Associate Professor Martina PAVLAČKOVÁ, PhD
(corresponding author)
E-mail: pavlackovam@centrum.cz
Department of Applied Mathematics
Moravian Business College Olomouc, Czech Republic

Assistant Professor Ondřej PAVLAČKA, PhD
E-mail: ondrej.pavlacka@upol.cz
Department of Mathematical Analysis and Applications of Mathematics
Palacký University Olomouc, Czech Republic

Bc. Tereza HORČIČKOVÁ, Master Student
E-mail: 7977@mail.mvso.cz
Department of Applied Mathematics
Moravian Business College Olomouc, Czech Republic

A FUZZY GROUP DECISION-MAKING MODEL FOR MEASUREMENT OF COMPANIES' PERFORMANCE

Abstract. *In the paper, a fuzzy multi-criteria group evaluation model suitable for the measurement of companies´ performance is developed. The constructed model considers in addition to the traditionally used quantitative financial point of view also nonfinancial qualitative criteria such as Corporate Social Responsibility, innovation, or service level containing a considerable degree of uncertainty that can be appropriately modeled by the tools of fuzzy sets theory. Since the evaluations are often done by a group of experts with different opinions, our approach enables one to find at first a set of alternatives that are good enough for the sufficient quantity of relevant experts, and therefore to reach a consensus among evaluators. The best alternative is subsequently chosen from this set using a fuzzy weighted average operation. The model can be used for the comparison of companies in the same field, in one region, or to compare a company before and after managerial interventions.*

Keywords*: Group Decision-making, Multi-criteria Evaluation, Fuzzy Consensus, Fuzzy Weighted Average Operation*

JEL Classification: D790, D810, M210

1. Introduction

In today's ever-changing and competitive environment, the correct evaluation of company performance is an important issue not only for customers and investors but also for companies from the same field or region themselves.

DOI: 10.24818/18423264/57.2.23.08

In this paper, a model suitable for such an evaluation of company performance will be developed; the inspiration for its construction is the competition the Olomouc Region Entrepreneur of the Year that has been serving for several years, for the comparison of the companies in the Czech Republic from the Olomouc region. This competition belongs to one of the Entrepreneurs of the Year events that was founded by EY (formerly known as Ernst & Young) in the United States of America in 1986. Since then, the Entrepreneurs of the Year events have spread to almost 60 countries on six continents.

Inspired by this event, the evaluation model based on the group decision-making methods will be developed in this paper. The prerequisite that the best alternative can be only the one that is good enough according to the sufficient number of important evaluators will be one of the key factors for the creation of this model. Moreover, since uncertainty is present in the evaluation process, it will be appropriate to implement the tools of fuzzy sets theory in the constructed model.

Let us note that it is possible to use the model also for other purposes, e.g., for the comparison of the companies from the same field or for the selection of the best supplier. In addition, this model can perform the comparison of the evaluations of one company before and after managerial interventions.

The model will be based on the concept of the soft consensus introduced by Kacprzyk and Fedrizzi (1986) and by Kacprzyk and Fedrizzi (1997) and will represent an application of an appropriate modification of the fuzzy group decision-making model developed in Sukač and Pavlačka (2018). It will enable one to handle the situation where experts have different competencies and knowledge. Since the experts can have also different attitudes to the evaluation of the companies' performance, they can set different weights describing the importance of the criteria.

Since fuzzy sets theory is a suitable tool for implementing qualitative criteria in the analysed model as well as the uncertainty of the inputs, it has been used in companies´ performance measurement models without group character several times. For example, in Magni et al. (2006) and Magni et al. (2020) models for rating companies and companies' default risk based on fuzzy logic and expert system were introduced. In Erturğul and Karakaşoğlu (2009), a fuzzy model for the evaluation of the performance of the companies by using financial ratios, based on the Fuzzy Analytic Hierarchy Process and TOPSIS methods, has been developed. The proposed method has been subsequently used in Ertuğrul and Karakaşoğlu (2009) for evaluating the performance of 15 Turkish cement companies. The same methods have been applied in Sun (2010) to help industrial practitioners with performance evaluation in a fuzzy environment. A multi-criteria model for the analysis and evaluation of services in manufacturing companies using the Analytic Hierarchy Process and fuzzy logic has been introduced in Oblak et al. (2017). Using possibility distribution-based multi-criteria decision analysis, a resilient supplier selection problem has been solved in Jiang et al. (2020) and applied to the case study for illustrating the mechanism of the proposed algorithm.

Summing up, many different fuzzy multi-criteria decision-making methods have been applied to solve both general companies' evaluation tasks and specific problems in concrete supplier branches. However, in none of the mentioned papers, the group decision-making approach has been considered; the evaluations of companies' performance have been done there always by the individual evaluator. But when the evaluation process is a part of some competition or decision of a group of managers, it is necessary to deal with it from the group decision-making perspective and optimally implement also the consensus reaching inside, as in our model.

The paper is organised as follows. In the next section, some preliminary definitions dealing with fuzzy sets, fuzzy numbers, and linguistic fuzzy modelling, in general, are briefly mentioned. Also, a fuzzy weighted average operation that will be used for the computing of the overall evaluations will be introduced there. In the subsequent section, the proposed multi-criteria group decision-making model for the companies' performance measurement evaluation will be described in detail and the effect of the consensus attitude will be analysed.

2. Preliminaries

A powerful tool for implementing uncertainty into the group decision-making models is the fuzzy sets theory that was introduced in 1965 by Lotfi A. Zadeh in Zadeh (1965). In this section, basic notions concerning fuzzy sets, fuzzy numbers, and linguistic variables will be mentioned. Also, the fuzzy weighted average operation that will be applied as one of the principal tools for the construction of the group multi-criteria evaluation model will be briefly described.

2.1 Fuzzy Sets and Fuzzy Numbers

Let U be a nonempty set. A fuzzy set A on U is characterised by its membership function $\mu_A: U \rightarrow [0,1]$, where, for all $x \in U$, $\mu_A(x)$ expresses the degree of membership of x in the fuzzy set A. The family of all fuzzy sets on the universe U will be denoted by $\mathrm{F}(U)$. For any $\alpha \in (0, 1]$, $A(\alpha)$ will mean an α-cut of A, i.e. $A(\alpha) = \{x \in U\colon \mu_A(x) \geq \alpha\}$. By Core($A$) and Supp($A$), a core of A and a support of A will be denoted, i.e., $\mathrm{Core}(A) = \{x \in U\colon \mu_A(x)=1\}$ and $\mathrm{Supp}(A) = \{x \in U\colon \mu_A(x) > 0\}$. If the support of A is a discrete set, $\mathrm{Supp}(A) = \{x_1, \ldots, x_k\}$, then the fuzzy set A will be denoted by $A = \{\mu_A(x_1)|x_1, \ldots, \mu_A(x_k)|x_k\}$.

A special type of fuzzy sets are fuzzy numbers. A fuzzy number is a fuzzy set A on the real axis $\mathbf{R}$ (or on its nonempty subset I) that fulfils the following conditions: 1) $\mathrm{Core}(A) \neq \emptyset$, 2) for all $\alpha \in (0, 1]$, $A(\alpha)$ are closed intervals, 3) Supp(A) is bounded. The set of all fuzzy numbers on I will be denoted by $\mathrm{F_N}(I)$ in the sequel.

Each fuzzy number A can be uniquely determined the system of its α-cuts (see, e.g., Dubois and Prade, 1988). Let, for $0 < \alpha \leq 1$, $\underline{a}(\alpha)$ and $\overline{a}(\alpha)$ denote the lower and the upper bound of the α-cut of a fuzzy number A, and let $[\underline{a}(0), \overline{a}(0)]$ be the closure of the support of A. Then, such a fuzzy number A will be denoted by $A = [\underline{a}, \overline{a}]$ in the paper.

Let us note that a real number a can be viewed as a fuzzy number $A = [\underline{a}, \overline{a}]$ of a special kind, where $\underline{a}(\alpha) = \overline{a}(\alpha) = a$, for all $\alpha \in [0,1]$. Therefore, it is possible to also handle the cases where some variables in the model are known precisely and some are fuzzy.

In the constructed multi-criteria group decision-making model, the triangular and the trapezoidal fuzzy numbers will be used. A trapezoidal fuzzy number A is described by the quartet $(a_1; a_2; a_3; a_4)$, where a_1 represents the smallest possible value, the interval $[a_2; a_3]$ the most possible values, and a_4 the largest possible value of the considered fuzzy variable. A trapezoidal fuzzy number is called a triangular if $a_2 = a_3$, i.e., if it is uniquely described by a triplet $(a_1; a_2; a_3)$.

As one of the steps of the model, the defuzzification by the centre of gravity will be used. By this process, a fuzzy number A is de-fuzzified into its centre of gravity $COG(A)$, i.e. into the real number given by the formula

$$COG(A) = \frac{\int_{-\infty}^{\infty} \mu_A(x) x \, dx}{\int_{-\infty}^{\infty} \mu_A(x) dx}, \quad \text{if } \int_{-\infty}^{\infty} \mu_A(x) dx \neq 0;$$

$$COG(A) = a, \quad \text{if } A \text{ represents a real number } a.$$

In the paper, a linguistic approach to group decision-making problems will be applied. Therefore, the basic facts about the linguistic variables will be mentioned now.

A linguistic variable is characterised by a quintuple $(V, T(V), U, G, M)$, where V is the name of the linguistic variable, $T(V)$ is the set of its linguistic terms, $U \subset \mathbf{R}$ is the universe on which fuzzy numbers expressing meanings of the linguistic terms are defined, G is the grammar used for generating of linguistic terms $T(V)$ and M is a semantic rule for associating each term $\mathcal{A} \in T(V)$ with its meaning $A = M(\mathcal{A})$, $A \in F(U)$.

For more details about fuzzy sets theory, we recommend Zadeh (1965) and for more information about linguistic variables, see, e.g., Zadeh (1975).

2.2 Fuzzy Weighted Average of Fuzzy Numbers

In group decision-making models, weights of criteria expressing their importance as well as the criteria values are usually set subjectively and are therefore uncertain. For this purpose, the models are more realistic if both are expressed by means of the tools of fuzzy sets theory. If the 'crisp' model is based on the l weighted average operation, the appropriate transformation into the fuzzy sets' environment is formed by fuzzy weights and fuzzy weighted average of fuzzy numbers (see, e.g., Pavlačka et al., 2017, Pavlačka and Pavlačková, 2021).

Let $I \subset \mathbf{R}$ be nonempty. A fuzzy weighted average of fuzzy numbers $U_1, U_2, \ldots U_m \in F_N(I)$, $U_i = [\underline{u}_i, \overline{u}_i]$, for all $i = 1, \ldots, m$, with fuzzy weights $W_1, W_2, \ldots W_m \in F_N([0,1])$ is a fuzzy number $A = [\underline{a}, \overline{a}]$ defined on I such that for all $\alpha \in [0,1]$:

$$\underline{a}(\alpha) = \min\left\{\frac{\sum_{i=1}^{m} w_i \underline{u}_i(\alpha)}{\sum_{i=1}^{m} w_i} : w_i \in W_i(\alpha), i = 1, \ldots, m\right\}, \quad (1)$$

$$\overline{a}(\alpha) = \max\left\{\frac{\sum_{i=1}^{m} w_i \overline{u}_i(\alpha)}{\sum_{i=1}^{m} w_i} : w_i \in W_i(\alpha), i = 1, \ldots, m\right\}. \quad (2)$$

The usual approach to computing the fuzzy weighted average is the α-cut decomposition method. The input fuzzy numbers are discretized into the set of selected α-cuts and the border values of corresponding α-cuts of the fuzzy weighted average are computed by solving optimisation problems (1) and (2). The resulting fuzzy weighted average is then given approximately by connecting the α-cuts together.

3. Companies´ Evaluation Model Based on the Fuzzy Group Decision-Making Methods

The proposed multiple criteria group decision-making model for companies´ comparison is inspired by the model described in Sukač and Pavlačka (2018), that represents the improvement of the model introduced in Sukač et al. (2016). The whole concept is based mainly on reaching a fuzzy consensus among evaluators. Such an approach enables to find at first a set of alternatives that are good enough for the sufficient quantity of relevant experts, and therefore to reach a required consensus among evaluators. The best alternative is subsequently chosen from this set using a fuzzy weighted average operation.

Let us note that the linguistic terms used in the model together with their descriptions by triangular or trapezoidal fuzzy numbers can be in concrete applications modified in order to properly express the experts´ recommendations.

The constructed fuzzy group decision-making for performance measurement of companies consists of several phases that are characterized in detail in this section. Each step contains a general introduction at first. Afterward, the concrete form of the step for our model for the evaluation and comparison of companies that have participated in the competition of the Olomouc Region Entrepreneur of the Year is described in detail.

1) *The statement of the multiple criteria group decision-making problem*

As a first step, it is necessary to specify the group decision-making problem itself. Generally, we assume that $m, n, l \in \mathrm{N} \setminus \{1\}$ and consider the problem in which the set of alternatives $X = \{X_1, \ldots, X_n\}$ is evaluated with respect to the set of criteria $C = \{C_1, \ldots, C_m\}$ by the group of experts $E = \{E_1, \ldots, E_l\}$ with possibly different competencies. The experts' competencies are given by the linguistic terms described by trapezoidal fuzzy numbers $K_j \in F_N([0,1])$, $j = 1, \ldots, l$ (see Table 1 and Figure 1).

Table 1. The linguistic terms describing the experts' competencies

Linguistic term	Trapez. fuzzy number K_j
Extremely low competence (EL)	(0.00; 0.00; 0.08; 0.15)
Very low competence (VL)	(0.08; 0.15; 0.23; 0.31)
Low competence (L)	(0.23; 0.31; 0.38; 0.46)
Average competence (A)	(0.38; 0.46; 0.54; 0.62)
High competence (H)	(0.54; 0.62; 0.69; 0.77)
Very high competence (VH)	(0.69; 0.77; 0.85; 0.92)
Extremely high competence (EH)	(0.85; 0.92; 1.00; 1.00)

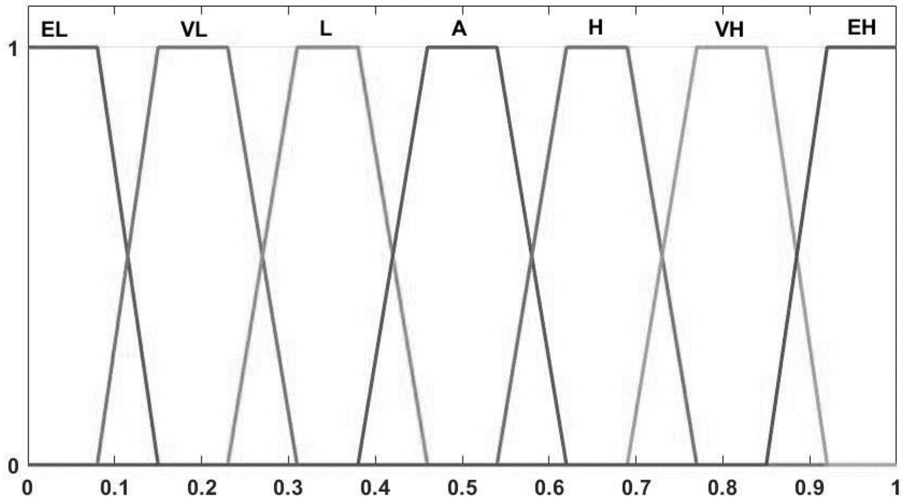

Figure 1. Meanings of the linguistic terms describing the experts' competencies

For the purposes of our companies´ evaluation model, we consider that m = 5, n = 8, and l = 6, i.e., that 6 evaluators participate in the competition with the different competencies described in Table 2.

Table 2. Competencies of experts

Expert	Expert´s competence
Expert 1	Extremely high (EH)
Expert 2	Average (A)
Expert 3	Low (L)
Expert 4	High (H)
Expert 5	Very high (VH)
Expert 6	Low (L)

Furthermore, each of the evaluators rates each of 8 competing companies, for reasons of anonymity denoted there by X_1, X_2, …, and X_8, according to 5 criteria, C_1 = Company's Financial Health, C_2 = Quality, C_3 = Service level, C_4 = CSR, and C_5 = Innovation.

The first criterion, financial health, covers the traditional metrics (Brealey et al., 2020 or Kieso et al., 2007) structured in four groups: liquidity, solvency, profitability, and stability. The combination of these is inspired by the Bankruptcy prediction model (Thomas and Indriaty, 2020 or Altman, 2013), which originated in 1968 and is still in use today (Ogachi et al., 2020). The second criterion, quality, deals with the quality of products, ISO and similar standards, certificates, etc., labour expertise, and operational excellence. The criterion service level is related to pre-sales, sales, and after-sales services, and on-time delivery. CSR relates to Corporate Social Responsibility and usually deals (Young and Thyil, 2009) with ecology, social impact, employees, etc.

Since the partial evaluations of variants are done by the physical persons, evaluators, with different experiences and different preferences, and since the criteria contain sometimes quite contradictory parts, the evaluations can be very variable, and quite different results of the same company can therefore occur.

2) Setting the degrees of fulfilment of the goals by alternatives

Each criterion C_i, $i = 1,\ldots, m$, is connected with the partial goal and each expert E_j, $j = 1,\ldots, l$, assigns the level of fulfilment of this partial goal by each of the alternatives X_k, $k = 1,\ldots, n$. This assignment is achieved through the triangular fuzzy number H^j_{ki} on [0,1] that expresses the fuzzy degree of fulfilment of the given *i*-th goal by the alternative X_k according to the expert E_j. The linguistic terms used for the description of the fulfilment of partial goals, together with the corresponding fuzzy numbers, can be found in Table 3 and Figure 2.

Table 3. The linguistic terms describing the fulfillment of partial goals

Linguistic term	Triang. fuzzy number H^j_{ki}
Very poor fulfillment (VP)	(0.00; 0.00; 0.15)
Poor fulfillment (P)	(0.00; 0.15; 0.35)
Substandard fulfillment (SS)	(0.15; 0.35; 0.50)
Standard fulfillment (S)	(0.35; 0.50; 0.65)
Above standard fulfillment (AS)	(0.50; 0.65; 0.85)
Very good fulfillment (VG)	(0.65; 0.85; 1.00)
Excellent fulfillment (E)	(0.85; 1.00; 1.00)

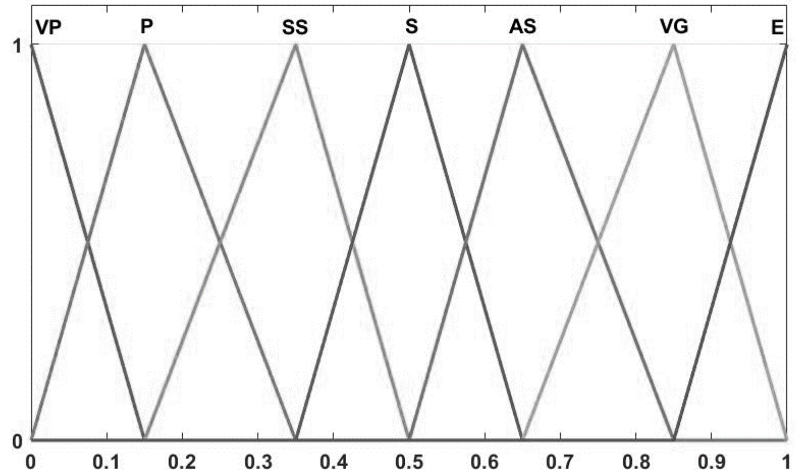

Figure 2. Meanings of the linguistic terms describing the fulfilment of partial goals

In our companies´ evaluation model, the evaluators have different attitudes and opinions about the studied decision-making problem and the fulfilment of partial goals. The experts´ evaluations of the fulfilment of the partial goals by particular variants are shown in Tables 4-9 below.

Table 4. Expert 1 partial evaluations

Exp.1	C_1	C_2	C_3	C_4	C_5
X_1	VG	E	AS	AS	VG
X_2	VG	E	E	E	E
X_3	AS	P	E	VG	E
X_4	VG	VG	P	P	P
X_5	S	AS	S	SS	AS
X_6	AS	VG	VG	VG	VG
X_7	E	SS	E	SS	P

Table 5. Expert 2 partial evaluations

Exp.2	C_1	C_2	C_3	C_4	C_5
X_1	E	AS	AS	AS	VG
X_2	VG	E	E	E	E
X_3	AS	SS	E	VG	E
X_4	AS	AS	VP	P	SS
X_5	VG	VG	S	SS	VG
X_6	AS	VG	AS	VG	VG
X_7	VG	AS	E	SS	P

Table 6. Expert 3 partial evaluations

Exp.3	C_1	C_2	C_3	C_4	C_5
X_1	E	AS	SS	S	VG
X_2	S	E	E	E	E
X_3	S	P	VG	E	E
X_4	VG	VG	P	VP	P
X_5	VG	VG	AS	S	VG
X_6	AS	AS	VG	VG	E
X_7	VG	AS	VG	P	VP

Table 7. Expert 4 partial evaluations

Exp.4	C_1	C_2	C_3	C_4	C_5
X_1	VG	VG	S	AS	VG
X_2	S	AS	VG	VG	S
X_3	VG	SS	VG	VG	E
X_4	VG	AS	P	P	P
X_5	VG	VG	AS	P	AS
X_6	AS	VG	VG	VG	VG
X_7	E	S	VG	P	P

Table 8. Expert 5 partial evaluations

Exp.5	C_1	C_2	C_3	C_4	C_5
X_1	VG	E	AS	AS	AS
X_2	S	AS	AS	VG	S
X_3	AS	SS	E	E	VG
X_4	AS	AS	VP	P	VP
X_5	S	AS	S	P	AS
X_6	AS	VG	VG	VG	VG
X_7	VG	S	VG	SS	VP

Table 9. Expert 6 partial evaluations

Exp.6	C_1	C_2	C_3	C_4	C_5
X_1	E	AS	S	VG	VG
X_2	E	E	E	E	E
X_3	AS	S	E	VG	E
X_4	VG	VG	P	VP	P
X_5	VG	VG	AS	SS	VG
X_6	AS	VG	AS	AS	E
X_7	E	AS	E	SS	P

3) Setting the fuzzy weights

As the next step, each expert E_j, $j = 1, \ldots, l$, assigns to the criteria C_i, $i = 1, \ldots, m$, the fuzzy weights $W^j{}_1, W^j{}_2, \ldots, W^j{}_m \in F_N([0,1])$ that express the importance of the considered criteria from his/her point of view. For this purpose, the linguistic terms very low importance, low importance, average importance, high importance, and very high importance are commonly used. Their meanings are shown in Table 10.

Table 10. The linguistic terms describing the importance of criteria

Linguistic term	Triang. fuzzy number $W^j{}_i$
Very low importance (VL)	(1.00; 1.00; 2.00)
Low importance (L)	(1.00; 2.00; 3.00)
Average importance (A)	(2.00; 3.00; 4.00)
High importance (H)	(3.00; 4.00; 5.00)
Very high importance (VH)	(4.00; 5.00; 5.00)

In the studied companies' fuzzy evaluation model, the experts set the fuzzy weights describing the importance of the criteria as shown in Table 11.

Table 11. Experts' fuzzy weights

	C_1	C_2	C_3	C_4	C_5
E_1	VH	H	A	L	A
E_2	VH	H	H	A	A
E_3	VH	A	A	VL	A
E_4	VH	H	A	A	A
E_5	VH	H	A	H	L
E_6	VH	H	L	H	A

4) Aggregation of partial evaluations for each expert

After the determinations of the fuzzy weights and the degrees of the fulfilment of the given goals by particular alternatives according to all experts, the partial evaluations should be aggregated. The aggregation is done using the fuzzy weighted average of fuzzy numbers with fuzzy weights applying formulas (1) and (2) using α-cut decomposition method.

The overall evaluation $H^j_k = [\underline{H}^j_k, \overline{H}^j_k]$, $j = 1, \dots, l$, $k = 1, \dots, n$, of the k-th variant according to the j-th expert is computed for all $\alpha \in [0,1]$ as follows:

$$\underline{H}^j_k(\alpha) = \min\left\{\frac{\sum_{k=1}^{m} w_k{}^j \underline{H}^j{}_{ik}(\alpha)}{\sum_{i=1}^{m} w_k{}^j} : w_k{}^j \in W_k{}^j(\alpha), k = 1, \dots, m\right\}, \quad (3)$$

$$\overline{H}^j_k(\alpha) = \max\left\{\frac{\sum_{k=1}^{m} w_k{}^j \overline{H}^j{}_{ik}(\alpha)}{\sum_{i=1}^{m} w_k{}^j} : w_k{}^j \in W_k{}^j(\alpha), k = 1, \dots, m\right\}, \quad (4)$$

where $H^j_{ik} = [\underline{H}^j_{ik}, \overline{H}^j_{ik}]$, $i = 1, \dots, m$, $j = 1, \dots, l$, $k = 1, \dots, n$, is an evaluation of the alternative X_k w.r.t. the i-th goal according to the expert E_j. The computed overall evaluation H^j_k describes the acceptability of the alternative X_k by the expert E_j and thus does not depend on the set of alternatives.

In the proposed companies´ evaluation model, the partial evaluations are computed by the Matlab software applying formulas (3) and (4) together with α-cut decomposition method. The results are shown in Tables 12 and 13 below.

Table 12. Partial evaluations of alternatives by experts 1-3

Alternative	E_1	E_2	E_3
X_1	(0.62; 0.83; 0.97)	(0.59; 0.77; 0.93)	(0.50; 0.74; 0.88)
X_2	(0.77; 0.96; 1.00)	(0.78; 0.96; 1.00)	(0.64; 0.83; 0.92)
X_3	(0.43; 0.68; 0.86)	(0.52; 0.75; 0.89)	(0.40; 0.63; 0.83)
X_4	(0.25; 0.52; 0.78)	(0.21; 0.39; 0.63)	(0.24; 0.51; 0.75)
X_5	(0.36; 0.54; 0.75)	(0.46; 0.70; 0.89)	(0.56; 0.79; 0.96)
X_6	(0.59; 0.79; 0.97)	(0.56; 0.76; 0.95)	(0.58; 0.77; 0.94)
X_7	(0.35; 0.62; 0.78)	(0.40; 0.65; 0.84)	(0.35; 0.59; 0.82)
X_8	(0.49; 0.71; 0.92)	(0.29; 0.49; 0.70)	(0.38; 0.59; 0.79)

Table 13. Partial evaluations of alternatives by experts 4-6

Alternative	E_4	E_5	E_6
X_1	(0.54; 0.76; 0.94)	(0.59; 0.78; 0.94)	(0.60; 0.81; 0.96)
X_2	(0.45; 0.65; 0.85)	(0.45; 0.64; 0.84)	(0.85; 1.00; 1.00)
X_3	(0.51; 0.76; 0.92)	(0.51; 0.74; 0.89)	(0.55; 0.76; 0.91)
X_4	(0.22; 0.46; 0.71)	(0.18; 0.36; 0.62)	(0.24; 0.47; 0.72)
X_5	(0.44; 0.67; 0.88)	(0.27; 0.47; 0.69)	(0.47; 0.72; 0.91)
X_6	(0.60; 0.79; 0.97)	(0.60; 0.79; 0.97)	(0.56; 0.75; 0.93)
X_7	(0.34; 0.58; 0.77)	(0.34; 0.57; 0.79)	(0.38; 0.64; 0.80)
X_8	(0.30; 0.53; 0.74)	(0.19; 0.40; 0.60)	(0.33; 0.54; 0.74)

5) Defining the linguistic variable expressing the level of acceptance of alternatives by experts

Since the proposed model is based on the concept of the soft consensus, it is necessary to find in the next 6 steps the set of alternatives that are good enough according to a sufficient quantity of important experts. For this purpose, additional linguistic variables have to be defined. First, the linguistic variable *A* expressing the level of acceptance of alternatives by experts will be considered. Its linguistic term set is described in Table 14.

Table 14. The linguistic terms describing the linguistic quantifiers

Linguistic term	Trapez. fuzzy number
A_1 = at least unacceptable (UN)	(0.00; 0.00; 1.00; 1.00)
A_2 = at least borderline (B)	(0.25; 0.35; 1.00; 1.00)
A_3 = at least good (G)	(0.50; 0.60; 1.00; 1.00)
A_4 = at least very good (VG)	(0.75; 0.80; 1.00; 1.00)
A_5 = excellent (E)	(0.90; 0.95; 1.00; 1.00)

6) The truth values of the statements

As the next step, the truth values $\theta^j{}_{rk}$ of the statements 'The acceptance of X_k by the expert E_j is A_r' are computed, for all $k = 1, \ldots, n$, $j = 1, \ldots, l$, $r = 1, \ldots, 5$, by the following formulas (5) or (6):

$$\theta^j{}_{rk} = \frac{\int_0^1 \min\left\{\mu_{A_r}(x), \mu_{H^j{}_k}(x)\right\} dx}{\int_0^1 \mu_{H^j{}_k}(x)\, dx} \qquad \text{if } H^j{}_k \text{ is not a real number,} \tag{5}$$

$$\theta^j{}_{rk} = \mu_{A_r}\left(h^j{}_k\right) \qquad \text{if } H^j{}_k \text{ represents a real number } h^j{}_k. \tag{6}$$

Let us note that it is necessary to distinguish these two cases since for $H^j{}_k$ being a real number, the denominator in (5) equals 0.

For example, $\theta^4{}_{32}$ is the truth values of the statement 'The acceptance of the second variant X_2 by the fourth expert E_4 is at least good', etc.

In the companies´ evaluation model, corresponding 240 values $\theta^j{}_{rk}$ are computed by the Matlab software and are used in the next steps of the model.

7) Linguistic quantifiers for expressing the quantity of important experts

Since the best alternative should be chosen in the proposed model only from the set of alternatives that are 'acceptable enough according to the sufficient quantity of important experts', it is necessary, among other things, to represent the desired quantity. This will be done through the linguistic quantifiers Q_1, Q_2, Q_3, Q_4 shown in Table 15.

Table 15. The linguistic terms describing the linguistic quantifiers

Linguistic term	Trapez. fuzzy number Q_i
Q_1 = at least minority (MI)	(0.00; 0.00; 1.00; 1.00)
Q_2 = at least approximately half (AH)	(0.35; 0.40; 1.00; 1.00)
Q_3 = at least majority (MA)	(0.60; 0.65; 1.00; 1.00)
Q_4 = almost all (AA)	(0.80; 0.85; 1.00; 1.00)

8) Importance levels of experts

In order to define the importance level of each expert, the fuzzy set of important experts must be defined. The fact of how the important expert should be described is very variable and depends on the specific situation. For the purposes of our model, the term important expert will be modelled by the trapezoidal fuzzy number B = (0.3; 0.8; 1; 1). By using this term together with the trapezoidal fuzzy numbers $K_j \in F_N([0,1])$, $j = 1,\ldots, l$, describing the experts' competencies, it is possible to specify the importance level ζ_j, $j = 1,\ldots, l$, of each expert by the following formulas:

$$\zeta_j = \frac{\int_0^1 \min\{\mu_B(x),\ \mu_{K_j}(x)\}\,dx}{\int_0^1 \mu_{K_j}(x)\,dx} \quad \text{if } K_j \text{ is not a real number,} \tag{7}$$

$$\zeta_j = \mu_B(k_{kj}) \quad \text{if } K_j \text{ represents a real number } k_j. \tag{8}$$

In the companies´ evaluation model, the importance level of experts computed by the Matlab software are the following: $\zeta_1 = 1.00$, $\zeta_2 = 0.50$, $\zeta_3 = 0.15$, $\zeta_4 = 0.81$, $\zeta_5 = 0.99$, and $\zeta_6 = 0.15$.

9) The truth values of the statements

Inspired by the concept of soft consensus (see, e.g. Kacprzyk and Fedrizzi, 1986, Kacprzyk et al., 1997), the truth values of the statements 'the alternative X_k is A_r with respect to the opinion of Q_s of the important experts' (denoted ξ_k^{rs}) is determined, for all k =1, …, n, r = 1, …,5, s =1, …, 4. They are given, for all k =1, …, n, r = 1, …,5, s =1, …, 4 by the formula

$$\xi_k^{rs} = \mu_{Q_s}\left(\frac{\sum_{j=1}^{l} \min[\zeta_j, \theta^j{}_{rk}]}{\sum_{j=1}^{l} \zeta_j}\right). \tag{9}$$

For instance, ξ_2^{43} is the truth values of the fact that the alternative X_2 is at least very good with respect to the opinion of at least the majority of the important experts.

In the companies´ evaluation model, 160 values ξ_k^{rs} are computed by the Matlab software and are used afterwards in the next step of the model.

10) Finding the sets of alternatives Υ_{rs} and exploring non-emptiness of Υ_{rs}

For all r = 1, …,5, s =1, …, 4, the sets Υ_{rs} are defined, which include such alternatives that are A_r according to Q_s of the important experts,

$$\Upsilon_{rs} = \{X_k \in X :\ \xi_k{}^{rs} = 1\}. \tag{10}$$

For example, Υ_{24} is the set of alternatives that are at least borderline according to almost all important experts.

After finding and ordering all 20 sets Υ_{rs}, the first non-empty one of them has to be determined. The ordering is done with respect to the decreasing sum of r and s, i.e., we first take into account the set Υ_{54} of alternatives that are *excellent by almost all of the important experts*. If Υ_{54} is empty, then the set Υ_{53} of variants that are *excellent by at least the majority of important experts* is explored. In case of its emptiness, $\acute{Y}_{44}$ of variants *very good by almost all of the important experts* are taken into account. If also this one is empty, we proceed with the sets $\acute{Y}_{52}$, $\acute{Y}_{43}$, and $\acute{Y}_{34}$ etc. The set *S* of the first non-empty set $\acute{Y}_{rs}$ obtained by this procedure includes those alternatives among which the most promising one will be chosen.

Let us note that the ordering is defined by the decreasing sum of the lower indexes and putting the smaller emphasis on the consensus, i.e., we take e.g., into account $\acute{Y}_{53}$ before $\acute{Y}_{44}$, or $\acute{Y}_{52}$ before $\acute{Y}_{43}$ and $\acute{Y}_{34}$. This ordering strategy is, in general, quite variable and can be changed according to a concrete decision-making problem.

Let us also note that *S* should contain only alternatives that are relevant to the problem studied. E.g., if we are looking for alternatives that are at least good, it is not appropriate to accept at least borderline alternatives. Similarly, we can reflect our requirements of a sufficient number of experts that agree with the evaluation.

In our companies´ evaluation problem, $\acute{Y}_{54} = \acute{Y}_{53} = \acute{Y}_{44} = \acute{Y}_{52} = \emptyset$. The first non-empty set in accordance with the defined ordering is $\acute{Y}_{43} = \{X_1, X_6\} := S$.

11) Computation of overall evaluations

After finding the set *S* of alternatives that are acceptable enough according to the sufficient quantity of important evaluators, the overall evaluations of the alternatives from *A* are computed. If $X_k \in A$, then its overall evaluation $H_k = [\underline{H}_k, \overline{H}_k]$, is calculated as the fuzzy weighted average of fuzzy numbers H^j_k, $j = 1, \ldots, l$ with the experts' competencies K_j, $j = 1, \ldots, l$ as fuzzy weights, i.e., for all $\alpha \in [0,1]$,

$$\underline{H}_k(\alpha) = \min\left\{\frac{\sum_{j=1}^{l} k_j(\alpha)\underline{H}^j{}_k(\alpha)}{\sum_{j=1}^{l} k_j(\alpha)} : k_j \in K_j(\alpha), j = 1, \ldots, l\right\}, \tag{11}$$

$$\overline{H}_k(\alpha) = \max\left\{\frac{\sum_{j=1}^{l} k_j(\alpha)\overline{H}^j{}_k(\alpha)}{\sum_{j=1}^{l} k_j(\alpha)} : k_j \in K_j(\alpha), j = 1, \ldots, l\right\}, \tag{12}$$

where $H^j_k = [\underline{H}^j{}_k, \overline{H}^j{}_k]$ is the overall evaluation of the alternative X_k according to the expert E_j.

The overall group evaluations H_k can be finally compared by the centre of gravity method (see, e.g., Dubois and Prade, 1988).

If we turn our attention to the companies´ evaluation model, we apply using the Matlab software the α-cut decomposition method. In this way, the overall evaluations of the alternatives X_1 and X_6 are obtained approximately in the form of the following trapezoidal fuzzy numbers:

$H_1 = (0.575; 0.785; 0.788; 0.949)$ and $H_6 = (0.584; 0.781; 0.783; 0.964)$.

The corresponding centres of gravity are 0.77 for X_1 and 0.78 for X_6, and therefore, the variant X_6 should be chosen as the best one in our multi-criteria decision-making problem.

Let us note that if we ignore the usage of the consensus and only directly compute fuzzy weighted averages of evaluations of variants, then the best variant would be X_2. This difference is caused by the fact that there is not sufficient consensus between the E_1 evaluator with extremely high competence and the E_4 and E_5 evaluators with high and very high competences for this variant.

4. Conclusions

A fuzzy multi-criteria group evaluation model suitable for the performance measurement of companies has been developed in the paper. The model takes into account besides quantitative financial criteria also non-financial qualitative ones such as CSR, innovation, or service level. Since such criteria contain a considerable degree of uncertainty and also since their importance described by the evaluators can be uncertain and vague, the tools of fuzzy sets theory have been applied in order to properly describe and analyse the studied multi-criteria evaluation model.

Even if the model itself is mathematically quite complex, it is easy to handle for the evaluators, since only the importance of the evaluators has to be set in Table 2 and the Tables 4 – 9, and 11 describing experts' evaluations of alternatives and experts' fuzzy weights have to be filled by suitable linguistic terms. Then, all other calculations can be computed by suitable mathematical software.

The biggest advantage of the constructed model lies in the fact that it takes into account (in comparison with previous models for the evaluation of companies performance) reaching the consensus and group character of studied multi-criteria problem.

As for further research, it will be interesting to combine the concept of fuzzy consensus with other methods of multi-criteria evaluation like the Fuzzy Analytic Hierarchy Process or TOPSIS.

Acknowledgments

This research was supported by European Structural and Investment Funds (Operational Programme Research, Development and Education) and by Ministry of Education, Youth and Sports of the Czech Republic under the Grant No. CZ.02.2.69/0.0/0.0/18_054/0014592 The Advancement of Capacities for Research and Development at Moravian Business College Olomouc and by IGA Project 1_2021/22 entitled Analysis of selected companies in the Olomouc region using multicriteria evaluation methods, funded by the Moravian Business College Olomouc.

REFERENCES

[1] **Altman, E.I. (2013),** ***Predicting financial distress of companies: revisiting the Z-Score and ZETA models.*** In A. R. Bell, CH. Brooks, & M. Prokopczuk (Eds.), *Handbook of Research Methods and Applications in Empirical Finance* (pp. 428–456). Edward Elgar Publishing;

[2] **Brealey, R.A., Myers, S.C., Allen, F. (2020),** ***Principles of corporate finance***. McGraw-Hill Education;

[3] **Dubois, D., Prade, H. (1988),** ***Possibility Theory: An Approach to Computerized Processing of Uncertainty***. Plenum Press, New York;

[4] **Erturğul, I., Karakaşoğlu, N. (2009),** ***Performance evaluation of Turkish cement firms with fuzzy analytic hierarchy process and TOPSIS methods*****.** *Expert Systems with Applications*, 36(1), 702-715;

[5] **Jiang, D., Hasan, M.M., Faiz, T.I., Noor-E-Alam, M. (2020),** ***A possibility distribution-based multicriteria decision algorithm for resilient supplier selection problems***. *Journal of Multi-Criteria Decision Analysis*, 27(3-4), 203-223;

[6] **Kacprzyk, J., Fedrizzi, M. (1986),** ***Soft consensus measures for monitoring real consensus reaching processes under fuzzy preferences***. *Control and Cybernetics*, 15, 309-323;

[7] **Kacprzyk, J., Fedrizzi, M., Nurmi, H. (1997),** ***Soft degrees of consensus under fuzzy preferences and fuzzy majorities*****.** In J. Kacprzyk, M. Fedrizzi, & H. Nurmi (Eds.), *Consensus Under Fuzziness* (pp. 55–83). Springer Publishing Company;

[8] **Kieso, D.E., Weygandt, J.J., Warfield, T.D. (2007),** ***Intermediate Accounting***. John Wiley & Sons;

[9] **Magni, C.A., Malagoli, S., Marchioni, A., Mastroleo, G. (2020),** ***Rating firms and sensitivity analysis***. *Journal of the Operational Research Society*, 71(12), 1940-1958;

[10] **Magni, C.A., Malagoli, S., Mastroleo, G. (2006),** ***An Alternative Approach to Firms' Evaluation: Expert Systems and Fuzzy Logic***. *International Journal of Information Technology and Decision Making*, 5(1), 195-225;

[11] **Oblak, L., Kuzman, M.K., Grošelj, P. (2017),** ***A fuzzy logic-based model for analysis and evaluation of services in a Manufacturing company***. *Journal of Applied Engineering Science*, 15(3), 258-271;

[12] **Ogachi, D., Ndege, R., Gaturu, P., Zoltan, Z. (2020),** ***Corporate Bankruptcy Prediction Model, a Special Focus on Listed Companies in Kenya***. *Journal of Risk and Financial Management*, 13(3), 47;

[13] **Pavlačka, O., Pavlačková, M. (2021),** ***On the properties of the fuzzy weighted average of fuzzy numbers with normalized fuzzy weights***. *Iranian Journal of Fuzzy Systems*, 18(4), 1-17;

[14] **Pavlačka, O., Pavlačková, M., Hetfleiš, V. (2017),** ***Fuzzy weighted average as a fuzzified aggregation operator and its properties***. *Kybernetika*, 53(1), 137-160;

[15] **Sukač, V., Pavlačka, O. (2018),** ***Fuzzy consensus in group decision-making model using absolute-type evaluations***. *Proceedings of the 36th International Conference Mathematical Methods in Economics*, 527-532;

[16] **Sukač, V., Talašová, J., Stoklasa, J. (2016),** ***A linguistic fuzzy approach to the consensus reaching in multiple criteria group decision-making problems***. *Acta Universitatis Palackianae Olomucensis, Facultas Rerum Naturalium, Mathematica*, 55(1), 133-150;

[17] **Sun, C.C. (2010),** ***A performance evaluation model by integrating fuzzy AHP and fuzzy TOPSIS methods***. *Expert Systems with Applications*, 37(12), 7745-7754;

[18] **Thomas, G.N., Indriaty, L. (2020),** ***Analysis of Bankruptcy Prediction Models in Determining Bankruptcy of Consumer Goods Companies in Indonesia***. *International Journal of Psychosocial Rehabilitation*, 24(6), 14387-14391;

[19] **Young, S., Thyil, V. (2009),** ***Governance, employees and CSR: Integration is the key to unlocking value.*** *Asia Pacific Journal of Human Resources*, 47(2), 167-185;

[20] **Zadeh, L.A. (1965),** ***Fuzzy Sets***. *Information and control*, 8(3), 338-353;

[21] **Zadeh, L.A. (1975),** ***The concept of a linguistic variable and its application to approximate reasoning - I.*** *Information Sciences*, 8, 199-245.