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INVESTIGATING THE EFFICACY OF ARIMA AND ARFIMA MODELS IN NIGERIA ALL SHARE INDEX MARKETS

Abstract. *The study aims to investigate statistical issues, simplified facts, and efficacy of methodological characteristics of long memory models in the monthly Nigerian All Share Index markets. Specifically, the study investigates descriptive and other distributive properties of long-memory models in order to test the efficient*

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market hypothesis proposed by Fame. The data used in this study is the Nigeria All Share Index. The data points totalled 356 and spanned from January 1992 to August 2021. The study used Autoregressive Moving Average and its Fractional Integrated model (ARFIMA) to capture the characteristics of long memory. In addition, a comparison is made between ARIMA and ARFIMA.

***Keywords**: Statistical Problems, Stylised Facts, ARIMA Model, ARFIMA Model, Nigerian Stock Market Forecasting*

JEL Classification: C22, C53, E27, G15, G17

1. Introduction

In statistics, economics, finance, and econometrics models, time data series derived from financial markets appear to exhibit some unique characteristics. These unique characteristics appear to be unpopular among researchers and policy makers (Koutmos & Knif, 2002; Scruggs & Gabadanidis, 2003). Some of these unique characteristics include properties of standard deviations, minimum and maximum it, skewness, kurtosis, correlational properties, ARCH effect, autocorrelation, distribution properties, tail analysis, extreme variability (Box, Jenkins, Reinsel & Ljung, 2016), (Deebom, Etuk & Nwikorga, 2021). These are not adequately captured and integrated into several studies. Also, unlike any other market, the Nigeria All Share Index financial market is the creation of money supply and the demand for it, and so, if proper attention is not given to these properties may lead to biased estimation.

Although, there are several distinct major types of financial markets which include; the stock market, the bond market, commodities, forex, and the real estate market, among several others (Deebom, Etuk & Nwikorga, 2021). These financial markets exhibit one statistical issue or another.

Furthermore, it is generally accepted as a strategy that financial time series must be different in order to be invariant before fitting ARMA model families. Although what is not generally accepted is the best way to determine the proper method to stabilise a time series data from transformation, either in an integrally or fractionally converted. This is because stationarity is an important concept in time series analysis with a huge impact on how data is perceived and predicted. It is used to eliminate the dependence of the series on time, the so-called temporal dependence, and to define its structures such as trends and seasonality. In some boundary cases, distinguishing between a stationary series that should not be distinguished from a nonstationary series that should be distinguished is not trivial. Also, there is another problem of over specification of ARMA and ARIMA models using ARFIMA models. In determining the efficacy of ARIMA and the ARFIMA model in time series analysis, it is important to investigate distributional properties of the time series data, extreme variability, and long-memory properties of the data. These are useful for predicting the future values of the series based on these characteristics. The study is structured as follows: the literature review, the materials and methods for conducting the analysis, the fourth section presents the results, discussion of results, and the final section of this study provides a summary and conclusion of the research.

2. Literature review

ARIMA and the ARFIMA model have been focused on in various survey and review studies to assess the accuracy of different statistical techniques (Anderson, 2008; Christodoulos, Christos, Dimitris, 2010; and Chatfield, 2013). Nowadays, most of the studies are based on the use of stock market trend prediction. Also, unlike any other market, the Nigeria All Stock Index Financial. The market is not randomly generated values; it can be treated as a discrete time series model, and its trend can be analysed accordingly, so it can also be predicted. The ARIMA and ARFIMA model procedure provide great flexibility in defining univariate time series models, parameter estimation, and prediction. The difference between ARIMA and ARFIMA model is that the ARIMA model converts non-stationary data to stationary for in an integral order before acting on it, while the ARFIMA model provides sparse parameters for long memory models that overlap with ARMA (autoregressive moving average) model, which is widely used for short memory operations. Long memory processes can be obtained by partial fusion of short memory processes as a result of Granger and Joyeux (1980) who point out that many of these chains are not apparently stable, however, the divergent chain usually presents clear evidence of over differencing. ARIMA models are often called Box-Jenkins models, since the ARIMA approach was first popularised by Box and Jenkins (Mondal, Shit & Goswami, 2014). Box and Tiao (1975) discussed the general transfer function model used by the ARIMA procedure. The ARFIMA model was developed by Hurst (1951) in the field of hydrology, while the interest in long-memory models of economic series arises from the studies of Granger and Joyeux (1980). This was done as a generalised version of the ARIMA models (Granger & Joyeux, 1980). There are various motivations for testing the efficacy of ARIMA and the ARFIMA Model in Nigeria All Share Index Markets, one of them is financial gain and forecasting. A system that can identify whether the Nigeria All Share Index financial market is doing well and which indicators in the market that are volatile. This will make it easier for investors, or market or finance professionals to make decisions.

Several studies have been carried out on the use of ARIMA and the ARFIMA Model in modelling financial time series data but no much attention has been given to the statistics, stylised facts, and characteristics of long memory models in the monthly index of all share in the Nigerian financial markets. Among some of the studies are; Meher et al. (2021) studies on the ARIMA Hybrid Model for Stock Market Price Prediction: A Case Study of Indian Pharmaceutical Companies. The aim of the study is to predict the stock prices of selected pharmaceutical companies in India, trading at NIFTY100, using the ARIMA model. The ADF test is used to check whether the data is stable or not. For the ARIMA model estimation, significant spikes were observed in the correlation plot for ACF and PACF, and several models were framed using different AR and MA terms for each specific company. Then, the 5 best models were selected, the necessary implementation of several AR and MA terms to adjust the models, and the selection of the best adjusted ARIMA model for each company based on volatility, the adjusted R-squared and the information criterion of Akaike. The results can be used to analyse stock prices and predict depth in future research efforts.

Raheem and Ezepue (2018) examine some simplified facts of the short-term share prices of selected Nigerian banks. The study examines the existence on some simplified facts of short-term share prices in the banking sector on the Nigerian Stock Exchange (NSM). The outcomes that the four banks' stocks act somewhat in an unexpected way, yet for the most part have the elegant real factors tracked down in different business sectors. Similarly, Al-Khair, Muhammad, Ranjan, Nicholas, Amandine, and Siobhan (2014), Anticipating BRICS Value Returns Utilising ARFIMA Models. The outcomes show proof that ARFIMA models assessed utilising an assortment of assessment methods give preferable expectation results over non-ARFIMA models (ARFIMA and GARCH) regarding estimating stock returns. These discoveries apply without fail to various nations whose economies vary in size, nature, and advancement.

Also, Emenike (2010) Predicting Nigerian Stock Market Returns: Evidence for Autoregressive Integrated Model of Moving Averages (ARIMA). The model was fitted to 12 period forecasts, and their statistics show that the predictions of the ARIMA (1,1,1) model outperformed the later model. Therefore, the deviations indicate that the global economic crisis destroyed the existing correlation between the NSE All-Share Index and its past. The results confirmed that higher maturity is greater than higher volatility for conventional bonds and vice versa for Islamic bonds. The existence of long memory for traditional bonds with short and medium maturity was also confirmed, while the study recorded long maturity using the ARFIMA model on it for skulk to establish the relations between maturity and long memory of bonds.

Meher et al. (2021) paper attempts to predict the share prices of selected pharmaceutical companies in India, listed under NIFTY100, using the ARIMA model. A sample size of 782 time-series observations from January 1, 2017, to December 31, 2019, for each selected pharmaceutical firm has been considered to frame the ARIMA model. However, there are several shortcomings associated with these studies.

The study differs from existing literature on the Nigerian All Stock Index by not focusing on spot prices as seen in Suleiman et al. (2015) but we use returns series to investigate long-memory properties of Nigerian All Share Index. The studies done earlier were not enough to provide an appropriate model for statistical issues, simplified facts, and efficacy through ARIMA and ARFIMA Model. None of the studies reviewed so far focused on statistical issues, simplified facts, the efficacy of ARIMA and the ARFIMA model in Nigeria All Share Index markets. This implies that not much research has been carried out on long-memory models. Therefore, this study seeks to fill this gap in the literature. The contribution of this study to the scarce literature on the financial markets of Nigerian all-share index focuses on the objective of assessing the efficacy of ARIMA and the ARFIMA model in Nigeria all share index markets as it includes recent data sets covering periods up to 2021.

3. Research methods and analysis

This study used the monthly Nigerian All Share index, covering the period from January 1992 to May 2021. Data was obtained from the Central Bank of Nigeria website at https://www.cbn.gov.ng. The data has been converted to a record return string. This was done to determine if the outside in the series had any statistical properties that, among other things, might have led to a biased estimate. The statistical software used in the analysis is Microsoft Excel and STATA version 16.

The calculation of chain returns is considered as changes in the logarithmic process, and we calculate the percentage as follows:

$$RASI = log\left(\frac{ASI_t}{ASI_{t-1}}\right) \times 100 \qquad (1)$$

Where ASI *t* represents the all-share index at time *t* and ASI *t-1* represents the all-share index at time *t*. In the statistical analysis of price dynamics, we generally use the logarithm of the price at the moment divided by the previous (late) time. This is used because stock prices are often correlated, and when the correlation is present in one variable, it leads to a biased estimate. Similarly, prices can also be unstable and have trends that may not fit into cluster volatility, which is a feature of the data that violates the homeostatic assumption, resulting in hetero-elasticity. Brief descriptive statistics is performed to check whether the data is normally distributed. This test is performed using the statistics of the Jarque-Bera test. Also, the unit root test is performed to determine whether the series is integrated in the first order (I (1)) or not. This is because in statistics, Mukhtar (2010) revealed that time series data are frequently non-stationary and thus conducive to spurious regression. Financial data which have a unit root is said to be non-stationary. In this manner, to direct a measurable examination, therefore, to investigate the stationary of the involved in financial time series. To test the time-series data stationary, a typical method is to apply the Augmented Dickey-Fuller test (ADF) (Dickey and Fuller, 1979) to test for unit root. To test the time series data for stationary a common method is to apply an Augmented Dickey-Fuller test (ADF) (Dickey and Fuller, 1979), Phillip Perron Test (PPT), and Kwiatkowski, Phillips, Schmidt, Shin (KPSS) test to test the unit root. Similarly, to determine the order of processes in a time series data like All Nigerian Stock Index to be model with fractional and integral model we use the Autocorrelation and partial autocorrelation. These are measures of association between current and past series values and indicate which past series values are most useful in predicting future values (Deebom, Essi & Amos, 2021). More specifically, Autocorrelation function at certain lag, this is the correlation between series values that are intervals apart and Partial autocorrelation function, at certain lag, this is the correlation between series values that are intervals apart, accounting for the values of the intervals between. The results section, as well as how to construct a correlation chart that includes confidence intervals.

The ARCH-LM test was also conducted. According to Sanusi et al. (2015), this test is used to check whether the residuals obtained from the preliminary estimation of the series violate the homoscedasticity assumption; in other words, it is used to test for ARCH effects by regressing the squared errors on its lags. In statistics, this is another form of stylised fact use to establish whether the variance of the errors in a regression model involving financial is, constant: that is for homoscedasticity (Deebom, Essi & Amos, 2021). This test, and an estimator for heteroscedasticity-consistent standard errors

Some of the statistical issues and stylised facts about financial time series data and the efficacy of the use of ARIMA and ARFIMA Models in modelling include the issues of non-integral stationarity, inability to model long-range dependence, excessive speculative interest, price instability in foreign markets, and lack of convergence between futures and present market (Deebom, Essi & Amos, 2021; Zhuravka et al. 2021). In order to determine persistent and long-lasting effect, the following tests have been carried out in all Nigerian stock index markets: Lo R/S Statistics, Robinson's test, and GPH test (Deebom, Essi &Amos, 2021). According to the specific objectives of this study, there are two categories of models used in the study which include; ARIMA and ARFIMA. The ARIMA model is derived from ARIMA *(p,q)*, the process where p is the order of the automatic regression and q the order of the moving average can be represented in general as:

$$\varphi(L)X_t = \Theta(L)\varepsilon_t \tag{2}$$

Where

$\varphi(L) = 1 - \varphi_1 L - \varphi_2 L^2 - \cdots \ldots \ldots \varphi_p L^p$ the $\Theta(L) = 1 - \Theta_1 L - \Theta_2 L^2 - \cdots \Theta_q L^q$ know that for (2) to be constant and reversible, the zeros of φ (L) and θ (L) must lie outside the unit circle, respectively (Deebom, Essi & Amos, 2021). Most real-time series are non-stationary. For such time series, Box and Jenkins (1976) suggested that the difference up to the d command can be made constant. Therefore, ARIMA is the appropriate way to describe this type of time series. Consider the non-stationary time series *Yt* and suppose that the d-th order constant *Yt* is denoted by this $\nabla^d Y_t$. Putting $\nabla^d = (1 - L)^d$, condition ∇^d in (2) the ARIMA model gives the order *p*, *d* and *q*, denoted by ARIMA *(p, d, q)*:

$$\varphi(L)\nabla^d Y_t = \Theta(L)\varepsilon_t \tag{3}$$

ARFIMA models like ARIMA have three parameters: *p, d*, and *q*. The parameter corresponding to the number of lags included in the autoregressive part of the series is *p*. Meanwhile, the lag parameter of the moving average is *q*. If the series is a fractional integral, where *d* takes a value in the period 0 << 1, then the model is called the ARFIMA model.

Now consider that $\{Y_t\}$, t = 1,…, is a non-stationary process with time-varying mean and variance. Then Y_t is said to follow a partial integration if

$$\varphi(B)(1 - L)^d Y_t = b(L)\varepsilon_t \tag{4}$$

is the autoregressive factor, b is the moving average factor, d is a real number parameter of the fractional integral, L indicates the delay factor and $\varphi(B)$ is a white noise residual; $(1-L)^d$ represents the variation of the partial delay factor.

Given two instances of "*d*", when d = 0 and d = 1. Granger and Joyeaux (1980) opined that the letter "*d*" used in the model is. According to Hosking (1981), the "*d*" is between (-0.5 and 0.5), that is, -0.5 < d < 0.5. When 0< d < 0.5, the autocorrelation is positive and decay hyperbolically to zero implies long memory, when 0.5 < d <0, the process is identified as having intermediate memory and for the series to be *d=1* follow a unit root process (Deebom, Essi &Amos, 2021)

In addition, we must consider an irreversible process, which simply means that the string cannot be determined by any automatic regression. That ARFIMA is only a fractional integral part of the ARMA model process.

4. Empirical results

The data covers the period from January 1992 to May 2021 and is compiled from the Central Bank of Nigeria's statistical database.

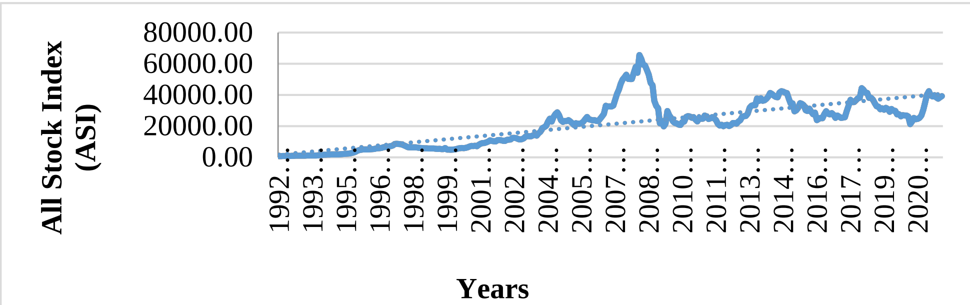

Figure 1. The raw series time plot for the Nigeria All-Share Index

Table 1. Descriptive Statistics and Normality Test for ASI and RASI

Test Statistics	ASI	RASI
Mean	21291.25	1.098549
Standard Deviation (Std. Dev)	145142.72	6.480165
Skewness	0.3772334	-0.459195
Kurtosis	2.512164	8.097591
Minimum	1259	14
Maximum	568496	1003
Shapiro Wilk	0.94666	0.94252
Probability	0.00000	0.00000

Table 1 shows the results of the descriptive statistic and normality test of the raw and yield series for the Nigeria All Stock Index. The raw and return series averages for all Nigerian stocks are positive, indicating that the series is reversing positively. The skewness coefficients for raw series are positive whereas negative for the return series, indicating a left-skewed distribution. The returns series shows that the value of the kurtosis (8.097591>3) is higher than that of the raw series. This means that the kurtosis is mesokurtic which is the same as the normal distribution (medium peak). The probability of the Shapiro wilk test is less than 5 percent level of significance. This shows that both raw and return series of Nigerian all share index are not normally distributed.

Table 2. Unit Root Test for Constancy in First Difference

Variable	Augmented Dickey Fuller Test (ADFT)				Phillip Peron (PPT) Test				KPSS Test			
	Test Statistics				Test Statistics				Test Statistics			
		1%	5%	10%		1%	5%	10%		1%	5%	10 %
ASI	-10261	-2337	- 1649	-1,284	-13371	-3,986	-3,426	-3,130	0.591	0.216	0.146	0.119

All results were tested at 1%, 5%, and 10% significance level, respectively.
Note: ASI represents All Share Index

Table 2 presents the results of the unit root test for stationarity. The probability of the test statistics for the ADFT, PPT and KPSS, respectively (Z < 0.05) then we conclude the raw series is stationary.

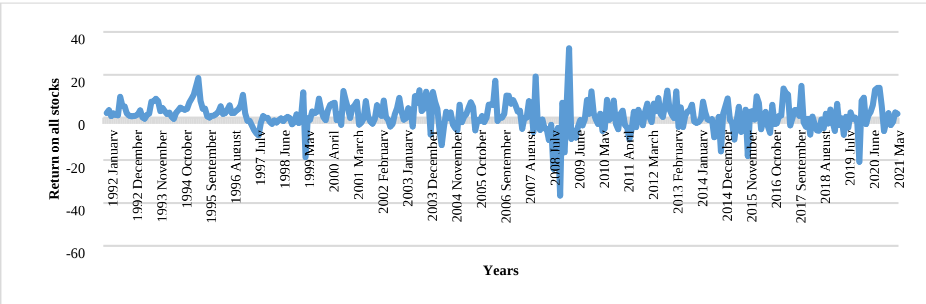

Figure 2. Time Plot on the Return and Volatility Time Series on Nigeria All Share Index

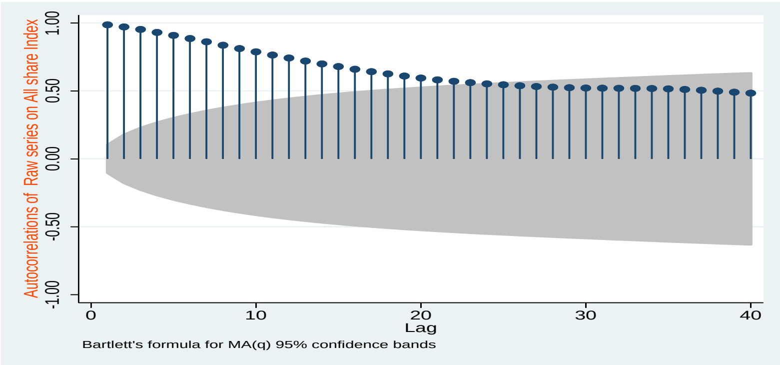

Figure 3. The autocorrelation function of the return series of the Nigeria All Share Index

The vertical axis of the graph indicates the autocorrelations of performance, which range from negative 1 to 1 in increments of 0.50, and the horizontal axis indicates lag, ranging from 0 to 40 in increments of 10. Also, Figure 4 is the Partial Correlation Function of the Nigerian All Share Index.

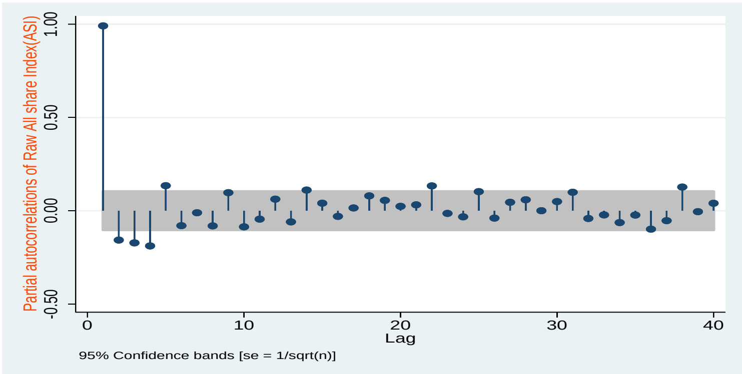

Figure 4. The result of the partial autocorrelation function in the Nigerian All Share Index

The vertical axis of the graph indicates the partial autocorrelation of the Nigerian all share index ranging from negative 0.50 to 1.0 in increments of 0.50, and the horizontal axis indicates the lag ranging from 0 to 40 in increments of 10.
The ARCH-LM test is also performed using a Lagrange Multiplier (LM) and the results are shown in Table 3 below.

Table 3. Jung-Box L-Statistic (Q) Estimation

All Stock Index (ASI)			**Returns on** All Stock Index (RASI)		
Ljung-Box(Q) statistic	**Q-statistics**	**Probability**	**Ljung-Box(Q) statistic**	**Q-statistics**	**Probability**
Q (5)	1630.808	0.000	QS (5)	64.2055	0.000
Q (10)	2914.7870	0.000	QS (10)	52.3151	0.000
Q (15)	3882.1563	0.000	QS (15)	57.0758	0.000
Q (20)	4622.7665	0.000	QS (20)	57.6984	0.000
Q (25)	5228.1442	0.000	QS (25)	63.7575	0.000

All results were tested at 1%, 5%, and 10% significance level, respectively.

Table 3 contains an estimate of the ARCH effect using the Ljung-Box (Q) statistic. The results are divided. The first part shows the L Jung-Box (Q) statistical estimates for the initial series of the Nigeria All-Share Index (ASI), while the second part contains the L Jung-Box (Q) statistical estimates for the series returns of the Nigeria All-Share Index (ASI). Thus, the Jung-Box L(Q) statistic shows the high significance of the p-values, which are all significantly different from zero. Since the estimated p-values have a significance level of less than 5%, there is an indication that the null hypothesis should be rejected while the alternative hypothesis should be accepted. In conclusion, the raw and return series have strong autoregressive conditional heteroskedasticity (ARCH) effects. In another development, in detecting long memory of the series, the result is shown in Table 4 below.

Table 4. Estimates for test of long memory in ASI and Its corresponding returns

Estimators	All Share Index (ASI)	Returns on All Share Index (ASI)
Test Statistics		
Lo R/S test	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098]
GPH Test		
$M = T^{0.5}$ $M = T^{0.6}$ $M = T^{0.7}$ $M = T^{0.8}$	[0.899***] [1099***] [1038***] [1,051***]	[0.240] [0.182] [0.201**] [0.293***]
Robinson Estimates		
0.5 0.6 0.7 0.8	[0.899***] [0.777***] [0.962***] [1059 1***]	[0.189] [0.181] [0.199**] [0.287***]

All results were tested at the 1%, 5%, and 10% significance level, while *, ** and *** represent the 1%, 5%, and 10% significance level.

Table 4 contains the results for test of long memory in ASI and Its corresponding returns. The results confirmed a long raw memory and returns in the Nigeria All Stock Index (ASI) using rescale Lo statistics, Robinson's test and GPH test, violating the weak form of the effective market hypothesis (EMH). To estimate the ARIMA and ARFIMA models, the study by Deebom et al. (2021) was followed. First, an estimation procedure carried out by Geweke and Porter-Hudak (1983), called GPH, based on the estimation of the long memory parameter d, was used. Second, we used the Robinson estimation procedure (1995). The third estimate used is the one made by Sowel (1992) based on the exact probability function and not on whittler's. Next, we continue to estimate Andrews and Guggenberger (2003), labelled 'GPH'. The choice of methods is stimulated by the literature, and in particular by the work of Fouquau and Spieser (2014) due to the sensitivity of the ARFIMA approach.

Table 5. Estimated ARIMA Models Used to Model Returns on the Nigerian All Stock Index

Models	Model Parameters						Model with lowest AIC/BIC
	A	Φ	Β.	ϒ	AIC	BIC	
ARIMA (1,1,1)	**108.74 (0.511)**	**0.773 (0.000)**	**-0.620 (0.000)**	**3250 (0.000)**	**6338552**	**6354.04**	**ARIMA (1,1,1)**
ARIMA (2,1,2)	109.62 (0.328)	0.357 (0.000)	0.571 (0.00)	3269 (0.000)	6340,553	6356.041	
ARIMA (3,1,3)	108.53 (0.908)	0.0900 (0.542)	0.136 (0.360)	3273 (0.000)	6340855	6356.344	
ARIMA (3,1,4)	105129 (0.158)	0.794 (0.000)	-0.864 (0.000)	6354.035 (0.000)	6354.035	6369.524	

All the results were tested at 1%, 5%, and 10% significance level.

The best comprehensive ARIMA model compatible with the Nigeria All-Share Index (ASI) is ARIMA (1,1,1) based on the model with the minimum Akaike information criteria.

Table 6. Estimation of Long Memory Models Used in Modelling ASI

Models	**Parameters of the ARFIMA Models**							**Model with lowest AIC**
	A	**Φ**	**Β.**	**D**	**ξ**	**AIC**	**BIC**	
ARFIMA (1, 0.411,1)	**20695.95 (9,128)**	**0.930 (0.000)**	**-0.011 (0.000)**	**0.411 (0.000)**	**3209019 (0.000)**	**6359.129**	**6378489**	
ARFIMA (2, 0.497.2)	**20418.77 (0.686)**	**0.772 (0.000)**	**-0.054 (0.000)**	**0.497 (0.000)**	**4495833 (0.000)**	**6480.221**	**6495.909**	
ARFIMA (1,0.178,3)	**21031.07 (0.000)**	**0.964 (0.000)**	**0.185 (0.003)**	**0.178 (0.00)**	**3161382 (0.000)**	**6,352,918**	**6372.279**	**ARFIMA (1, 0.178, 3)**
ARFIMA (2.0.497.3)	**20375.93 (0.522)**	**0.699 (0.000)**	**0.139 (0.000)**	**0.497 (0.000)**	**4428093 (0.000)**	**6474.514**	**6490.003**	

Models	**Parameters of the ARFIMA Models**							**Model with lowest AIC**
	A	**Φ**	**B.**	**D**	**ξ**	**AIC**	**BIC**	
ARFIMA (1, 0.248.4)	**21094.86 (0.120)**	**0.971 (0.000)**	**-0.196 (0.000)**	**0.248 (0.000)**	**3161616 (0.000)**	**6353.205**	**6372.566**	
ARFIMA (2, 0.496.4)	**20455.81 (0.775)**	**0.784 (0.000)**	**-0.009 (0.000)**	**0.496 (0.000)**	**4472184 (0.000)**	**6478345**	**6493,833**	

All results were tested at 1%, 5%, and 10% significance level.

We were able to conclude that the appropriate model estimated using partial variances is ARFIMA (1,0.178,3) based on the model with the minimum Akaike information criteria.

In order to confirm the validity of the results, the ARCH effect test is conducted and the result is shown in Table 7 below.

Table 7. Summary of the Diagnostic Verification of the ARFIMA Model (1,0.178,3)

	ARIMA (1,1,1)		**ARFIMA (1,0.178,3)**	
	(x^2)	**q value**	**($x^{2)}$**	**P-value**
LJJung -box	9.7814	0.0817	9.7814	0.2837
Jarque Bera	7.09	0.0288	7.09	0.4288

All results were tested at 1%, 5%, and 10% significance level

Table 7 contains a summary of the diagnostic examination for ARIMA (1,1,1) and ARFIMA (1, -0.021,1), done using Ljung-Box and Jarque-Bera test statistics. The results showed no arch effect for both models. Although the residuals of the ARIMA model (1,1,1) are not normally distributed. Also, the graph display below is the quantile-quantile (QQ-plot).

A QQ chart is a scatter plot plotted between two sets of measures against each other. This is done to check whether the two sets of quantities come from the same distribution, and if this is true, the two lines overlap and reveal that the estimated model residuals follow the standard order of the normal distribution.

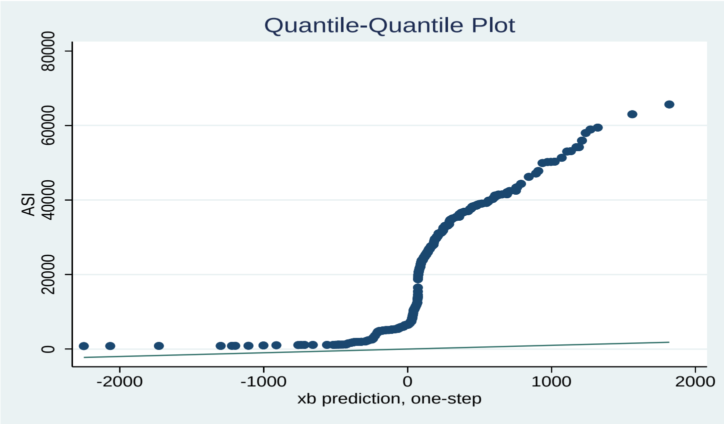

Figure 5. The result of the Quantile-Quantile plot for the prediction series of the ASI using ARIMA (1,1,1)

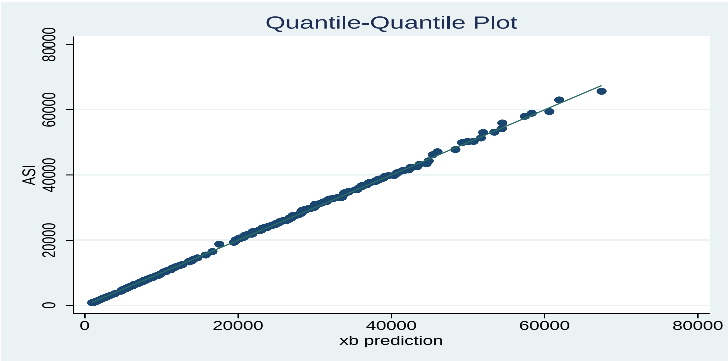

Figure 6. The result of Quantile-Quantile plot for ASI Series using ARFIMA(1,0.178,3)

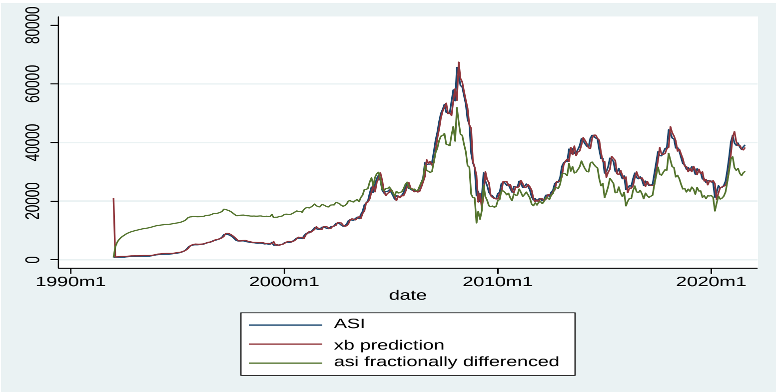

Figure 7. Nigeria All Stock Index Original Series, Expected and Fractional Difference Series ARFIMA (1,0.178,3)

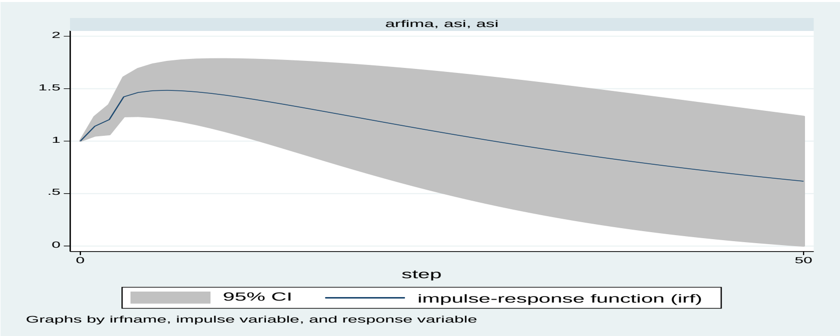

Figure 8. Plot of Impulse Response Functions (IRF) and Response Variable

The result shows a proof of the presence of long-memory in the markets.

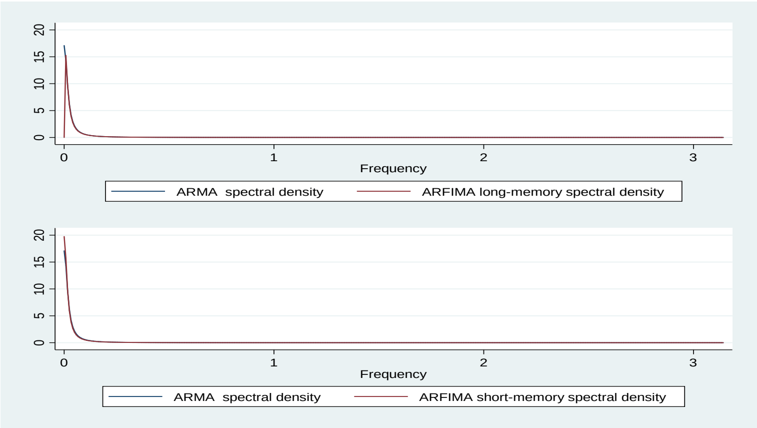

Figure 9. Line graph of Spectral Density and Short-memory spectral density

Table 8. Actual Observations and Expected Values using ARIMA (1,1,1) and ARFIMA (1,0.178,3)

Year(s)	Y_t **(Observed value of Nigeria All Share Index)**	**ARIMA (1,1,1) Model forecast**	ARFIMA (1,0.178,3) Model forecast
2021, September	39219.61	39146.8	39078.46
2021, October	40221.17	38988.43	39075.13
2021, November	40038.60	38831.42	39027.52
2021, December	43248.05	38675.76	38794.82
2022, January	42716.44	38521.43	38527.51
2022, February		38368.44	38241.78
2022, March		38216.75	37944.96
2022, April		38066.38	37641.39
2022, May		37917.29	37333.89
2022, June		37769.48	37024.77
2022, July		37622.95	36715.42
2022, August		37477.67	36407.12
2022, September		37333.64	36100.85

Table 8 contains the actual observations and results of the expected values for the ARIMA (1,1,1) and ARFIMA (1,0.178,3) models. Using these two estimated models, we made 13 out-of-sample predictions. The actual values from the Nigeria All Share Index published on the website of the Central Bank of Nigeria (www.cbn.org) and the expected values from the ARFIMA (1, 0.178,3) and ARIMA (1,1,1) models are shown in Table 3.7. From our visual checks, we see in Figure 6 that some time points of the results estimated using ARFIMA (1,0.178, 3) are closer to the actual values and, for others, ARIMA gives a better prediction.

5. Discussions

The raw series on Nigeria All-Share Index were plotted in time-plot as shown in Figure 1. Visual examination shows that the series is trending and fluctuating along the time plot. Also, the results in Table 1 show the descriptive statistic and normality test of the raw and return series for the Nigeria All Stock Index. The results obtained are in line with Lamouchi (2020). Research on long memory and stock market efficiency: the case of Saudi Arabia. In addition, the excess kurtosis to a distribution with values cantered around the middle and thicker tails of the two chains. Also, Table 2 presents the results of the unit root test for stationarity. The result test the null hypothesis of the presence of a unit root against the alternative hypothesis of absence of a unit root of the Dickey and Fuller (1981) augmented test, ADF, and Phillips and Byron's (1988) test, PP, which indicates the return of the steady process and variability chain. Since the probability of the test statistics for the ADFT, PPT, and KPSS, respectively, were less than the 5% ($Z < 0.05$) of significance then we conclude that the raw series is stationary. Also, Figure 2 shows the evolution of the time series of returns and the volatility of the Nigeria Index for all stocks. Shows periods of high volatility shaded by periods of high volatility, and periods of low volatility are manifested by periods of low volatility, showing volatility pools. Various spikes in volatility were observed during August 2007, July 2008 and June 2009.

Figures 3 and 4 show the autocorrelation and partial correlation function of the return series on the Nigeria All Share Index. The vertical axis of the graph contains autocorrelation and Partial Correlation function for the two graphs while their horizontal components indicate their respective lags. For Figure 3 the lags range from negative 1 to 1 in increments of 0.50, and the horizontal axis indicates lag, ranging from 0 to 40 in increments of 10. Their respective lags, indicating a clear pattern of persistence and slow decay, are typical cases of the long-memory process (Deebom & Essie, 2017). Figure 4, the partial autocorrelation is shown as positive. The first lag, the set of exponential decay, is an indication of the AR (P) spontaneous regression process. The plot shows a fairly large negative spike in the first interval, followed by rebounds between positive and negative values that are either not statistically significant or barely exceed the statistical significance threshold. However, from visual inspection, it appears that both autoregressive (AR) and moving average (MA) move in the opposite direction. Arivazhagan et al. (2019) found that the traditional static ARMA (autoregressive moving average) model has poor memory as a result of the autocorrelation function being significantly degraded. In contrast, ACF dies more slowly than theoretical autocorrelation with long delays. Arivazhagan et al. (2019) further demonstrated that conventional ARMA models typically produce a large number of parameters for model estimation, specifically when the autocorrelation decline history is slow. Based on our (unreported) empirical estimates and results, the appropriate short memory models used to describe Nigerian international markets can be traced back to autoregressive moving average (ARMA) models.

The estimate of the ARCH effect using the Ljung-Box (Q) statistic is shown in Table 3. The results show that there are strong autoregressive conditional heteroskedasticity (ARCH) effects in the two series.

Table 4 contains the results for test of long memory in ASI and its corresponding returns using rescale Lo statistics, Robinson's test and GPH test. The results confirmed the presence of long memory in the raw and returns on Nigeria All Stock Index (ASI). The estimates of ARIMA Models in modelling the returns on the Nigerian all share index are shown in Table 5. The best among the competing ARIMA model compatible with the Nigeria All-Share Index (ASI) is ARIMA (1,1,1) using the minimum Akaike information criteria. Table 6 also contains the results of estimation of long-memory models used in modelling ASI. It was found that the appropriate model estimated using the ARFIMA model is ARFIMA (1,0.178,3) based on the model with the minimum Akaike information criteria. The results of the confirmatory test for the absence of the ARCH in the ARIMA and ARFIMA model is contain in Table 7. Also, Figures 5 and 6 contain the results of the Quantile-Quantile plot for the prediction series of the ASI using ARIMA (1,1,1) and ARFIMA (1,0.178,3) while Figure 7 contains Nigeria all share index raw series, expected and fractional difference series for ARFIMA (1,0.178,3). All shows that both the ARIMA and ARFIMA model are adequate for forecasting. Figure 8 above shows that a shock to the Nigeria All-Share Index series can cause an underlying spike in the markets, after which the effect of the shock starts to gradually wear off. The actual observations and results of the expected values for the ARIMA (1,1,1) and ARFIMA (1,0.178,3) models are shown in Table 7. Two estimated models were estimated using 13 out-of-sample predictions; the results estimated using ARFIMA (1, 0.178, 3) are closer to the actual values, and for others, ARIMA gives a better prediction. The actual values from the Nigeria All Share Index published on the website of the Central Bank of Nigeria (www.cbn.org) and the expected values from the ARFIMA (1, 0.178,3) and ARIMA (1,1,1) models are shown in Table 8.

6. Conclusions

The objectives of this study include to evaluate statistical questions and to identify simple and stylised facts (such as mean, standard deviation, skewness statistic, kurtosis, ARCH effect, autocorrelation, and distribution properties) in relation to their effects on the financial time series under investigation, testing the efficient market hypothesis proposed by Fame, and to demonstrate the accuracy
of the prediction in the use of ARIMA and ARFIMA Model in Nigeria all stock index markets.

The mean of the gross series and returns in the Nigeria All Equity Index is a positive mean return by nature, although the standard deviation of the gross series is dispersedly distributed from the mean compared to the yield series, the primary series. (gross) is skewed to the right. While the return series exhibits left-skewed distributed characteristics. In addition, the kurtosis shows leptokurtic distribution characteristics with values centered in the middle and fat tail of the two shackles.

The ARCH effect was present in the series while the autocorrelation and the partial autocorrelation of the series showed distributive characteristics of the long-memory models. The implication of these simplified facts shows that the series exhibit tendencies that researchers must ultimately deal with in modelling financial markets. Based on these established patterns exhibited by some of the simplified facts in the raw and return series, the ARIMA and ARFIMA models have been found suitable for modelling the All-share index markets in Nigeria.

The gradual decline of the autocorrelation function demonstrates that sudden shocks to the index can trigger an initial shock in the market, after which the shock's impact slowly begins to die. This behaviour reveals evidence of long-memory processes, where there is a need to diversify investments because the recent shocks in the markets expose the economy and investors to risks. The results contradict the efficient market hypothesis proposed by Fame.

Also, the accuracy of the forecast showed that the two estimated models provide almost the same expected values as the actual values of the Nigerian All Share Index published by the Central Bank of Nigeria in its statistics database. This indicates that the ARIMA first-order process and the ARFIMA model can be used as an effective alternative for forecasting All Share Index markets in Nigeria.

Therefore, the effectiveness of ARIMA and the ARFIMA modelling financial series can be enhanced by incorporating other simplified facts for the provision of evidence, as well as other important features of long-memory time series models with the aim of improving the procedures for estimation of the models and their selection of criteria.

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