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MODEL OF OPTIMAL CONTROL OF LABOUR REPRODUCTION AND SAVING ENERGY IN UNDEFINED CONDITIONS OF THE CURRENT SITUATION

Abstract. *In this paper, we offer a mathematical model for the construction and implementation of a set of models for analysing the stability of labour reproduction processes and model of saving energy, evaluating the parameters that determine the state of the system that describes the processes of labour reproduction, optimal management of these processes; methodical recommendations for making management decisions to ensure the stability of labour reproduction processes have been developed.*

Keywords: *Stochastic space, Markovian Hamilton function, optimal control, model of labour reproduction and saving workers' energy, semi-Markovian process, differential equation with random coefficients.*

JEL Classification: C02, C69

1. Introduction

Recently, more and more attention has been paid to research for analysis of a model for detection of information attacks in computer networks (Dzhalladova et al., 2019), models which deal with the question of when the resource should be used on the condition that its use today might prevent it from being available to be used later. The analysis provides concepts, theory, applications, and distinctions to the market for understanding the strategic aspects of cyber conflict. Case studies include the same cyberattack and persistent cyber espionage applied by the same

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country's military (Dzhalladova et al., 2022). In this context, investigation, and construction of methods for stabilisation of different events and processes in the dynamic situation are relevant/topical. Moreover, recent events, such as COVID-19, the war in Europe, and other conflicts have shown that humans and their security in an undefined situation must be in the centre of science (Dzhalladova and Kaminskii, 2020). But along with these problems, there are always parallel economic aspects, some of which we consider below. Therefore, we have presented an improvement of a mathematical tool (Růžičková and Dzhalladova, 2011; Valeev and Dzhalladova, 1999) and its application in the investigation of some important economic problems in the current situation, particularly in Europe.

In our paper, we offer a mathematical model, to help analyse the difference between the projected number of employed and unemployed people registered with the State Employment Service. This difference is explained by the instability of the economic situation in the countries. This is also due to the problems of economic activity of the population, the quantitative and qualitative balance between supply and need for labour, illegal external labour migration, and the shadow labour market. The results obtained are correlated with the assessment of the stability of the system of differential equation, describing the process of labour reproduction (Dzhalladova, 2018). As one of the areas of investment, investment in housing construction is proposed, which can become an effective mechanism for providing the population with mass, affordable housing. The choice of direction is based on the fact that the stability of the processes of labour reproduction is directly related to the creation of optimal living conditions for workers, primarily with the improvement of living conditions, for example, refugees in condition war. The constructed model made it possible to determine the conditions for optimising the portfolio of the investor resident incident. Also, in our paper, we investigated the model to determine the advantages of optimal behaviour at work for the worker's organism ("problem of saved energy").

2. Main concepts and theorems

To construct our model, we introduce some theoretical concepts and prove theorems.

2.1. Statement of the problem

Let $\Omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be a collection of countable events. A σ- algebra is a collection of all subsets of Ω. A filtration, $\mathbb{F}$ is the smallest σ-algebra.

Later we introduce the Markovian random process, where the filtration is regarded as all the information up to the current time. In other words, filtration contains all and yet non-repeating information about the events. For example, if we look at the stock with a price $S(t)$ at time t, then write filtration as $\mathbb{F}_t$, this filtration should include all the past information of the stock where $\tau < t$ and all the current information about financial statements, publicly announced on the news, etc. A probability measure $\mathbb{P}$ is a function that maps a σ-algebra to a real number between 0 and 1, that is

$$\mathbb{P}: \mathbb{F} \to [0,1].$$

A general case for a probability measure is:

$$\mathbb{P}(A)[0,1] \text{ for any arbitrary } A \in \mathbb{F},$$

$$\int_\Omega \mathbb{P}(\omega) = 1.$$

Finally, we constructed a probability space is a triple: $(\Omega, \mathbb{F}, \mathbb{P})$ (Chen, 2009).

On probability space $(\Omega, \mathbb{F}, \mathbb{P})$ we consider a system of linear differential equations with random coefficients

$$\frac{d\Phi(\phi,t)}{dt} = G\big(t,\xi(t)\big)\Phi(t) + B\big(t,\xi(t)\big)\Xi(t) \tag{1}$$

$$\Phi(0) = \phi(\omega): \Omega \to \mathrm{R}^m, dim\Phi = \mathrm{m},$$

where $\xi(t)$ is a random Markov process that takes the finite number of values $\theta_1, \theta_2, .., \theta_n$ with probabilities

$$p_k(t) = P(\xi(t) = \theta_k), \qquad k = 1,2,\dots,n$$

that satisfy the following system of linear differential equations:

$$\frac{dp_k(t)}{dt} = \sum_{k=1}^{n} \alpha_{ks}(t)p_s(t), \qquad k = 1,2,\dots,n. \tag{2}$$

The coefficients of the system (2) satisfy the well-known conditions (Gardiner, 1983; Feller, 1991)

$$\alpha_{ks}(t) \geq 0, k \neq s, \textstyle\sum_{k=1}^{n} \alpha_{ks}(t) = 0,\ k,s = 1,2,\dots,n.$$

To describe the distribution density of the discrete-continuous random process $(\xi(t), \Phi(t))$, we can use a function of probability density in the form

$$F(t,\Phi,\xi(t)) = \sum_{k=1}^{n} f_k(t,\Phi)\delta(\xi - \theta_k)$$

where $\delta(\xi - \theta_k)$ - Dirac functions, w –or Dirac delta distribution (also known as the unit inpulse) $f_k(t,\Phi), k = 1,2.\dots.n,$ are called the partial probability densities.

We also introduce the partial mathematical expectations

$$m.e._k\,(\boxdot) = m.e._k\,(\varrho(t,\Phi(t),\xi(t)))$$

of an arbitrary function $\varrho(t,\Phi(t),\xi(t))$ as follows

$$m.e._k\,(\varrho(t,\Phi(t),\xi(t)) = \int_{R^n} \varrho(t,\Phi,\theta_k) f_k(t,\Phi) d\Phi, k = 1,2,\dots,n.$$

Here R^m is the standard Euclidean space of dimension m. Then the mathematical expectation of the function $\varrho(t,\Phi(t),\xi(t))$ is

$$m.e.\,(\boxdot) = m.e.\,(\varrho(t,\Phi(t),\xi(t))) = \int_{R^m}\int_{-\infty}^{+\infty} (\varrho(t,\Phi,\theta_k) f(t,\Phi)) d\xi d\Phi =$$

$$= \sum_{k=1}^{n} \int_{R^m} \varrho(t,\Phi,\theta_k) f_k(t,\Phi) d\Phi = \sum_{k=1}^{n} m.e._k\left(\varrho(t,\Phi(t),\xi)\right) = \sum_{k=1}^{n} m.e._k\,(\boxdot)$$

and then we consider the second-order partial moments as follows

$$p.m._k\,(\boxdot) = m.e._k\,\varrho(\Phi(t),\Phi(t)^*) \quad ,$$

or

$$p.m.\,(\boxdot) = \sum_{k=1}^{n} p.m._k\,(\boxdot).$$

Now we can formulate the problem of optimisation:
It is required to find the system of equations (1) of optimal control in the form

$$\Xi(t) = \mathbb{S}((t,\xi(t))\Phi(t)$$

that minimises the value of the functional

$$J(t) = m.e.\left(\int_t^{\infty} (\Phi^*(\tau) C(\tau,\xi(\tau))\Phi(\tau) + \Xi^*(\tau) D(\tau,\xi(\tau))\Xi(\tau)) d\tau\right) \tag{3}$$

2.2. Synthesis of optimal control for general case

Theorem 1. The necessary conditions for optimality of solutions of the system of equations (1) are expressed by the equalities

$$S_k(t) = D_k^{-1}(t) B_k^*(t) \psi_k(t), k = 1,2,\dots,n \tag{4}$$

where $\psi_k(t), k = 1,2.\dots.n$, can be found from the system equations

$$\frac{dp.m._{k}(t)}{dt} =$$
$$= \sum_{s=1}^{n} \alpha_{ks}(t)p.m._{s}(t) + \left(G_k(t) - B_k(t)D_k^{-1}(t)B_k^*(t)\psi_k(t)\right)p.m._{k}(t) +$$
$$+p.m._{k}(t)\left(G_k^*(t) - \psi_k(t)B_k(t)D_k^{-1}(t)B_k^*(t)\right) = 0, \mathrm{k} = 1,2,\dots,n$$
$$\frac{d\psi_k(t)}{dt} = -G_k^*(t)\psi_k(t) - \psi_k(t)G_k(t) - C_k(t) + \tag{5}$$
$$+\psi_k^*(t)B_k(t)D_k^{-1}(t)B_k^*(t)\psi_k(t) - \sum_{k=1}^{n} \alpha_{ks}(t)\psi_k(t),$$
$$G_k(t) = G(t,\theta_k), B_k(t) = B(t,\theta_k), C_k(t) = C(t,\theta_k),$$
$$D_k(t) = D(t,\theta_k), S_k(t) = S(t,\theta_k), k = 1,2,\dots,n$$

with the initial coditions that

$$\psi_k(0) = 0, k = 1,2.\dots.n.$$

Proof.

At first we use the well-known system of linear system equations for the second order partial moments $p.m._{k}(\boxdot)$:

$$\frac{dp.m._{k}(t)}{dt} = \sum_{s=1}^{n} \alpha_{ks}(t)p.m._{s}(t) +$$
$$+\left(G(t) + B_k(t)S_k\ (t))p.m._{k}(t) + p.m._{k}(t)(G_k^*(t) + S_k^*(t)B_k^*(t)\right)p.m._{k}(t),$$
$$k = 1,2.\dots.n.$$

Rewite expression for functional (4) in explicit form

$$J(t) = \sum_{k+1}^{n} \int_{t}^{+\infty} \left(C_k(\tau) + S_k^*(\tau)D_k(\tau)S_k(\tau)\right) \odot p.m._{k}(\boxdot)d\tau.$$

where operation $\odot$ denotes a scalar product of matrices of the same order N and S with ellements $v_{kj}, s_{kj}, k = 1,2,\dots,n; j = 1,2,\dots,m$: (Růžičková and Dzhalladova, 2011)

$$N \odot S = \sum_{k=1}^{n} \sum_{j=1}^{m} v_{kj}s_{kj}. \tag{6}$$

In the second step, we proved that (3) is a necessary condition for the optimality of solutions of the system of equations (1).

We introduce the Hamilton function (Růžičková and Dzhalladova, 2011; Stengel, 1994).

$$\begin{aligned} H = \sum_{k=1}^{n} \psi(t) \odot \Big(\big(G_k(t) + B_k(t)\big) S_k(t) \Big)\, p.m._k\,(\boxdot) + \\ + p.m._k\,(\boxdot)\big(G_k^*(t) + S_k^*(t) B_k^*(t)\big) + \textstyle\sum_{k=1}^{n} \alpha_{ks}(t) p.m._s\,(\boxdot) + \\ + \sum_{k=1}^{n} \big(C_k(t) + S_k^*(t) D_k(t) S_k(t)\big) \odot p.m._k\,(\boxdot), \\ k,s = 1,2,\dots,n, \end{aligned} \tag{7}$$

In accordance with the Pontryagin maximum principle, the necessary conditions of optimality have the form

$$\frac{DH}{DS_k} = 0, \qquad k = 1,2,\dots,n, \tag{8}$$

where we used to denote

$$\frac{DH}{DS_k} = \left\| \frac{\partial H}{\partial S_{kj}} \right\|, \qquad k,j = 1,2,\dots,n.$$

By using property

$$GMB \odot S = ASB \odot M$$

of scalar products (6), we transform the matrix function (7) to the form convenient for the differentiation with respect to the matrices $S_k(t), k = 1,2.\dots,n$:

$$\begin{aligned} H = \sum_{k=1}^{n} (2G_k^*(t)\psi_k(t) \circ p.m._k\,(\boxdot) + 2B_k^*(t)\psi_k(t) M_k(t) \circ S_k(t) + \\ + C_k(t) p.m._k\,(\boxdot) + D_k(t) S_k(t) p.m._k\,(\boxdot) \odot S_k(t) + \\ + \sum_{s=1}^{n} \alpha_{ks}(t)\psi_k(t) \odot p.m._k\,(\boxdot)\big). \end{aligned} \tag{9}$$

In view of property

$$\frac{D(N \odot S)}{DS} = N$$

of scalar products, the system of matrix equations (8) takes the form

$$\frac{DH}{DS_k} = 2B_k^*(t)\psi_k(t)p.m._k\,(t) + 2D_k(t)S_k(t)p.m._k\,(t) = 0, k = 1,2,\dots,n$$

whence we obtain equalities (3).

In the final step, we exclude the matrices $S_k(t), k = 1,2.\dots.n$, from (7) and (9) and using equalities (3), we get the expressions

$$H = \sum_{k=1}^{n} (G_k^*(t)\psi_k(t) \circ p.m._k\,(\boxdot) + \psi_k(t)G_k^*(t) \odot p.m._k\,(\boxdot) +$$

$$+C_k(t) \odot p.m._k\,(\boxdot) - \psi_k^*(t)B_k(t)D^{-1}(t)B_k^*(t) \odot \psi_k(t).m._k\,(\boxdot)) +$$

$$+\sum_{k=1}^{n} \alpha_{ks}(t)\psi_k(t) \odot p.m._k\,(\boxdot).$$

The variation in matrices $p.m._k\,(\boxdot), \psi_k(\boxdot), k = 1,2.\dots.n$, is determined by the system of matrix linear differential equations

$$\frac{dp.m._k\,(\boxdot)}{dt} = \frac{DH}{D\psi_k},$$

$$\frac{d\psi_k(\boxdot)}{dt} = -\frac{DH}{Dp.m._k}, k = 1,2,\dots,n. \tag{10}$$

The system of equations (10) for the matrices $\psi_k(\boxdot), k = 1,2.\dots.n$, does not depend on the matrices $p.m._k\,(\boxdot), k = 1,2.\dots,n$, and is a generalisation of the matrix Riccati equation to the stochastic case.

Assume that a solution such that $\psi_{kr}(\boxdot) = 0, k = 1,2.\dots.n$, is found. Then the matrices $\psi_{kr}(\mathrm{t}), k = 1,2.\dots.n$, minimising our functional

$$\min_{S(\tau)} \mathrm{J}(\boxdot) = \mathrm{m.}e. \int_t^{\infty} \Phi^*(\tau)\Big(C\big(\tau,\xi(\tau)\big)$$

$$+ \mathrm{S}^*\big(\tau,\xi(\tau)\big)D\big(\tau,\xi(\tau)\big)\mathrm{S}\big(\tau,\xi(\tau)\big)\Phi(\tau)\Big)\,d\tau =$$

$$= \sum_{k=1}^{n} \psi_{kr}(t) \odot p.m_k(\boxdot), \qquad \boxdot \le \tau \le \mathrm{T}\,.$$

Then the function $\psi(t), k = 1,2.\dots.n$, realising the minimal value of functional (4) can be approximately obtained from the relation

$$\psi_k(t) = \lim_{r\to\infty} \psi_{kr}(t).\, \mathrm{k} = 1, \dots, \mathrm{n}.$$

The theorem is proved.

2.3. Synthesis of optimal control for the stationary case with semi-Markov coefficients

Let us reformulate Theorem 1 for the case where the coefficients of the system of linear differential equations (1) and functional (4) are piecewise-constant functions.

Assume that the coefficients of the system of linear differential equations

$$\frac{d\Phi(\phi,t)}{dt} = \mathrm{G}\big(\xi(t)\big)\Phi(t) + B\big(\xi(t)\big)\Xi(t),$$
$$\Phi(0) = \phi(\omega)\colon \Omega \to R^n \tag{11}$$

depend on the semi-Markov process $\xi(t)$ that takes the finite number of values $\theta_1, \theta_2, \ldots, \theta_n$ with intensities matrix

$$Q(t) = \left[q_{sk}(t) : s,\ k = \overline{1,n}\ \right]\colon q_{sk}(t) \geq 0;\ \sum_{s=1}^{n}\int_0^\infty q_{sk}(t)dt = 1$$

$$q_k(t) = \sum_{s=1}^{n} q_{sk}(t)$$

$$\chi_k(t) = \int_{\mathrm{t}}^\infty q_k(t)dt$$

$$\chi_k(t) = P\{T_k > t\}\ k = 1,2,\ldots,n.$$

Theorem 2 Assume that there exists an optimal control of the form

$$\Xi(\boxdot) = S(\xi(\boxdot))\Phi$$

for the system (11) that minimises the functional (3). Then the matrices $\psi_k(t), k = 1,2.\ldots.n$, satisfy the system of matrix differential equations (5) with zero initial conditions

$$\psi_k(0) = 0, k = 1,2,\ldots,n.$$

Moreover, the minimal value of the functional J(t) is determined by the formula

$$\min_{S(\tau)} J(t) = \sum_{k=1}^{n} \psi_{kT}(\boxdot) \odot p.m.(\boxdot), \tau > T.$$

3. Application to practical problem

3.1. Model to determine the conditions for optimising the portfolio of the investor in the residential sector.

For the practical implementation of optimal management of directions of investment in the labor force, a model of housing lending is built, which is described by a linear differential equation of the first order:

$$\frac{d\phi(t)}{dt} = \mathrm{g}\big(\xi(t)\big)\phi(t) + b\big(\xi(t)\big) \tag{12}$$

where $\phi(t)$ – number of persons applying for credit, coeficients

$$g\big(\xi(t)\big) = \begin{cases} a_1, if\ \xi(t) = \Theta_1 \\ a_2, if\ \xi(t) = \Theta_2 \end{cases}, \; b\big(\xi(t)\big) = \begin{cases} b_1, if\ \xi(t) = \Theta_1 \\ b_2, if\ \xi(t) = \Theta_2 \end{cases} \tag{13}$$

Here

- in state Θ_1 persons repay the loan;
- in state Θ_2 persons accumulate the loan;
- a_1- a parameter characterising the intensity of the decrease in the number per unit of time at the expense of fully paid persons;
- a_2 - the average intensity of applications for accumulation;
- b_1- a parameter characterising the intensity of growth due to the transition of rollers to the group that repays the debt;
- b_2- parameter, which characterises the intensity of growth due to the transition of accumulators in the group of debt repayment; per unit of time at the expense of fully settled persons;
- θ_1, θ_2 with probability p_1 and p_2, satisfying the next system:

$$\frac{dp_1(t)}{dt} = -\lambda p_1(t) + \mathrm{c}p_2(t), \frac{dp_2(t)}{dt} = \lambda p_1 - \mathrm{c}p_2(t), \lambda + c = 0,$$
$$0 < \lambda, c < 1 .$$

As a mathematical toolkit, the method of momentary equations is proposed (Valeev and Dzhalladova, 1999). Taking into account (9) for equation (12), the moment equations of the first order are as follows

$$\frac{dp.m._1\,(t)}{dt} = (a_1 - \lambda)p.m._1\,(t) + \mathrm{c}p.m._2\,(t) + b_1,$$
$$\frac{dp.m._2\,(t)}{dt} = \big(a_2 - c)p.m._2\,(t) + \lambda p.m._1\,(t)\big) + b_2,$$

where

• $p.m._1$ – the average expected number of persons repaying the loan,

• $p.m._2$ –- the average number of persons accumulating,

• $M(t) = p.m._1\,(t) + p.m._2\,(t)$–- the average number of persons applying for a loan,

• $M_0 = M(t_0)$.

The conditions of the stationary mode of operation of the proposed system are obtained. If $a_1(t)$ increases with the number of applications, then $b_1(t)$ *and* $b_2(t)$ decreases and then $a_2(t)$ increases. The transition process ends, and the set mode in the system has stationary characteristics:

$$\frac{dp.m._1\,(t)}{dt} = \frac{dp.m._2\,(t)}{dt} = 0.$$

From this expression we obtain the number of applications in the system in the stationary mode (Fig.1-4):

$$p.m._1\,(t) = \frac{b_1a_2 - (b_1 + b_2)c}{a_1a_2 - (ca_1 + \lambda a_2)}\,M_0, p.m._2\,(t) = \frac{b_2a_1 - (b_1 + b_2)\lambda}{a_1a_2 - (ca_1 + \lambda a_2)}\,M_{0.}$$

For stability of existing banking system enough are holds equlity (Dzhalladova and Růžičková, 2020)

$$2c = 2b_1 - a_1 = 2b_2 - a_2, \text{ if } z = 1 \text{ (stabile situation).}$$

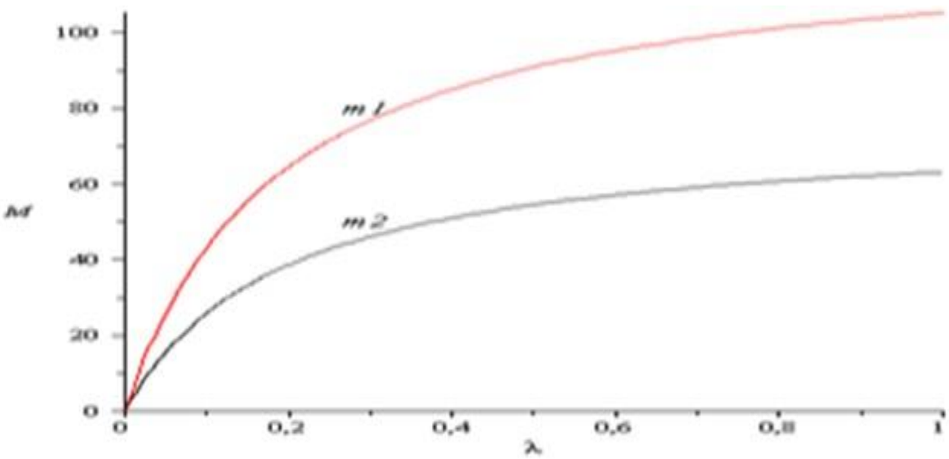

Figure 1: Dependence of the average number of loan repayers and accumulators on the parameters of the system at $M_0 = 200, a_1 = 0{,}5$, $a_2 = 0{,}3$.**; Source: own**

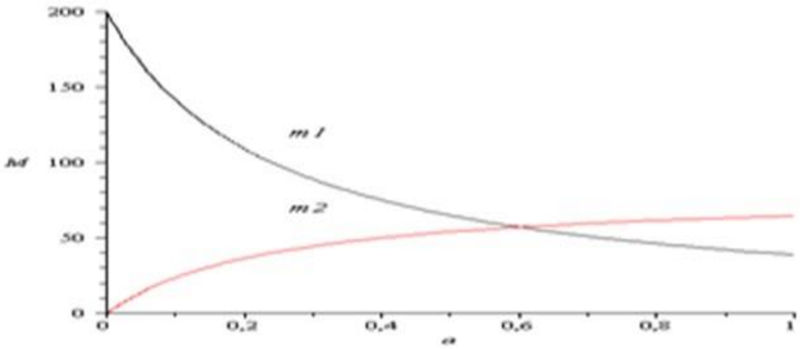

Figure 2: Dependence of the average number of loan repayers and accumulators on the parameters of the system at $M_0 = 200$**,** $\lambda = 0{,}3$, $b_1 = 0{,}6$ **; Source: own**

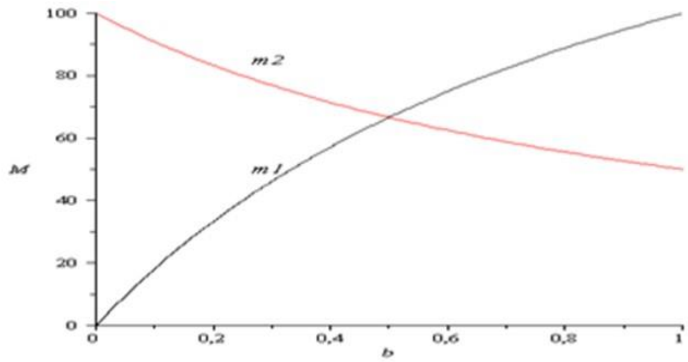

Figure 3: Dependence of the average number of loan repayers and accumulators on the parameters of the system at $M_0 = 200$, $\lambda = 0{,}2$, $a_1 = 0{,}5$**; Source: own**

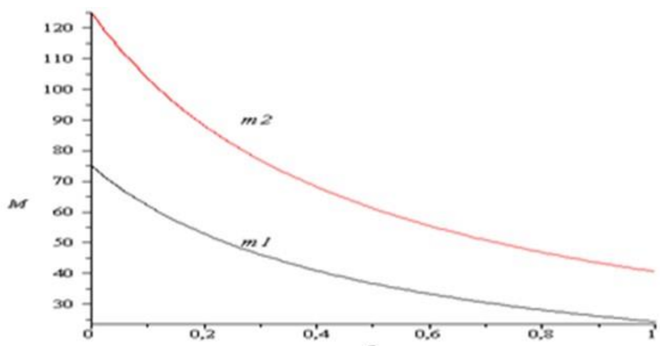

Figure 4: Dependence of the average number of loan repayers and accumulators on the parameters of the system at $M_0 = 200, \lambda = 0{,}3$, $\boldsymbol{b = 0,3}$**; Source: own**

3.2. Model to determine the optimal behaviour advantageous to the worker's organism

It is then reasonable to expect that, at least in natural and well-practised tasks, the observed behaviour will be close to optimal. This makes optimal control theory as effective mathematical instruments for studying the control of movement (Pandy, 2001). Optimal control is also a very successful framework in terms of explaining the details of observed movement. For example, Todorov, in his own research (Todorov, 2004), briefly summarises the existing optimal control models from a methodological perspective, and then lists some research directions considered. Most optimality models of biological movement assume deterministic dynamics and impose state constraints at different points in time. These constraints can, for example, specify the initial and final posture of the body in one step of moving, or the positions of a sequence of states, which the hand must pass through. Since the constraints guarantee accurate execution of the task, there is no need for accuracy-related costs, which specify what the task is. The single cost is a cost rate of the movement or as our model process of working. It has been as energy, or the squared derivative of acceleration (i.e., jerk), or the squared derivative of joint torque (Todorov, 2004).

The solution method is usually based on the maximum principle. Minimum-energy models are explicitly formulated as optimal control problems, while minimum-jerk and minimum-torque-change models are formulated in terms of trajectory optimisation. In our model, we easily transformed into optimal control of problem minimisation of the functional v(t). Let ϕ(t) be the vector of generalised coordinates of a human body move such as the human arm. Let v(t) be the vector of generalised forces. The equations of motion are describing by the linear differential equation.

$$\frac{d\phi(t)}{dt} = \mathrm{g}\big(\xi(t)\big)\phi(t) + b\big(\xi(t)\big)\Xi(t) \tag{14}$$

Let $g(\xi(t))$ and $b(\xi(t))$ be defined as (13).

Find the optimal control $\Xi(t) = s(t, \xi(t))\phi(t)$, from the condition of a minimum of a quadratic functional

$$v = \int_0^\infty \langle \phi^2(t) + \Xi^2(t) \rangle.$$

We assume that the semi-Markov process $\xi(t)$ acquires two states θ_1 and θ_2 with transition intensities $q_{11}(t) = q_{22}(t) \equiv 0$, $q_{12}(t) = q_{21}(t) \equiv \frac{1}{T}\Big(0 \le t \le \frac{1}{T}\Big)$, as well as, with each jump $\xi(t)$ from one state to another solution of the optimised system

$$\frac{d\phi(t)}{dt} = \Big(g\big(\xi(t)\big) + s\big(t, \xi(t)\big)\Big)\phi(t)$$

increases by a factor $h \neq 0$. Find the functions

$$\chi_1(t) = \chi_2(t) = \frac{T-t}{T}.$$

When $g(\theta_k) = g_k (k = 1{,}2)$ the system of equations (5) takes the form

$$\begin{aligned} \frac{dR_1(t)}{dt} &= -1 - 2a_1 R_1(t) + R_1^2(t) + \frac{R_1(t) - h^2 R_2(0)}{T-t}, \\ \frac{dR_2(t)}{dt} &= -1 - 2a_2 R_2(t) + R_2^2(t) + \frac{R_2(t) - h^2 R_1(0)}{T-t}. \end{aligned} \qquad (15)$$

The system of equations of the Riccati type (15) is integrated numerically on the interval $[0, T)$. This system has a special point $t = T$. To eliminate the feature, we take in the system of equations (15)

$$R_1(T) - h^2 R_2(0) = 0, R_2(T) - h^2 R_1(0) = 0,$$

and

$$y_k(z) = R_k(T - Te^z)(k = 1{,}2).$$

As a result, we obtain a system of differential equations

$$\begin{aligned} \frac{dy_1(z)}{dz} &= Te^z\left(1 + 2a_1 y_1(z) - y_1^2(z)\right) - \left(y_1(z) - c^2 y_2(0)\right), \\ \frac{dy_2(z)}{dz} &= Te^z\left(1 + 2a_2 y_2(z) - y_2^2(z)\right) - \left(y_2(z) - c^2 y_1(0)\right) \end{aligned} \qquad (16)$$

with initial conditions

$$y_1(-L) = h^2 y_2(0), y_2(-L) = h^2 y_1(0),$$

where $L > 0$ — quite a large number ($L = 20$).

The system of equations (15) is integrated numerically by the Runge-Kutta method in between $[-L, 0]$ at specified values $y_k(0)(k = 1{,}2)$. After finding new values $y_k(0)(k = 1{,}2)$ old values $y_k(0)(k = 1{,}2)$ are replaced by the new ones. This method of successive approximations is convergent, and therefore we find functions

$$s_k(t) = -R_k(t)(k = 1{,}2).$$

The results of calculations, as well as the values of functions $R_1(0{,}1kT), R_2(0{,}1kT)(k = 1{,}2,\ldots,10)$ are given in the table 1.

Table 1. Results of calculations

T = 0.5			T = 2		
k	R1	R2	k	R1	R2
1	6.549	7.0375	1	2.4478	2.8828
2	5.879	6.6516	2	2.2697	2.3117
3	5.395	5.3681	3	2.1502	1.9809
4	5.019	3.8630	4	2.0652	1.7632
5	3.714	3.4722	5	2.0021	1.6081
6	3.475	3.1592	6	1.9539	1.4915
7	3.2731	3.9019	7	1.9159	1.4003
8	3.1012	3.6858	8	1.8853	1.3269
9	3.9533	3.5013	9	1.8612	1.2665
10	3.8243	3.3415	10	1.8392	1.2157
T = 8			T = 32		
k	R1	R2	k	R1	R2
1	1.6510	1.6403	1	1.5991	0.9752
2	1.6522	1.2474	2	1.6086	0.8111
3	1.6415	1.0751	3	1.6118	0.7504
4	1.6357	0.9774	4	1.6133	0.7188
5	1.6322	0.9143	5	1.6142	0.6994
6	1.6299	0.8701	6	1.619	0.6862
7	1.682	0.8375	7	1.6154	0.6767
8	1.6269	0.8123	8	1.6157	0.6695
9	1.6259	0.7923	9	1.6159	0.6639
10	1.6252	0.7761	10	1.6161	0.6594

Thus, incorporating random control impact into an optimal control problem is equivalent to increasing the control energy cost. The cost increase required to make the two problems equivalent is, of course, impossible to compute solving the stochastic problem (since it depends on the unknown optimal value function).

By the analyse of the obtained values, we will assumed that with growth T value of functions $R_2(t)$ goes to the limit values $R_1 = 1{,}6180$, $R_2 = 0{,}6180$, that their coefficients acquire R_1, R_2 in the determined case in the absence of process transitions $\xi(t)$ from one state to another. Note also that the control coefficients are variable due to the non-stationary functions.

$$\chi(t) = (T - t)T^{-1}(k = 1{,}2).$$

The calculations were performed at the following values of the coefficients $a_1 = 0{,}5$; $a_2 = -0{,}5$; $h = 1{,}5$. Then the functional has a minimum value, or the human exerts minimum energy when working.

We have come to the conclusion that in order to preserve the energy of apparently his own strength, a person must work half the time, rest other half, and if work, then work with the coefficient of 1.5 of his capabilities.

4. Conclusions

Investing is less risky for those who provide a loan, the greater the number of people paying off the loan or having paid the loan off in full. The latter is associated with the level of wages as one of the main indicators of the quality of life of the population. Reducing risks affects the growth of investments in housing construction, which, in turn, leads to the growth of the country's economy and the solution to the employment issues of the economically active population. Thus, construction is one of the important areas that contribute to the economic development of the state, since it begins the chain of activity of many related industries and represents a very important problem related to human security. The authors hope to discuss this problem further in their future work.

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