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A NOVEL STATISTICAL PREDICTION TECHNIQUE BASED ON THE DYNAMIC RELATIONSHIP IDENTIFICATION ALGORITHM TO FORECAST SUPPLY CHAIN DEMAND

***Abstract**. A key driver for the success of the supply chain (SC) is effective customer demand forecasting. However, increasingly individualized demands and fierce competition between SCs create more uncertainty of predicted results in Mainland China nowadays. Thereby, appropriate forecasting techniques are critically important for the SC's decision-makers to analyze and uncover the patterns of historical correlative data of the demands and then project those patterns into the future. A novel statistical forecasting method based on the dynamic relationship identification algorithm (SF-DRIA) was presented in this paper. Combined with the adaptive modeling techniques, the dynamic relationship model reflecting the relational pattern between the forecasting result and its correlative influencing variables was constructed initially. Then, in accordance with the continuously optimized forecasting precision and the newly added correlative data, the parameter estimates and the structure coefficients of the model were regulated to achieve the optimal forecasting results. A case performed by the core enterprise in a supply chain reveals that the proposed method produces higher accurate forecasting and better tracking capability to the trend of the demands compared with several traditional statistical forecasting approaches. Moreover, the high robustness of the SF-DRIA method is guaranteed by the employment of the forecasting precision.*

***Keywords**: Supply chain demand forecasting; Dynamic relationship model; Identification algorithm; Forecasting precision; Adaptive modeling techniques.*

JEL classification: C44, C53, L22, M11

1. Introduction

In today's challenging Chinese markets, there is a growing recognition that individual businesses no longer compete as stand-alone entities, but rather as the supply chain (SC) in which respective strengths and competencies are leveraged to achieve greater responsiveness to the market demand. Once the SC is established, the first consideration of managers is usually the supply chain demand forecasting (SCDF), for it is the key to develop and implement the various plans, such as inventory & production plan, logistics plan and revenue plan to build the required capacity of the SC. Therefore, how to uncover the rules or patterns of numerous demand-relative data to procure the accurate, credible forecasting result is an important task for the decision-makers.

In general, two kinds of processes, push process driven by the forecast and pull process driven by the demand, are proposed in the SC (Zhao et al., 2008; Zhu et al., 2011; Roh et al., 2014). Figure 1 presents a common SC type in Mainland China nowadays (Yang et al., 2011), in which the push/pull boundary, also termed the de-coupling point in some literatures (Flynn et al., 2010; Tangpong et al., 2014), lies in the core enterprise. Usually, the manufacturer with good brand or reputation acts as the core enterprise, which locates and assembles the production lines by the customer orders, and more importantly, which also needs to arrange inventory, place component orders to upstream suppliers before arrival of the customer orders. Therefore, even for the core enterprise of the SC as shown in Figure 1 which has rapid response capability to the customer demands, conducting the SCDF is still a critical activity to improve efficiency and competition of the whole SC.

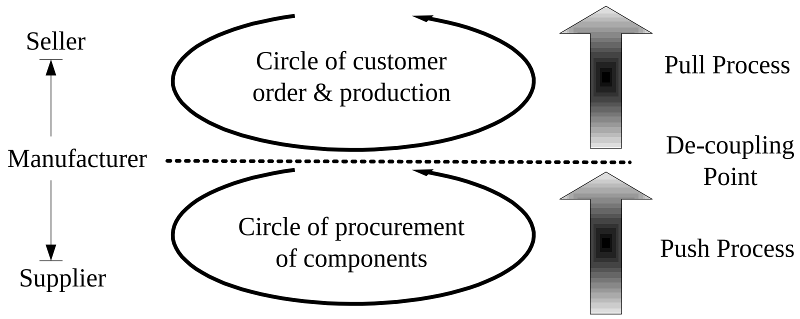

Figure 1. Brief structure of a supply chain in China

In Mainland China nowadays, with the rapid economic development and increasing personalized requirements, the characteristics of demand forecasting by the core enterprise of SC compared with that by the traditionally stand-alone manufacturer are as follows:

(1) Forecasting period is shortened, the demand pattern is treated with monthly,

weekly or even daily periodicities in accordance with increasingly individual demands;

(2) The trend of the actual demands is more volatile for intense competitions between SCs;

(3) Influencing factors to the SCDF increase due to rapidly changing market circumstance;

(4) More recent data are usually more important to the forecasting result than the earlier;

(5) More and more decision-makers pay attention to the multi-period-ahead forecasting.

The technical literature displays a wide range of methodologies for the SCDF, including the genetic approach (Yimer & Demirli, 2010; Sermpinis et al., 2014), nonlinear goal programming (Aghaei et al., 2013), neural network (Cheng et al., 2010), etc., among which the statistical technology has been intensively studied over the past years and still the most widely-used prediction tool by the core enterprise of SC (e.g. Jae, 2000; Hosoda & Disney, 2006; Pan et al., 2009; Reiner & Fichtinger, 2009; Thomassey, 2010; Acar & Cardner, 2012; Yan & Wang, 2012; Phuc et al., 2013; Lin & Wu, 2014). Statistical approaches generally employ two kinds of models: static models (Jae, 2000; Reiner & Fichtinger, 2009; Thomassey, 2010; Acar & Cardner, 2012; Önkal et al., 2012; Phuc et al., 2013; Tangsucheeva & Prabhu, 2013) and dynamic models (Jae, 2000; Perakis & Roels, 2007; Pan et al., 2009; Danese & Kalchschmidt, 2011; Ali et al., 2012; Lin & Wu, 2014; Zhao et al., 2014). As to static models, the demand is assumed to be a linear combination of certain functional elements which describe the basic features of demand behavior. In general, static models are easy to be applied, but such models are usually set up with a fixed structure (Reiner & Fichtinger, 2009; Tong, 2010; Acar & Cardner, 2012; Tangsucheeva & Prabhu, 2013). This reduces their adaptability and tracking capability to a changing environment.

Dynamic models treat the demand pattern as the time series signal with known yearly, monthly, weekly, daily or even hourly periodicities. In dynamic models, the difference between the actual and the predicted demand is considered as a stochastic process, and the analysis of this stochastic process leads to a more accurate prediction. However, many dynamic models are so complicated that human interventions are needed when running the programs developed by them (Chopra & Meindl, 2001; Hyndman, 2011; Karakaya & Bakal, 2013). In addition, some models have the potential weakness of low robustness due to divergence of the model parameter estimates or even the forecasting results (Jae, 2000; Danese & Kalchschmidt, 2011; Zhao et al., 2014).

Therefore, in view of the imperfection of existing statistical prediction methods mentioned above, a new statistical forecasting method based upon the dynamic relationship identification algorithm (SF-DRIA) is presented in this paper. Combined with the adaptive modeling techniques, a dynamic relationship model which extracted the relational pattern between the forecasting result and its correlative influencing variables was constructed at first. Then, in accordance with the continuously optimized forecasting precision which guaranteed the model's robustness, the model's parameter estimates and structure coefficients were adjusted to obtain the optimal predicted results in the next forecasting period.

The proposed SF-DRIA method was tested on mort than half-year historical demand-relative data of a security door manufacturer which was the core entity in a SC, and highly accurate forecasts were obtained. Compared with traditionally statistical forecasting algorithms such as the moving average (MA) and the exponential smoothing (ES) (Jae, 2000; Acar & Cardner, 2012), the accuracy of the SF-DRIA for forecasting is higher by about 20-55%. Meanwhile, the SF-DRIA method has also shown a high degree of adaptability and robustness.

The outline of the paper is as follows. The proposed SF-DRIA method is presented in Section 2. Properties of the SF-DRIA and its forecasting process are described in Section 3. Then, a case is introduced and detailed results are analyzed in Section 4. Conclusions are given in Section 5 and the proof for the recursive solution of the parameter estimate involved in the SF-DRIA method is given in the Appendix.

2. The introduction of the SF-DRIA method

An outline of the proposed SF-DRIA method based on the adaptive modeling technique includes construction of the dynamic relationship model, identification of model parameters and calculation of the optimal forecasting value. Details of the proposed SF-DRIA method are as follows.

2.1. Construction of the dynamic relationship model

In today's SCDF problems, shorter forecasting period leads to the more increased volatility of the predicted values. In addition, the influencing factors to the forecasting result are more varied than before. Therefore, it's better to make a dynamic relationship model to describe the relational pattern between the forecasting result and its correlative influencing factors. With new information added, the model's parameters and structure coefficients can be modified to acquire the more suitable or accurate developing trend of the forecasting result. The following is a difference equation to describe the dynamic relationship model,

$$A_j(z^{-1})z(k)=\sum_{i=1}^{n}B_{i,j}(z^{-1})u_i(k-d_{i,j})+e_j(k) \tag{1}$$

where $A_j\left(z^{-1}\right)=1+a_{1,j}z^{-1}+a_{2,j}z^{-2}+\cdots+a_{n_{a,j},j}z^{-n_{a,j}}$,

$B_{i,j}\left(z^{-1}\right)=b_{i,0,j}+b_{i,1,j}z^{-1}+b_{i,2,j}z^{-2}+\cdots+b_{i,n_{b_i,j},j}z^{-n_{b_i,j}}$. Thus, the demand can be expressed as

$$z(k) = -\sum_{r=1}^{n_{a,j}} a_{r,j} z(k-r) + \sum_{i=1}^{n} \sum_{s=0}^{n_{b_i,j}} b_{i,s,j} u_i\left(k-s-d_{i,j}\right) + e_j(k) \tag{2}$$

Herein we define the structure coefficient set $F_j = \left\{n_{a,j}, n_{b_1,j}, \cdots, n_{b_n,j}, d_{1,j}, \cdots, d_{n,j}\right\}$, $j = 1,2,\cdots,m$, where m means the number of the sets. The explanation of parameters and variables in Equation (2) follows:

$z(k)$: actual demand in the k period;

$u_i(k)$: actually observable value of the ith influencing variable in period k, $i = 1,2,\cdots,n$;

$n_{a,j}$: number of preceded successive demand values which are self-correlative to $z(k)$;

$n_{b_i,j}$: number of preceded successive values of the ith influencing variable which is correlative to $z(k)$;

$d_{i,j}$: pure delay period of the ith influencing variable, $d_{i,j} \in N$;

$a_{r,j}, b_{i,s,j}$: dynamic parameters of the model, $r = 1,2,\cdots,n_{a,j}$, $s = 0,1,2,\cdots,n_{b_i,j}$;

$e_j(k)$: process disturbance which is generally regarded as the white noise, $E\left[e_j(k)\right] = 0$.

Different structure coefficient set leads to different modeling, and different modeling leads to different expression of the relationship between the forecasting value and its influencing factors. The initial structure coefficient sets could be determined by experts' experience or certain searching algorithm. Then the different dynamic relationship models are obtained. We can also express Equation (2) as

$$z(k) = \varphi_j^T(k)\theta_j + e_j(k) \tag{3}$$

where $\varphi_j(k)$ is the regression vector comprised of the observably past demands and correlative variables, θ_j is the parameter vector of the model, they are expressed as follows:

$$\begin{aligned} \varphi_j^T(k) = \Big[& -z(k-1),\cdots,-z\left(k-n_{a,j}\right),u_1\left(k-d_{1,j}\right),u_1\left(k-1-d_{1,j}\right),\cdots,u_1\left(k-n_{b_1,j}-d_{1,j}\right), \\ & \cdots\ \cdots,u_i(k-d_{i,j}),u_i(k-1-d_{i,j}),\cdots,u_i(k-n_{b_i,j}-d_{i,j}), \\ & \cdots\ \cdots,u_n(k-d_{n,j}),u_n(k-1-d_{n,j}),\cdots,u_n(k-n_{b_n,j}-d_{n,j})\Big] \end{aligned} \tag{4}$$

$$\theta_j^T = \Big[a_{1,j}, a_{2,j}, \cdots, a_{n_a,j}, b_{1,0,j}, b_{1,1,j}, \cdots, b_{1,n_{b_1},j}, \cdots \cdots, b_{i,0,j}, b_{i,1,j}, \cdots, b_{i,n_{b_i},j}, \cdots \cdots, b_{n,0,j}, b_{n,1,j}, \cdots, b_{n,n_{b_n},j} \Big] \tag{5}$$

2. 2. Identification of model parameters

For the dynamic relationship model based on F_j, we define $\hat{\theta}_{L,j}$ as the parameter estimate vector which can be calculated from the observable values $\{z(k), u_i(k):\ i=1,2,\cdots,n;\ k=1,2,\cdots,L\}$, and define $\varepsilon_{L,j}(k)$ as the estimation deviation expressed by the difference between the actual demand $z(k)$ and the estimated demand $\hat{z}_{L,j}(k)$, i.e.

$$\varepsilon_{L,j}(k) = z(k) - \hat{z}_{L,j}(k) = \varphi_j^T(k)\theta_j + e_j(k) - \varphi_j^T(k)\hat{\theta}_{L,j} \tag{6}$$

Define the demand vector $z_L = \left[z(1), z(2), \cdots, z(L)\right]^T$, the regression matrix $\phi_{L,j} = \left[\varphi_j(1), \varphi_j(2), \cdots, \varphi_j(L)\right]^T$, the disturbance vector $e_{L,j} = \left[e_j(1), e_j(2), \cdots, e_j(L)\right]^T$ and the estimation deviation vector $\varepsilon_{L,j} = \left[\varepsilon_{L,j}(1), \varepsilon_{L,j}(2), \cdots, \varepsilon_{L,j}(L)\right]^T$. By using Equations (3) and (6), we get

$$z_L = \phi_{L,j}\theta_{L,j} + e_{L,j} \tag{7}$$

$$\varepsilon_{L,j} = z_L - \phi_{L,j}\hat{\theta}_{L,j} = \phi_{L,j}(\theta_{L,j} - \hat{\theta}_{L,j}) + e_{L,j} \tag{8}$$

In the dynamic relationship model for prediction, the more recent information usually plays more important role than the earlier to influence the forecasting result. To strengthen the effect of the later observable data on $\hat{\theta}_{L,j}$, we define the positive weighted matrix $w_L = diag\left(\rho^{L-1}, \rho^{L-2}, \cdots, \rho, 1\right)$ where ρ is a forgetting factor, $\rho \in [0,1]$. For the model based on F_j, the solution to the parameter estimate vector $\hat{\theta}_{L,j}$ is obtained by the following set of recursive equations:

$$\begin{cases} \hat{\theta}_{L,j} = \hat{\theta}_{L-1,j} + H_{L,j}\left[z(L) - \varphi_j^T(L)\hat{\theta}_{L-1,j} \right] \\ H_{L,j} = P_{L-1,j}\varphi_j(L)\left[\rho + \varphi_j^T(L) P_{L-1,j}\varphi_j(L)\right]^{-1} \\ P_{L,j} = \rho^{-1}\left[I - H_{L,j}\varphi_j^T(L)\right] P_{L-1,j} \end{cases} \tag{9}$$

The proof of Equation (9) sees the **Appendix**. For $j = 1,2,\cdots,m$, we initialize the calculation of $\hat{\theta}_{L,j}$ by using $\hat{\theta}_{0,j} = 0$ and $P_{0,j} = \beta I$, where β is a sufficiently large positive. This initialization approach (Ljung, 1987; Ali, 2012) to the solution $\hat{\theta}_{L,j}$ is valid for its ability to fast parameters convergence. In addition, define the begin-to-forecasting period $\underline{L} = \max\left\{n_{a,j}, n_{b_i,j} + d_{i,j} + 1 : i = 1,2,\cdots,n\right\}$, if $L \geq \underline{L}$, we consider that the obtained $\hat{\theta}_{L,j}$ is sufficiently effective.

2. 3. Calculation of the optimal forecasting value

Apply the parameter estimate vector $\hat{\theta}_{L,j}$ obtained in period $k = L$, $L \geq \underline{L}$, on the calculation of the forecasting value $\hat{z}_j(k = L+1)$, and consider the disturbance $e_j\left(L+1\right)$ uncorrelated to the forecasting value, we have

$$\hat{z}_j(L+1) = \varphi_j^T(L+1)\hat{\theta}_{L,j} \tag{10}$$

As we know, the dynamic relationship model which describes the relational pattern between the forecasting result and its correlative influencing factors is constructed based on the structure coefficient set F_j, $j = 1,2,\cdots,m$. Therefore, different F_j leads to different $\hat{\theta}_{L,j}$, and different $\hat{\theta}_{L,j}$ leads to the different forecasting result $\hat{z}_j(L+1)$. We define the forecasting precision $FP_{L,j}$ in period $k = L$ as

$$FP_{L,j} = 1 - \frac{1}{L - \underline{L}} \sum_{k=\underline{L}+1}^{L} \left| \frac{z(k) - \hat{z}_j(k)}{z(k)} \right| \tag{11}$$

It's obvious that, $FP_{L,j} \in \left(0,1\right]$, and the more the $FP_{L,j}$ approximates to 1, the better the forecasting result is.

Further, the optimal forecasting precision FP_L^* in period $k=L$ is determined as

$$FP_L^* = \max\left\{FP_{L,j}, j = 1,2,\cdots,m\right\} \tag{12}$$

Then, we define the optimal structure coefficient set F_L^* which leads to FP_L^*, and define the optimal parameter estimate vector $\hat{\theta}_L^*$ which is determined by F_L^*. Thereby, according to Equation (10), the optimal forecasting value $\hat{z}^*\left(L+1\right)$ is determined by $\hat{\theta}_L^*$ and $\varphi_j(L+1)$.

In view of two special conditions, the heuristic Formulas designed to obtain F_L^* are as follows.

Formula 1: While $L = \underline{L}$, the rule to determine $F_{\underline{L}}^{*}$ is: ① take the $F_{\underline{L},j}$ which satisfied with the $\max\left\{ n_{a,j} + \sum_{i=1}^{n} n_{b_i,j},\quad j = 1,2,\cdots,m \right\}$ as $F_{\underline{L}}^{*}$; ② if there exists more than one $F_{\underline{L},j}$ for the term ①, $F_{\underline{L}}^{*}$ can be determined on the following terms sequently: with the $\max\{n_{a,j}\}$, with the $\max\left\{\sum_{i=1}^{n} n_{b_i,j}\right\}$, with the $\min\left\{\sum_{i=1}^{n} d_{i,j}\right\}$, randomly.

Formula 2: While $L > \underline{L}$, if there exists more than one $F_{L,j}$ which lead to FP_{L}^{*}, ① take the one which produced the most numerous $\hat{z}^{*}(k : k = \underline{L},\cdots,L-1)$ as the F_{L}^{*}; ② if there exists more than one $F_{L,j}$ for the term ①, the further regulation refers to the Formula 1.

It should be pointed out that, generally, we could determine $F_{\underline{L}}^{*}$ by the Formula 1. However, the priority of the heuristic terms mentioned in the Formula 1 could also be regulated according to the characteristic of the realistic forecasting problem.

In addition, with the analogous process, the multi-period-ahead forecasting value can be obtained. We rewrite Equation (2) to the following,

$$z(k) = -\sum_{r=\lambda}^{n_{a,j}} a_{r,j} z(k-r) + \sum_{i=1}^{n} \sum_{s=\lambda-1}^{n_{b_i,j}} b_{i,s,j} u_i\left(k - s - d_{i,j}\right) + e_j(k) \tag{13}$$

where $\lambda \in N$, and the rest deduction is the similar as the above. Generally, the less the forecasting-ahead periods λ is, the better the forecasting result is.

3. Forecasting process of the SF-DRIA method and its features

A concise summary of forecasting steps based on the SF-DRIA method is shown as Figure 2, in which the $\underline{FP}_{L}^{*}$ is the threshold value of the optimal forecasting precision, usually, $\underline{FP}_{L}^{*} \in [0.85, 0.95]$.

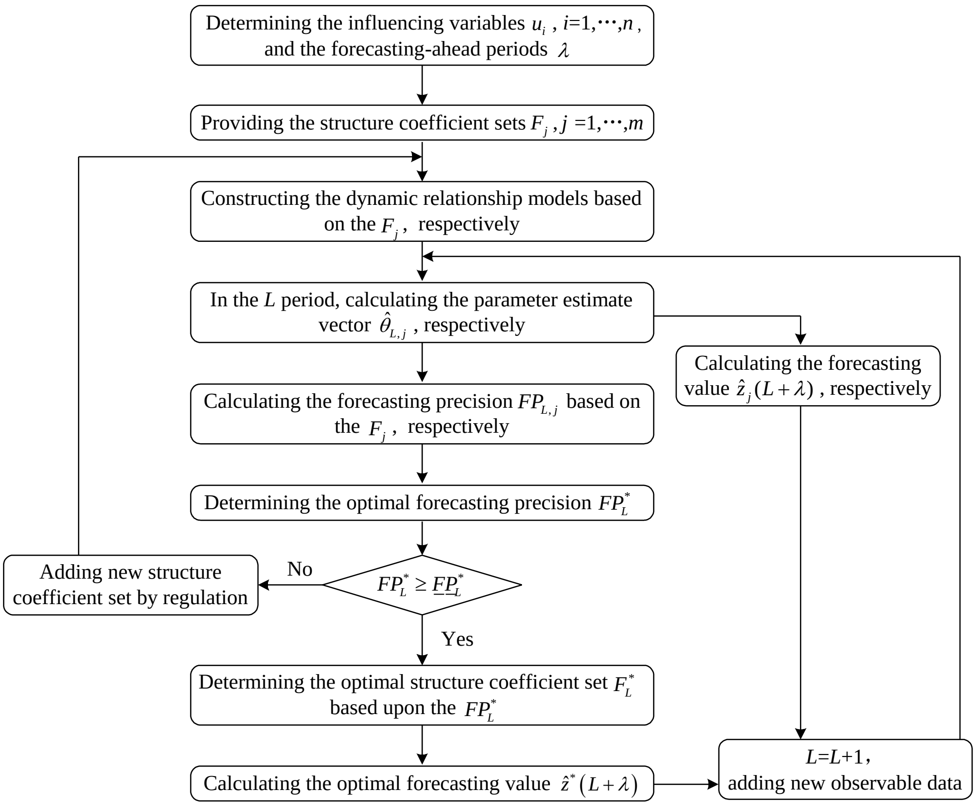

Figure 2. Concise forecasting process based upon the SF-DRIA method

The characteristics of the SF-DRIA method proposed in the previous section are summarized as follows.

(1) The dynamic relationship model is constructed according to the realistic forecasting problem, and the importance degrees of various influencing factors to the forecasting result are auto-identified by using the identification algorithm.

(2) The proposed relationship model is dynamic and adaptive to the continuously updating demand pattern, and the optimal forecasting value is obtained by updating the structure coefficient set and parameter estimate vector of the model.

(3) The number of influencing variables in the dynamic relationship model is not fixed, it can be readjusted by experts' experience or certain inspection algorithms if possible or necessary.

(4) The data of later periods are considered more importantly than those of earlier periods by the application of the forgetting factor, thus the capability of tracing the recent data is guaranteed and improved.

(5) The method is suitable to deal with the forecasting problem of multiple influence factors which reflect the actual situation of increment of customers' individual needs in Mainland China nowadays..

(6) The method provides more than one-period-ahead forecasting algorithm, which improves the ability to solving the realistic prediction problems.

(7) The robustness of the SF-DRIA method is guaranteed by the employment of the optimal forecasting precision.

(8) The forecasting process is normalized and reasonable, thus the forecasting system programmed based on the SF-DRIA method is easy to be applied.

Consider the features of the SCDF mentioned in Section 1, the SF-DRIA method is suitable and helpful to gain the useful data for SC managers to facilitate effective decision-making.

4. Case study

In this case, more than 30-week historical data from a security door manufacturer in Mainland China were used to evaluate the proposed SF-DRIA method against two traditional statistical forecasting approaches, the moving average (MA) and the exponential smoothing (ES).

The company lies in the Zhejiang Province where the market demand for security doors (mainly security metal doors) has been strong with the development of the real estate industry. In that region, there are about twenty security door manufacturers. In reaction to intense competition and increasingly individualized demand, combined with upstream suppliers and downstream sellers, a SC as Figure 1 shown was set up, in which the company is regarded as the core enterprise. To reduce the bullwhip effect and increase the efficiency of the whole SC as far as possible, the company carries out the SCDF weekly and then places orders of components to upstream suppliers according to the forecasting results.

Five types of security metal doors are produced by the company. We forecasted the demand of one type by the SF-DRIA method. The demand has little correlative to seasonal and regional changes, and the price of this type is relatively stable except that in holidays, such as New Year's Day, National Day and World Labor Day, when sellers usually give discount to enlarge sales volume. Hence, in the case, we didn't list the season, region and price as influencing factors to the SCDF. While we considered the advertisement expenses as an influencing variable which was represented by $u_1(k)$ in the regression vector, for the company has persistently advertised goods through newspaper, TV and Internet for a long time. We defined the structure coefficient set as $F_j = \left\{ n_{a,j}, n_{b_1,j}, d_{1,j} \right\}$. Thus, the regression vector $\varphi_j(k)$ and the parameter vector θ_j were expressed as follows,

$$\varphi_j^T(k) = \left[-z(k-1),\cdots,-z(k-n_{a,j}),u_1(k-d_{1,j}),u_1(k-1-d_{1,j}),\cdots,u_1(k-n_{b_1,j}-d_{1,j})\right] \tag{14}$$

$$\theta_j^T = \left[a_{1,j},a_{2,j},\cdots,a_{n_a,j},b_{1,0,j},b_{1,1,j},\cdots,b_{1,n_{b_1},j}\right] \tag{15}$$

According to the judgment of the sales, we primarily considered the range of $n_{a,j}$ is from 9 to 16, totally eight integers, $n_{b_1,j}$ from 2 to 4, totally three, and $d_{1,j}$ from 1 to 2, totally two. Thereby, the number of structure coefficient sets is equal to 8×3×2=48, i.e. $m = 48$, $j = 1,2,\cdots,m$. Moreover, giving the forgetting factor $\rho = 0.9$ and the initialization parameter $\beta = 10^8$, the proposed SF-DRIA method was implemented with Matlab and Excel Software.

This type of security metal door is a brand new product and begins to sell in mid-July 2013. Historical demand data and corresponding advertisement expenses from then to early October 2013, totally 12 weeks, are given in Table 1.

Table 1. Historical demands and corresponding advertisement expenses in the first 12 weeks

Week	1	2	3	4	5	6	7	8	9	10	11	12
Actual demand	302	346	372	368	390	407	425	455	491	453	425	420
Advertisement expenses ($ hundred)	8.2	7	7	7.7	7.7	8.2	8.2	8.2	8.2	7.5	7.5	7.5

Besides the data shown in Table 1, another 25-week actual data from middle October 2013 (the 13th week) to early March 2014 (the 37th week), as shown in Table 2, were applied to forecast demands. As we see, in this forecasting problem, the begin-to-forecasting period $\underline{L} = 12$, which is in accord with the effectiveness requirement of the parameter estimate. Because the actual demand data seems be irregular and elusive, we define the threshold value of the optimal forecasting precision $\underline{FP}_L^* = 0.88$. Table 2 also presents the optimal forecasting values and corresponding optimal structure coefficient sets obtained by the SF-DRIA method, as well as forecasting results calculated by the MA in which the forecasting value is regarded as the average of preceding six-week demands, and by the ES in which the smoothing is initialized using the average of demands in 7th-12th weeks, and smoothing constant $\alpha = 0.4$ and $\alpha = 0.8$ are used, respectively.

Table 2. Forecasting results by the SF-DRIA, MA and ES methods respectively

Week	Actual demand	Advertisement expenses ($ hundred)	Forecasting results				
			SF-DRIA		MA	ES (α=0.4)	ES (α=0.8)
			Optimal structure coefficient set F_L^*	Optimal forecasting value			
13	421	7.5	{11,4,1}	422	445	433	424
14	399	8	{11,4,1}	411	444	428	421
15	397	8	{11,4,1}	394	435	416	403
16	437	8.5	{11,4,1}	385	419	408	398
17	452	9	{11,4,1}	449	417	420	429
18	473	9	{11,4,1}	446	421	433	447
19	460	9	{11,4,1}	471	430	449	468
20	503	9	{11,4,1}	473	436	453	462
21	528	9	{14,4,1}	471	454	473	495
22	552	9	{13,4,1}	548	476	495	521
23	490	8.2	{13,4,1}	495	495	518	546
24	518	8.2	{13,4,1}	471	501	507	501
25*	517	8.2	{16,4,1}	532	509	511	515
26	471	8.2	{16,4,1}	470	518	513	517
27	436	9	{16,4,1}	464	513	496	480
28	430	9	{16,4,1}	464	497	472	445
29*	471	7.7	{16,4,1}	499	447	455	433
30	489	7.7	{15,4,1}	468	474	461	463
31	517	7.7	{15,4,1}	496	469	472	484
32	530	8.5	{15,4,1}	558	469	490	510
33	548	8.5	{15,4,1}	562	479	506	526
34	564	8.5	{16,4,1}	547	498	523	544
35	580	8.5	{16,4,1}	564	538	539	560
36	589	9	{16,4,1}	582	554	555	576
37	598	9	{16,4,1}	589	568	569	586

* New Year's Day is in the 25th week when the security doors were given discount to enlarge sale volume. The Spring Festival is in the 29th week while the sale volume is relatively low because more people pursued to stay with family members at home in those days. Due to lack of the historical data of this new type in the same period before, the demands shown in the table are the results after adjustment according to the historical data of other types in the same periods. The disposal of abnormal data refers to the literature (Jae, 2000).

Table 3 presents the statistics including the forecasting precise (*FP*), the mean absolute error (*MAE*) and the mean square error (*MSE*) by using these methods.

Table 3. Precision statistics by the SF-DRIA, MA and ES methods

Precise criteria	SF-DRIA	MA	ES （α=0.4）	ES （α=0.8）
FP	0.9599	0.9130	0.9315	0.9495
MAE	19.64	42.80	33.56	26.32
MSE	624.12	2311.04	1342.52	947.76

As Table 3 shown, highly accuracy forecasting is achieved with weekly average errors less than 5% by the SF-DRIA method. We consider such forecasting results by the SF-DRIA are valid and no other structure coefficient sets need to be added in the case. Table 3 also presents that all three precision statistics of the SF-DRIA are better than those of the MA and the ES. For example, the ratios of improvement for the *FP* from the SF-DRIA reach up to 53.9%, 41.5% and 20.6% compared with the MA, ES (α = 0.4) and ES (α = 0.8), respectively. The results reveal a high degree of accuracy and robustness of the SF-DRIA method for the SCDF.

For clear interpretation of other features of the SF-DRIA method, we plot the Figure 3 in which the trends of the actual demands and the forecasting results obtained by the SF-DRIA, MA and ES (α = 0.8) are presented respectively. Combined Figure 3 with Table 2, we get the following significant conclusions.

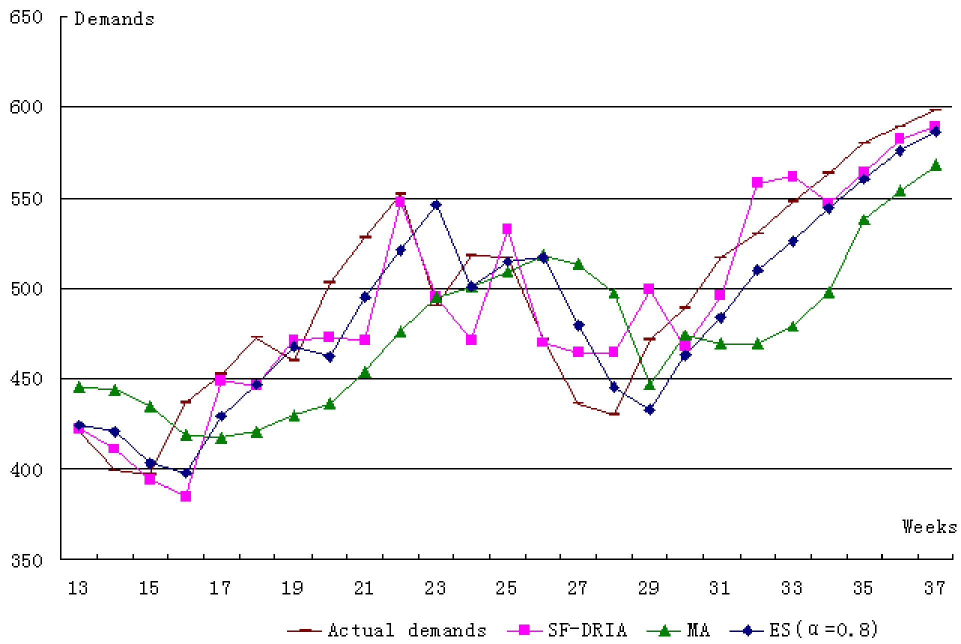

Figure 3. Trends of actual demands and according forecasting results by the SF-DRIA, MA and ES

(1) Better tracking ability

For example, due to continuous descent of actual demands from the 13th week to the 15th week, the forecasting value of the 16th week obtained by the SF-DRIA decreased to 385 according to the descending tendency. However, the actual demand in that period suddenly went up to 437. To follow the track of this abrupt ascending demand pattern, the forecasting result of the 17th week rose to 449, which nearly met the actual value 452. This reflects the attractive capability of the SF-DRIA method to track the changing pattern of demand.

(2) Highly adaptive capability

Some meaningful conclusions could be obtained from the data of the 19th-24th weeks. The trend of forecasting values during the 19th-21th weeks calculated by the SF-DRIA method kept relatively stable compared with that of actual demands in the same periods partially because of the descending trend of actual demands from the 18th week to the 19th week. Nevertheless, why the forecasting value of the 22th week went up to 548 abruptly? The reason was the adjustment of the optimal structure coefficient set, which changed from {11,4,1} of the 20th week to {14,4,1} of the 21th week, and finally located to {13,4,1} in the next three weeks. During the six-week period, the forecasting accuracy by the SF-DRIA method was better than those by the MA and the ES, because the relational pattern constructed in the

SF-DRIA method is dynamic and self-adjusting while those in the MA and the ES are fix-structured. The analysis of data from the 28th week to the 32th week is the similar as the above. These reconfirm the adaptive capability of the SF-DRIA method.

(3) Significant information obtained from the optimal structure coefficients

For instance, we notice that the $n_{b_1}^*$ in the optimal structure coefficient set was always equal to 4. This indicates that the advertisement expense is effective to enhance the sale, and the advertisement's influencing periods are more than three weeks.

5. Conclusions

In today's more challenging business circumstance where volatile market demand have become the norm, it's critically important for the SC managers to seek an adaptive, robust forecasting approach to procure the patterns of historical demand and then acquire the accurate, credible forecasts.

The approach proposed in this paper, a new statistical forecasting method based on the dynamic relationship identification algorithm (SF-DRIA), meets such requirements well. The most important invention of the proposed method for SCDF is that the parameter estimates and structure coefficients of the dynamic relationship model are continuously updated to procure the optimal forecasting results. The employment of the forecasting precision guarantees the robustness of the method. The case study has revealed that the SF-DRIA method has better capability to tracking the demand patterns and obtaining the accurate forecasts compared with some traditional statistical prediction approaches such as the moving average and the exponential smoothing. Another advantage of the method is that the forecasting software programmed based on the SF-DRIA method is easy to be applied for its normalized identification algorithm and reasonable forecasting process.

However, there exists an imperfection in the SF-DRIA method. To solve the long-term but short-period SCDF problems, it's better to combine the SF-DRIA method with other forecasting techniques which could extract the regular pattern of the demands in the certain cycle. That is the research work we need to do further.

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Appendix: The Proof for the recursive solution of the parameter estimate vector $\hat{\theta}_{L,j}$

Firstly, the following lemma on the solution of inverse matrix is introduced.

Lemma: Consider singular matrices *A*, *C* and *A*+*BCD*, the solution of inverse matrix *A*+*BCD* is

$$[A+BCD]^{-1}=A^{-1}-A^{-1}B[C^{-1}+DA^{-1}B]^{-1}DA^{-1} \tag{A.1}$$

With the actually observable values $\{z(k),u_i(k):\ i=1,2,\cdots,n;\ k=1,2,\cdots,L-1\}$, the parameter estimate vector $\hat{\theta}_{L-1,j}$ can be determined by the direct derivation algorithm. $\hat{\theta}_{L-1,j}$ should be satisfied with the minimization of the criterion J, which is described as

$$\begin{aligned}J&=\varepsilon_{L-1,j}^{T}w_{L-1}\varepsilon_{L-1,j}=\left(z_{L-1}-\phi_{L-1,j}\hat{\theta}_{L-1,j}\right)^{T}w_{L-1}\left(z_{L-1}-\phi_{L-1,j}\hat{\theta}_{L-1,j}\right)\\&=z_{L-1}^{T}w_{L-1}z_{L-1}-2\left(\phi_{L-1,j}^{T}w_{L-1}z_{L-1}\right)^{T}\hat{\theta}_{L-1,j}+\hat{\theta}_{L-1,j}^{T}\phi_{L-1,j}^{T}w_{L-1}\phi_{L-1,j}\hat{\theta}_{L-1,j}\end{aligned} \tag{A.2}$$

With respect to $\hat{\theta}_{L-1,j}$, we consider the partial derivative,

$$\frac{\partial J}{\partial\hat{\theta}_{L-1,j}}=-2\phi_{L-1,j}^{T}w_{L-1}z_{L-1}+2\phi_{L-1,j}^{T}w_{L-1}\phi_{L-1,j}\hat{\theta}_{L-1,j} \tag{A.3}$$

For w_{L-1} is a positive weighted matrix, it's obvious that

$$\frac{\partial}{\partial\hat{\theta}_{L-1,j}}\left(\frac{\partial J}{\partial\hat{\theta}_{L-1,j}}\right)=2\phi_{L-1,j}^{T}w_{L-1}\phi_{L-1,j}>0 \tag{A.4}$$

Thereby, when $\hat{\theta}_{L-1,j}$ is satisfied with the equation $\partial J/\partial\hat{\theta}_{L-1,j}=0$, we have the solution $\hat{\theta}_{L-1,j}$ which makes the criterion *J* minimized. In the realistic forecasting problem, $\phi_{L-1,j}$ is generally the singular matrix. Hence, the solution of $\hat{\theta}_{L-1,j}$ is

$$\hat{\theta}_{L-1,j}=\left(\phi_{L-1,j}^{T}w_{L-1}\phi_{L-1,j}\right)^{-1}\phi_{L-1,j}^{T}w_{L-1}z_{L-1} \tag{A.5}$$

The following is the discussion of the recursive algorithm to get the model's parameter estimate vector $\hat{\theta}_{L,j}$ when the new observable data $z(L)$ and $u_i(L)$ are added. Define $P_{L-1,j}=\left(\phi_{L-1,j}^{T}w_{L-1}\phi_{L-1,j}\right)^{-1}$, we get

$$P_{L,j}^{-1}=\phi_{L,j}^{T}w_{L}\phi_{L,j}=\sum_{k=1}^{L}\rho^{L-k}\varphi_j(k)\varphi_j^{T}(k)=\rho P_{L-1,j}^{-1}+\varphi_j(L)\varphi_j^{T}(L) \tag{A.6}$$

Using Equations (A.5) and (A.6), it follows that

$$\hat{\theta}_{L,j}=P_{L,j}\phi_{L,j}^{T}w_{L}z_{L} \tag{A.7}$$

$$\phi_{L-1,j}^{T}w_{L-1}z_{L-1}=P_{L-1,j}^{-1}\hat{\theta}_{L-1,j} \tag{A.8}$$

According to the Equations (A.6), (A.7) and (A.8), we obtain

$$\begin{aligned}\hat{\theta}_{L,j} &= P_{L,j}\phi_{L,j}^{T}w_{L}z_{L} = P_{L,j}\left[\rho P_{L-1,j}^{-1}\hat{\theta}_{L-1,j} + \varphi_{j}(L)z(L)\right] \\ &= P_{L,j}\left\{\left[P_{L,j}^{-1} - \varphi_{j}(L)\varphi_{j}^{T}(L)\right]\hat{\theta}_{L-1,j} + \varphi_{j}(L)z(L)\right\} \\ &= \hat{\theta}_{L-1,j} + P_{L,j}\varphi_{j}(L)\left[z(L) - \varphi_{j}^{T}(L)\hat{\theta}_{L-1,j}\right]\end{aligned} \quad \text{(A.9)}$$

Define the gain matrix $H_{L,j} = P_{L,j}\varphi_{j}(L)$, Equation (A.9) is rewritten as

$$\hat{\theta}_{L,j} = \hat{\theta}_{L-1,j} + H_{L,j}\left[z(L) - \varphi_{j}^{T}(L)\hat{\theta}_{L-1,j}\right] \quad \text{(A.10)}$$

Applying the ***Lemma*** into Equation (A.6), it yields that

$$P_{L,j} = \left[\rho P_{L-1,j}^{-1} + \varphi_{j}(L)\varphi_{j}^{T}(L)\right]^{-1} = \rho^{-1}\left[I - \frac{P_{L-1,j}\varphi_{j}(L)\varphi_{j}^{T}(L)}{\varphi_{j}(L)P_{L-1,j}\varphi_{j}^{T}(L) + \rho}\right]P_{L-1,j} \quad \text{(A.11)}$$

Then, introducing Equation (A.11) into the gain matrix $H_{L,j}$, we get

$$H_{L,j} = P_{L-1,j}\varphi_{j}(L)\left[\varphi_{j}^{T}(L)P_{L-1,j}\varphi_{j}(L) + \rho\right]^{-1} \quad \text{(A.12)}$$

According to the above deduction, the parameter estimate vector $\hat{\theta}_{L,j}$ is obtained by the following set of recursive equations,

$$\begin{cases}\hat{\theta}_{L,j} = \hat{\theta}_{L-1,j} + H_{L,j}\left[z(L) - \varphi_{j}^{T}(L)\hat{\theta}_{L-1,j}\right] \\ H_{L,j} = P_{L-1,j}\varphi_{j}(L)\left[\rho + \varphi_{j}^{T}(L)P_{L-1,j}\varphi_{j}(L)\right]^{-1} \\ P_{L,j} = \rho^{-1}\left[I - H_{L,j}\varphi_{j}^{T}(L)\right]P_{L-1,j}\end{cases} \quad \text{(A.13)}$$

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