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A FUZZY DYNAMIC MODEL FOR TOTAL QUALITY COST

Abstract. *This paper presents a dynamic Quality Cost Model capable of reconciling contributions of Juran's traditional Model with those of the Zero Defects Model. Here, the cost functions depend on a company's level of conformance in each period and at which stage of the Maturity Grid a company is positioned. To determine the stage at which the company is positioned on Crosby's Maturity Grid, expert's opinions will be considered using fuzzy logic to incorporate Hidden Quality Costs in the failure costs calculated by the company. The model will introduce exponential regression (ER) using Induced Ordered Weighted Average (IOWA). The results show that the learning process present in the proposed model leads it to evolve over time to positions that bring to mind the Zero Defects model. However, as appreciated in the case study, it can also regress and introduce the organisation into a level of saturation where the Cost of Quality is optimised before reaching perfection.*

Keywords: *Quality Management, IOWA, Fuzzy logic, Case Study, Quality cost*

JEL Classification: C02, M10, M49

1. Introduction

Today, there is an increasing number of organisations that have become aware of the importance of adequately measuring and managing Total Quality Cost (Glogovac and Filipovic, 2017). Moreover, highly reputed experts such as Crosby or Juran, who along with Deming are possibly the authors to have done most for the Quality Management movement, have devoted an extensive part of their work to analysing different aspects related to this cost. To be exact, an aspect that stands out among these is the analysis of the behaviour of Total Quality Cost in order to establish an optimum level of investment in quality. Bearing in mind the traditional

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goal of reducing costs, it is logical that any company could question the possible existence of an optimum level of quality which enables this cost to be minimised.

Traditionally, there have been two conflicting standpoints regarding optimum quality models: on the one hand, there is Juran's well-known traditional model (Juran et al., 1974), and on the other, there is the so-called Zero Defects model (Schneiderman, 1989).

Juran et al. (1974) sustain the importance of relating investment in prevention and appraisal to costs deriving from Non-quality (failure costs). These authors base themselves on a theoretical model that provides an optimum point where it is possible to minimise Total Quality Cost without minimising the Costs of Non-quality (Figure 1).

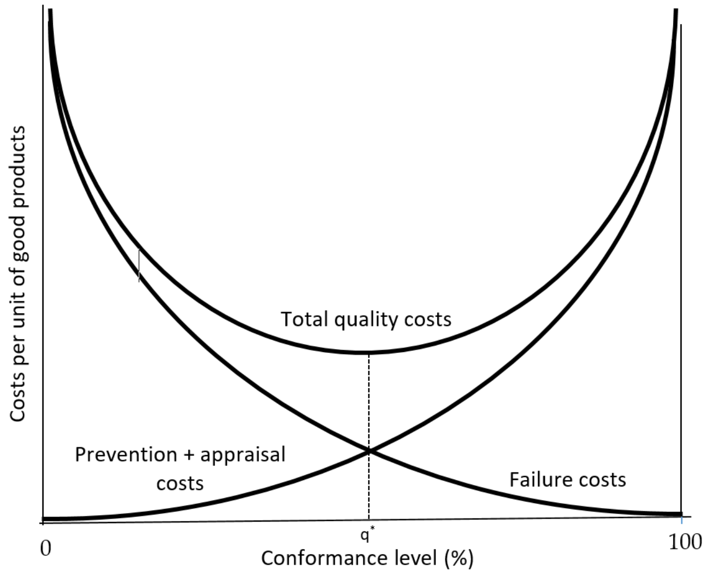

Figure 1. Juran's traditional model of optimum quality cost (Juran, et al., 1974)

The former approach seems to contradict the conceptual bases of continuous improvement and obtaining Zero Defects, as supported by different authors, among whom Crosby (1979) stands out. This author argues that one of his absolute principles is that there are no acceptable levels of failures, Zero Defects being the standard for quality performance; therefore, the idea of optimum quality cost is discarded. The introduction of the concept of Zero Defects into Juran's model resulted in a model where the optimum level of quality coincides with a hundred percent level of conformance (Figure 2).

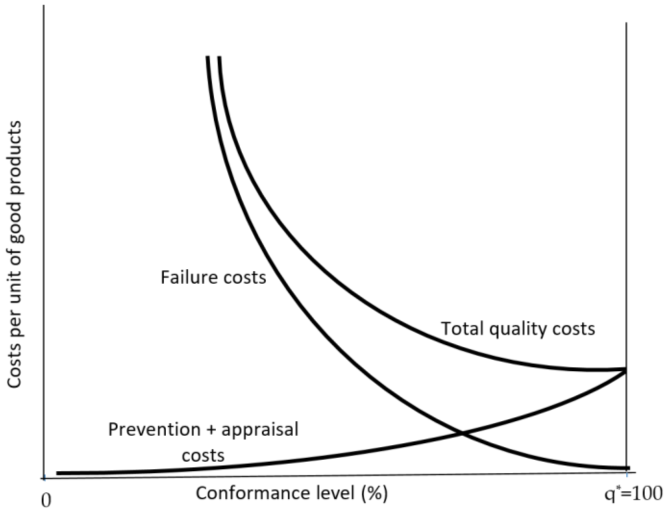

Figure 2. Zero Defects Model (Schneiderman, 1989)

The aim of this paper is to present a dynamic Quality Cost Model capable of reconciling contributions from Juran's traditional Model with those from the Zero Defects Model and allows further advances to be made in the study of this subject. Different authors have developed dynamic models, among which we highlight those by Li and Rajagopak (1998), Kim and Nakhai (2008), or Goswami et al. (2019), who also analysed the relation between Total Quality Cost and the level of quality existing in an organisation. However, the model proposed here makes some interesting contributions to the existing bibliography, such as the use of IOWAs induced according to the stage at which a company is located on Crosby's Maturity Grid. Although it is not new for IOWAs to be used in different areas of performance management involving multiple decision making approaches (Chen and Chen, 2003; Merigo and Gil-Lafuente, 2009; Yusoff et al., 2017), their use is completely innovative in the development of quality cost models. In addition, another innovative aspect of this study is the use of the Crosby's Maturity Grid as a means of formally introducing learning. In this respect, the use of fuzzy mathematics for defining the maturity of an organisation is of interest. Finally, the tool designed permits the estimation of Hidden Quality Costs and thereby enables their incorporation into the model. The paper ends with an application of the proposed model through a case study.

2. Proposed dynamic model

2.1. Presentation of the cost functions and the importance of Crosby's Maturity Grid in the proposed model

The use of the Cost of Quality as a management instrument is related to the degree of quality culture maturity that exists in an organisation (Glogovac and Filipovic, 2017). For this reason, it is important for optimum models to be able to incorporate dynamics that introduce quality management into the organisation. Authors like Pullen (2007) or Wendler (2012) point out that a maturity model provides an adequate framework for defining progress and measuring efforts towards an improvement in quality. Most maturity models take Crosby's Quality Management Maturity Grid (Crosby, 1979) as a reference, so in the dynamic model proposed, it has been chosen as an instrument to evaluate the level of quality management development in the organisation.

The table created by Crosby (1979) differentiates five stages of maturity (Uncertainty, Awakening, Enlightenment, Wisdom, Certainty), which describe the different phases that a company goes through, and consequently their costs. It develops from ignorance and complete distrust towards quality until reaching the ideal situation where quality administration is considered an essential part of the organisation.

In the proposed model, cost functions depend on a company's level of conformance in each period and at which stage of the Maturity Grid a company is positioned. Total Quality Costs for year s C_{TC}^{s} are obtained as the sum of the Costs of Quality (Prevention + Appraisal) for year s, $C_{C}^{s} = f\left(q_s, I_s\right)$ and of the Costs of Non Quality (Failure Costs) $C_{NC}^{s} = f\left(q_s, I_s\right)$, both functions of the level of conformance (q_s) and of the stage of Crosby's Maturity Grid (I_s)

$$C_{TC}^{s} = C_{C}^{s} + C_{NC}^{s} \tag{1}$$

However, one of the main criticisms made about traditional optimum models is that they did not take the so-called Hidden Quality Costs into account. The costs arising from failures can be differentiated by those that can be quantified on the basis of objective criteria, called visible costs; and those whose estimation requires resorting to essentially subjective and unconventional criteria, called hidden costs. In order to estimate these hidden costs, observation of the different stages of Crosby's Maturity Grid can once again play an important role. In this sense, a methodology is proposed based on the position of a company on this grid. Starting from the previous failure costs estimated by the company (C_{NC}^{s*}), thanks to a coefficient dependent on which stage of the Maturity Grid the company is situated, $c_s = f\left(I_s\right)$, it is possible to obtain some Costs of Non Quality C_{NC}^{s} that incorporate Hidden Quality Costs.

$$C_{NC}^{s} = C_{NC}^{s*}\left(1 + c_s\right) \tag{2}$$

As seen above, the proposed model requires the limits of the stage of Crosby's Maturity Grid to be defined for the estimation of the cost function parameters as well as for the incorporation of Hidden Quality Costs.

2.2. Estimation of the stage at which the company is situated on Crosby's Quality Management Maturity Grid

To determine the stage in which the company is positioned on the Crosby's Maturity Grid, a group of experts have to value which of the following stages $A = \left(A_j\right), j = 1,\ldots,5$ {Uncertainty, Awakening, Enlightenment, Wisdom, Certainty} the company belongs to. Given the uncertainty inherent in this valuation, the experts will use linguistic variables (Scherger et al. 2015) with 6 labels P = {p_1 = totally disagree, p_2 = strongly disagree, p_3 = disagree, p_4 = neutral, p_5 = true, p_6 = very true}. For each linguistic label a membership function is assigned $\mu_P = \{\mu_p\} = \{0;0.2;0.4;0.6;0.8;1\}$. That is to say, if an expert values a certain phase by indicating totally disagree', the membership function assigned is 0, if they value it as 'strongly disagree', 0.2 is assigned, and so on, with the corresponding membership function assigned according to the value given by the expert. If a_{ij} denotes the number of experts that value stage i with the label j, the company's membership function for each phase i is obtained as:

$$\mu_i = \sum_{j=1}^{6} a_{ij}\mu_j, i = 2,...,6 \tag{3}$$

The stage the company belongs to in year s can be obtained from the expression:

$$I_s = \sum_{i=1}^{6} i \cdot \mu_i \tag{4}$$

2.3. Determining the corrector coefficient (c_s) and the incorporation of Hidden Quality Costs in the failure costs calculated by the company

Table 1 shows the costs calculated by the companies, as well as the costs they would really have according to Crosby, for each of the stages of the Crosby's Maturity Grid, and the last column gives the coefficient that allows them to pass from calculated costs to real costs.

Table 1. Obtaining Crosby's Corrector Coefficient for the different Maturity Grid stages (in % sales)

Stage	(a_s^i) Calculated Costs (according to Crosby)	(b_s^i) Real Costs (according to Crosby)	($c_s^i = b_s^i / a_s^i$) Crosby's Corrector Coefficient
2. Awakening	3	18	6.00
3. Enlightenment	8	12	1.50
4. Wisdom	6.5	8	1.23
5. Certainty	2.5	2.5	1.00

Table 1 does not include the uncertainty stage since the companies included in this stage did not make any estimation for Total Quality Cost. Based on the company's membership function at each stage, the Crosby's corrector coefficient for each year s can be obtained as the product of the membership functions of stage i (μ_i) and the corrector coefficient of that stage (c_s^i).

$$c_s = \sum_{j=2}^{6} \mu_i c_s^i \tag{5}$$

On the basis of this corrector coefficient and in accordance with expression (2), it is possible to obtain the Costs of Non Quality for year s (C_{NC}^{s}) which incorporate both visible and hidden costs.

2.4. Estimation of the cost curves for the Costs of Quality and of Non-Quality and determining the optimum

Once the model has ensured the incorporation of Hidden Quality Costs in the rest of the cost categories, the method used for estimating the cost curves for the costs of Quality and of Non Quality Costs is outlined.

As in the case of the Juran's traditional model and following authors such as Abdul-Kader *et al.* (2010) or Dong *et al.* (2017), a positive exponential function is used for the Costs of Quality curve (C_{C}^{s}) while the Costs of Non Quality (C_{NC}^{s}) are represented by a negative exponential function.

As this is a dynamic model, the relation between costs and level of conformance is then estimated for each year s based on the information of that year and also from what has been learnt in previous p years. It is evident that a company evolves over time, therefore the cost curves will adapt to this evolution. As a result of the large number of unknown parameters, regression analysis is a fundamental instrument when modelling the behaviour of cost curves (Dong *et al.*, 2017).

Traditionally, exponential regressions have been used to analyse the relation between costs (dependent variable) and the level of conformance (independent variable). The exponential regression (Covrig and Badea, 2017) is bounded by some assumptions; for example, the unobserved error term is mutually independent and identically distributed. The parameters of this regression are usually estimated by ordinary least Squares (OLS) by minimising the sum of the squared errors. However, this methodology assigns the same importance to all the squared errors. The model's accuracy can be improved by assigning adequate weighting factors, which can be done using an extension of OWAs (Yager, 1988, 1992) Induced Ordered Weighted Average (IOWA) (Yager and Filev, 1999). In this way, the sum of squared errors will be minimised, weighted according to an induced variable.

Given its relevance to the proposed methodology, it is appropriate to introduce the definition of the exponential regression (ER) with IOWAs:

ER-IOWA operator of dimension n is a mapping ER-IOWA $ER-IOWA: R^{n} \rightarrow R$ given the variables q_k and C_k such that they have an associated weighting vector $W^{T} = (\omega_1, ..., \omega_K)$ where $\omega_k \in [0,1]$ and $\sum_{k=1}^{K} \omega_k = 1$ with an induced variable I_i for reordering . The formulation is:

$$z_j = \ln C_j = \alpha_{IOWA} + \beta_{IOWA} q_j \tag{6}$$

It is possible to develop β and α as follows:

$$\beta_{IOWA} = \frac{cov_{IOWA}(q,C)}{var_{IOWA}(q)} = \frac{\sum_{k=1}^{K}\omega_k (q_k - \mu)(z_k - \upsilon)}{\sum_{k=1}^{K}(q_k - \upsilon)^2} \tag{7}$$

$$\alpha_{IOWA} = \upsilon - \beta_{IOWA}\mu \tag{8}$$

Where μ and υ are the IOWA means for q and for C respectively: that is to say, the means for both variables using the weighting obtained based on the induced variable. The induced variable used is the difference between the stage at which the company is situated on the Crosby's Maturity Grid in the year of the calculation and each of the previous p years. In this way, the lower terms will be weighted to a greater extent; that is to say, those years whose stage on Crosby's Maturity Grid is closer to the current stage will therefore be the ones to have greater weight when obtaining the weighted sum of the squared errors.

Thus, the following methodology is proposed:

a) Starting from the estimated stage at which the company is situated on the Crosby's Maturity Grid, obtained from expressions (3) and (4).

b) For each of the years to be estimated, the induced variable considered is $\vartheta_{j,s}$, the difference between the stage of the Crosby's Maturity Grid for the year s to be estimated and the j stages of the immediate previous p-1 years,

$$\vartheta_{j,s} = \left|I_j - I_s\right|, j = s, s-1, ..., s-p+1 \tag{9}$$

c) The squared errors made for each year are ordered according to the induced variable (9), so that those corresponding to the years whose stage on the Crosby's Maturity Grid is closer to the year s (target estimation) are placed first.

d) Weighting for each j term has been considered as

$$\omega_j = \frac{2 \cdot j \cdot J}{1+J} \tag{10}$$

Where j=1 has been used for the first term of the order introduced in b), j=2 for the second, etc.

e) The application of expressions (6) to (8) permits the estimation of the curve parameters α^0_{IOWA} and β^0_{IOWA}.

Through the model's dynamic evolution, the evolution of the cost curves can be observed, and especially the point where total cost is minimised. Once the parameters of the cost functions (expressions (7) and (8)) are estimated, the total cost can be expressed as follows:

$$C^s_{TC} = C^s_C + C^s_{NC} = \exp\left(\alpha^C_{IOWA} + \beta^C_{IOWA} q_s\right) + \exp\left(\alpha^{NC}_{IOWA} + \beta^{NC}_{IOWA} q_s\right) \tag{11}$$

From this expression, it is possible to obtain the level of conformance level that permits Total Quality Costs to be minimised:

$$q^* = \frac{\ln\left(-\frac{\beta_{IOWA}^{C}}{\beta_{IOWA}^{NC}}\right) + \alpha_{IOWA}^{C} - \alpha_{IOWA}^{NC}}{\beta_{IOWA}^{NC} - \beta_{IOWA}^{C}} \tag{12}$$

By monitoring the minimum values, the company's evolution with respect to investment in quality can be analysed.

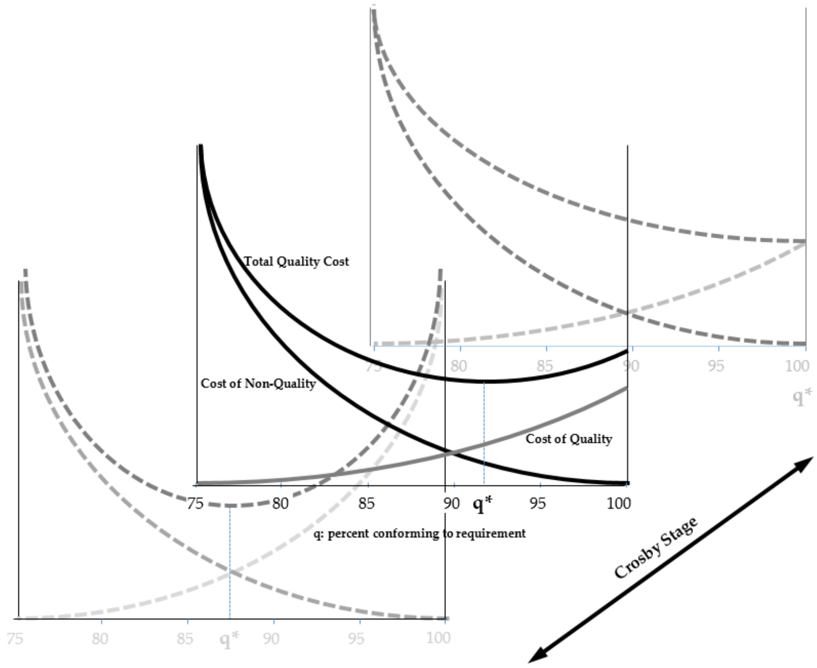

Figure 3. Proposed dynamic model

As illustrated, the proposed model will over time facilitate different cost curves that will adapt to the quality strategies followed by the company. These strategies are pursued in order to advance along the path towards Total Quality through continuous improvement and effective preventive actions that will position the company at more advanced stages on the Crosby's Maturity Grid. Consequently, and according to Figure 3, reaching the inflection point will be delayed, and it is even possible that it will not occur. All this implies reaching a minimum close to that established in the Zero Defects Model. However, the model foresees the possible involution of the system, whereby a change of attitude by the organisation or the application of less effective quality strategies will bring forward the inflection point; consequently, as Juran's traditional model suggests, this will provide an optimum level of conformance that will minimise Total Quality Cost before reaching perfection.

3. Application and discussion of the results

Below we present the results from the application of the proposed model to a Spanish SME dedicated to footwear manufacturing. The organisation analysed can be considered representative of the small and medium companies that operate in this sector in Spain.

Table 2 shows company data for a 9-year period as well as the simulation performed over the last two years. The second column gives the level of conformance for each of the periods of time analysed. The third column provides information on the stage at which the company is situated on Crosby's Maturity Grid according to the opinions expressed by the experts (Tables 3 and 4) and is adequately processed according to the methodology described in the model. The rest of the columns provide information about the different components of Total Quality Cost for each of the years. In order to facilitate the comparison of values in the different periods of time, the use of a basis of comparison is recommended. In this case, as it is straightforward and used extensively among companies, net sales are used, where the costs are expressed as a percentage of sales. Column 7 shows the corrective coefficients that facilitate the incorporation of hidden costs into the Costs of Non-quality, according to the proposed methodology, and in this way their inclusion in Total Quality Costs.

Table 2. Information on the components of Total Quality Cost, stage of Crosby's Maturity Grid and level of conformance

Year	Level of Conformance (%)	Maturity Stage	Prevention	Appraisal	Costs of Quality	External Failure	Internal Failure	Costs of Non Quality	Correction Coefficient	Real Costs of Non Quality (B)	Total Quality Costs (A+B)
Year 1	89.900	2.208	0.160	1.150	1.310	2.280	0.980	3.260	5.063	16.504	17.814
Year 2	91.400	2.400	0.490	1.490	1.980	1.900	1.010	2.910	4.200	12.222	14.202
Year 3	92.300	2.839	0.700	1.560	2.260	1.350	2.820	4.170	2.226	9.282	11.542
Year 4	93.800	3.074	0.790	1.480	2.270	1.140	3.910	5.050	1.480	7.474	9.744
Year 5	94.100	3.100	0.810	1.470	2.280	0.820	3.920	4.740	1.473	6.982	9.262
Year 6	95.200	3.103	0.970	1.350	2.320	0.770	2.410	3.180	1.472	4.681	7.001
Year 7	95.300	3.172	1.020	1.290	2.310	0.670	2.320	2.990	1.453	4.346	6.656
Year 8	95.200	3.198	1.040	1.290	2.330	0.670	2.310	2.980	1.447	4.311	6.641
Year 9	95.300	3.250	1.080	1.270	2.350	0.660	2.280	2.940	1.433	4.212	6.562
Simulation											
Year 8(s)	95.200	3.103	1.520	1.290	2.810	0.670	2.310	2.980	1.472	4.387	7.197
Year 9(s)	95.300	3.100	2.040	1.270	3.310	0.660	2.280	2.940	1.473	4.331	7.641

(s): Simulation performed assuming an involution in quality management

Table 3 illustrates the opinions of the five experts consulted about the company's possible membership to each of the stages of the Crosby's Maturity Grid for Year 1. As can be seen, with respect to the company being positioned at stage 2 of the Maturity Grid, two experts indicate neutral, two indicate true, and one indicates very true. Following the proposed methodology, the membership function for this stage is obtained as $2\times0.6+2\times0.8+1\times1=0.76$. The ratio between the product of the membership functions for each stage by the number assigned to each one and the sum of all the membership functions makes it possible to finally position the company on the Crosby's maturity grid, with the result being stage 2.21 for year 1 $\left(0.76\times2+0.20\times3\right)/\left(0.76+0.20\right)=2.21$. The estimation for the rest of the years has been performed in exactly the same way based on the opinions expressed by the experts, as shown in Table 4.

Table 3. Estimation of stage of Crosby's Maturity Grid for Year 1

Stage	Totally disagree	Strongly disagree	Disagree	Neutral	True	Very True	μ
1	0	0	0	0	0	0	0.000
2	0	0	0	2	2	1	0.760
3	1	3	1	0	0	0	0.200
4	5	0	0	0	0	0	0.000
5	5	0	0	0	0	0	0.000
						Maturity Stage	2.208

Table 4. Annual opinions expressed by the experts about the stages that the company belongs to on Crosby's Maturity Grid

	Stage on Crosby's Maturity Grid				
Year	1	2	3	4	5
1	{0;0;0;0;0;0}	{0;0;0;2;2;1}	{1;3;1;0;0;0}	{5;0;0;0;0;0}	{5;0;0;0;0;0}
2	{0;0;0;0;0;0}	{0;0;1;3;1;0}	{0;1;3;1;0;0}	{5;0;0;0;0;0}	{5;0;0;0;0;0}
3	{0;0;0;0;0;0}	{0;2;3;0;0;0}	{0;0;0;1;3;1}	{2;3;0;0;0;0}	{5;0;0;0;0;0}
4	{0;0;0;0;0;0}	{3;2;0;0;0;0}	{0;0;0;1;2;2}	{1;4;0;0;0;0}	{5;0;0;0;0;0}
5	{0;0;0;0;0;0}	{3;2;0;0;0;0}	{0;0;0;0;2;3}	{1;3;1;0;0;0}	{5;0;0;0;0;0}
6	{0;0;0;0;0;0}	{3;2;0;0;0;0}	{0;0;0;0;3;2}	{1;3;1;0;0;0}	{5;0;0;0;0;0}
7	{0;0;0;0;0;0}	{4;1;0;0;0;0}	{0;0;0;0;3;2}	{1;2;2;0;0;0}	{5;0;0;0;0;0}
8	{0;0;0;0;0;0}	{4;1;0;0;0;0}	{0;0;0;0;4;1}	{2;1;2;1;0;0}	{5;0;0;0;0;0}
9	{0;0;0;0;0;0}	{4;1;0;0;0;0}	{0;0;0;0;3;2}	{0;2;2;1;0;0}	{5;0;0;0;0;0}

Table 2 shows that the evolution of the different components of the Total Quality Cost as the company advances on the Crosby's Maturity Grid is as indicated by authors such as Sower, Quarles and Broussard (2007) or Moschidis, Chatzipetrou and Tsiotras, (2018), who emphasise the inverse synergistic effect that the Costs of Quality can have on the Costs of Non-quality. It is widely accepted, especially at the early stages of maturity models when quality of conformance is low, that committing to appraisal and prevention costs can result in large reductions in costs from internal and external failures. Everything will evidently depend on whether the investment in quality is adequate, since, as Fu et al. (2015) point out, the implementation of quality management does not always achieve the desired results. However, it is a fact that prevention, appraisal, and failure costs affect each other, and investment in one of the components will influence the others (Shafiei et al., 2020).

Although authors like Plunkett and Dale (1988) support the exchange relationship between Costs of Quality and Costs of Non Quality in static environments as established in Juran's traditional model, other authors like Kim and Nakhai (2008), Lim et al. (2015) or Alglawe et al. (2017) also accept this relationship in dynamic models as developed in this paper.

Based on the information in Table 2, the proposed model then estimates the curves for costs of Quality and of Non-quality, first for Year 7 and then for Year 9. To do so, the induced variable is calculated: 'the difference in absolute value between the stage of Crosby's Grid for the target year of the study and the stage of each of the 6 previous years'. The coefficient to be used in the ER-IOWA regression is obtained by ordering the results and applying expression (10). As can be observed in Table 5, the years whose induced variable has low values will be weighted more.

Table 5. Estimation of the ER_IOWA regression coefficients of the Costs of Quality Costs over the level of conformance for Year 7

Year (j)	Maturity stage	Induced variable ($\vartheta_{j,s}$)	Order	Coefficient (ω_j)
1	2.208	0.964	1	0.250
2	2.400	0.772	2	0.500
3	2.839	0.334	3	0.750
4	3.074	0.098	4	1.000
5	3.100	0.072	5	1.250
6	3.103	0.069	6	1.500
7	3.172	0.000	7	1.750

As seen in Figure 4, through the application of the methodology developed, the cost curves for the desired years are obtained: in this case, Year 7, and the last one of the time series with available data Year 9. In line with authors such as Porter and Rayner (1992), the costs are expressed on the axis of ordinates as percentages of net sales. As indicated by Freiesleben (2004), income is expected to increase in a competitive market as the levels of quality increase. If a client assumes that the company's products are unlikely to be faulty, it can be expected that they prioritise their acquisition before other competitors do. Therefore, sales will increase as either prices or the volume of units sold increases. All this implies that it is important to also consider income, and not only costs or costs per well-manufactured unit of products, as in the traditional optimum models.

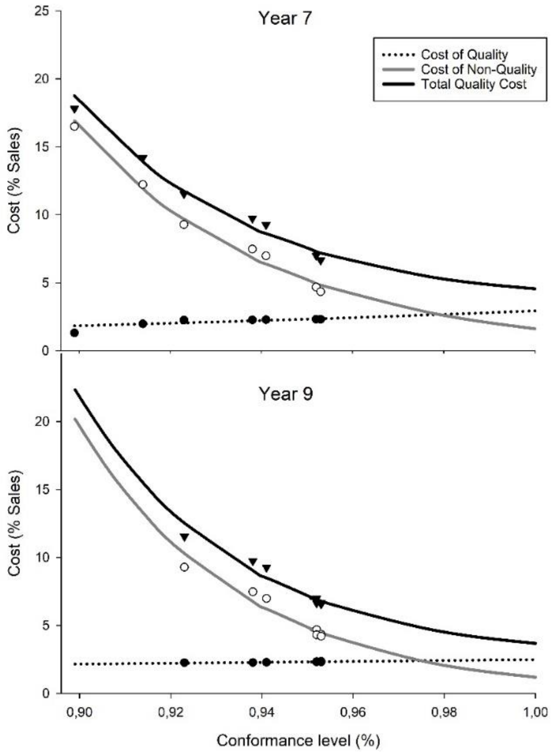

Figure 4. Application of the dynamic model proposed for Year 7 and Year 9

No substantial differences are apparent between Year 7 and Year 9 for this case study (Figure 4), where the curves appear to coincide with the Zero Defects model whose level of optimum quality is 100%. However, in the dynamic model proposed, the contributions from Juran's traditional model can be reconciled with those of the Zero Defects model. To demonstrate this, a simulation will be performed for Year 9. We are going to assume that from Year 7 and during the following two years the company has suffered an involution in quality management, moving back to the level of Year 6 and afterwards to that of Year 5. In this case, and according to the last two rows of Table 2, the main consequence from a loss of effectiveness in quality strategies is an annual increase of half a point in the Costs of Quality without it affecting the level of conformance initially observed in Years 8 and 9. As can be observed in Figure 5, the dynamic model adapts to the new circumstances and, taking into account the new information facilitated, it provides some curves that are much closer to those proposed by Juran's traditional model.

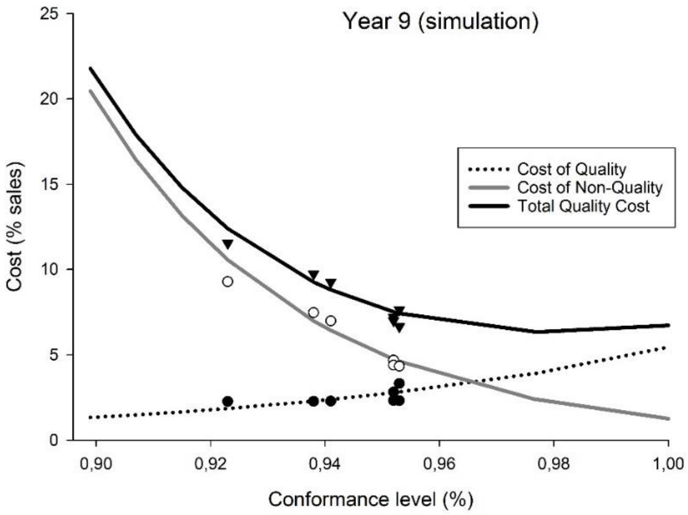

Figure 5. Application of the dynamic model proposed for the simulation of Year 9

In Table 6, the results obtained for Years 7 and 9 are mathematically described, as well as those in the simulation for Year 9. As can be seen, according to the estimations made, the optimum is obtained for a level of conformance of 100% for Year 7 as well as Year 9. On the contrary, for the simulation, the optimum is reached for a level of conformance of 98.10%, where, as in Juran's traditional model, errors are still made. Consequently, the learning process present in the proposed dynamic model leads it to evolve over time to positions that bring to mind the Zero Defects model. However, as appreciated in the simulation, it also regresses, moving backward on the Maturity Grid, and thereby introduces the organisation into a level of saturation where the Cost of Quality is optimised before reaching perfection. In this sense, and in line with other authors such as Li and Rajagopalan (1998) or Kim and Nakhai (2008), the dynamic model proposed provides a balanced vision of Total Quality Cost that does not contradict the central premise of optimum quality or total quality.

Table 6. Final results for Year 7, year 9 and for the simulation of Year 9

	Year 7	Year 9	Year 9 (Simulation)
Costs of Quality C_C^s	$\exp(-3.56+4.64\text{q})$	$\exp(-0.49+1.40\text{q})$	$\exp(-12.36+14.06\text{q})$
Costs of Non Quality C_{NC}^s	$\exp(23.70-23.22\text{q})$	$\exp(28.22-28.05\text{q})$	$\exp(27.81-27.57\text{q})$
Optimum level of conformance (%) (12)	100	100	98.10

4. Conclusions

In the specialised literature, two opposing standpoints can be highlighted with respect to the suitability of seeking optimum quality costs: on the one hand, there is Juran's well-known traditional model, and, on the other, there is the so-called Zero Defects model.

However, between the case proposed by the Juran's traditional Model and the one proposed by the Zero Defects Model, we have to assume that there are other curves that reflect the advances or setbacks of a company in its quality strategy. The dynamic model proposed manages to provide improvements to the traditional models without renouncing the premises they contain. The analytical focus used is related to studies by authors like Li and Rajagopalan (1998), Kim and Nakhai (2008), or Goswami *et al.* (2019), who also examine the dynamic relation between Total Quality Costs and the level of quality. However, what can be considered completely innovative is the way of formally introducing quality management as well as the use of IOWAs for the estimation of the cost curves. The use of induced variables to estimate the curves for Costs of Quality and of Non-quality enables them to adapt faster to the evolution that a company experiences on the Crosby's Maturity Grid. When a change in the stage on the Maturity Grid takes place, the new regression weights the values corresponding to the years whose stage is found closer to the current one more. The proposed methodology permits the periodical reestimation of the cost curves, and consequently, of the optimum level of conformance, taking into account the quality strategy the company follows. Furthermore, the definition of the company's level of maturity is of particular relevance as it allows the incorporation of Hidden Quality Costs, thus overcoming an important limitation that exists in the traditional optimum models. It should be noted that the tools from fuzzy mathematics applied to the opinions expressed by the experts facilitate the estimation of the company's position on the Maturity Grid. Undoubtedly, it is necessary for the models to be able to incorporate aspects arising from the uncertainty generated by unknown parameters into their formal approach. The use of all these tools makes this a robust model, improving its accuracy and predictive capacity, and allows managers to better evaluate the impact of their efforts in quality improvement over time in different dynamic scenarios to which the components Total Quality Cost are assigned.

The final section of this paper provides an application of the proposed Model through a case study. The use of a case study as a research method is especially recommended in situations where the phenomenon being studied cannot be understood in isolation and where numerous elements also interact. The definition of the quality philosophy existing in an organisation, a core aspect of the proposed methodology, is one of these situations. Moreover, as Yin (1989) indicates, case study is an appropriate method when the aim is to illustrate a theoretical model, as in this paper. In this way, the case presented makes it possible to observe the learning process present in the dynamic model proposed and the response to situations of evolution as well as setbacks in quality management. It is evident that the methodology developed facilitates adapting cost curves to the quality strategies pursued by the company.

Despite being of interest, the proposed model has one drawback which considerably limits its use among companies in general. For its application, it is necessary to rely on experts who are able to define the latent quality philosophy in the organisation. This could be excessively cumbersome, especially for organisations with fewer resources. In this sense, a future research line could be to seek practical solutions to reduce the cost of the practical application of the proposed model, and, therefore, improve its accessibility to a greater number of organisations.

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