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SUSTAINABLE ECONOMIC PRODUCTION QUANTITY (SEPQ) MODEL FOR INVENTORY HAVING GREEN TECHNOLOGY INVESTMENTS - PRICE SENSITIVE DEMAND WITH EXPIRATION DATES

Abstract. *Carbon emission in environment by the human activities is current key issue globally; the investment in green technology and adoption of government carbon reduction policy will be help to minimize the carbon emission and global warming. In this article, we developed a manufacturer's sustainable economic production quantity inventory model (SEPQ) with green technology investment and selling price-sensitive demand. Furthermore, products have a time varying deterioration rate, which also depends on the expiration date of the products. Set up the production system, manufacturing process, holding inventory, deterioration of products, environmental impact are sources of carbon emissions are considered in proposed model. Carbon tax and cap policy, green technology investment is implemented for achieve sustainability. The main objective of the study is to find optimal replenishment time, optimal green investment and optimal selling price by considering manufacture's profit maximization using classical optimization. A numerical example has studied to validate the sustainable economic production quantity model. Sensitivity analysis has provided with the managerial implication of the optimal feasible solution with respect to some key parameters. Finally, some concluding remarks along with future scopes are discussed.*

Keywords: *Inventory, Sustainable economic production quantity, expiration dates, Carbon tax and cap, green technology investment.*

JEL Classification: C05

1. Introduction

Now a day, global warming is a key issue for the environment. Many countries work on minimizing the effect of global warming using different steps. The main reason for global warming is the carbon emission from human activities

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like the burning of oil, coal, and gas as well as deforestation and the unsuitable establishment of industries. Control the global warming is a big challenge for developing countries, for the control of carbon emissions and reduce the effect of climate change many authorities adopted the carbon trading policy. As per the report of World bank in 2020, there are more than sixty one emissions trading schemes in use around the worldwide, includes carbon taxes and cap-and-trade. Green technology or sustainable technology is a part of science-technology which reduces the human impact on the environment. The products produced by green technology are non-polluting, and often include energy savings, preservation, safety, and health concerns. The CFL bulb, solar cell, wind turbines, wastewater electricity generator, and electric automobiles are examples of green technology. The involvement of deterioration affects revenue and reduces the firm's total profit. Deterioration rate is not always constant; It has been observed that perishable products decay continuously, with the product completely deteriorating as the expiration date gets closer. Demand for products is affected by the selling price; thus, the more practical scenarios should include selling price as a decision variable. The investment in green technology, result to produced green products and customer inspired to buy green products. Our research tries to build a sustainable production inventory model for perishable products by investing in green technologies and reducing carbon emissions under a carbon tax and carbon cap by considering demand as a function of green investment and selling price to address such kinds of problems.

2. Literature review

Economic Production Quantity model is adopted by the manufacturing industries for fulfil the different criteria's and maximize their total profit. EPQ model with partial backordering developed by Pentico *et al.* (2009). Hua *et al.* (2011) first took carbon emission into consideration in inventory models under carbon cap-and-trade regulation. Glock *et al.* (2012) developed a new SEPQ model with demand depends on price and quality of product. Bouchery *et al.* (2012) identified a sustainable model with carbon tax policy in which carbon emissions are minimized to a single object not preferable. Sarkar (2012) introduced an EOQ model by incorporating the fact that the deterioration rate of a perishable product increases over time and reaches 100% at its sell-by date. Benjaafar *et al.* (2013) investigated that how emission reductions influenced the cost of emissions. A model on manufacture's joint carbon emission reduction and pricing policy under a carbon emission trade developed by Hu and Zhou (2014). Chen and Teng (2014) espoused the deterioration rate linked to expiration date to develop an EOQ model with upstream credit payment system. Toptal *et al.*(2014) studied an inventory model with different carbon reduction policy and green investment concept. Qin *et al.* (2015) developed a trade-credit inventory model under demand depending on credit-period for a carbon tax, and carbon cap and trade policy. They found from their model that a carbon cap has a negative effect of the credit period under the case of carbon cap and trade policy, and a carbon tax and carbon trade price have negative effects on the credit period. Shi et.al (2015) developed a inventory model with carbon tax and different payment system under expiration dates. Lou *et al.* (2015) devolved two sustainable inventory models to find the maximum profits with green technology investment for reducing emissions under the carbon tax policies without shortages. Tiwari, Daryanto, *et al.* (2018) explored a sustainable model for perishable and imperfect items. Taleizadeh et al., (2018) introduced SEPQ model with partially backlogged shortages and analyzed impact of lost sales. Datta (2017) and Lin (2018) examined the the effect of green technology investment including the carbon tax system, while Lou *et al.* (2015) worked under the cap-and- trade system. Bhattacharyya and Sana (2019) identified the effect of green technology investment of green manufacturing process. Zand *et al.* (2019) discussed a supply chain model with greening level and price sensitive demand. Lu *et al.* (2020) discussed the influence of carbon cap-and-trade and carbon offset policies on a perishable inventory model where carbon reduction technology is used to minimize emissions from the supply chain. Yang and Lin (2020) created a sustainable supply chain model in which they identified the role of green innovation performance. Mishra *et al.* (2020) developed SEPQ model with constant demand including shortages and without shortages case. They used a green investment for reducing carbon emission with carbon cap and tax mechanism. Shah *et al.*(2020) developed a manufacturer-retailer supply chain model with retailer's payment time and customer's fixed credit period point of view. Yadav and Khanna (2021) examined the effect of expiration date with carbon tax and price reliant demand. Hasan *et al.* (2021) introduced inventory model with green investment and promotion dependent demand. Paul *et al.* (2021) developed their model with price and greening level based demand pattern with carbon tax regulation. Recently, Shah *et al.* (2022) examined the EPQ model with price-stock sensitive demand under carbon emission with carbon tax-cap mechanism including preservation and green investment.

Research gap and our contribution

The existing literature above highlights numerous methodologies published in the field of sustainable inventory management. Yet, there are significant concerns and obstacles in this novel field of green systems that need to be solved. As a result, the current research intends to create a sustainable economical production quantity model for products that incorporates several practical features such as time-varying deterioration rate, expiration date, as well as carbon cap and tax policy. The demand is considered to be green investment and price-sensitive. Furthermore, it is acknowledged that carbon emissions are produced as a result of the setup of production, inventory holding, manufacturing process, environmental impact, and deteriorating items when managing inventory systems. Consequently, carbon emission tax and cap policy and green technology has been established to minimise carbon footprints in order to achieve environmental sustainability. A mathematical model is developed to find the optimal green technology investment, selling price and cycle time that optimise the manufacturer's total profit. Numerical and sensitivity analyses have revealed several important managerial insights.

Table 1. Contribution of different author(s) related to our study

Author(s)	Model type	Demand Pattern	Deterioration	Expiration date	Carbon reduction policy	Green investment
Datta(2017)	EPQ	Selling price	–	–	Carbon tax	Yes
Bhattacharya and sana (2019)	SEPQ	Random variable and service level	–	–	–	Yes
Zand et al. (2019)	Supply chain	Selling price and green level	–	–	–	Yes
Mishra et al.(2020)	SEPQ	Constant	–	–	Carbon tax and cap	Yes
Yadav and Khanna(2021)	EOQ	Selling price	Time dependent	Yes	Carbon tax	–
Hasan et al.(2021)	EOQ	Green investment and promotion	–	–	Carbon tax, cap-trade, limit	Yes
Paul et al.(2021)	SEPQ	Greening degree and selling price	Constant	–	Carbon tax	Yes
Shah et al.(2022)	SEPQ	Stock and selling Price	Constant	–	Tax and cap	Yes
This model	SEPQ	Green investment and selling price	Time dependent	Yes	Tax and cap	Yes

A concise review of literature described in section 2, Section 3 give the notations and assumptions used in the model development; Section 4 presents the mathematical formulation; Section 5 discusses optimality of profit function. Numerical example, solution methodology and graphically validation presents Sections 6 and sensitivity analysis, observations and managerial insights has been discussed in 7 and lastly, Section 8 summarizes the paper with concluding remarks and future direction.

3. Assumptions and notations

3.1 Assumptions:

1. The manufacturer produced a single type of product.
2. Lead time to be considered negligible. Replenishment rate is instantaneous.
3. Rate of production is constant and more than a demand rate, shortages are avoided.
4. To reduce the effect of carbon emission, investment in green technology to be considered. The fraction of reduction of average emission is

$F(G)=\xi(1-e^{-\eta G})$; where, $0<\xi<1$ is the fraction of carbon emission after investing in green technology, $\eta>0$ efficiency of greener technology in reducing emission and $G>0$ is the green investment cost.

5. Demand function is defined as $D(G,p)=\alpha+\beta F(G)-\gamma p$, where $\alpha>0$ is the scale demand, $\beta>0$ is the constant coefficient of $F(G)$ and $0<\gamma<1$ is the price elasticity.
6. Product deteriorate continuously with time, product cannot be sold after expiration date *m*, and we assumed that the rate deterioration $\theta(t)=\dfrac{1}{1+m-t}, 0\le t\le T\le m$, if $t\to 0$ then deterioration rate is minimum, and $t\to m$ then all products deteriorate as its expiration date.
7. Replacement, repair, salvage value of deteriorate products is avoided.
8. The carbon footprint of the setup production system, manufacturing process, inventory holding operation, inventory deterioration, and environmental impact are all taken into account.
9. Carban tax and cap strategies applied for managing carbon emission.

3.2 Notations

Table 2. Notations used in model

A	Set up cost per cycle per order
P	Constant production rate (unit/year)
C_1	Production cost per cycle
C_2	Carbon emission for inventory holding per cycle (kg/year)
B	Carbon emissions unit associated in setup cost (kg/year)
l	Carbon emission from manufacturing process (kg/year)
r	Environmental impact carbon emissions for inventory (kg/year)
δ	Carbon tax per cycle
c_{cap}	Carbon emissions cap
Q	Maximum inventory level when production stops at $t=T_1$.
G	Green technology investment cost/unit/Cycle (a decision variable).
$F(G)$	The fraction of carbon reduction.
p	Selling Price (a decision variable)
T	Length of inventory cycle (years) (decision variable).
T_1	Point of time at which production stops (year)
$D(G,p)$	Demand function.
$I_1(t)$	Inventory level during $0\le t\le T_1$ (units).

$I_2(t)$	Inventory level during $T_1 \le t \le T$ (units).
m	The time to expiration date or the maximum shelf life in units of time, $m > 0$
ξ	fraction of carbon emissions after green technology investment $0 < \xi < 1$
η	Efficiency of greener technology in reducing emission $\eta > 0$.
κ	Emission per deteriorated item(kg/unit)

4. Mathematical formulation

In this section, a sustainable EPQ model is developed in which deterioration of item is time dependent. The manufacturing process start at time $t=0$ and goes up to time $t=T_1$, where the inventory level goes to its highest level. At time T_1, production stops, and inventory level decline due to the demand and deterioration. Inventory level after $t=T_1$ goes down to zero at $t=T$. It is observed that the rate of inventory level increases due to the production rate and decreases due to demand and deterioration rate. The following differential equation formulates the changing of inventory level.

$$\frac{dI_1(t)}{dt}+\left(\frac{1}{1+m-t}\right)I_1(t)=-(D(G,p)-P),0\le t\le T_1 \tag{1}$$

with initial condition $I_1(0)=0$.

Now, duration the period $[T_1,T]$, noticed that the inventory level consumed due to demand of item and deterioration effect on produced item, so the governing differential equation in this non production period is given by;

$$\frac{dI_2(t)}{dt}+\left(\frac{1}{1+m-t}\right)I_2(t)=-D(G,p),T_1\le t\le T \tag{2}$$

With the end inventory level $I_2(T)=0$.

The solution of the differential equations (1) and (2) with given conditions is respectively,

$$I_1(t)=(D(G,p)-P)(1+m-t)\ln\left(\frac{1+m-t}{1+m}\right),0\le t\le T_1, \tag{3}$$

$$I_2(t)=(D(G,p))(1+m-t)\ln\left(\frac{1+m-t}{1+m-T}\right),T_1\le t\le T \tag{4}$$

The function $I(t)$ is a continues function, the relation between T_1 and T is

$$T_1=(1+m)\left(1-\left(\frac{1+m-T}{1+m}\right)^{\frac{D}{P}}\right) \tag{5}$$

With the initial condition, $I_1(T_1)=Q$, the maximum produces items are

$$Q=\left(1+\eta-(1+\eta)\left(1-\left(\frac{1+\eta-T}{1+\eta}\right)^{\frac{D}{P}}\right)\right)(D-P)\ln\left(\frac{1+\eta-(1+\eta)\left(1-\left(\frac{1+\eta-T}{1+\eta}\right)^{\frac{D}{P}}\right)}{1+\eta}\right) \tag{6}$$

Our objective is to maximize the manufacturer's total profit. The total annual profit consists of the following components.

Net sales revenue from the selling the product is

$$SR=\frac{p}{T}\left(\int_0^T D(G,p)dt\right)=pD(G,p) \tag{7}$$

The production cost is over the cycle is

$$CP=\frac{C_1}{T}\left(\int_0^T D(G,p)dt\right)=C_1D(G,p) \tag{8}$$

The annual fixed setup cost has calculated as $SC=\frac{A}{T}$ (9)

The holding cost per cycle is given by

$$HC=\frac{C_h}{T}\left[\int_0^{T_1} I_1(t)dt+\int_{T_1}^{T} I_2(t)dt\right] \tag{10}$$

Green technology investment cost per year is $GT=\frac{GT}{T}=G$ (11)

The emission associated in setup production is $e_{sp}=\frac{B}{T}$ (12)

The emission from holding inventory is

$$e_h=\frac{C_2}{T}\left[\int_0^{T_1} I_1(t)dt+\int_{T_1}^{T} I_2(t)dt\right] \tag{13}$$

The emission from manufacturing is

$$e_m=\frac{l}{T_1}\left(\int_0^{T_1} D(G,p)dt\right)=l\,D(G,p) \tag{14}$$

The emissions due to environmental impact

$$e_{ei}=\frac{r}{T}\left(\int_0^T D(G,p)dt\right)=r\,D(G,p) \tag{15}$$

Number of deteriorated items is

$$DI=\frac{1}{T}\left(Q-\left(\int_0^{T_1} D(G,p)dt+\int_{T_1}^{T} D(G,p)dt\right)\right)$$

Emission due to deteriorating of product during the cycle is

$$e_{dp}=\frac{\kappa}{T}\left(Q-\left(\int_{0}^{T_1} D(G,p)dt+\int_{T_1}^{T} D(G,p)dt\right)\right) \tag{16}$$

Total carbon emission from Eq. (12) to Eq. (16) is

$$e_c=e_{sp}+e_h+e_m+e_{ei}+e_{dp} \tag{17}$$

The fraction of carbon reduction function $F(G)=\xi(1-e^{-\eta G})$ is taken as per Lou at al.(2015), the total carbon emission after applying green technology investment is,

$$\hat{e}_c=e_c(1-\xi(1-e^{-\eta G})) \tag{18}$$

Hence, the annual profit of the manufacturer is under a carbon cap and tax functions as below:

$$TP=(SR-CP-SC-HC-GT)+\delta[c_{cap}-e_c(1-\xi(1-e^{-\eta G}))] \tag{19}$$

$$=\left[pD(G,p)-C_1D(G,p)-\frac{A}{T}-\frac{C_h}{T}\left[\int_{0}^{T_1} I_1(t)dt+\int_{T_1}^{T} I_2(t)dt\right]-G\right]$$

$$+\delta[c_{cap}-\left(\begin{array}{l}\frac{B}{T}+\frac{C_2b}{T}\left[\int_{0}^{T_1} I_1(t)dt+\int_{T_1}^{T} I_2(t)dt\right]+lD(G,p)+rD(G,p)\\ +\frac{\kappa}{T}\left(Q-\left(\int_{0}^{T_1} D(G,p)dt+\int_{T_1}^{T} D(G,p)dt\right)\right)\end{array}\right)(1-\xi(1-e^{-\eta G}))]$$

Here notice that the manufacturer total carbon emission cost is $\delta[c_{cap}-\hat{e}_c]$ included green technology investment and carbon cap-tax policy. The value of $\delta[c_{cap}-\hat{e}_c]$ is positive then manufacturer should sell the remaining carbon quota and earn extra revenue. If increased the total carbon emission from his/her total carbon quota then the manufacturer must buy a extra carbon quota.

5. Optimality of profit function

To find optimal value of G, p and T and to prove the concavity of equation (19) with respected to G, p and T, the following methodology to be adopted.

Theorem 1: For any given distinct value of T and fixed positive value of p, then

(a) The equation (A11) has a unique solution.

(b) The solution in (a) satisfies the second order condition for maximum.

Proof: See Appendix 1.

Theorem 2: For any distinct value of T and fixed positive value of G, then

(a) The equation (A21) has a unique solution.

(b) The solution in (a) satisfies the second order condition for maximum.

Proof: See Appendix 2.

Theorem 3: For any positive value of p and fixed positive value of G, then

(a) The equation (A31) has a unique solution.

(b) The solution in (a) satisfies the second order condition for maximum.

Proof: See Appendix 3.

Theorem 4: For any fix positive value of G, the total profit of manufacturer $TP(G,p,T)$ expressed in Eq. (19) is maximum value if determinant of Hessian matrix is greater than zero.

Proof : See Appendix 4

6. Numerical example and solution procedure

We consider following example to validate the mathematical formulation. The numerical values of the parameter in correct units were used as input for the model's numerical, graphical, and sensitivity analyses.

$\alpha=150, \beta=10, \gamma=0.7, C_1=6, C_2=4, C_h=6, c_{cap}=900, l=40, r=60,$

$\delta=0.33, P=700, A=80, \kappa=30, B=60, a=0.2, \xi=0.2, \eta=0.8, m=1$

Step 1: Using Maple 18 software (or Matlab) and taking initial parameters mentioned above in (19).

Step 2: Set $G=0$

Step 3: Evaluate p from equation (A21), T from equation (A31).

Step 4: Check sufficient conditions $H_{11}<0$, $H_{22}<0$ and $H_{11}H_{22}-H_{12}H_{21}>0$. Otherwise choose different parametric value in step 1.

Step 5: Increase the value of G from 0 and repeat Step 3 until to get maximum value of $TP(G,p,T)$.

Step 6: Evaluate total carbon emission from equation (18) and production quantity from equation (6)

Step 7: Obtain manufacturer's total profit using equation (19)

Step 8: Stop

Table 3. Output value as per above procedure

G^*	T^*	p^*	$\hat{e}_c$	TP^*	Q^*
0	0.3881456182	125.3513237	2099.67636	5351.19940	24
5	0.4013775110	123.7673693	1763.46649	5946.66697	25
7.77	**0.4016376947**	**123.7416265**	**1757.56279**	**5954.10798** ←	**26**
9	0.4016576952	123.7396542	1757.10994	5953.66084	26
12	0.4016685541	123.7385838	1756.86405	5951.08582	27

***the local solution; ← the optimal solution; bold value indicates the optimal results;** $H_{11}=-1.35<0, H_{22}=-3318<0$ **and** $H_{11}H_{22}-H_{12}H_{21}=4474>0$**.**

Optimum value of green investment is $G^*=7.7702$,manufacture's selling price p^*=123.7416 and production cycle time is T^*=0.4016 year.

The maximum profit of manufacturer's is **5954.11**. The optimum production quantities produced by manufacturer are $Q^* = 26$ **units**. Total carbon emission after investing in green technology is **1757.56** kg/year/unit and without investing in green technology the carbon emission is **2099.68** kg/year/unit.

Graphical representation of profit function

The concavity behaviour of profit function is shown in Figure 1,2,3 as below:

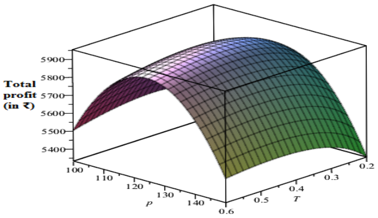

Figure 1. Total profit with respect to p and T

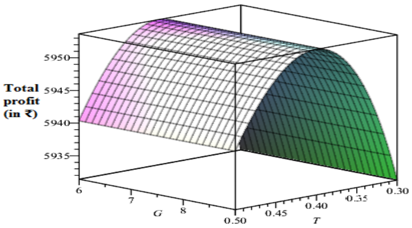

Figure 2. Total profit with respect to G and p

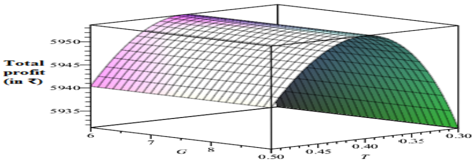

Figure 3. Total profit with respect to G and T

7. Sensitivity analysis and observations

We now analyze the effect of changes in system parameters on the optimal values base on numerical example taken in section 6, sensitivity analysis is performed by changing each parameter values in relative steps of -20%,-10%,+10%,+20%, taking one parameter at a time and the remaining values of the parameters are unchanged.

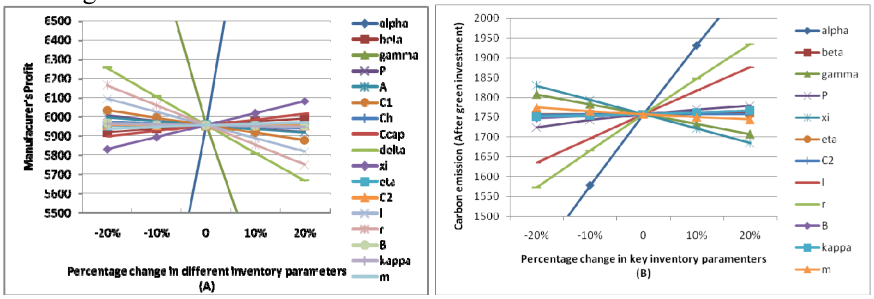

Figure 4. Effect of inventory parameters on manufacturer's profit and carbon Emission

Figure 4(A) explored the impact of the parameters on total profit. If the scale demand α and the constant coefficient of emission reduction function are sensitive to the total profit, increases the α and β result into increases the total profit. Increasing the product rate then the profit is decreases. It is obvious that the profit will be decreased due to the increases the value of cost parameters A, C_1 and C_h. The higher value of carbon cap will be positive effect in profit but higher value of carbon tax may decreases the profit. If the increases the carbon reduction function parameter ξ and η then carbon reduction is decreased, consequently profit is increases. On other hand, profit will be decreases the higher impact of emission parameters. It is notice that the higher value of m will be positive effect on total profit.

From **figure 4(B)**, it is observed that the total carbon emission increases heavily, when the parameters α, β, P, η, C_2, l, r, B and κ. If the increases the value of γ, ξ and all inventory cost parameters C_1, C_h, A then carbon emission decreases. Other parameters have a minor effect on carbon emission.

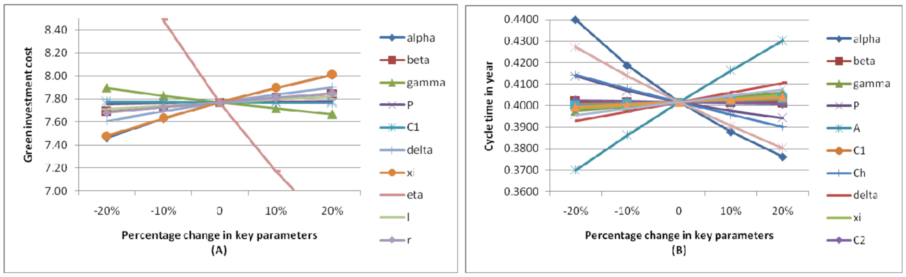

Figure 5. Effect of inventory parameters on Green investment cost and Cycle time

Figure 5(A) show that the changes in key parameters can influence the green investment cost. The parameters α, β, γ, P, χ, η have a significant effect on green investment cost. Increases in α, β, P, δ, χ, l and r result into increase in green investment cost. On the other side, the green investment cost decreases with increases in the parameters γ, C_1 and η. The changes of parameters A, C_h, Z, B, κ and m have no major effect on green investment cost.

Figure 5(B) shows how the changes in parameters can affect the cycle time. Cycle time increase significantly when γ, A, δ and χ are increases but the parameters α, P, C_h and κ increases the cycle time decreases. For other parameters, a very minor effect is observed in cycle time.

Figure 6(A) show that impact of the changes in key parameters on selling price. It is clearly shows that the selling price is very sensitive to the parameters α, γ and

δ .With increase in α , δ , l, A, C_1 and r, then the selling price increases significantly. Further, the increases in parameters γ, χ and κ also result into the decrease in selling price. Selling price has negligible sensitive to the parameters β , C_h , Z , η and B .It is observe from figure 6(B), production quantities are highly influence with changing the parameters α, A, C_h and κ. If the increases the value of α, A then production quantity also increase but C_h and κ increases the production quantity also decrease , other parameters have a no effect on production quantity.

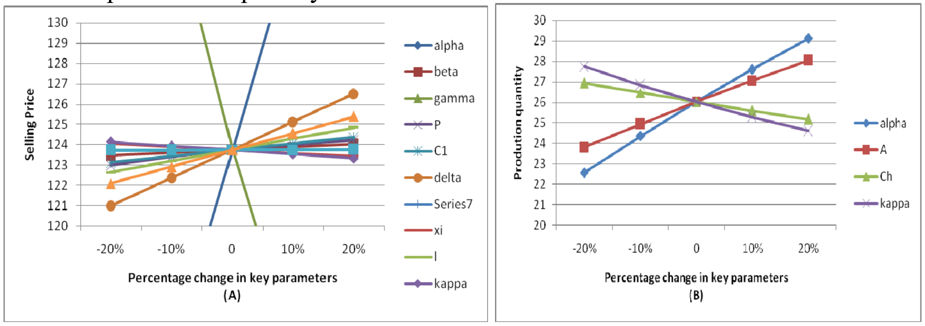

Figure 6. Effect of inventory parameters on selling price and production quantity

Discussion about managerial insights:

Based on the behavioural changes as reflected in sensitivity analysis, the following managerial insights derived.

Manufacturer's total profit will be decreases due to the higher value of carbon emission parameters, it is indicated that the higher carbon emission will be harmful for environment and negative effect on profit earning. A decision maker should be try to reduced the carbon emission and increase the profit.

The scale demand and constant coefficient of reduction function indicated that the higher scale demand and higher value of carbon emission reduction increases the profit with increases the selling price and green investment. It is shown that the more investment in green technology result to increased the profit. The higher value of selling price may decrease the profit but decreased the carbon emission. It is indicated that the manufactures should try to restrict on selling price such that earn profit. Manufacture should maintain the product rate; the higher production rate may decrease the profit and increases the carbon emission.

As per government resolution, manufacturer adopted carbon tax and cap mechanism for sustainability. In our study it is suggested that the higher value of carbon cap, use to sell it and earn more profit but the manufacturer paid higher carbon tax to government; it may decreases the his/her profit. It is recommended that the manufacturer should be maintaining the proper carbon cap and minimizes the carbon tax.

In the our study, it is shown that the fraction of carbon emission after investing in green technology parameters ξ and efficiency of greener technology in reducing emission η increases then the manufacture's total profit increased and total carbon reduction is reduces. That means green technology investment increases optimum profit and reduces emission; manufacturer should be more investing in green technology for gain more profit. The company's desire to reduce emissions such as investing in green technology, using renewable energy, or redesigning the logistic network is greater with a carbon cap and tax system.

Manufacturer should be reducing the inventory cost for gain more profit. From the above analysis, it is clear that the higher value of ordering cost, purchase cost, holding cost, production cost are negatively proportional to total profit. Hence, a manufacturer must keep the lower rate of inventory cost parameter.

By utilizing the proposed model, a decision maker can undoubtedly decide optimum selling price to accomplish a margin in profit. An optimization in the selling price gives an escalation in the demand of the customers and, thusly, to the total profit. The age of perishable product is more important regarding the business point of view. The product whose shelf life is higher, consequently it may useful to acquire more profit because the owner getting more period for sell the product. Hence the manager should be choosing the product whose shelf life is longer.

8. Conclusion and future scope

Manufacturer's optimal replenishment cycle time, optimal green investment cost and optimal selling price have been determined. Mishra et al.(2020) developed a SEPQ model with constant demand and not considered product deterioration., it is not always possible market demand is constant. Here, demand is considered as green investment and selling price dependent which is novelty of this article, higher investment in green technology and increase the carbon reduction rate cause to increase market demand. Our study proposes a sustainable inventory model for managing perishable products, in which the product's deterioration fluctuates with time and is determined by its expiration date. Because the value of products depreciates throughout period, consumers are highly conscious about the product's expiration date. A sustainable carbon tax and cap-based production model considered for a controllable carbon emission rate by investing in green technology initiatives and identified the roll of green technology investment. Classical optimization method is used to find global maximum solutions. Sensitivity analysis has been conducted to provide some managerial insights. This model can be useful for the pharmaceutical, chemical and/or pesticides manufacturing industries. For the future study, model will be extending for the different payment system and another extension use the preservation technology for reduces the effect of deterioration.

REFERENCES

[1] **Benjaafar, S., Li, Y. & Daskin, M. (2012),** ***Carbon Footprint and the Management of Supply Chains: Insights from Simple Models.*** *IEEE transactions on automation science and engineering*, *10*(1), 99-116;

[2] **Bhattacharyya, M. & Sana, S. S. (2019),** ***A Mathematical Model on Eco-Friendly Manufacturing System under Probabilistic Demand***. *RAIRO-operations Research*, *53*(5), 1899-1913;

[3] **Bouchery, Y., Ghaffari, A., Jemai, Z. & Dallery, Y. (2012),** ***Including Sustainability Criteria into Inventory Models.*** *European Journal of Operational Research*, *222*(2), 229-240;

[4] **Datta, T. K. (2017),** ***Effect of Green Technology Investment on a Production-Inventory System with Carbon Tax.*** *Advances in operations research*, *2017*. https://doi.org/10.1155/2017/4834839;

[5] **Glock, C. H., Jaber, M. Y. & Searcy, C. (2012),** ***Sustainability Strategies in an EPQ Model with Price‐and Quality‐Sensitive Demand.*** *The International Journal of Logistics Management*;

[6] **Hasan, M. R., Roy, T. C., Daryanto, Y. & Wee, H. M. (2021),** ***Optimizing Inventory Level and Technology Investment under a Carbon Tax, Cap-and-Trade and Strict Carbon Limit Regulations.*** *Sustainable Production and Consumption*, *25*, 604-621;

[7] **Hua, G., Cheng, T. C. E. & Wang, S. (2011),** ***Managing Carbon Footprints in Inventory Management.*** *International Journal of Production Economics*, *132*(2), 178-185;

[8] **Hu, H. & Zhou, W. (2014),** ***A Decision Support System for Joint Emission Reduction Investment and Pricing Decisions with Carbon Emission Trade***. *International Journal of Multimedia and Ubiquitous Engineering*, *9*(9), 371-380;

[9] **Lu, C. J., Yang, C. T. & Yen, H. F. (2020),** ***Stackelberg Game Approach for Sustainable Production-Inventory Model with Collaborative Investment in Technology for Reducing Carbon Emissions.*** Journal of Cleaner Production, 270, 121963;

[10] **Lin, B. & Jia, Z. (2018),** ***The Energy, Environmental and Economic Impacts of Carbon Tax Rate and Taxation Industry: A CGE Based Study in China***. Energy, 159, 558-568;

[11] **Lou, G. X., Xia, H. Y., Zhang, J. Q. & Fan, T. J. (2015),** ***Investment Strategy of Emission-Reduction Technology in a Supply Chain.*** Sustainability, 7(8), 10684-10708;

[12] **Mishra, U., Wu, J. Z. & Sarkar, B. (2020),** ***A Sustainable Production-Inventory Model for a Controllable Carbon Emissions Rate under Shortages***. *Journal of Cleaner Production, 256,* 120268;

[13] **Paul, A., Pervin, M., Roy, S. K., Maculan, N. & Weber, G. W. (2022),** ***A Green Inventory Model with the Effect of Carbon Taxation***. *Annals of Operations Research, 309*(1), 233-248;

[14] **Pentico, D. W., Drake, M. J. & Toews, C. (2009),** ***The Deterministic EPQ with Partial Backordering: A New Approach.*** *Omega, 37*(3), 624-636;

[15] **Qin, J., Bai, X. & Xia, L. (2015),** ***Sustainable Trade Credit and Replenishment Policies under the Cap-and-Trade and Carbon Tax Regulations.*** *Sustainability, 7*(12), 16340-16361;

[16] **Sarkar, B. (2012),** ***An EOQ Model with Delay in Payments and Time Varying Deterioration Rate***. *Mathematical and Computer Modelling, 55*(3-4), 367-377;

[17] **Shah, N., Shah, P. & Patel, M. (2020),** ***Inventory Policies with Retailer's Flexible Payment Time and Customer's Fixed Credit Time for Manufacturer-Retailer Supply Chain.*** *Economic Computation & Economic Cybernetics Studies & Research, 54*(4),87-102;

[18] **Shah, N. H., Patel, E. & Rabari, K. (2022), EPQ** ***Model to Price-Sensitive Stock Dependent Demand with Carbon Emission under Green and Preservation Technology Investment.*** *Economic Computation & Economic Cybernetics Studies & Research,56*(1), 209-222;

[19] **Shi, Y., Zhang, Z., Chen, S. C., Cárdenas-Barrón, L. E. & Skouri, K. (2020),** ***Optimal Replenishment Decisions for Perishable Products under Cash, Advance, and Credit Payments Considering Carbon Tax Regulations.*** *International Journal of Production Economics, 223,* 107514;

[20] **Taleizadeh, A. A., Soleymanfar, V. R. & Govindan, K. (2018),** ***Sustainable Economic Production Quantity Models for Inventory Systems with Shortage***. *Journal of cleaner production, 174,* 1011-1020;

[21] **Tiwari, S., Daryanto, Y. & Wee, H. M. (2018),** ***Sustainable Inventory Management with Deteriorating and Imperfect Quality Items Considering Carbon Emission***. *Journal of Cleaner Production, 192,* 281-292;

[22] **Toptal, A., Özlü, H. & Konur, D. (2014),** ***Joint Decisions on Inventory Replenishment and Emission Reduction Investment under Different Emission Regulations***. *International journal of production research, 52*(1), 243-269;

[23] **Yadav, S. & Khanna, A. (2021),** ***Sustainable Inventory Model For Perishable Products with Expiration Date and Price Reliant Demand under Carbon Tax Policy.*** *Process Integration and Optimization for Sustainability, 5*(3), 475-486;

[24] **Zand, F., Yaghoubi, S. & Sadjadi, S. J. (2019),** ***Impacts of Government Direct Limitation on Pricing, Greening Activities and Recycling Management in an Online to Offline Closed Loop Supply Chain.*** *Journal of cleaner production, 215*, 1327-1340.

Appendix 1:

Proof: any discrete value of T and fixed positive value of p, The first and second order partial derivative of Eq. (19) with respect to G; the following results can be found:

$$
\begin{aligned}
\frac{\partial TP}{\partial G} &= \left[p\beta\xi(\eta e^{-\eta G}) - C_1\beta\xi\eta e^{-\eta G} - 0 - C_h \frac{\partial}{\partial G}\left[\frac{X}{T}\right] - 1 \right] \\
&+ \left[\delta Z(-\eta e^{-\eta G}) - \left[\begin{array}{l} \frac{B}{T}\xi(-\eta e^{-\eta G}) + C_2 b \frac{\partial}{\partial G}\left[(1-\xi(1-e^{-\eta G}))\left[\frac{X}{T}\right]\right] + \\ (l+r)\frac{\partial}{\partial G}\left[(1-\xi(1-e^{-\eta G}))(\alpha + \beta\xi(1-e^{-\eta G}) - \gamma p)\right] \\ +\kappa\frac{\partial}{\partial G}\left[(1-\xi(1-e^{-\eta G}))\left[\frac{Y}{T}\right]\right] \end{array} \right] \right] = 0
\end{aligned}
\tag{A11}
$$

Where,

$$X = \left[\int_0^{T_1} \left\{ -(P - D(G,p))(1+m-t)\ln\left(\frac{1+m-t}{1+m}\right)\right\} dt + \int_{T_1}^{T} \left\{ (D(G,p))(1+m-t)\ln\left(\frac{1+m-t}{1+m-T}\right)\right\} dt \right]$$

and $Y = Q - \left(\int_0^{T_1} D(G,p)dt + \int_{T_1}^{T} D(G,p)dt \right)$

$$
\begin{aligned}
\frac{\partial^2 TP}{\partial G^2} &= \left[p\beta\xi(-\eta^2 e^{-\eta G}) + C_1\beta\xi\eta^2 e^{-\eta G} - C_h \frac{\partial^2}{\partial G^2}\left[\frac{X}{T}\right] \right] \\
&+ \left[\delta Z(\eta^2 e^{-\eta G}) - \left[\begin{array}{l} \frac{B}{T}\xi(\eta^2 e^{-\eta G}) + C_2 b \frac{\partial^2}{\partial G^2}\left[(1-\xi(1-e^{-\eta G}))\left[\frac{X}{T}\right]\right] + \\ (l+r)\frac{\partial^2}{\partial G^2}\left[(1-\xi(1-e^{-\eta G}))(\alpha + \beta\xi(1-e^{-\eta G}) - \gamma p)\right] \\ +\kappa\frac{\partial^2}{\partial G^2}\left[(1-\xi(1-e^{-\eta G}))\left[\frac{Y}{T}\right]\right] \end{array} \right] \right]
\end{aligned}
$$

But $p > C_1, \frac{\partial^2}{\partial G^2}\left[\frac{X}{T}\right] > 0, \frac{\partial^2}{\partial G^2}\left[(1-\xi(1-e^{-\eta G}))\left[\frac{X}{T}\right]\right] > 0, \frac{\partial^2}{\partial G^2}\left[(1-\xi(1-e^{-\eta G}))\left[\frac{Y}{T}\right]\right] > 0$

$\Rightarrow \frac{\partial^2 TP}{\partial G^2} < 0$, Hence the G have a unique value and profit function is concave at G.

Appendix 2:

Proof: For any discrete value of T and fixed positive value of G, the first and second order partial derivatives of Eq. (19) with respect to p; the following results can be found:

$$\frac{\partial TP}{\partial p}=\left[\alpha+\beta\xi(1-e^{-mG})-2\gamma p+C_1\gamma-C_h\frac{\partial}{\partial p}\left[\frac{X}{T}\right]\right]+\left[\begin{pmatrix}\delta C_2 b\frac{\partial}{\partial p}\left[\frac{X}{T}\right]-\delta\gamma(l+r)\\ +\delta\kappa\frac{\partial}{\partial p}\left[\frac{Y}{T}\right]\end{pmatrix}(1-\xi(1-e^{-\eta G}))\right]=0 \tag{A21}$$

Where, X and Y as per appendix 1.

$$\frac{\partial^2 TP}{\partial p^2}=\left[-2\gamma-C_h\frac{\partial^2}{\partial p^2}\left[\frac{X}{T}\right]\right]+\left[-\left(\delta C_2 b\frac{\partial^2}{\partial p^2}\left[\frac{X}{T}\right]+\delta\kappa\frac{\partial^2}{\partial p^2}\left[\frac{Y}{T}\right]\right)(1-\xi(1-e^{-\eta G}))\right]<0, \tag{A22}$$

$\frac{\partial^2}{\partial T^2}\left[\frac{X}{T}\right]>0$, $\frac{\partial^2}{\partial T^2}\left[\frac{Y}{T}\right]>0$ and $0<1-\xi(1-e^{-\eta G})<1$. It's shown that selling price have a unique value exist and TP is concave at p.

Appendix 3:

Proof: Take the first and second order partial derivative of profit function with respect to T.

$$\frac{\partial TP}{\partial T}=\left[\frac{A}{T^2}-C_h\frac{\partial}{\partial T}\left[\frac{X}{T}\right]\right]+\left[\begin{pmatrix}-\frac{\delta B}{T^2}+\delta C_2 b\frac{\partial}{\partial T}\left[\frac{X}{T}\right]\\ +\delta\kappa\frac{\partial}{\partial T}\left[\frac{Y}{T}\right]\end{pmatrix}(1-\xi(1-e^{-\eta G}))\right]=0 \tag{A31}$$

Where, X and Y as per appendix 1.

$$\frac{\partial^2 TP}{\partial T^2}=\left[-\frac{2A}{T^3}-C_h\frac{\partial^2}{\partial T^2}\left[\frac{X}{T}\right]\right]+\left[-\begin{pmatrix}\frac{2\delta B}{T^3}+\delta C_2 b\frac{\partial^2}{\partial T^2}\left[\frac{X}{T}\right]\\ +\delta\kappa\frac{\partial^2}{\partial T^2}\left[\frac{Y}{T}\right]\end{pmatrix}(1-\xi(1-e^{-\eta G}))\right] \tag{A32}$$

For the any positive value of p and G, $\frac{\partial^2}{\partial T^2}\left[\frac{X}{T}\right]>0$, $\frac{\partial^2}{\partial T^2}\left[\frac{Y}{T}\right]>0$ and $0<1-\xi(1-e^{-\eta G})<1$. Hence, $\frac{\partial^2 TP}{\partial T^2}<0$

Appendix 4:

Proof: From Equation (19) with respect to p and T, we have $\frac{\partial^2 TP}{\partial p \partial T} = [C_h X_1] + \left[\left(\delta C_2 b X_1 + \delta \kappa Y_1\right)(1-\xi(1-e^{-\eta G}))\right]$,Where $X_1 = \frac{\partial^2}{\partial p \partial T}\left[\frac{X}{T}\right]$ and $Y_1 = \frac{\partial^2}{\partial p \partial T}\left[\frac{Y}{T}\right]$, First, the Hessian matrix,

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 TP(G,p,T)}{\partial p^2} & \frac{\partial^2 TP(G,p,T)}{\partial p \partial T} \\ \frac{\partial^2 TP(G,p,T)}{\partial T \partial p} & \frac{\partial^2 TP(G,p,T)}{\partial T^2} \end{bmatrix}$$

is determined to check for total profit is concave. The determinant of Hessian matrix should be calculated by numerically and checked positive or not when G considering fixed value.

From the equations (A22) and (A33), it is notice that $H_{11} < 0$ and $H_{22} < 0$ .The determinant of the Hessian matrix is for the positive value of G

$$\frac{\partial^2 TP(G,p,T)}{\partial p^2} \frac{\partial^2 TP(G,p,T)}{\partial T^2} > \left(\frac{\partial^2 TP(G,p,T)}{\partial p \partial T}\right)^2 \Rightarrow \det(H) > 0$$

Hence the total profit is maximum and unique exist. The total profit is concave with respect to all decision variable has shown in numerical example section.