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BIVARIATE AND MULTIVARIATE COINTEGRATION AND THEIR APPLICATION IN STOCK MARKETS

Abstract. *The purpose of this article is to introduce the basic elements of what is considered to be the missing piece in the time series puzzle, cointegration. We begin by defining the spurious regression and the notion of nonstationary, the departure point for cointegrated series. We explain the meaningful core of cointegration, as equilibrium process, as well as the connection with the error correction model and with the Granger Representation Theorem. As the proof of the mentioned theorem offers a deeper understanding of the mechanisms lying behind the presented phenomena,, we decided to also provide a sketch of the proof. The article continues with procedures used to test for the existence of cointegration and to estimate the cointegration vectorial space. In order to support this, we will revise, as methodology and as well as logical deduction, the Engle Granger two stage procedure, mainly utilized in bivariate systems, and the Johansen procedure, utilized in multivariate systems. As an applicative study, we have chosen to set up a study on the stock market. The capital market having the stock exchange series evolving as Random Walk processes proves itself being an excellent candidate in testing the cointegrated systems. We have chosen three series of stock exchange indexes, from Romania, France and US, series with daily frequencies. Using unit root tests ,Dickey Fuller, we find that the three series are nonstationary, moreover each of the series is integrated of first order. Afterwards we test the existence of cointegration with the Engle Granger procedure. We find that the series BET and CAC40 are cointegrated, thus we can estimate an error correction model, and we find that about 2% of the distance between the two series is corrected daily, as we have daily observations. Eventually we run the Johanses procedure – the three series form a cointegrated system, the dimension of the cointeration space is one.*

Key Words: *cointegrated systems, spurious regression, Engle Granger two stages procedure, multivariate cointegration, Johansen procedure.*

JEL Classification: **C01, C59**

1. INTRODUCTION

If mathematics discovers relationships between deterministic at the aid of rigorous arguments, scientists soon found out that many objects are not deterministic and have to be described using probabilities. That is how mathematics developed a subfield Statistics. The birth of Econometrics came naturally, like Biometrics or Psychometrics, with time series one of its most important domains.

The article starts with reviewing the spurious regression notion (mainly a regression estimated with non-stationary data and residuals), then we introduce the concept of cointegration.

Box and Jenkins methodology (1970), suggested that removing unit roots through successive differencing ought to be necessary in order to "prepare" the series for regression analysis. This approach was criticized, as difficulties arose in inferring the long run equilibrium form the estimated model. *If deviations from the equilibrium relationship affect future changes in a set of variables, estimating a differenced model would entail a misspecification error.* (Dolado, Gonzalo, 1999).

Starting from the above mentioned idea, Granger proposed an innovative approach, pointing out that a vector of variables, stationary after differencing, could have linear combinations that are stationary in levels. In 1987, Engle and Granger were the first to formalize the idea of integrated variables, that is variables sharing an equilibrium relation which turns out to be either stationary or of a lower degree of integration than the original series. They denoted this property as "co-integration", which can be broadly defined as *"co-movements among trending variables which could be exploited to test for the existence of equilibrium relationship within a fully dynamic specification framework".* (Dolado, Gonzalo, 1999)

After sketching the proof of the Granger representation theorem and then briefly outlining the error correction model, the study continues with the estimation models. By applying the estimation procedures on the capital market, we find that stock indexes from different countries show a comovement behavior.

2. SPURIOUS REGRESSION

For many years, equations involving nonstationary variables modeling macroeconomic or financial relationships were estimated and processed using simple/multiple linear regression. But, they soon found out that testing hypothesis on coefficients using standard regression lead to a statistically significant dependence where in fact there is none. The first ones who pointed out the misleading effects of using regression on nonstationary time series were Clive Granger and Paul Newbold.

In 1974, they introduced the idea of (what will be called) spurious regression. By generating independent nonstationary series (random walks), regressing them on each other, they obtained the value of the t-statistic of the coefficient, calculated under the hypothesis that the true value of the coefficient equals zero (that is, the series are independent, as mentioned above). The authors found that the null hypothesis is rejected, that is the results show that the two series are correlated, thus not independent, as we might have expected. The experiment was conducted under the form of Monte Carlo simulation and the null hypothesis of a zero coefficient was rejected much more frequently than permitted by standard theory. These results are of great importance. They show that many relationships between nonstationary economic variables may well be spurious.

A first solution to this problem was proposed by statisticians working with time-series models. They suggested using first difference levels in regression, because most of the series used in macroeconomics are first order integrated (that is after differencing the series once, they become stationary). However, a problem arises, as economic theories are generally formulated for levels of variables rather than for differences. For example, theories of consumption are based on levels of consumption, income, etc and not on their growth rates. If one uses a model based on the first differenced of these variables will not make full use of the theories.

An alternative approach suggests removing the linear trend from the variables. Still, removing time trends assumes that the variables we study follow separate trends, not plausible giving the long-run implications.

The spurious regression is on of the implications in dealing with integrated variables, closely related with the notion of unit-roots. The presence of unit-roots gives rise to stochastic trends, characterized by that of making innovations permanent rather than transitory.

3. COINTEGRATION

Many time series are rather smooth, moving with local trends, or with long, irregular swings. However, time series that are smooth are very difficult to analyze with standard statistical methods, most of which assume that series are stationary.

Macroeconomic theory relies on forces that, although seem to have divergent trajectories, tend on the long term to have resembling trends. Econometric studies focus on equilibrium relation or the equilibrium error, that is the distance between the economic variables and the curve that gives the equilibrium relation.

Integration

The starting point for the co-integration theory is Wold's representation theorem, that is a stationary time series can be written as an infinite moving average series, generally approximated by a finite ARMA process.

We start our theoretical presentation by defining some basic elements. First, a series integrated of order d, I(d) has a stationary, invertible ARMA representation after differencing it d times. Let's note that an I(0) process has finite variance, innovations have only a temporary effect on the series' values and the autocorrelation function decreases steadily in magnitude, whereas I(1) processes have infinite variance, innovations have permanent effect on the series' values; ACF has values close to 1, even when the series goes to infinite. An I(1) series is rather smooth, with dominant long swings, as compared to an I(0) series.

Theoretical infinite variance from I (1) series comes completely from the contribution of low frequencies, that is the long run part of the series. We also note that:

$$if \quad x_t \to I(d), \quad then\ a + bx_t \to I(d), \qquad where\ a, b\ are\ constants$$

This gives:

if x_t, y_t are both $I(d)$, then the linear combination $z_t = x_t - ay_t$ will also be $I(d)$.

For x_t, y_t to be cointegrated, then the linear combination of them is stationary. In the stated relation, if $a = 1$, it means that x_t and y_t cannot drift too far apart and their difference will be *I (0)*.

We can broadly say that if the difference between a pair of integrated series can be stationary, then the two series are cointegrated: two smooth series, properly scaled can move and turn, slowly, in similar but not identical fashions, but the distance between them can be stationary. Clive Granger, in a lecture delivered in Stockholm, in Dec.2003, with the occasion of receiving the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, makes the following suggestive comparison between the concept of co-integration and a string of pearls:

"Suppose that we had two similar chains of pearls and we threw each on the table separately, but for ease of visualization, they do not cross one another. Each would represent a smooth series but would follow different shapes and have no relationship. The distance between the two sets of pearls would also give a smooth series if you plotted it.

*However, if the pearls were set in small but strong magnets, it is possible that there would be an attraction between the two chains, and that they would have similar, but not identical, smooth shapes. In that case, the distance between the two sets of pearls would give a stationary series and this would give an example of cointegration" (*C.Granger, 2003).

But the definition of cointegration in general sense is as follows:

The components of the vector x_t are cointegrated of order *d, b*, noted as *CI(d,b)* if:

i). All components of x_t are *I(d)*

ii). There exists a vector α (not zero) so that $z_t = \alpha' x_t \to I(d-b), b > 0.$

In the above relation, α is called the cointegrating vector. If x_t has N components (considering an N-dimensional space vs. two-dimensional space as above), there may be more than one cointegrating vector α, mathematical aspect that sustains the economic feature that it is possible for the variables to be governed by more than one equilibrium relations.

We will assume that there are *r* linearly independent cointegrating vectors, with $r \leq N - 1$. In this case, we will have a cointegration matrix, with *N x r* dimensions of rank *r*.

4. COINTEGRATION AND ERROR CORRECTING MODELS

The idea of the error-correction models is the following: a proportion of the disequilibrium from one period is corrected in the next period. For example, the change in price in one period might depend upon the excess demand in the previous period.

In a simple, two variable system, the classical error correction model would make connections between the change in one variable, past equilibrium errors and past changes in both variables.

In a multivariate system however we can define a general error correction representation in terms of L, the lag or backshift operator. We say that a vector x_t has an error correction representation if it can be expressed as:

$$A(L)(1-L)x_t = -\gamma z_{t-1} + \mu_t$$

where μ_t is a stationary multivariate disturbance

A(0) = I, A(1) – has finite elements, A(L) is a polynomial matrix.

$$z_t = \alpha' x_t$$

z_{t-1} is the explanatory variable, that is the disequilibrium in the previous period.

The above relation is the base of the error correction representation model, meaning that the differenced x_t can be explained starting from the disequilibrium of the variable.

Granger Representation Theorem

Let's suppose that each component of x_t is *I(1)*, this is the most common and often case analyzed. It means that changes in each component of x_t are zero mean purely nondeterministic stationary stochastic processes. The Wold representation of the process is:

$$(1-L)x_t = C(L)\varepsilon_t$$

The left side of the equation is the differenced x_t, an *I(0)* process, while its right side is a polynomial in *L*, a $MA(\infty)$ process and ε_t - white noise.

The Granger representation, first stated by Granger in 1981, it's a relationship between the error correction model and cointegration showing that cointegrated series can be represented by error correction models.

In the next paragraph we will state the theorem:

Let x_t be an *N x 1* vector with the representation:

$$(1-L)x_t = C(L)\varepsilon_t$$

x_t cointegrated with *d=1, b=1, CI(1,1)*, that is :

$$\exists z_t = \alpha' x_t \to I(0), \ \ with\ rank(\alpha') = r$$

Then, we will have:

1) Rank of C(1) is N-r.

2) There exists a vector ARMA representation:

$$A(L)x_t = d(L)x_t$$

where $A(0) = I_N$ and $A(1)$ has rank *r*,

$d(L)$ is a scalar lag polynomial with $d(1)$ finite

3) There exist *N x r* matrices α, γ of rank *r* such that:

$$\alpha' C(1) = 0; \quad C(1)\gamma = 0; \quad A(1) = \gamma\alpha'$$

4) There exists an Error correction representation with $z_t = \alpha' x_t$, an *r x 1* vector of stationary random variables:

$$A'(L)(1-L)x_t = -\gamma z_{t-1} + d(L)\varepsilon_t$$

with $A'(0) = I_N$.

5) Vector z_t is given by:

$$z_t = K(L)\varepsilon_t$$
$$(1-L)z_t = -\alpha'\gamma z_{t-1} + J(L)\varepsilon_t$$

where $K(L)$ is a *(r x N)* matrix of lag polynomials given by $\alpha' C'(L)$ with all elements of $K(1)$ finite with rank *r* and $\det(\alpha'\gamma) > 0$.

6) If a finite vector autoregressive representation is possible, it will have the form given by:

$$A(L)x_t = d(L)x_t$$
$$A'(L)(1-L)x_t = -\gamma z_{t-1} + d(L)\varepsilon_t,$$

with $d(L) = 1$ and $A(L)$ and $A'(L)$ - matrix of finite polynomials.

We choose to present here also the guiding lines of the theorem's proof , as it is relevant for a deeply understanding of its meaning.

First, we state the following lemma on determinants and adjoints of polynomial matrix. Let $G(\lambda)$ a matrix polynomial *N x N* on $\lambda \epsilon [0,1]$. The representation of the matrix could be:

$$G(\lambda) = G(0) + G(1)\lambda + \cdots + G(p)\lambda^p + \cdots$$

The rank of $G(0) = N - r,\ with\ 0 \leq r \leq N$ and if $G^*(0) \neq 0$ in

$$G(\lambda) = G(0) + \lambda G^*(\lambda)$$

Then:

$$\det\big(G(\lambda)\big) = \lambda^r g(\lambda) I_N, \qquad g(0)\ finite$$

$$adj\big(G(\lambda)\big) = \lambda^{r-1} H(\lambda)$$

where I_N is the $N\ x\ N$ identity matrix, $1 \leq rank\big(H(0)\big) \leq r, H(0)\ finite$

Proof of the Lemma:

$\mathrm{Det}\big(G(\lambda)\big)$ can be expressed in a power series in λ as

$$\mathrm{Det}\big(G(\lambda)\big) = \sum_{i=0}^{\infty} \delta_i \lambda^i$$

each δ_i is a sum of finite number of products of elements of $G(\lambda)$ and therefore is itself finite valued, having some terms from $G(0)$ and some from $\lambda G^*(\lambda)$. As rank of $G(0) = N - r$, any product with more then $N - r$ terms from $G(0)$ will be zero because this will be the determinant of a submatrix of larger order than the rank of $G(0)$. The non-zero terms will have *r* or more terms from $\lambda G^*(\lambda)$, so we can write:

$$\mathrm{Det}\big(G(\lambda)\big) = \sum_{i=0}^{\infty} \delta_i \lambda^i = \sum_{i=r}^{\infty} \delta_i \lambda^i = \lambda^r \sum_{i=r}^{\infty} \delta_i \lambda^{i-r} = \lambda^r g(\lambda)$$

and demonstrate the first part of the lemma.

For the second statement, we will express the adjoint matrix (constructed by replacing the elements of the transposed matrix with its algebraic complements) in a power series in λ:

$$\mathrm{adj}\big(G(\lambda)\big) = \sum_{i=0}^{\infty} \lambda^i H_i$$

Because the adjoint matrix is composed of order *N-1* determinants, the above relation establishes that the first *r-1* terms must be zero :

$$\mathrm{adj}\big(G(\lambda)\big) = \lambda^{r-1} \sum_{i=r-1}^{\infty} \lambda^{i-r+1} H_i = \lambda^{r-1} H(\lambda)$$

As the elements of H_{r-1} are products of finite numbers, *H(0)* must be finite.

The product of a matrix and its adjoint will always give the determinant, so:

$$G(\lambda) * \text{adj}(G(\lambda)) = \det(G(\lambda)) \Rightarrow \lambda^r g(\lambda) I_N = (G(0) + \lambda G^*(\lambda))\lambda^{r-1} H(\lambda) \Rightarrow \lambda^r g(\lambda) I_N$$
$$= G(0)\lambda^{r-1} H(\lambda) + \lambda^r G^*(\lambda) H(\lambda)$$

by equating powers of λ we get: $G(0)H(0) = 0$, that is rank of $H(0) \leq r$.

With that we have demonstrated the lemma. The proof of Granger's representation theorem is starting from the Wold representation theorem:

$$(1 - L)x_t = C(L)\varepsilon_t,$$

for a vector of *N* random variables x_t which are co-integrated: $z_t = \alpha' x_t$, α' the cointegration vector. By multiplying the above relation to α' we obtain:

$$(1 - L)\alpha' x_t = (\alpha' C(1) + (1 - L)\alpha' C^*(L))\varepsilon_t \Leftrightarrow (1 - L)z_t = (\alpha' C(1) + (1 - L)\alpha' C^*(L))\varepsilon_t$$

For z_t to be *I(0)*, $\alpha' C(1)$ must have rank *N – r* with a null-space containing all co-integrating vectors, thus $\alpha' C^*(L)$ must be an invertible Moving Average representation, we demonstrated with this the first statement of the theorem.

For the second statement, one uses the lemma with:

$$\lambda = 1 - L\,;\; G(\lambda) = C(L)\,; H(\lambda) = A(L)\,; g(\lambda) = d(L)\,.$$

We will also sketch the proof of the fourth statement. By rearranging terms in $A(L)x_t = d(L)\varepsilon_t$ we obtain:

$$[\tilde{A}(L) + A(1)](1 - L)x_t = -A(1)x_{t-1} + d(L)\,\varepsilon_t$$
$$A^*(L)(1 - L)x_t = -\gamma z_{t-1} + d(L)\,\varepsilon_t$$

5. COINTEGRATION IN A MULTIVARIATE SYSTEM

Let's review what we found out until now: a *N*-dimensional purely non-deterministic stochastic process x_t is called cointegrated of order one-one, if each component of x_t is integrated of order one and if there exist *r* linearly independent *N*-dimensional vectors α_i, such that $\alpha_i' x_t$ are stationary. The number *r* is referred to as the dimension of the cointegration space spanned by $\alpha_1, \alpha_2, \dots, \alpha_r$.

In the context of a bivariate system, in 1987 Engle and Granger proposed a two-step procedure for inferring the existence of a linear combination and for estimating a basis vector. After normalization (with an arbitrary procedure), the procedure consists of two stages. In the first stage, we run an OLS regression of one variable against the other. The estimated coefficient vector of this regression then gives a basis of the cointegrating space.

The test for cointegration is based upon the residuals of this preliminary regression: if the residuals still contain a unit root, the null hypothesis of non-cointegration cannot be rejected.

The second stage consists in estimation the corresponding error correction model where the estimated residuals represent the disequilibrium terms. Although it seems possible to extend this method to higher dimensional systems, there appears a problem in choosing cointegrating vectors which are not sensitive to normalization. There are several approaches to this problem in the literature.

Stock and Watson (1988), propose a test on the hypothesis that there are *r* cointegrating relations against the alternative that there are *r+1* on the OLS estimates of the first order serial correlation matrix of the first $N - r$ principal components of x_t.

However, in the same year Soren Johansen, based on maximum likelihood, offers an alternative, unified approach for estimating as well as testing. His idea is to analyze the canonical correlation between levels and first differences corrected for lagged differences and deterministic components like constants terms.

Let's consider an *N*-dimensional Gaussian vector autoregression of order *k+1* with constant term μ:

$$x_t = \sum_{i=1}^{k+1} a_i x_{t-i} + \mu + e_t$$

with a non-singular covariance matrix. If we re-parameterize the process, we obtain:

$$\Delta x_t = \sum_{i=1}^{k} A_i \Delta x_{t-i} + A_{k+1} x_{t-k-1} + \mu + e_t$$

with A_i defined as $-I + a_1 + \cdots + a_i$. The rank of A_{k+1} gives the dimension of the cointegration space. Under the null hypothesis that the dimension is *r*, the matrix A_{k+1} when decomposed into:

$$-A_{k+1} = \alpha\beta'$$

The matrix of cointegrating vectors β is not unique, but the space spanned by its columns vector is. The dynamics of the vector autoregression therefore depend on the "error correction" or "disequilibrium" vector $\beta' x_{t-k-1}$.

The strength of Johansen's procedure is the possibility of constructing likelihood ratio tests for the hypotheses that the space spanned by empirically determined cointegrating vectors contains a certain subspace spanned by the columns of a $N \times m$ matrix K, $1 \leq m \leq r$, or is itself contained in a space spanned by the columns of matrix H. The test statistics are distributed as chi-squared.

Economic models can then be tested not only by exploring the number of cointegrating relations but also the implications about the cointegrating space itself.

6. APPLICATION

Our goal is to test for cointegration using the Engle-Granger procedure and to test for multivariate cointegration. We take 497 observations representing values of stock indexes from Romania (BET), France (CAC40) and US (Dow-Jones).

The series are on a daily frequency and they span the period 03.01.2006 – 20.12.2007. First we will generate the natural log of the series, which we will use in our analysis. We will test stationarity and obtain that all three series are I(1) variables. The graph of the three series before differencing suggests AR(1) representations:

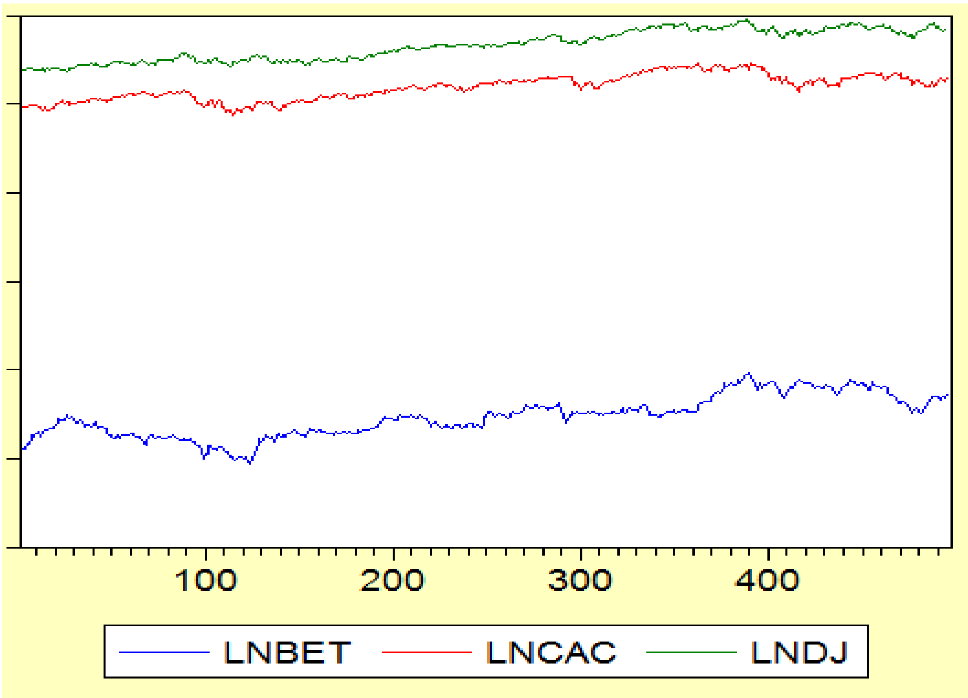

Figure 1 – Graphs of the three stock indexes series, before differencing

Performing Dickey-Fuller tests on each of the series, we cannot reject the hypothesis that every series has a unit root:

Augmented Dickey-Fuller Unit Root Test on LNBET

Null Hypothesis: LNBET has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=17)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.755873	0.4025
Test critical values:	1% level	-3.443307	
	5% level	-2.867147	
	10% level	-2.569818	

Figure 2 – Output table of the Dickey-Fuller unit root test for BET index

One observes that the values for the probability and t-statistic are very strong in accepting the null hypothesis, that is the series have unit roots. Consequently, the differenced series are stationary. This affirmation is sustained by the Dickey Fuller test (probability zero – the null hypothesis is rejected, the first differences of the series are stationary. This result is also sustained by the graphic of the autocorrelation and the partial autocorrelation function, without peaks or values statistically significant from zero:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.067	0.067	2.2165	0.137
		2	-0.041	-0.046	3.0641	0.216
		3	0.042	0.049	3.9660	0.265
		4	-0.036	-0.045	4.6307	0.327
		5	-0.019	-0.009	4.8107	0.439
		6	-0.001	-0.005	4.8114	0.568
		7	0.016	0.019	4.9459	0.667
		8	-0.007	-0.010	4.9711	0.761
		9	-0.022	-0.020	5.2137	0.815
		10	-0.073	-0.074	7.9318	0.635
		11	0.090	0.102	12.047	0.360
		12	0.097	0.080	16.886	0.154
		13	0.035	0.038	17.521	0.177
		14	-0.036	-0.052	18.189	0.198
		15	0.043	0.051	19.160	0.207
		16	0.000	-0.005	19.160	0.260
		17	0.058	0.080	20.882	0.232
		18	-0.034	-0.060	21.476	0.256
		19	0.007	0.020	21.502	0.310

Figure 3 – Autocorrelation Function and Partial Autocorrelation Function of the differenced BET index series

In the next section we will test for cointegration using the basic Engle Granger procedure for two series. We will take two stock indexes: ln(BET) and ln(CAC40), both I(1). We form a group with the two series and perform the first stage regression, that is regressing one series over the other:

Dependent Variable: LNBET
Method: Least Squares
Date: 03/07/00 Time: 23:38
Sample: 1 497
Included observations: 497

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNCAC	1.274973	0.044493	28.65530	0.0000
C	-4.222256	0.382390	-11.04175	0.0000

R-squared	0.623896	Mean dependent var	6.734875
Adjusted R-squared	0.623136	S.D. dependent var	0.114555
S.E. of regression	0.070324	Akaike info criterion	-2.467379
Sum squared resid	2.448036	Schwarz criterion	-2.450443
Log likelihood	615.1436	F-statistic	821.1260
Durbin-Watson stat	0.071411	Prob(F-statistic)	0.000000

Figure 4 – Output table of the regression between BET and CAC40 indexes

The results in the output table show that the evolution of BET index can be explained by that of the CAC40. This is only the first part of the test. From this regression we save the first-stage residuals.

The next step in the procedure is to run an ADF test on the residuals to determine if it is stationary. The results show that the residual itself is stationary:

Augmented Dickey-Fuller Unit Root Test on RES

Null Hypothesis: RES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=17)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.965305	0.0390
Test critical values:	1% level	-3.443307	
	5% level	-2.867147	
	10% level	-2.569818	

Figure 5 – Output table of the Dickey-Fuller Unit Root test for the residuals of the above regression

The probability under 5% threshold determines the rejection of the null hypothesis (the series of residuals does not have a unit root), thus the residuals are I(0) and LNBET and LNCAC are cointegrated.

The relationship between the two can be expressed as an error correction model (ECM), in which the error term from the OLS regression, lagged once, represents the error correction term. The ECM, considering DLNBET the dependent variable is:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLBET(-1)	0.077753	0.044781	1.736286	0.0831
DLCAC	0.095431	0.058155	1.640992	0.1014
DLCAC(-1)	-0.055071	0.058367	-0.943519	0.3459
RES(-1)	-0.021414	0.008907	-2.404065	0.0166
C	0.000547	0.000620	0.882129	0.3781

Figure 6 – Estimation of the Error Correction Model

The coefficient of the lagged residuals acts as a measure of the speed of adjustment back to equilibrium following an exogenous shock. From the output table we can deduce that about 2% of the “disequilibrium”, the “distance” between the two indexes is corrected each day, as we have daily data.

Multivariate Cointegration

In this case, one of the approaches to test for cointegration is by using a Vector Autoregressive (VAR) approach.

We assume by this that all the variables in the model are endogenous, although it is possible also to include exogenous variables, even though these do not have the role of dependent variables. We will use the Johansen Maximum Likelihood procedure, which gives the possibility to have more than a single cointegration relationship and determine the number of cointegrating vectors. For this test we will use all three indexes.

We will start by determining the dimension of the cointegration space, using the cointegration Johansen test, using the trend assumption version. We find that the hypothesis that there is no cointegration vector is rejected at 1% level, that is with a 99% probability, very accurate result. Also, the hypothesis that there are at most two cointegrating vectors is rejected, this at time at 5% level.

The output table gives the following result: both at 5% and 1% significance levels, there is one cointegrating vector for the three series, that is $r = 1$. We have the matrix made up by the eigenvectors (corresponding to the largest eigenvalue). Below it is also represented the estimated alpha matrix:

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.086551	44.35866	23.78	28.83
At most 1	0.012229	6.029215	16.87	21.47
At most 2 *	0.008659	4.261361	3.74	6.40

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s) at both 5% and 1% levels

Unrestricted Adjustment Coefficients (alpha):

D(LNBET)	-0.000479	-0.001452	-0.000282
D(LNCAC)	0.002469	-1.93E-05	-0.000582
D(LNDJ)	-0.001335	0.000119	-0.000662

Figure 7 – Johansen's output result

7. CONCLUSIONS

In practice, many pairs of macroeconomic series appear to have the property of cointegration, as suggested by economic theory. The cointegrated variables can be considered to be generated by the error correction model, in the sense that changes of one of the series is explained in terms of the lag of the difference between the series possibly after scaling, and lags of the difference of each series. Data generated form such a model are sure to be cointegrated. The error correction model is important in making the concept of cointegration practically and useful.

We tested for cointegration (both bi- and multivariate cointegration) three stock indexes daily series, spanning a period of three years. We found that the series are cointegrated, thus there is evidence of a long-run relationship between the variables, as one might expect considering the specific capital market behavior.

In the end, we may conclude, without exaggerate, that cointegration is "*the missing piece in the approach to modeling groups of series*" (C. Granger, 2003). These models have been used with success to provide short and medium-term forecasts for important macro-variables, such as consumption, income, investments, unemployment, all integrated series.

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