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A HYBRID BA - VNS ALGORITHM FOR SOLVING THE WEAPON TARGET ASSIGNMENT CONSIDERING MOBILITY OF RESOURCES

Abstract. *This study discusses the modelling, allocating, and scheduling in weapon target assignment (WTA) considering the mobility of weapons and targets. We provide a two-level mathematical model for this problem. A model is provided for each level, each of these two models has its objective function, and the two decision makers (defender and attacker) are unaware of each other's objective function. To solve the mathematical model in small sizes, IBM ILOG CPLEX software is used. For solving medium- and large-sized problems, a hybrid BA-VNS algorithm, which is a combination of the Bees Algorithm (BA) and Variable Neighbourhood Search (VNS) algorithm, is proposed. The Taguchi method is used to set parameters of the BA-VNS algorithm. The computational results show that the BA-VNS algorithm has the least error with the exact solution results. Also, the BA-VNS Algorithm has a good performance in solving medium and large sizes of problems.*

Keywords*: Competition Mechanism, Weapon Target Assignment, Mathematical Model, Bees Algorithm, Variable Neighbourhood Search*

JEL Classification: C61, C44, C02

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1. Introduction

Recent world events indicate that facilities are vulnerable to both attacks and dangers. Identifying critical system infrastructures and planning to increase and strengthen their security are key elements for the sustainability and efficiency of the service system in case of deliberate attacks and natural disasters (Hien et al., 2020). Critical infrastructures include the physical assets of a system, the loss of which may lead to a major disruption in the operating and application systems (e.g., transportation links like bridges, tunnels, railways, terminals, power plants, warehouses, emergency response centres (ERCs), hospitals, critical vaccine, medicine and food warehouses, key personnel like water system operators, and national signs whose loss can severely affect the public morale) (Xiao et al., 2020). Although the experience and skill of managers have an irreplaceable and indisputable role in the success of operations, the stressful conditions of the battle scene (which may disrupt optimal decision-making and even change the fate of operations) cannot be overlooked (Kline et al., 2019). Due to the high importance of manpower in military operations, one of the novel approaches in most of the world's armies is the use of automatic self-control systems; for example, nowadays drones are manufactured in most military industries (Truong et al., 2021). It is worth noting that to guide and tactically control these weapons (e.g. drones, unmanned armoured vehicles, etc.), there is a dire need for mathematical modelling to maximise their combat power on the scene (Silav et al., 2021). Therefore, along with the experience and wisdom of managers, using a series of decisions by mathematical models is logical to bring about better results (Wu et al., 2021). In the real world, the immobility of military equipment in field operations in one place has no military or tactical justification at all, and most military equipment is built with the approach of increasing mobility and movement; so, all the models presented in this field are modelled by assuming that one or both parties involved are fixed. Now, considering that in this research, we attempt to model weapon target assignment (WTA) based on the mobility of both weapons and targets, and since each weapon on the battlefield at the moment is engaged with one single target or numerous targets, first the targets assigned to the weapon must be prioritised according to their distance and proximity to the weapon, the range of the weapon loaded on the target and the type of target; thus, in addition to WTA, the scheduling of targets must be done, too. In this research, an attempt is made to discuss the allocation of a number of mobile weapons to a number of mobile targets. It should be noted that according to the conducted research, in the weapon target assignment, mobility of the weapon and the target has not been researched and studies can be done in this field. The purpose of this research is to model the problem of assignment of the weapon to the target, taking into account mobility of both the weapon and the target, and solving it using meta-heuristic algorithms. In this matter, each of these critical infrastructures must satisfy the demand of society in the conditions before and after the attack. To strengthen these critical infrastructures, a limited budget has been considered to protect critical infrastructures in the event of an attack. On the other hand, the attacker also has a limited budget to attack critical infrastructures. This problem can be considered as a leader-follower game, which facility should be served by each facility in pre-attack conditions, and how many defenders should be assigned to each facility in post-attack conditions? The main focus of this article is on how to model, allocate, and schedule insider fire bases in a network-oriented situation, and the variety of weapons in different sites by applying objective functions and operational limitations. The type of target function, taking into account the damage caused to the targets, the protection of sensitive areas, and the effective use of weapons - and the types of operational limitations - such as taking the damage caused to the targets to prevent ineffective shots, taking into account Conditions for using the model in multiple combat periods, applying restrictions to protect internal areas, the possibility of using guided weapons, taking into account volleys of fire, as well as maneuvering time for targets - which are examined in this article and in none It has not been investigated in previous researches. Weapon target assignment (WTA) is a fundamental problem arising from the defence-related applications of research in operations and means the allocation of n weapons to m known targets to minimise the possibility of target saving (survival). Many exact problem solving approaches have been examined so far to solve the problem of WTA in certain situations. For example, Ahuja et al. (2007) proposed a branch-bound method, Karasakal (2008) used the Lagrange release approach, and many other researchers used other exact methods like Losada et al. (2009) used Benderz analysis, Quadros et al. (2018) used branch and limit algorithm and etc. Additionally, as well as the complexity of equipment and resource assignment problems, the use of heuristic and meta-heuristic solution methods seems inevitable (Cao and Fang, 2020). The review of the literature shows that no study has fully considered the targets and protection of sensitive areas as a competitive game. In addition, the efficient use of weapons and the consideration of all kinds of restrictions such as the destruction of targets to prevent ineffective firing have not been comprehensively reviewed so far. Also, mobile targets and firebases, as well as the manoeuvring time for targets have not been considered yet. Kim et al. (2022) presented practical weapon target assignment (WTA) algorithms for a defence system to counter multiple targets concentrated within a narrow area, such as low-altitude rocket threats or drone swarms. Huang Fu et al. (2022) presented a weapon-target assignment method to achieve effective interception for all targets and avoid over-allocation of missiles. The genetic algorithm with elite retention strategy was used to solve the optimal allocation problem in consideration of adaptive grouping and field-of-view angle constraints. Liu et al. (2022) proposed a scheme based on decentralised peer-to-peer architecture and adapted artificial bee colony (ABC) optimisation algorithm. In the decentralised architecture, the peer computing node is distributed to each weapon units and the packet loss rate is used to simulate the unreliable communication environment. The rest of this paper is organised as follows: Section 2 contains problem definition, game strategy, and mathematical modelling. Section 3 describes the proposed BA-VNS meta-heuristic algorithm and Parameter adjustment. Section 4 presents the computational results of the research. Section 5 describes the conclusion and future research.

2. Problem Definition

The problem considered in this paper can be stated as follows: allocation of n weapons to m new targets to minimise the possibility of targets saving (survival). This study discusses the WTA and scheduling of several mobile weapons to several mobile targets. In this problem, there are a set of indoor fire bases, $n = \{1, \dots, N\}$, a set of targets $m = \{1, \dots, M\}$,, and a Set of candidate points of fire bases, $l = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2), \dots, (\alpha_n, \beta_n)\}$, and Set of a sensitive points for protection $p = \{(\alpha'_1, \beta'_1), (\alpha'_2, \beta'_2), \dots, (\alpha'_n, \beta'_n)\}$, so that each indoor fire bases n_i, If the suggested location is allowed for the base I, consists of a sequence of m_i target, as result maximum of protection from sensitive areas p created by attacking targets that have the possibility of hitting these areas. We should have a mechanism for competition between the two actors (defender and attacker). This method is necessary to minimise the information shared between the two decision makers. So, this study used bi-level programming. Each of these two models has its objective function, and both decision makers (defender and attacker) are unaware of each other's objective function. In this study, the decision maker at the upper level is attacker (Model 1) and the decision maker at the lower level is defender (Model 2).

2.1. Game strategy

As shown in Figure 1, the game mechanism begins with model 2 (defender model). After solving the model 2, if the time of the last shot is not positive, it means that the facility will not be attacked and the algorithm will stop. Otherwise, if the facility is attacked, the other variables of the attacker model are calculated. The decision variable of model 2 related to allocation and scheduling will be considered as the input of the model 1 (attacker model). After solving the model 1, if the average amount of damage to all targets reaches a certain amount (complete or predetermined destruction), the game will stop. The method of connecting the two levels of the model depends on a competitive game. In this method, two models will be presented from the perspective of two decision- makers; then these two models will be linked to each other by the competition mechanism. This method is necessary to minimise the information shared between the two decision-makers. Each of these two models has its objective function, and each of the two decision-makers (defender and attacker) is unaware of the other party's objective function. The competition mechanism between the two levels of the model and how the two models are transformed and connections are illustrated in Figure 1.

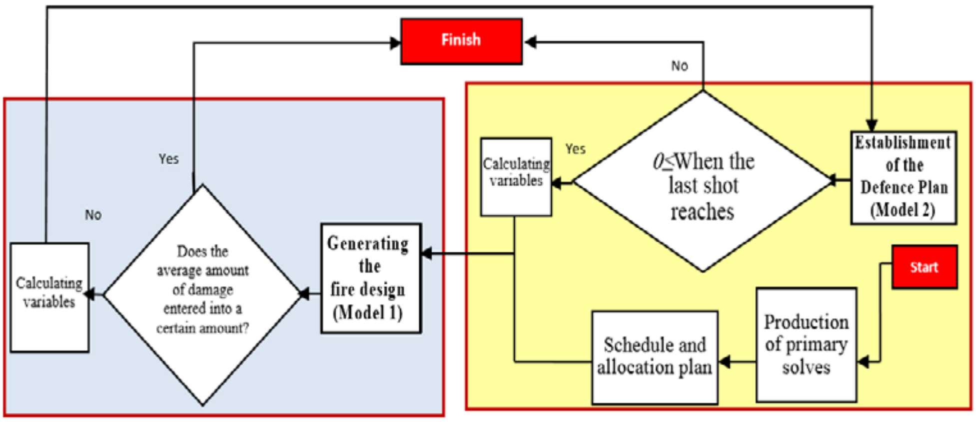

Figure 1. Competition mechanism of a two-level model

2.2. Mathematical modelling

Indexes

$N = \{1, \dots, n\}$	Set of indoor fire bases
$i \in n$	Index of indoor fire bases
$M = \{1, \dots, m\}$	Set of targets
$j \in m$	Index of targets
$T = \{1, \dots, t\}$	Time assigned for battle
$k \in K$	Index of weapons
K_i	Fire base weapons *i*
$L = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2), \dots, (\alpha_n, \beta_n)\}$	Set of candidate points of fire bases
$P = \{(\alpha'_1, \beta'_1), (\alpha'_2, \beta_{\prime 2}), \dots, (\alpha'_n, \beta'_n)\}$	Set of sensitive points for protection

Parameters

η_{il}	If the *suggested* location is allowed for base *i*, the value is 1; otherwise, it is equal to zero
δ_{ij}	Risk coefficient of target j for firebase *i*
b_j	Significance coefficient of target *j*
w'_{jp}	Risk coefficient of target *j* for sensitive area *p* according to the importance of the area
s	Time to prepare the weapon for firing (hr)
fl_{ik}	Weapon flight speed *k* from firebase *i* (hr)
mt_j	Manoeuvre time of target *j* (hr)
ut	Time to identify the location of a shot (hr)
v_i	Movement speed/velocity of firebase *i* (km)
x_{it}	Longitude of firebase *i* at moment *t*
y_{it}	Latitude of target *j* at moment *t*

χ_{jt}	Longitude of target *j* at moment *t*
w_{jt}	Latitude of target *j* at moment *t*
υ_j	Target speed *j* (km/hr)
β_j	Movement angle of target *j*
dis_{ijt}	Distance of firebase *i* from target *j* at moment *t* (km)
r_{ik}^{min}	Minimum firing range for weapon *k* from firebase *i* (km)
r'_j	The maximum range of target *j* (km)
φ_{ik}	Equals 1 if weapon *k* is guided from the firebase *i*; otherwise zero
ε_{ijt}	Equals 1 if the path of the direct line of fire *i* to target *j* passes over a minimum of an area; otherwise zero
d_{ijk}	Average destruction rate of target *j* if hit by weapon *k* from firebase *i*
h_j	Approximate weight of target *j* (in tons)
e_j	Destruction rate of target *j*
a_{ik}	Accuracy of weapon *k* from firebase *j* to destroy targets
γ	The average weight of the enemy's weapons and munitions in the battle scene
τ	The percentage of reduction in weapon accuracy at its maximum range
B	Resistance coefficient
R	The Earth radius
G	Big number

Variables

z_{ikjt}	If weapon *k* fires at target *j* from firebase *i* at moment *t*, it is equal to 1; otherwise, it is equal to zero
v_{pjt}	If the protection zone p at moment t at target *j* is equal to 1; otherwise it is equal to zero
$w_{ikpjt} = v_{pjt}.z_{ikjt}$	If a weapon *k* fires at target *j* from base *i* at moment *t* while area p is within range of target *j* at this point, it is equal to 1, otherwise it is zero.
A_{jt}	The cost amount of destruction created for target *j* at moment *t*
TS_j^{min}	When the first shot hits target *j*
TS_j^{max}	When the last shot reaches target *j*
TS_i^{f}	Time of the first firing from firebase *i*
TS_i^{l}	Time of the last firing from firebase *i*

Model 1

$$Min\ F_1 = \frac{\sum_{j\epsilon M}\sum_{t\epsilon T} b_j A_{jt}}{\sum_{j\epsilon M} b_j} \quad (1)$$

$$\sum_{t\in T} A_{jt} \le \sum_{i\in N}\sum_{t\in T}\sum_{k\in Ki} a_{ik}\,(1 - \frac{\tau(dis_{ijt} - r_{ik}^{min})}{(r_{ik} - r_{ik}^{min})} \times \frac{\gamma d_{ikt}}{h_j}) z_{ikjt} \qquad \forall j \in M \quad (2)$$

$$\sum_{t\in T} A_{jt} \leq 1 - e_j \qquad \forall j \in M \qquad (3)$$

$$\sum_{i\in N}\sum_{t\in T} z_{ikjt} \leq 1 \qquad \forall j \in J, k \in K_i \qquad (4)$$

$$v_{pjt} > (r'_j - dis_{pjt})/G \qquad \forall p \in P, j \in M, t \in T \qquad (5)$$

$$v_{pjt} \leq 1 + (r'_j - dis_{pjt})/G \qquad \forall p \in P, j \in M, t \in T \qquad (6)$$

$$\sum_{i\in N}\sum_{k\in Ki} w_{ikpjt} \leq G.v_{pjt} \qquad \forall p \in P, j \in M, t \in T \qquad (7)$$

$$TS_j^{max} - TS_j^{min} \leq mt_j \qquad \forall j \in M \qquad (8)$$

$$, z_{ikjt}, w_{ikpjt}, v_{pjt} \in \{0,1\} \qquad A_{jt} \geq 0 \qquad \forall j \in M, t \in T \qquad (9)$$

Function (1) is the Objective function of model 1 (F_1). This target function represents the average amount of damage inflicted on all targets. Therefore, the attacker tries to minimise the damage to facilities. Constraint (2) shows the amount of damage caused to each target during the entire planning period. In this regard, the accuracy of weapon k from the firebase i when target j is within a kilometer of this weapon is equal to 0.8 and when it is within the possible distance from this weapon, it is equal to 0.8. Also, the amount of destruction is calculated based on the relationship because the parameter is equal to the average amount of destruction inflicted on a target of 3500 tons, and the average amount of destruction decreases as the weight increases.

Constraint (3) limits the amount of destruction that is determined in constraint (2) to the value, that is, the percentage of the target's health, because allocating more weapons is useless and causes an ineffective increase in costs. constraint (4) states that each weapon from each firebase is capable of firing only once because it only carries one missile. Constraints (5) and (6) specify the definition of the variable. Constraint (7) is used to determine the relationship between the variables. In constraint (8), to inflict maximum destruction on the targets and prevent them from maneuvering, different shots must be fired at the same target consecutively and not exceed a maximum of the one-time unit. Constraints (9) shows the type of decision variables.

Model 2

$$Max\ F_2 = BF_{2.1} - (1 - B)F_{2.2} \qquad (10)$$

$$F_{2.1} = \frac{\sum_{i\in N}\sum_{k\in Ki}\sum_{p\in P}\sum_{t\in T}\sum_{j\in M} w'_{pj} w_{ikpjt}}{\sum_{i\in N}\sum_{k\in Ki}\sum_{p\in P}\sum_{t\in T}\sum_{j\in M} w'_{pj}} \qquad (11)$$

$$F_{2.2} = \frac{\sum_{i\in N}\sum_{k\in Ki}\sum_{j\in J}\sum_{t\in T}(r_{ik} - dis_{ijt}) \quad z_{ikjt}}{\sum_{i\in N}\sum_{k\in Ki}\sum_{j\in J}\sum_{t\in T}(r_{ik} - dis_{ijt} + \varepsilon)} \qquad (12)$$

$$\sum_{j\in N}\sum_{t\in T} z_{ikjt} \leq 1 \qquad \forall i \in N, k \in K_i \qquad (13)$$

$$\sum_{k\in Ki}\sum_{j\in M}\sum_{t'\in t+s_{ilt}+1} z_{ikjt'} \leq G.(1-\sum_{k\in Ki}\sum_{j\in M} z_{ikjt}) \quad \forall i \in N, t=1,\dots|T|-s_{ilt}, l \in L \quad (14)$$

$$r_{ik} - dis_{ijt} \geq G(z_{ikjt}-1) \quad \forall i \in N, k \in K_i, j \in M, t \in T \quad (15)$$

$$dis_{ijt} - r_{ik}^{min} \geq G(z_{ikjt}-1) \quad \forall i \in N, k \in K_i, j \in M, t \in T \quad (16)$$

$$\sum_{p\in P} w_{ikpjt} \leq G.z_{ikjt} \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (17)$$

$$w_{ikpjt} \geq z_{ikjt} + v_{pjt} - 1 \quad \forall i \in N, p \in P, k \in Ki, j \in M, t \in T \quad (18)$$

$$z_{ikjt} \leq G(1-\varepsilon_{ijt}) + G.\varphi_{ik} \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (19)$$

$$z_{ikjt'} \leq 1 + \frac{(1-e_j) - \sum_{k\in K_i}\sum_{i\in N_i}\sum_{t\in T} a_{ik}\,(1-\frac{\tau(dis_{ijt}-r_{ik}^{min})}{(r_{ik}-r_{ik}^{min})}\times\frac{\gamma d_{ikt}}{h_j})z_{ikjt}}{G} \quad (20)$$
$$\forall i \in N, k \in Ki, j \in M, t' \in T$$

$$TS_j^{min} \leq \left(t+\frac{dis_{ijt}}{fl_{ik}}\right) z_{ikjt} + G(1-z_{ikjt}) \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (21)$$

$$TS_j^{max} \geq \left(t+\frac{dis_{ijt}}{fl_{ik}}\right) z_{ikjt} \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (22)$$

$$TS_i^f \leq z_{ikjt} + G(1-z_{ikjt}) \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (23)$$

$$TS_i^l \geq z_{ikjt} \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (24)$$

$$TS_i^l - TS_i^f \leq ut \quad \forall i \in N \quad (25)$$

$$v_{pjt}, z_{ikjt}, w_{ikpjt} \in \{0,1\} \quad \forall i \in N, k \in Ki, j \in M, t \in T \quad (26)$$

$$A_{jt} \geq 0 \quad \forall j \in M, t \in T \quad (27)$$

$$TS_j^{min}, TS_j^{max}, TS_i^l, TS_i^f \in Z^+ \quad \forall j \in M, i \in N \quad (28)$$

Also Function (10) is the Objective function of model 2 (F_2). In the objective function (11), $F_{2.1}$ indicates the average amount of protection from sensitive areas by attacking targets that have the possibility of hitting these areas. In the objective function (12), $F_{2.2}$ represents the efficient use of the weapons of each base. This objective function allows weapons to be assigned to targets that have a close distance target range to the target range; this allows the weapons to be used as efficiently as possible and long-range weapons not fired at near targets. Constraint (13) means that each base can fire at a target at any time of maximum time. Constraint (14) also indicates the time interval between the firing of a firebase according to the time interval required to prepare the firebase for the next fire. According to Constraint (15), a specific weapon from a firebase can only fire when the target is within range of the weapon. Constraint (16) implies that if a shot can be assigned that its distance is more than r_{ik}^{min}Constraint (17) determines the relationship between variable $w_{ikpjt}$and variable z_{ikjt}Constraint (18) states that if to increase the destruction of targets F_1, one fire at a target such as j and on the other hand a protected area such as p is in the range of this target this firing can also be considered as defending area p. Constraint (19) describes the use or non-use of guided missiles as a weapon. Constraint (20) prevents the over-assignment of missiles to targets in the event of destruction. Constraints (21) and (22), respectively, determine the time of the first and last shots hitting the target. Constraints (23) and (24) specify the first and last shots fired from the firebase, respectively. Constraint (25) states that because the location of a firebase is detected shortly after the first shot at time ut by the targets on the battlefield, the distance between the first and last shots from this location should not be more than ut. Constraints (26), (27) and (28) Represents the type of decision variables.

3. Solution approach

The weapon target assignment (WTA) problem is known to be NP-complete (Lu & Chen, 2021). In recent years, there has been a lot of interest in using meta-heuristic algorithms to NP-complete problems (Mohammadi et al., 2013; Mehdizadeh and Fatehi Kivi, 2014; Aghajani-Delavar et al., 2015; Mehdizadeh et al., 2016). This section describes the proposed BA-VNS meta-heuristic algorithm which is a combination of bees algorithm (BA) and Variable neighbourhood search (VNS) method. Finally, the method of adjusting parameters is also stated.

VNS algorithm (Hansen and Mladenovic, 2001; Yazdani et al. 2010; Lv et al., 2023) is a modern meta-heuristic based on systematic changes of the neighbourhood structure within a search to solve combinatorial optimization problems. The main idea of the VNS algorithm is to use predefined neighbourhood structures to search for better solutions. Also, various versions of optimization algorithms have so far been proposed based on the group behaviour of bees. One of the most popular versions of bee-based algorithms is the BA method (Hussein et al., 2017) that mimic the intelligent behaviour of bees in search of food. This method was first proposed by Pham et al. (2006). The pseudo-code of the proposed BA-VNS algorithm is displayed in Figure 2.

Initialisation:
Initialize the initial population and Evaluate fitness;
Calculate the initial cost function value, f(Sol);
Set best solution, Solbest _ Sol;
Set maximum number of iteration, NumOfIte;
Set the population size;
//where population size = OnlookerBee = EmployeedBee;
iteration _ 0;
Improvement:
do while (iteration < NumOfIte)
for i=1: EmployeedBee
Select a random solution and apply random

neighbourhood structure;
Initialization. Select a set of neighbourhood structures Nk, k = 1,…, kmax, that will be used in the search; find an initial solution x; choose a stopping condition.
Repeat the following until the stopping condition is met:
(1) *Set* k = 1.
(2) *Until* k = kmax, repeat the following steps:
(a) Shaking. Generate a point x' at random from the k-th neighbourhood of x (x' ∈ Nk(x));
(c) Move or not. If this local optimum x' is better than the best incumbent, move to x=x', and continue the search with N_1, k=1; otherwise, set k=k+1.
End
Sort the solutions in ascending order based on the
Penalty cost;
Determine the probability for each solution, based
on the following formula:

$$p_i = \frac{\sum(\frac{1}{fit_i})^{-1}}{fit_i}$$

end for
for i=1: OnlookerBee
Sol* _ select the solution who has the higher
probability;
Sol** _ Apply a random Nbs on Sol*;
if (Sol** < Solbest)
Solbest=Sol**;
end if
end for
Scoutbee determines the abandoned source
and replace it with the new source.
iteration++
end do

Figure 2. Pseudo-code of proposed BA-VNS algorithm

To design the structure of the combined algorithm, the Random-Key (RK) technique has been used. The problem vector in the meta-heuristic algorithm presented in this research is displayed in Figure 3, the integer encoding method is used, in which the number of integers created is equal to the sum of the number of weapons (W) and the target solve (Ta) minus It is one (W+TA-1). To encrypt the generated vector after randomly generating the code, among the numbers generated in the vector, the number of TA-1 is created. It will be assigned to each firebase. To allocate 11 Target to four fire stations, the solution was 11+4-1= 14, in which the numbers 12, 13, and 14 will be the separators for each of the fire stations, which is below how they are located. Display the solve vector and decode the vector, where the targets 3 and 5 are to the firebase number 1 and the targets 2, 6, and 9 are to the firebase 2 and the exact solve is 11, 7, and 10 to the firebase number 3 and the exact solve is 1 and 8. It is assigned to fire station number 4.

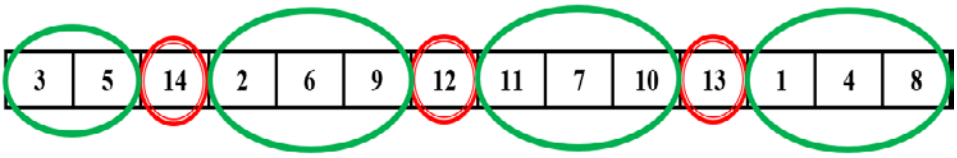

Figure 3. Decoding the relevant algorithm

As shown in Figure 3, first, a string of integers from 1 to 14 is randomly generated, in the generated string numbers 12, 13, and 14 are separators. According to this numerical string, targets 3 and 5 are assigned to fire station number 1, targets 2, 6 and 9 to fire station number 2, targets 11, 7 and 10 to fire station number 3, and targets 1, 4 and 8 to fire station no. 4 are allocated.

In the present study, the extracted features begin by giving the initial random population of the search space:

$$x_{mi} = x_i^{min} + rand * (x_i^{max} - x_i^{min}) \quad (29)$$

Where, x_{mi} is a solution vector for the optimisation problem $i = 1, \dots, n$ and $m = 1, \dots, S_N$. Also, S_N represents the number of initial populations, which is an n-dimensional vector in each x_i. Then the fitting function of each solution is calculated as below:

$$P_m = \frac{Fit_m(x_m)}{\sum_{m=1}^{SN} Fit_m(x_m)} \quad (30)$$

Where, $Fit_m(x_m)$ is a function of fit x_m, which is a ratio of the nectar volume of the food source where there in location m, and SN is the number of food sources. In the present study, a two-point intersection was used. The mechanism of this operator is such that two points are selected randomly and replaced with each other (Figure 4).

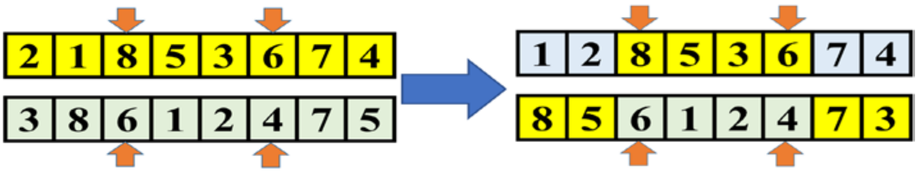

Figure 4. Two-point intersection

Figure 5 indicates the mutation operator. For this purpose, an allele is selected desirably; then it is reversed.

Figure 5. Mutation operator

In this study, two types of stop criteria have been considered for the algorithm. If either of these two conditions happens, the algorithm will stop. These two conditions are:

- Stop after reaching a certain number of repetitions (maximum repetition).
- Stop after a number of repetitions when there is no improvement in the solution.

The parameters of the proposed BA-VNS algorithm are adjusted by using the Taguchi method, and the results are reported in Table 1.

Table 1. Parameter adjustment

MaxIt	Max number of repetition	300
nPop	Max number of bee population	100
nOnlooker	Number of observer bees	50
L	Food source release limit	0.6
nVNS	VNS algorithm stop criterion	10

4. Computational results

To conduct the experiments, we use a PC with Intel® Core™ i7-4200M CPU and 6GB RAM for performing computational study. The small sized-problems are solved by using the exact solution tool of IBM ILOG CPLEX software. The research problem is NP-Hard, we will not reach the optimal solution of the medium- and large-sized problems in a reasonable time by using exact method of software, and with the increase in the size of the problem, the time increases exponentially. Therefore, to solve the problem in medium and large sizes, the BA-VNS meta-heuristic algorithm is proposed. In order to conduct the experiment, we implement the meta-heuristic algorithm in MATLAB R2020b. In this section, the results obtained by exact and meta-heuristic methods for generated test problems are presented and compared. To design sample problems, 10 samples with small sizes, 10 samples with medium sizes, and 10 samples with large sizes were prepared. Table 2 indicates the parameter values which are calculated with regard to uniform functions and used for generating test problems.

Table 2. Parameter values (u=uniform)

Parameter	Value	Parameter	Value	Parameter	Value
η_{il}	$u\sim$ [0, 1]	w_{jt}	$u\sim$(15, 45)	e_j	$u\sim$(15, 100)
δ_{ij}	$u\sim$(0, 1)	dis_{ijt}	$u\sim$(20, 200)	a_{ik}	$u\sim$(10, 150)

Parameter	Value	Parameter	Value	Parameter	Value
b_j	u~(0, 1)	r_{ik}^{min}	u~(20, 200)$	R	u~(50, 100)
w'_{jp}	u~(0, 1)	r'_j	u~(5, 100)$	G	100000000
s	u~(10, 50)	φ_{ik}	u~ [0, 1]	υ_j	u~(10, 70)
fl_{ik}	u~(100, 700)	ε_{ijt}	u~ [0, 1]	mt_j	u~(1, 4)
x_{it}	u~(15, 45)	d_{ijk}	u~(15, 80)	ut	u~(1, 3)
y_{it}	u~(20, 65)	h_j	u~(20, 500)	v_i	u~(5, 50)
χ_{jt}	u~(25, 45)	β_j	u~(15, 45)		

In the first part, the results of solving the problem in small sizes, in order from sample S1 to S10, are solved by using the exact solution tool of IBM ILOG CPLEX software and hybrid meta-heuristic algorithm BA-VNS; then the results are compared. In a way, the efficiency and quality of the meta-heuristic algorithm solutions used in the research are evaluated and checked. The numerical results obtained are presented in Table 3.

Table 3. The results of solving in small sizes

Problem No.	BA-VNS algorithm solution				exact solution				Error 1	Error 2
	Objective function Model 1 F_1	time (seconds)	Objective function Model 2 F_2	time (seconds)	Objective function Model 1 F_1	time (seconds)	Objective function Model 2 F_2	time (seconds)		
S1	376843.6	20	2392241.1	35	376843.6	15	2392241.5	45	0	1.67E-05
S2	427316.1	57	2192411.2	66	427313.0	97	2192416.3	55	0.0007	0.0002
S3	526216.8	55	2037911.5	63	526212.3	267	2037914.8	66	0.0008	0.0001
	532719.8	65	1973456.8	72	532684.2	388	1973459.5	79	0.0066	0.0001
S4	666705.8	174	1824631.5	237	666702.8	589	1824635.5	597	0.0004	0.0002
S5	669627.3	197	1782184.9	315	669622.7	678	1782218.8	753	0.0006	0.001
S6	675628.5	203	1765673.0	320	675618.7	1894	1765677.4	874	0.0014	0.0002
S7	698725.8	236	1722009.4	410	698720.0	2734	1722015.5	2157	0.0008	0.0003
S8	775823.0	279	1697177.7	428	775814.6	3832	1697182.3	3864	0.001	0.0002
S9	795659.9	383	1652474.0	434	795643.5	4682	1652480.0	4907	0.002	0.0003
S10	845945.5	411	1627568.3	456	845941.1	7934	1627574.6	6985	0	1.67E-05

To clarify the results of Table 3, the problem S6 is considered. The objective function value of model 1 is equal to 675628.5 and the objective function value of model 2 is equal to 1765673.0 and the computational time is equal to 203 and 320 seconds (for the meta-heuristic approach) for problem S6. Meanwhile, the objective function of model 1 in the exact solution is equal to 675618.7 and the objective function of the second model is equal to 1765677.4 and the solution time is equal to 1894 and 874 seconds. Also, the amount of all errors was below one percent, so we can trust the proposed solution approach to solve medium- and large-sized problems.

In the following, taking into account that the time to solve the problems increases exponentially with the increase in the sizes of the problem and it was not possible to reach the optimal solution in a reasonable and appropriate time using the exact solver of the IBM ILOG CPLEX software, the hybrid meta-heuristic method of the BA and VNS algorithms (BA-VNS) has been used to solve the problems in the medium and large sizes. The numerical results of solving the problems are presented in Table 4. As can be seen, samples M1 to M10 are for medium sizes, and samples L1 to L10 are for large sizes.

Table 4. The results of solving on medium and large-sized of problems

Problem No.	BA-VNS algorithm solution			
	Objective function Model 1 F_1	time (seconds)	Objective function Model 2 F_2	time (seconds)
M1	959992.1	471	1497578.9	466
M2	969585.4	498	1475662.2	489
M3	975546.7	519	1461148.7	486
M4	977536.4	555	1447733.3	497
M5	989945.1	573	1424667.1	510
M6	997277.7	584	1414567.7	533
M7	1074781.9	685	1407568.8	568
	1275433.8	795	1377586.5	589
M8	1376579.0	1190	1357669.9	643
	1474775.6	1490	1355567.7	673
M9	1554781.8	1984	1342323.6	717
M10	1668970.0	2154	1322745.4	1792
	1699766.8	2795	1320037.8	1895
L1	2125734.4	2964	1252590.0	2093
L2	2186738.9	3118	1246115.2	2574
L3	2227834.8	3257	1234473.7	2794
L4	2315475.0	3364	1208725.8	2893
L5	2336736.6	3463	1172583.1	2984
L6	2375734.9	3574	1162844.7	3011
	2401765.8	3588	1160921.4	3062
L7	2415642.0	3599	1159376.4	3183
L8	2472768.8	3794	1132643.3	3232
L9	2515787.5	4010	1105765.3	3342
L10	2668568.8	4234	1062565.1	4243

Figure 6 shows the problem-solving computational time for all research sample problems with two approaches, exact solution and meta-heuristic solution. As seen from the presented figure, the computational time of the problems responses increases exponentially with the increase in the problem sizes.

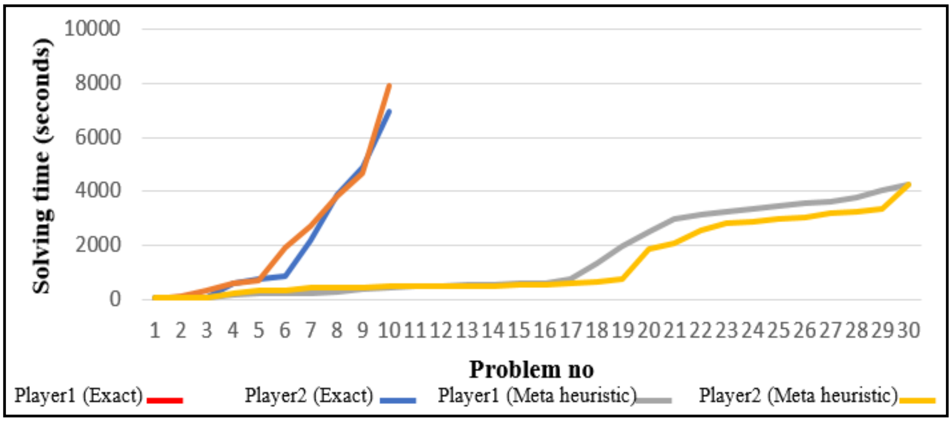

Figure 6. Computational times of meta-heuristic and exact methods based on problems

5. Conclusions and Future Research

In the real world, the immobility of military equipment in the battle scene has no military or tactical justification, so that, nowadays, most military equipment is built with the approach of increasing mobility and displacement. However, all models presented in these domains are modelled by assuming that one or both parties involved are immobile. We attempted to model the WTA problem according to mobility of both weapons and targets, and since each weapon on the battlefield at the moment is engaged with the target and/or targets, the assigned targets to the weapon must first be prioritised based on the distance and proximity of the target to the weapon, the range of weapon loaded on the target, and the type of target; hence, in addition to WTA, the scheduling of targets must have also been done in this research.

Accordingly, a mathematical model for managing the war scene was presented. The model consists of two actors as firebase and target. Each of these two models has its objective function, and each of the two decision-makers (defender and attacker) is unaware of the other party's objective function. According to the test problems solved to validate the model, the results showed that the proposed two-level model can correctly and effectively model WTA problems. The results of calculations on small-sized problems showed that by increasing the size of problems, problem-solving computational time will increase exponentially. Therefore, the studied problem is NP-Hard. In order to solve the model at a reasonable time in medium and large sizes, BA-VNS meta-heuristic algorithm was used. We implemented the Taguchi approach to adjust the parameters. The small-sized problems were solved by an exact solution approach (IBM ILOG CPLEX Software) and the BA-VNS algorithm. The results on small sizes showed that the proposed BA-VNS algorithm, like the exact solution approach, could achieve a global optimum solution with minimum error. Also, the BA-VNS algorithm achieved efficient solutions in a reasonable time for solving medium and large sizes of problems. In the end, the following suggestions are provided for future studies: considering the flight targets to confront the targets; presenting a scenario-based mathematical model and solving the model using robust optimisation; considering other meta-heuristics or machine learning approaches to solve the proposed model.

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