the score function

Step 1 Write PFAP in tabular form

Step 2 Convert the Pythagorean fuzzy assignment problem into a crisp assignment problem by using the score function.

Step 3 Examine whether the problem is balanced or not. If it is not balanced, then add dummy variables so that the problem is converted into a balanced assignment problem.

Step 4 The higher cell value of the matrix will indicate the preference of j th job to the i th person

Illustrative Examples:

Here, we have solved the assignment problem by using similarity measure and score function.

Example 4.1: A manufacturing company decides to make six subassemblies through six contractors. One contractor has to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor and is shown in Table 1 in Pythagorean fuzzy number. The problem is how to assign subassemblies to contractors to get the optimal assignment.

Table 1. Assignment problem based on PFN

S, Q	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
S_1	(0.7,0.6)	(0.7,0.7)	(0.8,0.5)	(0.7,0.6)	(0.6,0.7)	(0.6,0.5)
S_2	(0.63,0.67)	(0.9,0.5)	(0.8,0.53)	(0.8,0.3)	(0.9,0.2)	(0.45,0.59)
S_3	(0.83,0.4)	(0.5,0.7)	(0.6,0.7)	(0.5,0.7)	(0.20,0.81)	(0.5,0.8)
S_4	(0.63,0.55)	(0.71,0.63)	(0.66,0.35)	(0.9,0.3)	(0.4,0.8)	(0.73,0.4)
S_5	(0.7,0.5)	(0.65,0.35)	(0.32,0.7)	(0.8,0.5)	(0.4,0.9)	(0.85,0.18)
S_6	(0.45,0.75)	(0.83,0.3)	(0.35,0.7)	(0.55,0.8)	(0.5,0.6)	(0.3,0.8)

Solution:

Table 2. S (L, L $^+$) column-wise

S, Q	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
S_1	0.43	0.39	0.60	0.43	0.29	0.33
S_2	0.33	0.76	0.59	0.63	0.8	0.18
S_3	0.67	0.19	0.29	0.20	0.02	0.17
S_4	0.36	0.43	0.42	0.80	0.11	0.51
S_5	0.46	0.41	0.08	0.60	0.09	0.72
S_6	0.15	0.68	0.09	0.21	0.22	0.06

Table 3. S (L, L $^-$) column-wise

S, Q	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
S_1	0.29	0.39	0.17	0.29	0.43	0.22
S_2	0.38	0.23	0.19	0.05	0.02	0.33
S_3	0.10	0.46	0.43	0.46	0.65	0.60
S_4	0.26	0.31	0.10	0.05	0.62	0.12
S_5	0.20	0.10	0.48	0.17	0.15	0.02
S_6	0.54	0.06	0.48	0.58	0.33	0.63

Table 4. Relative similarity matrix R (column-wise)

S, Q	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
S_1	0.59	0.5	0.77	0.59	0.40	0.6
S_2	0.46	0.76	0.75	0.92	0.97	0.35

$\mathcal{S},\mathcal{Q}$	$\mathcal{Q}_1$	$\mathcal{Q}_2$	$\mathcal{Q}_3$	$\mathcal{Q}_4$	$\mathcal{Q}_5$	$\mathcal{Q}_6$
$\mathcal{S}_3$	0.87	0.29	0.4	0.3	0.02	0.22
$\mathcal{S}_4$	0.58	0.58	0.80	0.94	0.15	0.80
$\mathcal{S}_5$	0.69	0.8	0.14	0.77	0.37	0.97
$\mathcal{S}_6$	0.21	0.91	0.15	0.26	0.4	0.08

Similarly

Table 5. Relative similarity matrix S (row-wise)

$\mathcal{S}\mathcal{Q}$	$\mathcal{Q}_1$	$\mathcal{Q}_2$	$\mathcal{Q}_3$	$\mathcal{Q}_4$	$\mathcal{Q}_5$	$\mathcal{Q}_6$
$\mathcal{S}_1$	0.34	0.25	0.59	0.34	0.16	0.36
$\mathcal{S}_2$	0.21	0.57	0.56	0.84	0.94	0.12
$\mathcal{S}_3$	0.75	0.08	0.16	0.09	0.00	0.04
$\mathcal{S}_4$	0.33	0.33	0.64	0.88	0.02	0.64
$\mathcal{S}_5$	0.47	0.64	0.01	0.59	0.13	0.94
$\mathcal{S}_6$	0.04	0.82	0.02	0.06	0.16	0.0

Now compute the composite matrix T=R× $\mathcal{S}$=$r_{ij} \times s_{ij}$. This matrix T represents the preference the $\mathcal{S}^{th}$ subassembly to $\mathcal{C}^{th}$ contractor

Table 6. the preference the $\mathcal{S}^{th}$ subassembly to $\mathcal{C}^{th}$ contractor

$\mathcal{S},\mathcal{Q}$	$\mathcal{Q}_1$	$\mathcal{Q}_2$	$\mathcal{Q}_3$	$\mathcal{Q}_4$	$\mathcal{Q}_5$	$\mathcal{Q}_6$
$\mathcal{S}_1$	0.11	0.06	0.34	0.11	0.02	0.12
$\mathcal{S}_2$	0.04	0.32	0.31	0.70	0.88	0.01
$\mathcal{S}_3$	0.56	0.0	0.02	0.0	0.0	0.0
$\mathcal{S}_4$	0.10	0.10	0.4	077	0.0	0.4
$\mathcal{S}_5$	0.22	0.40	0.0	0.34	0.01	0.88
$\mathcal{S}_6$	0.0	0.67	0.0	0.0	0.02	0.0

The optimal assignment policy is: subassembly 1→contractor 3, subassembly 2→contractor 5, subassembly 3→contractor 1, subassembly 4→contractor 4, subassembly 5→contractor 6, subassembly 6→contractor 2.

Next, we consider the same Example 4.1 as given above and apply the proposed score function for the Pythagorean fuzzy number (Definition 3.2). We get the Table 7 corresponding to Table 1.

Solution for PFAP:

Table 7. Value calculated by score function

$\mathcal{S},\mathcal{Q}$	$\mathcal{Q}_1$	$\mathcal{Q}_2$	$\mathcal{Q}_3$	$\mathcal{Q}_4$	$\mathcal{Q}_5$	$\mathcal{Q}_6$
$\mathcal{S}_1$	0.689	0.632	0.758	0.689	0.612	0.725
$\mathcal{S}_2$	0.636	0.775	0.743	0.851	0.908	0.648
$\mathcal{S}_3$	0.811	0.592	0.612	0.592	0.439	0.524
$\mathcal{S}_4$	0.704	0.675	0.808	0.866	0.500	0.795
$\mathcal{S}_5$	0.742	0.806	0.552	0.758	0.418	0.910
$\mathcal{S}_6$	0.548	0.856	0.559	0.536	0.652	0.474

The optimal assignment policy is: subassembly 1→contractor 3, subassembly 2→contractor 5, subassembly 3→contractor 1, subassembly 4→contractor 4, subassembly 5→contractor 6, subassembly 6→contractor 2.

5. Comparative analysis and conclusions

The examples mentioned above have vividly demonstrated the proposed similarity measure and score function as a potential tool for solving the assignment problem in Pythagorean fuzzy sets. From the analysis, it is observed that the results obtained by the implementation of the developed similarity measure and score function are more accurate and reliable. Compared to the existing methodologies in the literature, the novel score function proposed in the present paper has the following advantages:

(i) The proposed method has a simple presentation such that it can significantly avoid the information loss that may have previously occurred in the score function defined by Peng & Yang (2016) and Garg's (2017). It is envisioned that there exist certain values where Peng & Yang (2016) and Garg (2017) score function failed to give valid results.

(ii) We have also observed that Example 4.1 cannot be solved by using the score function given by Garg (2017) as the score values of the cell (2,2) representing $\mathcal{S}_2\mathcal{Q}_2$ (0.9, 0.5).

(iii) The diversity and fuzziness of the decision maker's assessment information can be well reflected and modelled using the proposed similarity measure and score function.

(iv) The result offered by using the novel similarity measure and score function is consistent with the result obtained in the existing work, Mukherjee and Basu (2012), and Kumar and Bajaj (2014). Therefore, the proposed method becomes more flexible and convenient for solving the Pythagorean fuzzy assignment problem.

In this paper, we have proposed a methodology to solve the Pythagorean fuzzy assignment problem. We have solved the problem using the similarity measure and the score function to test the optimality of the problem. It is anticipated that the proposed methodology is capable of managing the uncertainty persisting within the intricate assignment problem. The working of proposed technique has been illustrated via examples to test the validity. We further provide a comparison with the existing methods in the literature. From the comparative study and analysis, it can be concluded that the proposed method overcomes the limitations present in the existing work. Table 8 provides a comparative analysis of the proposed score function. Additionally, it would be engrossing to explore the application of the developed approach to picture fuzzy sets, spherical fuzzy sets and interval-valued picture fuzzy sets, etc., also to deal with other linear programming problems.

Table 8. Comparative analysis of the present work

Problem	**Score Function by Garg (2017)**	**Score Function by Peng & Yang (2016)**	**Proposed Score Function & Similarity Measure**
PFAP	Failed	This is not valid when score values & accuracy are the same	Solution exists

REFERENCES

[1] **Agheli, B., Adabitabar Firozja, M., Garg, H. (2022),** ***Similarity measure for Pythagorean fuzzy sets and application on multiple criteria decision making.*** *Journal of Statistics and Management Systems*, 25(4), 749-769;

[2] **Atanassov, K. (1984),** ***Intuitionistic fuzzy sets****. Fuzzy Sets and Systems,* 20 (1), 87-96;

[3] **Boran, F. E., Boran, K., Menlik, T. (2012),** ***The evaluation of renewable energy technologies for electricity generation in Turkey using intuitionistic fuzzy TOPSIS*****.** *Energy Sources, Part B: Economics, Planning, and Policy*, 7(1), 81-90;

[4] **Chanas, S., Kołodziejczyk, W., Machaj, A. (1984),** ***A fuzzy approach to the transportation problem*****.** *Fuzzy sets and Systems*, 13(3), 211-221;

[5] **Ejegwa, P. A. (2019),** ***Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition.*** *Complex & Intelligent Systems*, 5(2), 165-175;

[6] **Fei, L., Deng, Y. (2020),** ***Multi-criteria decision making in Pythagorean fuzzy environment*****.** *Applied Intelligence*, 50(2), 537-561;

[7] **Garg, H. (2017),** ***A new improved score function of an interval-valued Pythagorean fuzzy set based TOPSIS method***. *International Journal for Uncertainty Quantification*, 7(5), 463-474;

[8] **Gurukumaresan, D., Duraisamy, C., Srinivasan, R., Vijayan, V. (2020),** ***Optimal solution of fuzzy assignment problem with centroid methods*****.** *Materials Today: Proceedings*, 37, 553-555;

[9] **Kumar, A., Gupta, A. (2011),** ***Methods for solving fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions***. *Fuzzy Information and Engineering*, 3(1), 3-21;

[10] **Kumar, G., & Bajaj, R. K. (2014).** ***On solution of interval valued intuitionistic fuzzy assignment problem using similarity measure and score function*****.** *International Journal of Mathematical and Computational Sciences*, *8*(4), 715-720.

[11] **Mukherjee, S., Basu, K. (2012),** ***Solution of a class of intuitionistic fuzzy assignment problem by using similarity measures*****.** *Knowledge-Based Systems,* 27, 170-179;

[12] **Peng, X., Yang, Y. (2015),** ***Some results for Pythagorean fuzzy sets***. *International Journal of Intelligent Systems,* 30(11), 1133-1160;

[13] **Peng, X., Yang, Y. (2016),** ***Fundamental properties of interval‐ valued Pythagorean fuzzy aggregation operators*****.** *International Journal of Intelligent Systems,* 31(5), 444-487;

[14] **Roseline, S., Amirtharaj, H. (2015),** ***Methods to find the solution for the intuitionistic fuzzy assignment problem with ranking of intuitionistic fuzzy numbers*****.** *International Journal of Innovative Research in Science, Engineering and Technology,* 4(7), 10008–10014;

[15] **Shahzadi, G., Akram, M., Al-Kenani, A. N. (2020),** ***Decision-making approach under Pythagorean fuzzy Yager weighted operators*****.** *Mathematics,* 8(1), 70;

[16] **Thakre, T. A., Chaudhari, O. K., Dhawade, N. R. (2018),** ***Placement of staff in LIC using fuzzy assignment problem.*** *International Journal of Mathematics Trends and Technology (IJMTT),* 53(4), 259-266;

[17] **Tsai, C. H., Wei, C. C., Cheng, C. L. (1999),** ***Multi-objective fuzzy deployment of manpower*****.** *International Journal of the Computer, the Internet and Management,* 7(2), 1-7;

[18] **Verma, R., Merigó, J. M. (2019),** ***On generalized similarity measures for Pythagorean fuzzy sets and their applications to multiple attribute decision‐ making*****.** *International Journal of Intelligent Systems,* 34(10), 2556-2583;

[19] **Yager, R. R. (2013),** ***Pythagorean membership grades in multi-criteria decision making*****.** *IEEE Transactions on Fuzzy Systems,* 22(4), 958-965;

[20] **Yuen, K. K. F., Ting, T. O. (2012),** ***Textbook selection using fuzzy PROMETHEE II method*****.** *International Journal of Future Computer and Communication,* 1(1), 76;

[21] **Zadeh, L. A. (1965),** ***Fuzzy sets*****.** *Information and Control,* 8(3), 338-353;

[22] **Zhang, X., Xu, Z. (2014),** ***Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets.*** *International Journal of Intelligent Systems,* 29(12), 1061-1078.