Ngo Thai HUNG, PhD
E-mail: hung.nt@ufm.edu.vn
Department of Economics and Law
University of Finance-Marketing
Ho Chi Minh City, Vietnam

TIME-FREQUENCY LINKAGES BETWEEN INTERNATIONAL COMMODITIES AND THE BRICS EQUITY MARKETS

Abstract. *This paper aims to explore the time-frequency relationship between international crude oil, gold prices and financial markets in the BRICS countries. Using MGARCH-DCC and Wavelet Coherence transform frameworks, our findings reveal that the return correlations are time-varying in terms of different phases in the five nations except for the cases of the crude oil, gold prices and SSE, USDBRL, RTS, JSE, USDZAR, and exhibit both negative and positive relationship between the pairs of return series throughout the sample period. The graph of the values of conditional correlations shows higher correlations between the Chinese, Indian, Brazilian currencies and the international commodities market. The short periods of negative connectedness between crude oil, gold prices and the exchange rates are also observed. Furthermore, the results of wavelet coherence show that the international commodities market has a significant influence on the BRICS financial markets in the short run, but crude oil and gold variables impacted by stock and exchange markets in the BRICS in the long term. In addition, the findings of this paper have significant implications for portfolio management, as well as the financial risk management in the BRICS equity markets and provide straightforward insight for monetary and fiscal policies by taking into account the pressure of the international crude, gold prices generating on the stock and exchange rate markets.*

Keywords*: Crude oil price, gold price, stock market, exchange rate, MGARCH-DCC, Wavelet technique, BRICS.*

JEL Classifications : G1, G11, G15, E40, Q48

1. Introduction

With the rapid growth of economic globalization, the five main emerging national economies, including Brazil, Russia, India, China, and South Africa (BRICS), rising interrelatedness between equity markets and the international commodity markets (Jain and Biswal, 2016). According to Chkili (2016), BRICS countries are the pivotal recipients of international investments and major trading partners with industrial nations. Their financial markets are closely connected with global

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commodity assets as well. Therefore, investigating intercorrelations between gold price, crude oil price and the BRICS equities could be helpful to investors and policy makers.

The financial markets in BRICS have grown rapidly in both value and volume in recent years Jain and Biswal (2016). This has also made available huge investment and trading opportunities for market participants. The BRICS markets, known as an emerging market, provide significant opportunities to gain high returns in comparison to what can be earned by investing in developed markets. Several investigations highlight that BRICS countries are a key driver of the world commodity markets, especially oil and gold, owing to its consistent and high economic growth (Hussain et al., 2017). It is widely acknowledged that international crude oil and gold prices move in the opposite direction. Increasing oil and gold prices both have a remarkable influence on the international markets and the global economy (Beckmann et al. (2015).

The interactive linkages between oil, gold prices and financial markets have been extensively examined in the extant literature. The general consensus is that commodity shocks can significantly impact the economic activity of the country (Singhal et al. 2019). Recently, a number of studies have extended the interconnectedness between commodities and financial markets. For instance, Jain and Biswal (2016) suggest fall in gold and crude oil prices cause fall in the value of the Indian Rupee and the benchmark stock index. Singhal et al. (2019) show that gold prices positively impact the stock market of Mexico while oil prices influence them negatively.

This study aims to examine whether time-frequency interconnected exist between BRICS equity markets (stock and currency markets) and the global commodity markets (crude oil and gold markets). This study contributes to the present international finance theories in the BRICS region, and provides evidence on the influence of oil and gold prices on BRICS financial markets in novel approaches. First, the multivariate generalized autoregressive conditional heteroskedasticity dynamic conditional correlation (MGARCH-DCC) model is employed to capture the evolution of volatilities and correlations between international commodities and BRICS equities over time. This model has been extensively utilized in related research (Jain and Biswal, 2016; Nagayev et al. 2016). Second, we use the wavelet coherence analysis for the investigation of time series in the time-frequency domain to uncover the dynamics of correlations between international commodity markets and financial markets in the BRICS area. Furthermore, the wavelet approach uncovers complex price correlation patterns without resorting to frequency frameworks (Nagayev et al. 2016). This model has been used in some of the recent studies such as Nagayev et al. (2016), Raza et al. (2017) and Raza et al. (2019). Some of the reasons why we incorporate MGARCH-DCC model into wavelet coherence techniques are because MGARCH-DCC can capture variations in correlations and volatilities in higher frequency level in a more productive and better snapshot. Wavelet can evaluate the co-movement between the assets on medium and high scales. As a result, these models formally issue a more in-depth and robust analysis that reinforces better understanding of the study under examination. This type of analysis may be extremely important that investors, portfolio managers, and policymakers do understand the dynamic interactions between international commodity markets and financial assets in the BRICS countries.

This paper is organized as follows. The next section briefly reviews the existing literature in related areas of study. Section 3 explains the data and methodology. Section 4 discusses the empirical results. Section 5 concludes with research implications.

2. Literature review

The interconnectedness between oil price and exchange rate, oil price and the stock market, gold price and exchange rate, gold price and stock returns have been researched extensively in the literatures. In this section, recent studies that centered primarily on a pair of the variables under examination has been reviewed. Also, we discuss some of the findings and conclusion of investigations that have been taken into consideration in financial markets in the BRICS nations.

Different investigations have shed light on the interconnectedness between oil price and exchange rate markets and provide evidence of the existing relationship between them in Asian countries. For instance, Nusair and Olson (2019) study the effects of oil price shocks on Asian exchange rates using quantile regression analysis and conclude that that positive and negative oil price shocks have asymmetrical influences on exchange rate returns, which vary in significance, size, and sign throughout the distribution of exchange rate returns. Hussain et al. (2017) use a detrended cross-correlation specification to examine the co-movements of the oil price and exchange rate in 12 Asian countries (China, Hong Kong, India, Indonesia, Japan, Korea, Malaysia, Pakistan, the Philippines, Singapore, Sri Lanka, Taiwan). The findings highlight a weak negative cross-correlation between oil price and the exchange rate for most Asian countries included during the sample period. Chen et al. (2016) present the impacts of oil price shocks on the bilateral exchange rates of the US dollar against currencies in 16 countries, and put forward that the responses of dollar exchange rates to oil price shocks differ greatly depending on whether changes in oil prices are driven by aggregate demand.

Some scholars have also explored the interplay between gold price and exchange rate. Beckmann et al. (2015) also suggest that exchange rate depreciation initially have a negative effect on the gold market after one day which turns out to be positive after two days in most of the cases.

So far as oil price and stock market relationship is concerned, it has already received increasing attention by practitioners and policymakers. Kang et al. (2016) find that a positive U.S oil shock has a positive influence on U.S real stock returns as they conduct an examination of the impact of both U.S and non-U.S oil supply shocks on U.S stock returns. Kang et al. (2015a) also reveal that oil price shocks contain information for forecasting real stock return. In the same vein, Kang et al. (2015b) highlight that positive shocks to aggregate demand and oil market specific demand are associated with negative impacts on the covariance of return and volatility. Fang and You (2014) put forward that oil price shocks influence stock prices in emerging economies.

Literature indicates a significant relationship between the gold market and stock prices. Nguyen et al. (2016) focus on the nexus between international stock markets and gold prices. According to the findings, gold may be a safe haven asset during the market crash for the case of Malaysia, Singapore, Thailand, the UK and the US markets but not for the Indonesian, Japanese and the Philippines markets. Chkili (2016) employs the Asymmetric DCC model to estimate the time-varying correlations between the gold and stock markets, and indicates that the dynamic conditional correlations switch between positive and negative values over the period under study.

There have been studies taking into consideration more than two of these variables as well. For example, Singhal et al. (2019) examine the dynamic relationship among crude oil prices, gold prices, exchange rate and the stock market in Mexico. Raza et al. (2017) concentrate on the dependence structure between crude oil prices and stock market indices, as well as the exchange rates in a number of economies categorized with respect to their status as developing/emerging markets, and oil importer/exporter countries. Jain and Biswal (2016) investigate the relationship between global prices of gold, crude oil, the USD-INR exchange rate, and the stock market in India.

As a short summary, very limited research has been conducted on the BRICS financial markets (Nguyen et al. 2016). Furthermore, the most used techniques for connectedness analysis in communities and financial markets are cointegration tests and vector error correction models, which do not imply the underlying time-varying change in the lead-lag structure. In this paper, we examine the time-frequency relationship between international commodities and primary financial markets in the BRICS countries using MGARCH-DCC model alongside with wavelet coherence framework. As a result, the purpose and primary contribution of this current investigation is to fill this gap.

3. Methodology and Data

The dynamic conditional correlation model (MGARCH-DCC)

The dynamic conditional correlation (DCC) is employed. Engle (2002) introduced this estimator to capture the dynamic time-varying behavior of conditional covariance. The conditional covariance matrix H_t is now defined as,

$$\mathrm{H}_t = D_t R_t D_t \tag{1}$$

where $D_t = diag\sqrt{\{\mathrm{H}_t\}}$ is the diagonal matrix with conditional variances along the diagonal, and R_t is the time-varying correlation matrix.

A GARCH (1,1) specification of each conditional variance can be written as,

$$h_{ii,t} = c + a_i\varepsilon_{i,t-1}^2 + b_i h_{ii,t-1} \quad (2)$$

$$h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}},\ i, j = \overline{1,n} \quad (3)$$

where c is a $n \times 1$ vector, a_i and b_i are diagonal $(n \times n)$ matrices.

Equation (1) can be re-parameterized with standardized returns as follows, $e_t = D_t^{'}\varepsilon_t$

$$\mathrm{E}_{t-1}e_t e_t^{'} = D_t^{-1}H_t D_t^{-1} = R_t = \left[\rho_{ij,t}\right] \quad (4)$$

Engle (2002) suggests the following mean-reverting conditionals with the GARCH(1,1) specification:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} \quad (5)$$

where $q_{ij,t} = \bar{\rho}_{ij}(1-\alpha-\beta) + \alpha e_{i,t-1}e_{j,t-1} + \beta q_{ij,t-1}$ (6)

And $\bar{\rho}_{ij}$ is the unconditional correlation between $e_{i,t}$ and $e_{j,t}$. Scalar parameters α and β must satisfy, $\alpha \geq 0,\ \beta \geq 0,$ and $\alpha + \beta < 1$

The value of $(\alpha+\beta)$ close to one reveals high persistence in the conditional variance. In matrix form, $Q_t = \bar{Q}(1-\alpha-\beta) + \alpha e_{t-1}e_{t-1}^{'} + \beta Q_{t-1}$ (7)

where $\bar{Q} = Cov\left[e_t, e_t^{'}\right] = E\left[e_t, e_t^{'}\right]$ is unconditional covariance matrix of the standardized errors $\bar{Q}$ can be estimated as, $\bar{Q} = \frac{1}{T}\sum_{t=1}^{T} e_t e_t^{'}$ (8)

R_t is then obtained by $R_t = \left(Q_t^*\right)^{1/2} Q_t \left(Q_t^*\right)^{1/2}$ (9)

where $Q_t^* = diag\left\{Q_t\right\}$

Wavelet coherence

A brief note on wavelet coherence is defined as follows:

$$R_n^2(S) = \frac{\left|S\left(s^{-1}W_n^{XY}(s)\right)\right|^2}{S\left(s^{-1}\left|W_n^X(s)\right|^2\right).S(s^{-1}\left|W_n^Y(s)\right|^2} \quad (10)$$

where S is a smoothing operator. Smoothing is achieved by convolution in time and scale.

$$S(W) = S_{scale}\left(S_{time}\left(W_n(s)\right)\right) \tag{11}$$

Where S_{scale} and S_{time} illustrate smoothing on the wavelet scale axis and in time, respectively. Smoothing operator we use in this study is the Morlet wavelet, so the more suitable definition is as follows:

$$S_{time}(W) = \left.\left(W_n(s) * c_1^{\frac{-t^2}{2s^2}}\right)\right|_s \text{ and } S_{time}(W)_s = \left.\left(W_n(s) * c_2\Pi\left(0.6s\right)\right)_s\right|_n \tag{12}$$

where c_1 and c_2 are normalization constants and Π is the rectangle function, the scale decorrelation length for the Morlet wavelet is 0.6

The wavelet coherence coefficient measures the local linear correlation between two stationary time series at each scale and ranges $R_n^2(s) \in [0,1]$.

Data

In this study, we look into the association of international oil and gold prices with stock and exchange markets in BRICS countries (Brazil, Russia, India, China and South Africa). West Taxes Intermediate (WTI) crude spot price is taken as a proxy for international oil price movements. The international gold spot price is used as a proxy, which is represented by GOLD. This paper uses daily data for all series for analysis. The sample period ranges from February 2008 to February 2019, and the data is obtained from the Bloomberg database. Because the BRICS countries have various trading days, we synchronized the data by omitting non-overlapping trading days. Detailed information is shown in Table 1. We calculate the returns of the above markets: $R_t = 100 \times \ln(P_t / P_{t-1})$ where P_t is the price level of an index at time t.

Table 1. Sample selection

Country	Exchange rate	Stock index	Observations
Brazil	USDBRL	BOVESPA	2875
Russia	USDRUB	RTS	2875
India	USDINR	SENSEX	2875
China	USDCNY	SSE	2875
South Africa	USDZAR	FTSE/JSE	2875

We report summary statistics for daily returns for the full sample, as well as statistics testing for normality and stationarity in Table 2. On average, exchange rate returns are lower than the returns on stock prices for all countries, but they are less volatile as indicated by the associated standard deviations. WTI and SSE also have a similar trend. The exchange rate USDINR is the least volatile (0.03%) among the time series under consideration, while crude oil volatility is (2.39%). The measures for kurtosis and skewness illustrates that all return series are highly leptokurtic with respect to the normal distribution, which is also confirm by the Jarque-Bera test. ADF unit root tests report that all return series are stationary at the 1% significance level. Furthermore, based on the wavelet-based unit root test, Raza et al. (2019) confirm that series are free from the unit root problem and can be used for long run interaction. Finally, the ARCH test shows the presence of autocorrelation and heteroskedasticity issues in data. We can see that there is strong evidence of the existence of the persistence of ARCH effect in all concerned variables. Hence, they are appropriate for further statistical analysis. Volatility property represents the persistence of conditional heteroscedasticity in the variance process of the return series. Thus DCC-GARCH specification is employed to adequately model the interrelatedness between the international commodities (WTI, GOLD) and financial markets in BRICS nations.

Table 2. Descriptive statistics of the return series

Variable	Mean	Std.dev	Skewness	Kurtosis	ADF	JB	ARCH
Brazil							
Exchange rate changes	-0.032	0.062	-8.216	120.078	-5.177*	1674.369*	63.954*
Stock returns	0.015	1.679	0.003	10.675	-34.30*	7056.949*	113.81*
Russia							
Exchange rate changes	-0.040	0.1312	-11.375	188.774	-4.731*	4275.294*	23.372*
Stock returns	0.032	1.021	0.286	17.000	-54.60*	4196.281*	797.85*
India							
Exchange rate changes	-0.017	0.0309	-7.864	117.662	-7.623*	1604.593*	792.218*
Stock returns	-0.002	1.1162	44.308	2226.53	-54.49*	5931.020*	0.0259*
China							
Exchange rate changes	-0.001	0.1616	0.434	15.974	-30.14*	2025.438*	117.46*
Stock returns	0.034	1.7678	-0.534	7.0101	-53.40*	2063.090*	0.0375*
South Africa							
Exchange rate changes	-0.030	0.047	-5.996	58.416	-10.83*	3851.135*	188.225*
Stock returns	0.0255	1.302	-0.053	6.926	-53.21*	1848.221*	126.021*
WTI	-0.020	2.398	0.1445	7.699	-56.60*	2653.113*	178.251*
GOLD	0.0143	1.211	-0.1963	8.182	-56.20*	3231.879*	82.2257*

Notes: ADF illustrate the Augmented Dickey. ARCH test is used to test the presence of ARCH effect in the datasets. JB denotes Jarque-Bera test. * represents significant at 1%.

4. Results

We first investigate the connectedness between commodities and BRICS equities using MGARCH-DCC model to track the time-varying characteristics of the variables in terms of volatility and correlations. We then apply the wavelet coherence technique to investigate the interplay between the series on a multi-scale, time-frequency domain.

MGARCH-DCC model

We employ MGARCH-DCC model with Gaussian distribution to capture the time-varying volatilities and correlations. Table 3 reports the MGARCH-DCC results for the pairs of assets during the study period. The volatility persistence is measured by $(\alpha+\beta)$. The estimated α and β parameters are statistically significant in all cases except for the cases of South Africa. It is clear that $\beta>\alpha$, indicating that the current variances are more influenced by the past return innovations. The coefficient of α reflects the effect of the past shocks on current conditional correlation, while the β captures the impact of past correlation. The sum of the parameters α and β is close to one. This means that the process described by the model is not mean reverting. Put differently, after the innovations occurred in the markets, the dynamic correlation will not return to the long-run unconditional level.

Table 3. Estimation results of MGARCH-DCC model

		WTI-SSE	WTI-USDCNY	GOLD-SSE	GOLD-USDCNY
DCC	α	0.0521*	0.0121138*	0.000354	0.449935**
	β	0.1945	0.50000*	0.504026**	0.569807*
	ARCH-LM	19.846 (0.4675)	39.601 (0.69924)	11.643 (0.92780)	50.370 (0.26928)
	$Q^2(20)$	18.8672 (0.5305)	16.78388 (0.6670)	21.712 (0.3563)	42.291228 (0.5862)
	$\text{Hosking}^2(20)$	46.7083 (0.9989)	63.53790 (0.91142)	44.5163 (0.99955)	103.2628 (0.2443)
	McLeod-$\text{Li}^2(20)$	31.095 (0.5039)	17.533882 (0.6181)	22.5712 (0.3103)	0.274244 (1.0000)
		WTI-BOVESPA	WTI-USDBRL	GOLD-BOVESPA	GOLD-USDBRL
DCC	α	0.0102*	0.00164	0.01949*	0.002146535*
	β	0.9222*	0.05000	0.62238*	0.825283253*
	ARCH-LM	43.23 (0.547)	38.29 (0.75014)	52.48 (0.20657)	71.39 (0.00737)
	$Q^2(20)$	24.942 (0.2036)	13.78188 (0.8414)	22.0189 (0.3395)	21.593607 (0.3630)
	$\text{Hosking}^2(20)$	81.2097 (0.44124)	104.657 (0.7928)	89.20765 (0.22543)	18.33918 (0.4975)
	McLeod-$\text{Li}^2(20)$	19.306 (0.5020)	14.7091 (0.7928)	23.0779 (0.2850)	24.630386 (0.2159)
		WTI-SENSEX	WTI-USDINR	GOLD-SENSEX	GOLD-USDINR
DCC	α	0.0395***	0.017538*	0.01227*	0.0133***
	β	0. 364*	0.5157665*	0.93537*	0.8984*
	ARCH-LM	43.15 (0.5503)	124.86 (0.3024)	34.20 (0.87963)	32.67 (0.91459)
	$Q^2(20)$	46.277	17.40988	21.5197	25.186465

		(0.1318)	(0.6262)	(0.3671)	(0.1944)
	Hosking^2(20)	81.95537 (0.8382)	39.11757 (0.8017)	86.40282 (0.29264)	140.5269 (0.4575)
	McLeod-Li^2(20)	12.625 (1.000)	17.35141 (0.6301)	23.1240 (0.2827)	26.550182 (0.1484)
		WTI-JSE	WTI-USDZAR	GOLD-JSE	GOLD-USDZAR
DCC	α	0.0134	0.0498	0.011136	0.027219
	β	0.05000	0.0500	0.05000	0.8172
	ARCH-LM	34.33 (0.8762)	34.35 (0.87569)	52.04 (0.2188)	109.35 (0.2102)
	$Q^2(20)$	14.955 (0.7790)	15.05416 (0.7733)	22.956 (0.2909)	39.97757 (0.1050)
	Hosking^2(20)	88.02096 (0.25258)	34.01731 (0.6701)	125.5511 (0.1009)	174.4967 (0.3489)
	McLeod-Li^2(20)	18.415115 (0.5601)	19.528163 (0.4878)	22.018740 (0.3395)	20.00691 (0.6160)
		WTI-RTS	WTI-USDRUB	GOLD-RTS	GOLD-USDRUB
DCC	α	0.0009711*	0.0626*	0.006740985	0.09112*
	β	0.0665684**	0.6200***	0.0500000	0.9500*
	ARCH-LM	191.31 (0.2013)	27.74 (0.9784)	42.2 (0.5914)	44.03 (0.5128)
	$Q^2(20)$	14.636919 (0.7968)	15.504597 (0.7468)	21.853729 (0.3485)	23.909745 (0.2464)
	Hosking^2(20)	111.6022 (0.1013)	154.6292 (0.1084)	99.90670 (0.60538)	127.7873 (0.2023)
	McLeod-Li^2(20)	18.238259 (0.5717)	8.53048 (0.9877)	18.868796 (0.5304)	26.962143 (0.1363)

Notes: Q^2 (20) is the Ljung-Box test statistics employed to the squared standardized residuals. Hosking^2(20) and McLeod-Li^2(20) multivariate Portmanteau statistics test the null hypothesis of no serial correlation in the squared standardized residuals (20 lags). JB denotes Jarque-Bera test. p-values are given in parentheses. *,**,*** present the significance level at 1%, 5% and 10%.

It can be seen from Table 3 that both coefficients of β, α in the pair of WTI-SSE, GOLD-SSE, WTI-USDBRL, GOLD-RTS, WTI-JSE, WTI-USDZAR, GOLD-JSE and GOLD-USDZAR are not statistically significant, respectively. As a result, our findings do not support the hypothesis of dynamic conditional correlation but are in favor of constant conditional correlation. It indicates that the correlations between these pairs are constant and stable without many outliers. This is formally confirmed by Figure 1.

Figure 1 plots the daily dynamic conditional correlation for all pairs of return indices under investigation. In all cases, the plot depicts that the DCC falls within the range of -0.7 to 0.8, exhibiting both negative and positive relationship between the pairs of return series throughout the sample period. In general, quite a constant movement is evident in case of the conditional correlations between gold, crude oil prices and SSE, USDBRL, BOVESPA, SENSEX, USDINR, RTS, USDRUB and there are varying trends apparent with the case of gold and exchange rate. Namely, for the pair of (GOLD,USDCNY), dynamic conditional correlation coefficients are found to be relatively high in the period of 2010-2012 and decrease gradually between 2012 and 2014 as the European stock market collapse and the bank save the euro, whereas the rest of the sample period is found to be highly fluctuated. The WTI-USDCNY correlation remains stable within a band of -0.2-0.3. This reveals the co-dependency of both the crude oil market and the exchange rate. For the rest of the other pairs under consideration, these coefficients are found to be slightly fluctuated during the research period. This research has results consistent with the previous studies (Nusair and Olson 2019; Hussain et al. 2017).

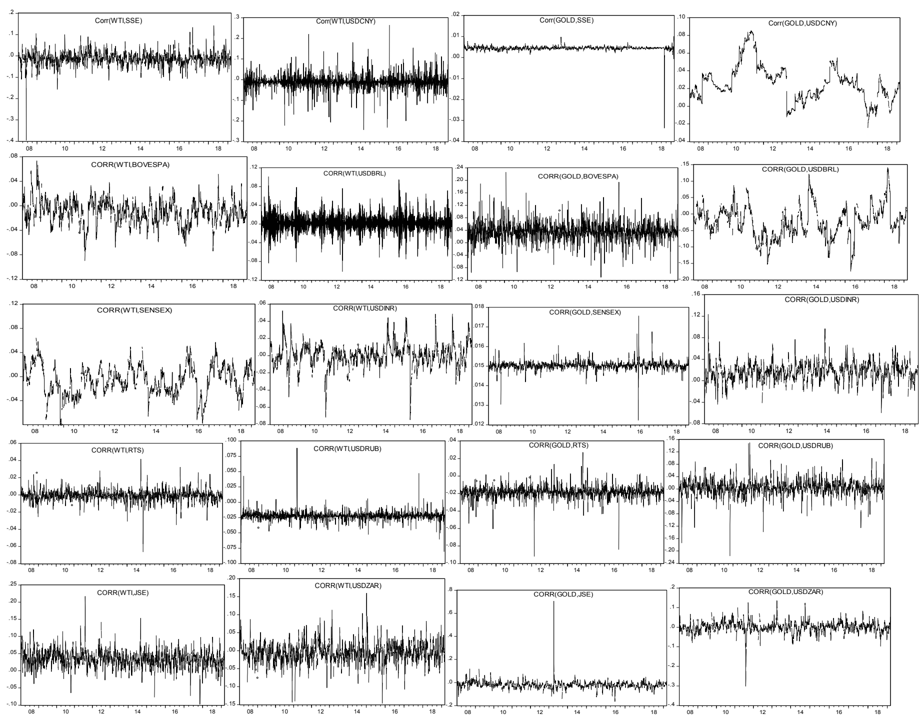

Figure 1. **Daily dynamic conditional correlations**

As for the robustness of the estimations of the MGARCH-DCC model, we have used the multivariate ARCH-LM test on the residuals of each model to determine whether the ARCH effect still exists in the model. As we can see from the estimates, there are no problems of ARCH effect for all selected variables during the study period. The Ljung-Box test statistics for the squared standardized residuals suggest that the null hypothesis of no serial correlation is not rejected for all cases, providing some indications of good specification in each model. In addition, the Hosking and McLeod and Li test results indicate acceptance of the null hypothesis of no serial correlation in the conditional variances estimated by the MGARCH-DCC model. Hence, we can conclude that there is no evidence of statistical misspecification in the MGARCH-DCC model. Obviously, the MGARCH-DCC model is employed to examine the time-varying contemporaneous correlation amongst the selected variables. Nevertheless, it does not illustrate the lead or lag structure amongst variables (Jain and Biswal, 2016). To supplement our analysis and address this issue, we employed the cross-wavelet transform to seek for regions in time-frequency space in which the time series display high common power. The different interrelatedness structures are elaborated in the following sections.

Wavelet coherence

In order to analyze the causal association among the variables, the wavelet coherence transform is employed. It offers the prevalent power and comparative phase of different time sequences in present time-frequency space. Fig. 3 below illustrates the estimated wavelet coherence and the phase difference for all examined pairs under investigation. Time is exhibited on the horizontal axis and frequency is shown on the vertical axis - regions in time-frequency space where two concerned variables co-vary are located by wavelet coherence. Regions with significant interconnection are presented by warmer colors(red), while lower dependence between variables is signified by colder colors (blue). Cold regions beyond the significant areas show frequencies and time with no dependence in the series. Both the frequency and the time intervals where the pairs of concerned variables move together significantly can be identified. An arrow in the wavelet coherence plots displays the lag phase connections between the examined variables. A zero phase difference explains that the two variables move together on a particular scale. Arrows point to the right (left) when the return series are in phase (anti-phase), simply meaning that they move in the same direction when the two series are in phase, and they move in the opposite direction when the two series are in anti-phase. Arrow pointing to the right-down or left-up describe that crude oil and gold returns lead the BRICS financial markets, while arrows pointing to the right-up or left-down show that crude oil and gold returns are leading (Nagayev et al. 2016**).**

To assess whether the dependencies vary across different frequencies, we find in Figure 2 that in the short run, we have an out-phase situation from 2015 to 2017 in which crude oil is leading (WTI has a causal impact over SSE, BOVESPA, SENSEX, RTS, JSE). In the long run from 2008 to 2013, we see an anti-phase situation where USDCNY, USDINR, USDRUB are leading (USDCNY, USDINR and USDRUB have a causal influence on WTI). Similarly, gold is leading in the short run from 2013 to 2015 (GOLD has a casual effect on SSE, BOVESPA, SENSEX, JSE).

Again, we see an anti-phase situation in the long run from 2008 to 2012, the exchange rate return is leading (USDCNY, USDINR, USDBRL and USDRUB have a causal influence over GOLD). The wavelet coherence estimation indicates that crude oil and gold have a positive effect on the financial markets in the BRICS countries and BRICS versus the US dollar in the short run, whereas stock and exchange markets in the BRICS region negatively impact the two above commodities. In addition, there is a unidirectional effect of crude oil and gold on the financial markets in this area in the long run. However, no causal linkage is found in the medium run.

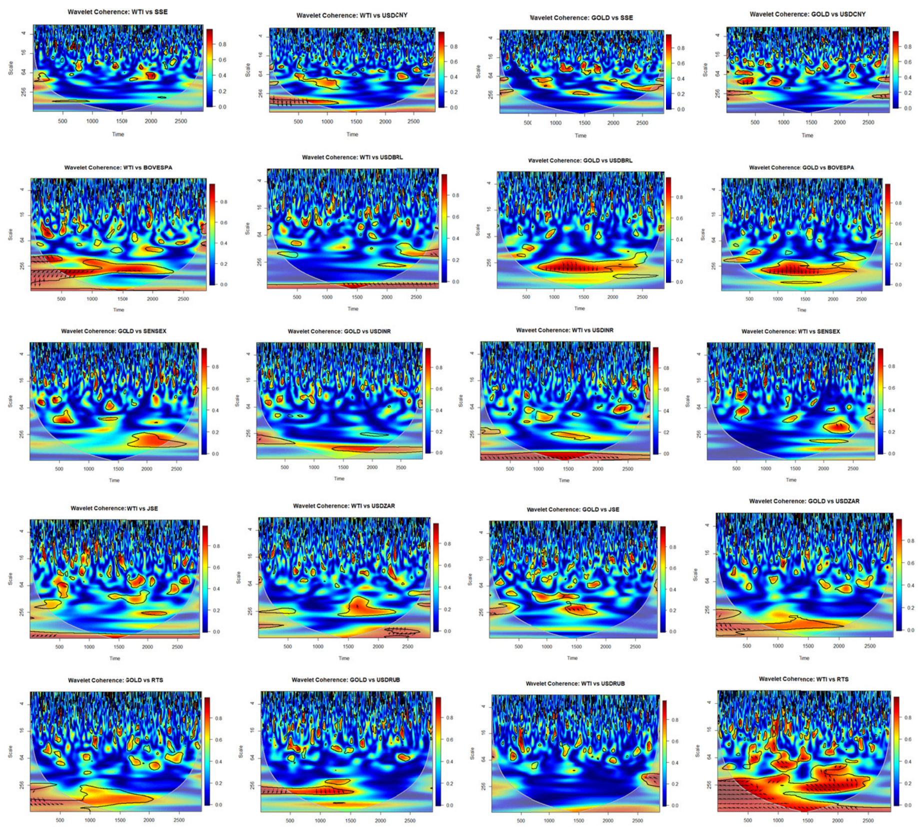

Figure 2. Wavelet coherence results

Notes: This figure represents the Wavelet coherence of commodities (WTI, GOLD) and BRICS financial markets (exchange rate and stock markets) pairs. Time and frequency are presented on the horizontal (time period from January 2008 to February 2019, with 500 = 2009-2010, 1000 = 2011-2012, 1500 = 2013-2014, 2000 = 2015-2016, 2500 = 2017-2018) and the vertical axis, respectively.

The warmer the color of a region, the greater the coherence is between the pairs. The black solid line isolates the statistical significance area at the level of 5 %.

These findings reveal that increasing oil prices do not benefit the nation via its oil import. At the same time, the interdependency between the exchange rate and oil prices indicates that oil price impacts the exchange rate in the long run. This provides evidence that an increase in oil price prompts currency depreciation of oil importing countries. In the context of the association between the gold price and financial markets in BRICS countries indicates that gold price significantly affects the stock price and exchange rate. In both cases, the interrelation is due to causality from varying over time. On observing the various periods from the figure, it can be inferred that the fall in gold prices causes depreciation of the BRICS exchange rates. Fall in the international gold market may cause an increase in demand for gold in these nations. Therefore, increasing its imports and causing depreciation of the currency. The findings support the previous studies of Kang and Ratti (2015) and Fang and You (2014).

Economic implications of the model

Portfolio weights

We then illustrate how empirical findings of the DCC model considered can be employed to calculate the optimal weights and hedge ratios of the global commodity – BRICS equity portfolio. We follow the applications proposed by Kroner and Ng (1998) by taking into consideration a portfolio that minimizes risk without lowering expected returns. The portfolio weight of holdings of two equity indices in a country is given by:

$$\omega_{12,t} = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}} \quad \text{and} \quad \omega_{12,t} = \begin{cases} 0, & if\ \omega_{12,t} < 0 \\ \omega_{12,t}, & if\ 0 \le \omega_{12,t} \le 1 \\ 1, & if\ \omega_{12,t} > 1 \end{cases}$$

where $\omega_{12,t}$ is the portfolio weight for the oil and gold assets with respect to financial equity indices at time t. $h_{12,t}$ is the conditional covariance between international commodities and equity indices, and $h_{22,t}$ is the conditional variance of the international asset index. The weight of the financial market index in the one dollar portfolio is $1-\omega_{12,t}$.

Hedge ratios

We estimate the risk-minimizing hedge ratios for two markets by employing the estimations of the multivariate GARCH model as developed by Kroner and Sultan (1993). In order to minimize the risk of several portfolios, a long position of one dollar taken in the oil and gold indices should be hedged by a short position of \$ β_t in financial market index in a country at time t. The $\beta_{12,t}$ is written as:

$\beta_{12,t} = -\frac{h_{12,t}}{h_{22,t}}$,where $\beta_{12,t}$ is the risk-minimizing hedge ratio for the two selected assets.

The average values of optimal weights reported in the Table 4 reveal the amount of oil and gold should be purchased or sold so as to hedge a \$1 of BRICS equities at divergent time periods. As shown, the estimated coefficients demonstrate that the optimal weights for the global assets in the hedged portfolios vary remarkably across nations.

Applying a MGARCH-DCC model, the highest average value of optimal weights for Russia and the lowest average optimal weight for Brazil. For instance, on average, the value of $\omega_{12,t}$ of a portfolio of (JSE-OIL) in South Africa is about 0.78, while the average risk-minimizing hedge ratio between two markets is 0.018. This shows that the optimal holding of the oil index in one dollar of JSE/OIL index portfolio for South Africa is 78 cents compared to 22 cents for the oil index. Further, these optimal portfolio weights indicate that investors should own more financial equities than oil and gold in their portfolios. The results are the same for the rest of the nations.

Table 4. Optimal portfolio weights and hedge ratios

Portfolio	**Weight ($\omega_{12,t}$)**	**Beta ($\beta_{12,t}$)**
Brazil		
BOVESPA/OIL	0.681	0.100
BOVESPA /GOLD	0.350	0.420
USDBRL/OIL	0.0013	-0.0071
USDBRL/GOLD	0.003	0.212
Russia		
RTS/OIL	0.848	0.0001
RTS /GOLD	0.587	0.0123
USDRUB/OIL	0.795	0.0030
USDRUB/GOLD	0.803	-0.0020
India		
SENSEX/OIL	0.5551	0.0006
SENSEX/GOLD	0.2408	-0.0004
USDINR/OIL	0.8009	-0.0007
USDINR/GOLD	0.8790	-0.0003
China		
SSE/OIL	0.639	-0.0004
SSE/GOLD	0.309	0.0218
USDCNY/OIL	0.794	-0.0007
USDCNY/GOLD	0.782	0.0015
South Africa		

JSE/OIL	0.783	0.0186
JSE/GOLD	0.530	0.1686
USDZAR/OIL	0.879	0.0004
USDZAR/GOLD	0.708	0.0002

5. Conclusion

The core aim of this paper is to investigate the extent to which the international commodities market co-moves with the BRICS equity markets using MGARCH-DCC model alongside with wavelet coherence frameworks. In order to examine the association between variables, daily data from the year of 2008 (February) to 2019 (February) were used. Our findings highlight that the return correlations are time-varying in terms of different phases except for the cases of the crude oil, gold prices and SSE, USDBRL, RTS, JSE, USDZAR, and exhibit both negative and positive relationship between the pairs of return series throughout the sample period.

The graph of the values of conditional correlations shows higher correlations between the Chinese, Indian, Brazilian currencies and the international commodities market. The short periods of negative correlations between crude oil, gold prices and the exchange rates are also observed. This phenomenon might be indicative of investors shifting from risky assets like the stock market to the perceived safe haven like gold (Jain and Biswal, 2016). The decay factors of time-varying correlations show mean reverting correlations between concerned variables moving within specific bands. These findings tally with the study of Chen et al. (2016). The results of wavelet coherence show that the international commodities market has a significant influence on the BRICS financial markets in the short run, but crude oil and gold variables impacted by stock and exchange markets in BRICS in the long run. In addition, there is a unidirectional effect of crude oil and gold on the financial markets in this area in the long run. However, no causal linkage is found in the medium run.

There seem to be a considerable opportunity for investors and portfolio managers to invest in the BRICS markets with a better understanding of the correlation between them. The findings of this research also have significant implications for BRICS portfolio management, as well as the financial risk management and provide straightforward insight for monetary and fiscal policies by taking into account the pressure of the international crude, gold prices generating on the stock and exchange rate markets. Further, they help in a clear understanding of the influence of fluctuations of the international commodities market, which play a pivotal role in making pressure on the financial market indices of the BRICS countries.

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