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A NEW MATHEMATICAL MODEL FOR STAFF SCHEDULING IN THE OPERATING ROOM BASED ON THEIR PREFERENCES

Abstract. *Staff scheduling in operating rooms same as nurse scheduling is a crucial part of medical activities which are key parts of human resource planning. The staff's satisfaction can be considered as a key objective to roster the staff in the core part of hospitals same as operating rooms. For this purpose, we address the monthly staff roster problem in the multi-operating theater to maximize staff's preferences. In this problem, staffs have preferences such as being off for some days, being on for some shifts, etc. On the other hand, there are some requirements such as the number of staff per shift that should not be violated. Considering these practical hypotheses, an integer programming model is made concerning staff's preferences. In this model, hard constraints for requirements, and soft constraints for staff's preferences are defined. Also, the deviations from goals are defined as objectives according to the preferences. The multi-objective model is then converted to a single objective using the goal programming method and then is solved in GAMS software and the results are validated by using real data from a hospital located in Iran. Consequently, the computational experiments state that the method outperforms the current practice used in the hospital to optimize staff's preferences.*

Keywords*: Goal programming, Staff scheduling problem, Staff roster, Operating room, Integer programming, Staff's preference.*

JEL Classification: C61

1. Introduction

For many companies including hospitals, the ability to possess enough staff to serve their customers is important. Nurse scheduling is known as a significant scheduling problem in health care systems. This problem can be modeled with mixed integer programming (MIP) techniques considering

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optimization objectives to minimize or maximize a certain criterion. Many scholars have reviewed the different types of staff scheduling problems including uncertainty of a fuzzy or stochastic environment to optimize the number of nurses, etc. Some researchers have focused more on solution approaches such as evolutionary computations or exact algorithms. Commonly we observe that the nurse scheduling problem is a kind of pure integer mathematical optimization problem. In this kind of problem, there are priorities such as preference shift, preference day off, etc. Nurse scheduling and rostering problem in the operating room involve the assignment of the shifts and days to the nurses so that the maximum preferred on-days and off-days are satisfied. This problem is called the operating room staff scheduling (ORSS). In this problem, each nurse has her/his wishes and restrictions, as does the hospital. The problem is described as finding a schedule that both respects the constraints on the nurses and fulfills the objectives of the hospital. In this problem, we must search for a solution satisfying as many wishes as possible while not compromising the needs of the hospital. In our case study, the increasing population in the hospitals of Iran results in an increasing demand to access to healthcare. Thus, nurse rostering is among the key parts of human resources planning. The important objective, here, is providing a suitable condition which could lead to good roster. In this paper, ORSS is formulated as an integer programming model (IP). For this aim, a new goal programming (GP) is employed. The goal of our problem is to fulfill maximum staff's preferences during the monthly planning. While tackling the ORSS problem, we offer the following contributions to the literature:

- The extension of the staff scheduling problem using the factor of the prior staff in the operating room of a case study for assigning or non-assigning.
- The extension of the mathematical model using soft and hard constraints or goal programming for providing new factors in the special case.
- A new analysis is presented to validate the IP model ORSS.

This paper is structured as follows: In section 2, we gave a brief overview of the previous studies on the staff scheduling problem. In section 3, the problem is stated and then in section 4, a goal programming is extended. In section 5, illustrative examples are provided and the computational experiments are presented, and lastly, conclusions, as well as some suggestions for future research, are given in section 6.

2. Background and related work

In the literature on the nurse rostering/scheduling problem, there are various researches so that the studies are categorized based on the extensions of the problem, the mathematical models, the proposed algorithms, and the objective functions. The researches up to 2022 are shown in Table 1.

Table 1 – The researches up to 2022

Authors (year)	Extension	N [a]	OS [b]	Objective	Algorithm
Dowsland&Thompson (2000)	IP	●		PSCV [c]	TS [d]
Aickelin&Dowsland (2000)		●		PSCV	GA [e]
Burke et al. (2001)		●		PSCV	MA [f]+TS
Burke et al (2008)		●		PSCV	VNS [g]
Maenhout&Vanhoucke (2010)	MIP	●		PSAN [h]	B&P [i]
Mobasher et al (2011)	IP		●	DG [j]	GP [k]
Wu et al (2015)	IP	●		DG	PSO [l]
Santos et al (2016)	IP	●		PSCV	MIP-H [m]
Rajeswari et al (2017)	MIP	●		NP [n]	BCO [o]
Aktunc&Tekin (2018)	MIP		●	PSCV+NC [p]	GP
El Adoly et al (2018)	IP	●		NP+NC	B&B [q]
Ikeda et al (2019)		●		NP	QA [r]
Ala (2019)	IP	●		NP	NSGA-II [s]
Alade&Amusat (2019)		●		NP	CP [t]
Legrain et al (2020)	IP	●		PSCV	R-B&P
Khalili et al (2020)	IP+FF [u]	●		NP+FF	LP-metric
Amindoust et al (2021)	IP+FF	●		NC + FF	HGA
Zhuang&Yu (2021)	IP+LL [w]	●		NP+NW [x]	GP
Guo&Bard (2022)	MILP	●		NP+O [y]	CG [z]
Turhan&Bilgen (2022)	IP+UAC [aa]	●		NP+skill	D-PSO
This Research	IP+PC [ab]		●	SP [ac]+WE [ad]	GP

a) Nurse b) Operating room staff c) Penalty of soft constraint violation d) Tabu search e) Genetic algorithm f) Memetic algorithm g) Variable neighborhood search h) Penalty cost of assignment of nurses i) Branch & Price j) Deviations of goals k) Goal Programming l) Particle swarm optimization m) Heuristic n) Nurse preference o) Bee colony optimization p) Nurse cost q) Branch & Price r) Quantum Annealing s) Non-sorting genetic algorithm t) Constraint programming u) Fatigue Bound w) Labor law x) Nurse workload y) Overtime z) Column Generation aa)

Unit assignment constraint ab) Priority constraints ac) Staff preferences ad) Working experience

There are some studies, in whose new exact or heuristic algorithms and solutions were developed to tackle the basic problem, for instance, modified genetic algorithm (Aickelin & Dowsland, 2000), hybrid of memetic algorithm and tabu search (Burke et al, 2001), the hybridization of heuristic ordering and variable neighborhood search algorithm (Burke et al, 2008), quantum annealing (Ikeda et al, 2019), constraint programming (Alade & Amusat, 2019) were proposed to optimize the nurse preferences or the penalty of the soft constraint violations. Despite the proposing novel algorithms in mentioned studies, the mathematical model for the problem has not been extended. In some studies, not only new algorithms are developed but also, mathematical programming models are proposed. Also, tabu search (Dowsland & Thompson, 2000) was developed to optimize the penalty of the soft constraint violation in a knapsack and network flow model. In another research, branch & price was developed so that Maenhout & Vanhoucke, (2010) modeled a formulation to minimize the penalty cost of the assignment of the nurses, and Legrain et al, (2020) modeled an integer programming to minimize the penalty of the soft constraint violation. In other studies, particle swarm optimization (Wu et al, 2015), mixed integer programming heuristics (Santos et al, 2016), and bees colony optimization (Rajeswari et al, 2017) were developed to optimize the nurse preferences or the penalty of the soft constraint violations. Also, few researchers proposed goal programming to minimize the penalty of the soft constraint violation (Mobasher et al, 2011) and to minimize the cost of the nurses despite the preferences (Aktunc & Tekin, 2018) in their integer programming model. In another study, El Adoly et al, (2018) constructed a multi-commodity network flow, and then the authors proposed a branch & bound to minimize both costs and nurse preferences. In a different study, non-sorting genetic algorithm-II (Ala, 2019) was developed to minimize the nurse preferences, and lastly, column generation (Guo & Bard, 2022) was applied to minimize the nurse preferences and overtime in a mixed integer linear programming model. Although new algorithms have been developed to solve the mathematical programming models, the problem has not been extended in mentioned studies. In recent studies, some new constraints according to the new parameters or factors are considered and thereby, the mathematical programming model is extended and a method is proposed to tackle the model. There are few studies in the literature on the problem that extended the model by additional constraints or objective functions. For instance, fatigue factor as the new objective is considered in a few studies (Khalili et al, 2020; Amindoust et al, 2021) and new integer programming models are extended. In a study (Zhuang & Yu, 2021), part-time and full-time nurses are considered to model the problem, and then goal programming is proposed based on nurse workload and their preferences. In another study, Turhan & Bilgen (2022) considered the skill of the nurses, and then the authors extended the integer programming taking into account unit assignment constraints. To improve this work, the priority of the nurses for the assignment could be considered a new constraint. Moreover, many studies in the literature have focused on nurse scheduling in the ward, while a few studies (Mobasher et al, 2011; Aktunc & Tekin, 2018) consider the nurses in the operating room. More studies considered patient scheduling in operating rooms or multi-resource scheduling without considering staff's preferences (Behmanesh et al, 2020, Behmanesh & Rahimi, 2021). The conceptual framework of the problem is presented in Fig. 1. As it is shown, the relationship between parameters and variables constructs the constraints and objective functions so that the timetable of operating room staff is displayed using goal programming.

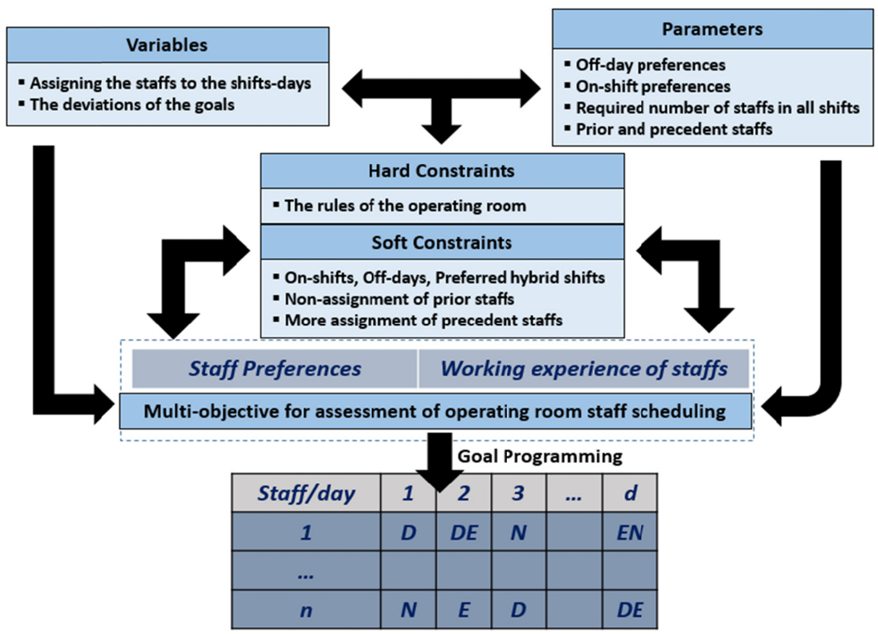

Figure 1 – Conceptual framework of operating room staff scheduling

To the best of our knowledge, considering the priority of the staff and working experience of the staff simultaneously for the nurse rostering problem has not been studied in any problem and no study in the literature takes into account the operating room staff/nurses as we study. On the other hand, an integer programming model subjected to the novel soft constraints related to the priority of the staff and the precedent staff using goal programming has not been extended in any research. This paper addresses the nurse rostering problem in Iran, and none of the current works could provide the appropriate mathematical model for this special case. This paper studies and analysis the issue practically, and extend a model for enhancing the satisfaction of the operating room nurses.

3. Operating room staff scheduling problem statement

Staff roster in an operating room is an assignment problem in which staff is assigned to days in a month based on some criteria. In our problem, the staff is divided into two groups; anesthetist technicians, and the personnel of the operating room. Staffs have some preferences such as being off for someday or being on for some shifts that are allowed to be violated with the penalty. On the other hand, there are some requirements of management such as the number of staff per shift that should not be violated. It should be noted that each preference that is violated must be penalized based on score or weight. Therefore, we consider some weights for each preference according to its priority. Moreover, there are three shifts (morning/evening/night) per day that must be filled by the staff and we called these shifts (D/E/N) respectively. We illustrate the problem with a small example. Assume staff (A, B, C) in an operating room should be assigned to 5 days periodically. We assume the requirements for managers (a, b) and the preferences for staff (c, d) as the following rules:

a. Three shifts (morning/evening/night) per day should be assigned completely.
b. The single staff that is assigned to the night shift should not be assigned to the next morning shift immediately.
c. The maximum number of days that staff prefers to be off is respected (if it is violated, is penalized with a score of 4).
d. The maximum number of shifts that staff prefers to be on is respected (if it is violated, is penalized with a score of 3).

Also, Table 2 shows the staff's preferences list:

Table 2 – Staff's preferences

Staff	Day/Preference
A	4/off
B	1, 5/on/night
C	3, 4/off

So, we represent two different rosters that consider some staff's preferences to respect all requirements. As it is indicated in Tables 3 and 4, some preferences are violated. For example, in roster #1 all preferences of staff #B and one preference of staff #C are violated, while in roster #2 only one preference of staff #C is violated. So, we obtain the score of (-10) for roster #1 and it is obtained (-4) for roster #2 according to weights and the results show the second roster respects more preferences than the first roster. It means that there are some solutions (roster) that improve the score of preferences and also we can obtain optimum roster using operation research approaches. The next section provides a mathematical model for the staff assignment problem. Then this model is applied to a real case (the hospital in Iran) and the results, and discussions are depicted in the following section.

Table 3 – Roster #1

Staff	1	2	3	4	5
A	N	---	N	---	N
B	D	DE	DE	N	---
C	E	N	---	DE	DE

Table 4 – Roster #2

Staff	1	2	3	4	5
A	D	DE	N	---	DE
B	N	---	DE	DE	N
C	E	N	---	N	---

4. Operating room staff scheduling model – goal programming

In this section, we formulate an integer programming model for goal programming. The assumptions of the problem are presented below.

- There are three working shifts for assigning the staff to days; the D starts at 8:00 AM, the E starts at 14:00, and the N starts at 20:00
- The number of days in the planning horizon is determined before scheduling.
- The number of staff on the planning horizon is determined before scheduling.
- It is assumed that each staff prefers to be assigned to one or some shifts in a day.

The problem has n staffs, d days in a period of planning, and three shifts per day. The Sets/indices, parameters, and decision variables used in the mathematical model are described by notations in the next sections.

4.1 Problem size parameters

In this subsection, the sets, subsets, and indices are introduced in Table 5:

Table 5 – Indices and sets for the IP model

Sets	Description
$\mathcal{I}$: $\{1,2,\dots i \dots, n\}$	Set of all the staff (operating room and anesthetist technician)
$\mathcal{J}$: $\{1,2,\dots j \dots, d\}$	Set of all days on the planning horizon

$\boldsymbol{m(i)}$: $\{1,\dots,m\}$	The subset of all staff (men)
$\boldsymbol{w(i)}$: $\{1,\dots,w\}$	The subset of all staff (women)
$\boldsymbol{pp(i)}$: $\{1,\dots,pp\}$	The subset of all precedent personnel
$\boldsymbol{pw(i)}$: $\{1,\dots,pw\}$	The subset of all prior female
$\boldsymbol{nh(j)}$: $\{1,\dots,nh\}$	The subset of all non-holidays
$\boldsymbol{h(j)}$: $\{1,\dots,h\}$	The subset of all holidays
$\boldsymbol{off(j)}$: $\{1,\dots,off\}$	The subset of all off-days for each staff$\in \{I_{off}\}$
$\boldsymbol{onm(j)}$: $\{1,\dots,onm\}$	The subset of preferable on-morning shifts for each staff$\in \{I_{onm}\}$
$\boldsymbol{one(j)}$: $\{1,\dots,one\}$	The subset of preferable on-evening shifts for each staff$\in \{I_{one}\}$
$\boldsymbol{onn(j)}$: $\{1,\dots,onn\}$	The subset of preferable on-night shifts for each staff$\in \{I_{onn}\}$
$\boldsymbol{onme(j)}$: $\{1,\dots,onme\}$	The subset of preferable on-morning-evening shift for each staff$\in \{I_{onme}\}$
$\boldsymbol{onmn(j)}$: $\{1,\dots,onmn\}$	The subset of preferable on-morning-night shift for each staff$\in \{I_{onmn}\}$
$\boldsymbol{onen(j)}$: $\{1,\dots,onen\}$	The subset of preferable on-evening-night shift for each staff$\in \{I_{onen}\}$
$\boldsymbol{onmen(j)}$: $\{1,\dots,onmen\}$	The subset of preferable on-morning-evening-night shift for each staff$\in \{I_{onmen}\}$
$\boldsymbol{Std(j)}$: $\{1,\dots,std\}$	The subset of all days that should not be assigned to students $\in \{I_{std}\}$
$\boldsymbol{foff(j)}$: $\{1,\dots,foff\}$	The subset of days that should not be assigned to staff$\in \{I_{foff}\}$, because they are forced to be off

4.2 Parameters

Also, there are four parameters consisting of three total parameters for all staff and one parameter for each staff (vacation days) in the problem that are defined in Table 6.

Table 6 – Parameters for IP model

Parameters	Description
$offpr$:	Total off-day for staff's preference on all days
$onpr$:	Total on-shift for staff's preference on all shifts

dd:	Duty days per month (the constant threshold is varied for each month)
mr_i:	Number of staff's vacation (part of duty days)
n_{nhm}:	Required number of staff on non-holidays per morning shift
n_{hm}:	Required number of staff on holidays per morning shift
n_{nhe}:	Required number of staff on non-holidays per evening shift
n_{he}:	Required number of staff on holidays per evening shift
n_{nhn}:	Required number of staff on non-holidays per night shift
n_{hn}:	Required number of staff on holidays per night shift

4.3 Decision variables

The purpose of this problem is to determine which staff must be allocated to which shift per day of the month according to requirements and his/her preference. Since the model is multi-objective, goal programming is employed to maximize the nurse preferences. Therefore, the deviations from the goals are defined to model integer programming. The deviations are the variables that are used in soft constraints and are classified into underachievement (d^-) and overachievement (d^+) deviations. The first is undesirable for the constraints greater than the right-hand side, and the second is undesirable for the constraints less than the right-hand side. These variables are defined in Table 7.

Table 7 – Decision and auxiliary variables for IP model

Variables	Description
x_{ij}:	$\begin{cases}1, & \text{if staff } i \text{ is assigned to day } j \text{ in morning shift}\\ 0, & \text{otherwise}\end{cases}$
y_{ij}:	$\begin{cases}1, & \text{if staff } i \text{ is assigned to day } j \text{ in evening shift}\\ 0, & \text{otherwise}\end{cases}$
z_{ij}:	$\begin{cases}1, & \text{if staff } i \text{ is assigned to day } j \text{ in night shift}\\ 0, & \text{otherwise}\end{cases}$
d_i:	The number of on-days per month for each staff without calculation of vacations
g:	A (d^+) to minimize the assigned preferred off-days to the staff (it is the preferable value if equals 0)
p:	A (d^-) to minimize the non-assigned preferred on-days to the staff (it is the preferable value if equals 0)
ky_i:	A (d^+) to minimize the assigned evening shifts to the prior staff like pregnant women (it is the preferable value if equals 0)

kz_i:	A (d^+) to minimize the assigned night shifts to the prior staff pregnant women (it is the preferable value if equals 0)
kp_i:	Maximizing assigning all shifts to the precedent staff based on his/her working experience (it is preferable value if greater than 0)
de_{ij}:	A (d^-) to minimize the non-assigned morning-evening shift/day to the staff (is preferable value if equals 0)
dn_{ij}:	A (d^-) to minimize the non-assigned morning-night shift/day to the staff (is preferable value if equals 0)
en_{ij}:	A (d^-) to minimize the non-assigned evening-night shift/day to the staff (is optimal if equals 0)
den_{ij}:	A (d^-) to minimize the non-assigned morning-evening-night shift/day to the staff (is preferable value if equals 0)

4.4 Constraints

There are two sets of constraints corresponding to this problem that is called hard and soft constraints. As it is said in the problem statement, hard constraints correspond to manager requirements and cannot be violated. Soft constraints are related to staff's preferences and can be violated with a penalty. Therefore, soft constraints construct the objective function and are called objective constraints.

Hard Constraints: Equations (1-15) below are hard constraints.

$$\sum_{i=1}^{n} x_{ij} = n_{nhm} \qquad \forall\, j \in \boldsymbol{nh} \qquad (1)$$

$$\sum_{i=1}^{n} x_{ij} = n_{hm} \qquad \forall\, j \in \boldsymbol{h} \qquad (2)$$

$$\sum_{i=1}^{n} y_{ij} = n_{nhe} \qquad \forall\, j \in \boldsymbol{nh} \qquad (3)$$

$$\sum_{i=1}^{n} y_{ij} = n_{he} \qquad \forall\, j \in \boldsymbol{h} \qquad (4)$$

$$\sum_{i=1}^{n} z_{ij} = n_{nhn} \qquad \forall\, j \in \boldsymbol{nh} \qquad (5)$$

$$\sum_{i=1}^{n} z_{ij} = n_{hn} \qquad \forall\, j \in \boldsymbol{h} \qquad (6)$$

$$\sum_{i\in w} x_{ij} \geq 1 \qquad \forall\, j \in \boldsymbol{\mathcal{J}} \qquad (7)$$

$$\sum_{i\in w} y_{ij} \geq 1 \qquad \forall\, j \in \boldsymbol{\mathcal{J}} \qquad (8)$$

$$\sum_{i\in w} z_{ij} \geq 1 \qquad \forall\, j \in \boldsymbol{\mathcal{J}} \qquad (9)$$

$$x_{ij} + y_{ij} + z_{ij} = 0 \qquad \forall\, i \in I_{std}\,, j \in \boldsymbol{Std} \qquad (10)$$

$$\sum_{j\in foff} x_{ij} + \sum_{j\in foff} y_{ij} + \sum_{j\in foff} z_{ij} = 0 \qquad \forall\, i \in \boldsymbol{\mathcal{I}}_{foff} \qquad (11)$$

$$\sum_{j\in J} x_{ij} + \sum_{j\in J} y_{ij} + 2\sum_{j\in J} z_{ij} = d_i \qquad \forall\, i \in \boldsymbol{\mathcal{I}} \qquad (12)$$

$d_i + mr_i \geq dd \qquad \forall\, i \in \mathcal{I} \qquad (13)$

$x_{ij} + z_{i(j-1)} \leq 1 \qquad \forall\, i \in \mathcal{I}, j \in \{2, \ldots, 31\} \qquad (14)$

$x_{ij} + y_{ij} + z_{ij} \leq 1 \qquad \forall\, i \in \boldsymbol{pw}, j \in \mathcal{J} \qquad (15)$

Constraints (1-6) guarantee the required number of staff per shift. In our case, the parameters of $n_{nhn}, n_{hm}, n_{he}, n_{hn}$are equal to 2, n_{nhe} is equal to 3, and n_{nhm} is equal to 6. Constraints (7-9) demand that at least one female must be allocated to working shifts. Constraint (10) makes sure that students are not assigned to shifts per determined day. Constraint (11) demands that some staff are forced to be off for some determined days. Constraints (12-13) specify that staff's duty days must be more than the constant threshold. Constraint (14) enforces that an exceptional staff that is assigned to a late shift should not be assigned to the next early shift immediately. Constraint (15) ensures that only one shift per day must be assigned to the prior female (pregnant/married).

Soft (Objectives) Constraints: Constraints (16-33) are soft constraints. These are to be satisfied based on staff's preferences but these may be violated and some penalties will be imposed to maximize total staff's preferences. It is noted that each preference is divided into two subsequent terms: the main constraint is formulated in the first term, and the penalty variables or the deviations of the goals are defined in the second term and referred to as objective functions. The main constraints (16,20,22) are less than the right-hand side, and the others are greater than the right-hand side.

$\sum_{i\in I_{off}} \sum_{j\in off} x_{ij} + \sum_{i\in I_{off}} \sum_{j\in off} y_{ij} + \sum_{i\in I_{off}} \sum_{j\in off} z_{ij} - g = 0 \qquad (16)$

$\boldsymbol{\mathcal{OFP}} = (offpr - g) \qquad (17)$

$\sum_{i\in I_{onm}} \sum_{j\in onm} x_{ij} + \sum_{i\in I_{one}} \sum_{j\in one} y_{ij} + \sum_{i\in I_{onn}} \sum_{j\in onn} z_{ij} + p = onpr (18)$

$\boldsymbol{\mathcal{ONP}} = (onpr - p) \qquad (19)$

$\sum_{j\in J} y_{ij} - ky_i = 0 \qquad \forall\, i \in \boldsymbol{pw} \qquad (20)$

$\boldsymbol{\mathcal{EWP}} = -\sum_{i\in pw} ky_i \qquad (21)$

$\sum_{j\in J} z_{ij} - kz_i = 0 \qquad \forall\, i \in \boldsymbol{pw} \qquad (22)$

$\boldsymbol{\mathcal{NWP}} = -\sum_{i\in pw} kz_i \qquad (23)$

$\sum_{j\in J} x_{ij} + \sum_{j\in J} y_{ij} + 2\sum_{j\in J} z_{ij} - kp_i = 0 \qquad \forall\, i \in \boldsymbol{pp} \qquad (24)$

$\boldsymbol{\mathcal{VIP}} = \sum_{i\in pp} kp_i \qquad (25)$

$x_{ij} + y_{ij} + de_{ij} = 2 \qquad \forall\, i \in I_{onme}, j \in \boldsymbol{onme} \qquad (26)$

$$\boldsymbol{\mathcal{DE}} = -\sum_{i\in I_{onme}}\sum_{j\in onme} de_{ij} \tag{27}$$

$$x_{ij} + z_{ij} + dn_{ij} = 2 \qquad \forall\ i \in I_{onmn}, j \in \boldsymbol{onmn} \tag{28}$$

$$\boldsymbol{\mathcal{DN}} = -\sum_{i\in I_{onmn}}\sum_{j\in onmn} dn_{ij} \tag{29}$$

$$y_{ij} + z_{ij} + en_{ij} = 2 \qquad \forall\ i \in I_{onen}, j \in \boldsymbol{onen} \tag{30}$$

$$\boldsymbol{\mathcal{EN}} = -\sum_{i\in I_{onen}}\sum_{j\in onen} en_{ij} \tag{31}$$

$$x_{ij} + y_{ij} + z_{ij} + den_{ij} = 3 \qquad \forall\ i \in I_{onmen}, j \in \boldsymbol{onmen} \tag{32}$$

$$\boldsymbol{\mathcal{DEN}} = -\sum_{i\in I_{onmen}}\sum_{j\in onmen} den_{ij} \tag{33}$$

$$x_{ij}, y_{ij}, z_{ij} \in \{0,1\} \tag{34}$$

$$d_i, ky_i, kz_i, kp_i, p, g \in \mathbb{Z} \tag{35}$$

As it is shown, all deviations in the second terms exception of (25) are expressed in negative form, and thereby these should be maximized. Equation (25) reflexes assigning more shifts to the precedent staff with high working experience that is presented in the conceptual framework. Constraints (16-17) respect off-day preferences for staff. In these equations, the first constraint is violated with variable g and the penalty variable $\boldsymbol{\mathcal{OFP}}$ is defined to maximize off-day preferences for staff. Constraint (18-19) respects on-shift preferences for staff that first equation is violated with variable p and the penalty variable $\boldsymbol{\mathcal{ONP}}$ is defined to maximize on-shifts preferences to staff. Constraints (20-23) are related to the prior staff that is presented in the conceptual framework and these make sure that prior female staff (pregnant/married) is not assigned to night and evening shifts preferably. So, equations are violated with variables ky_i and kz_i and then penalty variables $\boldsymbol{\mathcal{EWP}}$ and $\boldsymbol{\mathcal{NWP}}$ are defined to maximize non-allocating evening and night shifts to determined women sets. Constraint (24-25) specifies that more shifts are assigned to prior/precedent personnel preferably. Therefore, variable $kp_i$is to violate the equation, and then penalty $\boldsymbol{\mathcal{VIP}}$ is to maximize allocating preferred shifts (D/E/N) to precedent personnel. Constraints (26-33) guarantee more hybrid shifts such as morning-evening and morning-evening-night based on staff's preferences. Thus, variables $de_{ij}, dn_{ij}, en_{ij}, den_{ij}$ are defined to violate the constraints, and, penalty variables $\boldsymbol{\mathcal{DE}}$, $\boldsymbol{\mathcal{DN}}$, $\boldsymbol{\mathcal{EN}}$, and $\boldsymbol{\mathcal{DEN}}$ are defined to maximize morning-evening, morning-night, evening-night, and morning-evening-night shifts, respectively. Constraints (34-35) are binary and integer variables.

4.5 Objective function

In this model, the objective function contains nine terms as penalty variables or deviations as follows:

(1) $\boldsymbol{\mathcal{VIP}}$ number of assigned shifts to precedent staffs $i \in \boldsymbol{pp}$

(2) $\boldsymbol{\mathcal{OFP}}$ number of not-assigned off-days to staffs $i \in \boldsymbol{I_{off}}$

(3) $\boldsymbol{\mathcal{ONP}}$ number of assigned on-shifts to staffs $i \in \boldsymbol{I_{onm}} \cup \boldsymbol{I_{one}} \cup \boldsymbol{I_{onn}}$

(4) $\boldsymbol{\mathcal{EWP}}$ number of non-assigned evening shifts to female staffs$i \in \boldsymbol{pw}$

(5) $\boldsymbol{\mathcal{NWP}}$ number of non-assigned night shifts to female staffs $i \in \boldsymbol{pw}$

(6) $\boldsymbol{\mathcal{DE}}$ number of allocated morning-evening shifts to staffs $i \in \boldsymbol{I_{onme}}$

(7) $\boldsymbol{\mathcal{DN}}$ number of allocated morning-night shifts to staffs $i \in \boldsymbol{I_{onmn}}$

(8) $\boldsymbol{\mathcal{EN}}$ number of allocated evening-night shifts to staffs $i \in \boldsymbol{I_{onen}}$

(9) $\boldsymbol{\mathcal{DEN}}$ number of allocated all shifts to staffs $i \in \boldsymbol{I_{onmen}}$

These deviations are the multi objective of goal programming and therefore, the total objective function based on the deviations of the model is constructed according to the considered weights for each objective term as the following equation:

$$Max\ z = w_{vip}\boldsymbol{\mathcal{VIP}} + w_{ofp}\boldsymbol{\mathcal{OFP}} + w_{onp}\boldsymbol{\mathcal{ONP}} + w_{ewp}\boldsymbol{\mathcal{EWP}} + w_{nwp}\boldsymbol{\mathcal{NWP}} + w_{de}\boldsymbol{\mathcal{DE}} + w_{dn}\boldsymbol{\mathcal{DN}} + w_{en}\boldsymbol{\mathcal{EN}} + w_{den}\boldsymbol{\mathcal{DEN}} \quad (36)$$

so that the single objective model is made by using the weighting method. The objective function is maximized since the negative deviations must be maximized.

5. Computational Experiments

5.1 Illustrative examples

To evaluate the proposed approach, we collected real data (staff's preferences) from a Hospital located in Iran, in the course of previous annual from February 2021 to 2022. Moreover, we conducted eight scenarios applying the different simulated weight values as displayed in Table 8. The randi() function was used to generate random integer weights in scenarios. We provided some parameters similar to off-day for staff's preference, on-shift for staff's preference, duty days per month, and the number of staff's vacations, monthly from data to solve ORSS.

Table 8 – Simulated weights for eight scenarios

Weights / Scenarios	w_{vip}	w_{ofp}	w_{onp}	w_{ewp}	w_{nwp}	w_{de}	w_{dn}	w_{en}	w_{den}
S1	6	4	3	2	6	1	1	1	2
S2	1	1	1	1	1	1	1	1	1
S3	1	4	3	1	1	1	1	1	4
S4	1	5	4	1	1	1	1	1	1
S5	1	2	6	1	1	1	1	1	1
S6	1	6	1	1	1	1	1	1	1
S7	1	1	1	3	1	1	1	1	1
S8	1	1	1	1	1	1	1	1	5

5.2 Evaluation of proposed method's performance

After executing the GP on problems, results are collected in the table (8×12) where rows show scenarios and columns show a problem or monthly plan. The performance indicator in this study is the objective function obtained from solving IP model using GP in GAMS software. The computational results corresponding to the objective function for all problems which were solved by our proposed method (GP) and hospital's plan (H) are displayed in Table 9. We designed two-factor factorial experiments for examining the effect of the two solution methods and 12 test instances on total objective function (equation 36), with 8 times runs (different scenarios) to analyze these results.

Table 9 – Computational results for comparison of the results of methods

Method	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
GP-S1	644	662	738	576	607	531	519	424	364	497	519	464
GP-S2	127	130	150	107	107	94	85	54	40	93	97	74
GP-S3	384	401	425	396	384	289	229	220	144	248	264	168
GP-S4	516	540	543	549	519	400	307	308	212	323	371	224
GP-S5	546	552	567	574	577	391	319	296	202	312	489	273
GP-S6	347	375	379	365	302	297	205	214	154	238	157	119
GP-S7	118	121	140	95	93	86	80	51	37	86	88	70
GP-S8	114	117	150	86	102	68	72	37	18	86	83	69
H-S1	440	491	434	415	478	455	396	414	418	323	526	427
H-S2	55	41	69	33	58	4	11	30	29	21	66	10
H-S3	108	107	162	65	114	52	-6	158	114	13	152	50
H-S4	177	173	251	115	180	147	33	212	221	83	268	119
H-S5	179	125	293	147	182	180	25	169	213	45	210	135
H-S6	125	161	139	53	128	59	31	175	149	91	236	60
H-S7	35	19	39	1	36	-58	-29	-32	-21	-31	44	-54
H-S8	11	1	25	1	18	-64	-33	26	-39	-51	-14	-38

The ANOVA test results for objective function as a response variable are presented in Table 10. According to the *P*-value for the method's main effect, it is seen that the effect of this factor is significant. It means that there is a significant difference between the mean values obtained for the two approaches. However, the problem's main effect and interaction effect of problems and methods are not significant.

Table 10 – ANOVA result for comparison of the objective function

Source of variation	DF	SS	MS	F	*P*-value
Method	1	1105651	1105651	40.38	0.000
Problem	11	465217	42292	1.54	0.120
Interaction	11	242030	22003	0.80	0.636
Error	168	4600001	27381		
Total	191	6412899			

The total objectives of H-S7 and H-S8 for test problems P6 to P12 are almost negative and these results indicate that the hospital's plan is not good evaluated by single objectives such as $\boldsymbol{\mathcal{EWP}}$, $\boldsymbol{\mathcal{NWP}}$, $\boldsymbol{\mathcal{DE}}$, $\boldsymbol{\mathcal{DN}}$, $\boldsymbol{\mathcal{EN}}$, and $\boldsymbol{\mathcal{DEN}}$ because the best value for these objectives is zero. Therefore, these values in comparison to the results of GP-S7 and GP-S8 indicate that the hospital's plan is unable to decrease the allocation of the prior staff such as pregnant or married female staff to the evening and night shifts. On the other hand, it is inferred from these results that the hospital's plan is unable to fulfill the hybrid shifts based on the staff's preferences. To obtain a good timetable, it is necessary to maximize the negative deviations of the mentioned goals to zero so that the proposed GP provides this optimized tool. The average of objective functions corresponding to the test problems for GP and hospital's plan is shown in Fig. 2, and this graph shows that the proposed approach obtains better solutions (solutions with better objective function) than the hospital's plan for each problem and therefore, our method outperforms timetable of the hospital.

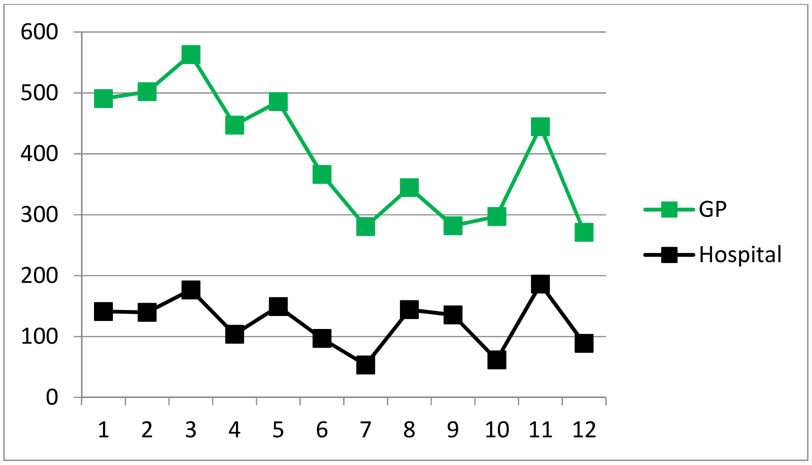

Figure 2 – Interaction between the methods and problems for the objective

6. Conclusion

In this paper, the monthly staff scheduling problem in the multi-operating theaters is addressed to maximize staff's preferences. So, a new mixed integer linear programming model is extended and goal programming is proposed to tackle this problem. A new goal programming was developed and run in GAMS software to analyze the extended MILP model. Therefore, to illustrate our methodology, we provide real data (staff's preferences) from a hospital located in Iran, and then the results were compared with a real plan in the hospital. To evaluate and compare the performance of methods, the objective function of the model on several instances was applied. As a consequence, the computational experiments state that goal programming outperforms hospital planning, and so the results indicate the efficiency and capability of our methodology for optimizing staff's preferences. At last, we suggest some directions as opportunities for future research in this area. To construct robust staff scheduling, it would be essential to consider uncertain conditions such as emergency conditions for staff such as changes in the staff's preferences or inserting new staff into the schedule during the month. Also, developing Pareto-based approaches is proposed to solve multi-objective ORSS problem. On the other hand, using and validating new meta-heuristic approaches can be taken into account to solve the large-scale problem (more staff /more days).

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