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THE INFLUENCE OF THE ABSOLUTE RISK AVERSION COEFFICIENT ON CHOOSING THE OPTIMAL PORTFOLIO

Abstract. *The paper examines changes in the optimal proportions of income or wealth invested in a safe active and in a risky active by an expected utility maximizing economic agent (investor). We will use some local measures of risk aversion to derive the necessary and sufficient conditions for the problem of choosing the optimal portfolio. We will present the relationship between the coefficient of absolute risk aversion and the return of the safe asset and we will derive some results concerning this relationship. We will show that, if the absolute risk aversion coefficient is an increasing function of income, then the return of the safe asset and the amount invested in the risky asset change in opposite directions. Finally, we will present an alternative way of analyzing agent's behavior toward risk, the non-neutrality measure of risk aversion and we will derive a measure of the global approach to the neutrality.*
Keywords: *uncertainty, risk aversion, absolute risk aversion, optimal portfolio, risky and safe assets, non-neutrality measure of risk aversion.*

JEL Classification: D 81, G 11

1. Introduction

Since Arrow (1965) first posed and studied the two-asset portfolio problem, much work has been done on the decision problems under uncertainty. A major problem - in the theory of portfolio selection - concerns the relationship between an investor's optimal mix of assets and changes in the probability distribution of return for one or more of the assets in the portfolio.

In an earlier paper, Cass and Stiglitz (1972) established theorems concerning the relationship between relative risk aversion, the relative allocation to different assets, statistical properties of the rate of return and the certainty equivalent rate of return. They also established corresponding theorems concerning absolute risk aversion, the absolute allocation to certain assets, statistical properties of the total return to a portfolio and the certainty equivalent value of the final wealth. In the same paper, there are presented theorems describing the effect of changes in wealth on portfolio for the special case of two assets, one of which is perfectly safe; these theorems are extended to situations where there are more than two assets.

Fishburn and Porter (1976) examined changes in the optimal mix of investment capital between one safe asset and one risky asset in response to changes in the rate of return of the safe asset and shifts in the distribution function of return for the risky asset for a risk averse expected utility maximizing investor. Given an initial position in which both assets have positive allocations, an increase in the rate of return of the safe asset will increase the allocation to the safe asset if absolute risk aversion is nondecreasing or if proportional risk aversion never exceeds unity.

Kira and Ziemba (1980) investigated the effect on the demand for a risky asset when there are changes in either initial wealth or one of the asset return distributions in two asset expected utility portfolio problems. They presented the necessary and sufficient conditions for first, second and third degree stochastic dominance shifts to yield increasing demand for the risky asset, when choosing between a risky and a safe asset. Their analysis extended and generalized the results of Arrow, Cass and Stiglitz and Fishburn and Porter.

Cheng et al. (1987) analyzed a class of problems where the agent' choices among uncertain prospects can be represented by a von Neumann-Morgenstern utility function, and the agent's random wealth is determined by a decision variable, an exogenous parameter and an exogenous random variable that is chosen so as to maximize the expected utility of wealth. They presented a complete solution of the comparative statics problem for the basic two-asset portfolio problem and gave necessary and sufficient conditions for qualitative results both in the case of parameter changes and in the case of stochastically dominant shifts in the random variable. In fact, the conditions required are reduced to restrictions on the behavior of one of the three local measures of risk aversion associated with a von Neumann-Morgenstern utility function – absolute, relative or partial relative risk aversion.

Hadar and Seo (1988) considered a portfolio with two risky assets and derived the conditions (which are both necessary and sufficient conditions) under which the proportion of a given asset in the optimal portfolio of a risk averse agent is at least as large as some given proportion. They analyzed portfolios in which one asset stochastically dominates another asset, either in first degree or in a mean-preserving spread. An optimal portfolio should contain at least as much of the dominating asset than the other asset if and only if the investor's utility function satisfies certain restrictions. The main results were extended to the case of n-asset portfolio.

In a recent paper, Marinescu et al. (2008) analyzed the influence of the changes in taxation when choosing the optimal portfolio with two assets, one risky asset and one safe asset. They showed that when the absolute risk aversion coefficient is decreasing the amount invested in the risky active increase as a result of the income and substitution effects.

The paper is organized as follows. In Section 2 we briefly present some concepts regarding agent's attitude toward risk; these definitions of some local measures of risk aversion will be used in the next sections. Section 3 describes the central elements of the model: we consider a two-asset portfolio problem, where one asset is safe, and the other one is risky. This section also presents our analysis of choosing the optimal portfolio, the features of the optimal solution, deriving the necessary and sufficient conditions for the mentioned problem. In Section 4, we study an alternative way of measuring the risk aversion, using the non-neutrality measure. We also define a measure of the global approach to the risk neutrality. Section 5 discusses the main findings and concludes the paper.

2. Some Local Measures of the Risk Aversion

In the following sections we will use some local measures of the risk aversion, namely: the coefficient of absolute risk aversion, the coefficient of relative risk aversion, risk premium and the non-neutrality measure of risk aversion. We will define below these concepts.[6, 9, 11]

Definition 1. Given a (twice-differentiable) utility function $U(\cdot)$ for money, the (Arrow-Pratt) *coefficient of absolute risk aversion* at a given level of wealth, $\bar{x}$, is defined as $r_a(\bar{x}) = -\dfrac{U''(\bar{x})}{U'(\bar{x})}$.

Definition 2. Given a (twice-differentiable) utility function $U(\cdot)$ for money, the (Arrow-Pratt) *coefficient of relative risk aversion* at a given level of wealth, $\bar{x}$, is defined as $r_r(\bar{x}) = -\dfrac{\bar{x}U''(\bar{x})}{U'(\bar{x})}$.

Definition 3. For a given random variable *z*, the associated *risk premium* denoted by ρ_z satisfies the following relation:

$$U[E(z) - \rho_z] > E[U(z)]$$

Definition 4. The *distance from the neutrality* (the *non-neutrality measure*) for a risk averse agent with the initial endowment x_0 is:

$$\rho_d(x_0) = \frac{U(x_0)}{U'(x_0)} - x_0$$

The risk aversion indexes (coefficients) are derived using a utility function $U(\cdot)$, but they are invariant to any linear and positive transformation of the utility function: $U \to aU + b$, with $a > 0$ and $b \in R$.

Indeed, if we consider the utility function $V(x) = aU(x) + b$, then $V'(x) = aU'(x)$ and $V''(x) = aU''(x)$. Therefore:

$$-\frac{U''(x)}{U'(x)} = -\frac{V''(x)}{V'(x)}, \forall x > 0,$$

or

$$r_a(U, x) = r_a(V, x)$$

This means that these coefficients of risk aversion are intrinsically associated with the agent's preferences and not with the particular form that was chosen for the utility function.

On the other hand, the risk aversion is defined by the concavity of the utility function. There is a qualitative differentiation regarding risk aversion (an agent could be more risk averse than another one). Then, the functions concavity differs: we could say that "The utility function $U^1(\cdot)$ is *more concave* (*more-risk-averse-than*) than the utility function $U^2(\cdot)$." Mathematically, this property is written as:

Definition 5. The utility function $U^1(\cdot)$ is *more concave* (*more-risk-averse-than*) than the utility function $U^2(\cdot)$ if there exists a concave function $h : R \to R$ so that $U^1(x) = h(U^2(x)), \forall x \geq 0$.

Arrow and Pratt (1964) argued that there are three different ways for ordering two economic agents with respect to the risk aversion.

Theorem 1. (ARROW-PRATT Theorem), [6]***:*** Given two utility functions, $U^1(\cdot)$ and $U^2(\cdot)$, twice differentiable, strictly increasing and concave, the following properties are equivalent:

i) $r_a^1(x) \geq r_a^2(x)$, for any $x > 0$

ii) $\rho_1(x) \geq \rho_2(x)$, for any $x > 0$

iii) There exists a concave function $h : R \to R$ so that $U^1(x) = h(U^2(x)), \forall x > 0$

Remark*:* Note that the original proof can be found in [6], but another interesting proof is given by Marinescu et al. [11]

3. The Problem of Optimal Portfolio

We will consider an investor (an economic agent) with an initial income (endowment) s_0. He has two investment opportunities: he can invest either in one active without risk, or in one risky active. We will denote by *r* the rate of return for the riskless active. The other active, the risky one, yields a random return represented by the random variable *X*, with mean and variance finite.[8]

The values of the random variable belong to the support $[x_1, x_2]$, and the density function is $f(\cdot)$.

We will also consider that the proportion invested in the risky active is *y*. Then, at the end of the period, the agent will get from this investment:

$$s_0 y(1+x),\ x \in [x_1, x_2] \tag{1}$$

Investing in the active without risk the amount $(1-y)s_0$, this yields at the end of the period:

$$(1-y)s_0(1+r) \tag{2}$$

The final wealth or income (at the end of the period) is obtained by simply adding the two expressions from (1) and (2). So the agent will obtain:

$$S(x,y) = s_0 y(1+x) + (1-y)s_0(1+r) = s_0[1+r+y(x-r)] \tag{3}$$

where $x \in [x_1, x_2]$ is one of the possible values of the random variable *X*.

The investor's attitude toward risk is represented by the von Neumann-Morgenstern utility function $U(\cdot)$, with the well-known properties.

Suppose that the amount invested in the risky active is strictly positive; hence *y* satisfies the condition $0 < y < 1$. Therefore, we will find an interior optimum for the optimization problem.

The economic agent's problem is finding the optimal portfolio – that is maximizing the expected value of the final amount $S(x,y)$ with respect to *y*. Thus, the following concave nonlinear optimization program must be solved:

$$\underset{0<y<1}{Max} \int_{x_1}^{x_2} U\big(S(x,y)\big) f(x)dx \tag{4}$$

Now, we can state the following result:

Proposition 1. The necessary and sufficient conditions for optimum are that the return of the riskless active *r* satisfies the following property:

$$\int_{x_1}^{x_2} (x-r)U'\big(S(x,y)\big) f(x)dx = 0 \tag{5}$$

or

$$\frac{\int_{x_1}^{x_2} xU'\big(S(x,y)\big) f(x)dx}{\int_{x_1}^{x_2} U'\big(S(x,y)\big) f(x)dx} = r \tag{5'}$$

Proof:

We first note that the first order conditions (the necessary conditions) are also sufficient conditions in this situation, because of the utility function's properties. We will use the theorem for differentiation with respect to a parameter in order to derive the necessary conditions. Hence, we have:

$$\frac{d}{dy}\left(\int_{x_1}^{x_2} U\left(S(x,y)\right) f(x)dx\right) = 0$$

or

$$\int_{x_1}^{x_2} S'_y \cdot U'\left(S(x,y)\right) f(x)dx = 0$$

Using the relation (3), by differentiation with respect to *y*, we have $S'_y = s_0(x-r)$ and inserting this expression in the above relation we obtain:

$$\int_{x_1}^{x_2} s_0(x-r) \cdot U'\left(S(x,y)\right) f(x)dx = 0$$

Therefore, rearranging the terms, we have:

$$\int_{x_1}^{x_2} xU'\left(S(x,y)\right) f(x)dx = r\int_{x_1}^{x_2} U'\left(S(x,y)\right) f(x)dx$$

This is equivalent to the condition (5) or (5').

The monotonicity property of the absolute risk aversion coefficient induces, at the optimum, an inferior limit for the rate of return obtained when investing a part of the initial wealth in the safe active. The result is summarized below.

Proposition 2. If the coefficient of absolute risk aversion, $r_a(\cdot)$, is an increasing function, than, at the optimum, the rate of return *r* verifies the following inequality:

$$r \geq \frac{\int_{x_1}^{x_2} xU''\left(S(x,y)\right) f(x)dx}{\int_{x_1}^{x_2} U''\left(S(x,y)\right) f(x)dx} \tag{6}$$

Proof:

From *Definition 1*, the coefficient of absolute risk aversion at a given level of money $S(x,y)$, is $r_a(S) = -\frac{U''(S)}{U'(S)}$.

We consider the set $A = \{x | x - r \geq 0,\ x \in [x_1, x_2]\}$.

Then $s_0(1+r) \leq s_0\left(1 + r + y(x-r)\right)$ and using the monotonicity property of the absolute risk aversion's coefficient, we get:

$$r_a\left[s_0(1+r)\right] \geq r_a\left[s_0\left(1+r+y(x-r)\right)\right] \qquad (7)$$

for any $x \in A$.

If $x \in C_{[x_1,x_2]}(A)$, or $x-r<0$ we have:

$$r_a\left[s_0(1+r)\right] < r_a\left[s_0\left(1+r+y(x-r)\right)\right] \qquad (8)$$

Then, for any $x \in [x_1, x_2]$, using the relations (7) and (8) we obtain:

$$(x-r)r_a\left[s_0(1+r)\right] \geq (x-r)r_a\left[s_0\left(1+r+y(x-r)\right)\right]$$

or

$$(x-r)\frac{-U''\left(s_0(1+r)\right)}{U'\left(s_0(1+r)\right)} \geq (x-r)\frac{-U''\left(S(x,y)\right)}{U'\left(S(x,y)\right)}$$

Rearranging the terms in the above inequality and multiplying both sides by $f(x)$ yields:

$$(x-r)U''\left(S(x,y)\right)f(x) \geq -(x-r)r_a\left[s_0(1+r)\right]U'\left(S(x,y)\right)f(x) \qquad (9)$$

We will next use the monotonicity property of integration and the result of *Proposition 1* in (9), to finally get:

$$\int_{x_1}^{x_2}(x-r)U''\left(S(x,y)\right)f(x)dx \geq -r_a\left[s_0(1+r)\right]\int_{x_1}^{x_2}(x-r)U'\left(S(x,y)\right)f(x)dx = 0$$

Therefore, we can rewrite the above relation as:

$$\int_{x_1}^{x_2} xU''\left(S(x,y)\right)f(x)dx \geq r\int_{x_1}^{x_2} U''\left(S(x,y)\right)f(x)dx$$

or

$$r \geq \frac{\int_{x_1}^{x_2} xU''\left(S(x,y)\right)f(x)dx}{\int_{x_1}^{x_2} U''\left(S(x,y)\right)f(x)dx}.$$

Next, we are interested in determining how the proportion of the amount invested in the risky active changes with respect to changes of the rate of return. We will discuss these aspects in the following two theorems.

Theorem 2. If the absolute risk aversion coefficient is a decreasing function of income, then the derivative of the amount invested in the risky active, *y*, with respect to the rate of return *r* (of the active without risk) is given by:

$$\frac{dy}{dr}=\frac{\int_{x_1}^{x_2} U'\big(S(x,y)\big)f(x)dx - s_0(1-y)\int_{x_1}^{x_2}(x-r)U''\big(S(x,y)\big)f(x)dx}{s_0\int_{x_1}^{x_2}(x-r)^2U''\big(S(x,y)\big)f(x)dx}$$

Proof:

We will use the necessary and sufficient conditions (5), derived in *Proposition 1*:

$$\int_{x_1}^{x_2}(x-r)U'\big(S(x,y)\big)f(x)dx=0$$

By differentiation with respect to r and y, we obtain:

$$\left[\int_{x_1}^{x_2}(x-r)U''\big(S(x,y)\big)s_0(x-r)f(x)dx\right]dy+\left[\int_{x_1}^{x_2}(-1)U'\big(S(x,y)\big)f(x)dx+\right.$$
$$\left.+\int_{x_1}^{x_2}(x-r)U''\big(S(x,y)\big)s_0(1-y)f(x)dx\right]dr=0$$

Thus,

$$\frac{dy}{dr}=\frac{\int_{x_1}^{x_2} U'\big(S(x,y)\big)f(x)dx - s_0(1-y)\int_{x_1}^{x_2}(x-r)U''\big(S(x,y)\big)f(x)dx}{s_0\int_{x_1}^{x_2}(x-r)^2U''\big(S(x,y)\big)f(x)dx}$$

The sign of this derivative $\frac{dy}{dr}$ could not be well-defined, although the denominator is strictly negative. This is due to the fact that the term $\int_{x_1}^{x_2}(x-r)U''\big(S(x,y)\big)f(x)dx$ could be either positive, or negative, its sign being dependent on the relation between the two returns: the one of the risky active and the return of the active without risk.

Theorem 3. The sufficient condition for the sign of the derivative $\frac{dy}{dr}$ to be negative is that the absolute risk aversion coefficient must be an increasing function.

Proof:

Using the monotonicity property of the function $r_a(\cdot)$, the relation (9) can be written as:

$$(x-r)U''\big(S(x,y)\big)f(x) \le -(x-r)r_a\left[s_0(1+r)\right]U'\big(S(x,y)\big)f(x)$$

Integrating over the interval $\left[x_1, x_2\right]$ and using the result in *Proposition 1*, we obtain:

$$\int_{x_1}^{x_2}(x-r)U''\big(S(x,y)\big)f(x)dx \le 0$$

Hence, the numerator of the derivative $\frac{dy}{dr}$ is positive, and therefore $\frac{dy}{dr}<0$ holds.

4. The risk aversion and the non-neutrality measure

Usually, the agent's attitude toward risk and money was studied using the total utility function, $U(\cdot)$. If we consider a certain level of initial endowment (a fixed level of income), then using only the total utility function it is enough for deriving the agent's attitude. On the other hand, if we consider the possibility of a change in the income's level, then this approach is no more useful. This is because the total utility function misrepresents the attitude toward risk and money with respect to the initial income.

There are two different approaches:

i) *The static approach* – where there is no possibility for varying the level of initial income. In this case, using the total utility function is the appropriate way for characterizing the agent's behavior.

ii) *The dynamic approach* – where the level of the initial income can vary. In this case, it is necessary to use also the marginal utility function, $U'(\cdot)$, for characterizing the agent's attitude toward changes in the level of the initial income (endowment).

The above analysis intuitively yields to a definition for the "equivalence" of the behavior or attitude for two agents, referring to the initial endowment level. This definition corresponds to the definition of marginal utility.

We will consider two economic agents, having the preferences represented by the corresponding utility functions, $U^1(\cdot)$ and $U^2(\cdot)$, strictly increasing and strictly concave.

If the first economic agent is more risk averse than the second one (in the sense of *Theorem 1 -ARROW-PRATT Theorem*), then the distance toward neutrality for the first agent is greater than the corresponding distance for the second agent. Before proving this result, we need one more result given in the next Proposition.

Proposition 3. Let $f: R_+ \to R_+$ be a strictly concave and strictly increasing function, with $f(0) = 0$. Then, $f'(x) < \dfrac{f(x)}{x}$.

Proof:

Using Lagrange's Theorem of finite increases, there exists a $\xi \in (0, x)$, such that, $f(x) - f(0) = (x - 0) f'(\xi)$, for every $x > 0$

or
$$f'(\xi) = \frac{f(x)}{x}.$$

But the function $f'(\cdot)$ is strictly decreasing (note that *f* is strictly concave, i.e. $f''(\cdot) < 0$) and as $\xi < x$, it follows that $f'(x) < f'(\xi)$. Therefore:

$$f'(x) < \frac{f(x)}{x}, \forall x > 0.$$

We now turn to the problem of characterizing the agents' risk aversion using the non-neutrality measure.

Theorem 4. We consider two economic agents, 1 and 2, with the utility functions $U^1(\cdot)$ and $U^2(\cdot)$, and assume that the first agent is more averse than the other agent (in the sense of the Arrow-Pratt Theorem). Then $\rho_d^1(x) > \rho_d^2(x), \forall x > 0$.

Proof:

Let $h: I \subseteq R \to R, h = U^1 \circ \left(U^2\right)^{-1}$ be a strictly increasing and concave function, i.e. satisfying the following properties:

$h'(x) > 0, h''(x) < 0, \forall x \in I$. We assume also that $h(0) = 0$.

By definition, we then have $U^1(x) = h\left(U^2(x)\right), \forall x > 0$ and:

$$\left(U^1\right)'(x) = h'(y) \cdot \left(U^2\right)'(x), \text{ where } y(x) = U^2(x).$$

The considered function *h* verifies the assumptions in *Proposition 3*. Therefore, we must have:

$$h'(y) < \frac{h(y)}{y}, \forall y \in I$$

and so:

$$\left(U^1\right)'(x) < \frac{h(y)}{y} \cdot \left(U^2\right)'(x), \forall x > 0$$

Noting that $y = U^2(x)$, we obtain $U^1(x) = h(y)$. The above relation can be written as:

$$\left(U^1\right)'(x) < \frac{U^1(x)}{U^2(x)} \cdot \left(U^2\right)'(x), \forall x > 0$$

Rearranging the terms, we obtain:

$$\frac{U^1(x)}{\left(U^1\right)'(x)} - x < \frac{U^2(x)}{\left(U^2\right)'(x)} - x, \forall x > 0$$

meaning that: $\rho_d^1(x) > \rho_d^2(x), \forall x > 0$.

Turning back to the problem of measuring risk aversion using the distance toward neutrality, there is another way of doing this, but not for a single value x_0. We will further investigate this problem for a whole interval $[0, x_0]$.

We consider two economic agents:

- the risk averse agent A, who has an initial endowment x_0 and whose concave utility function is $U(\cdot)$;
- the risk neutral agent B, who has a linear utility function $V(\cdot)$ satisfying also:

$$V(x) = \frac{U(x_0)}{x_0} x$$

The two considered utility functions satisfy also the following relation:

$$U(x_0) = V(x_0)$$

The figure below illustrates the two utility functions.

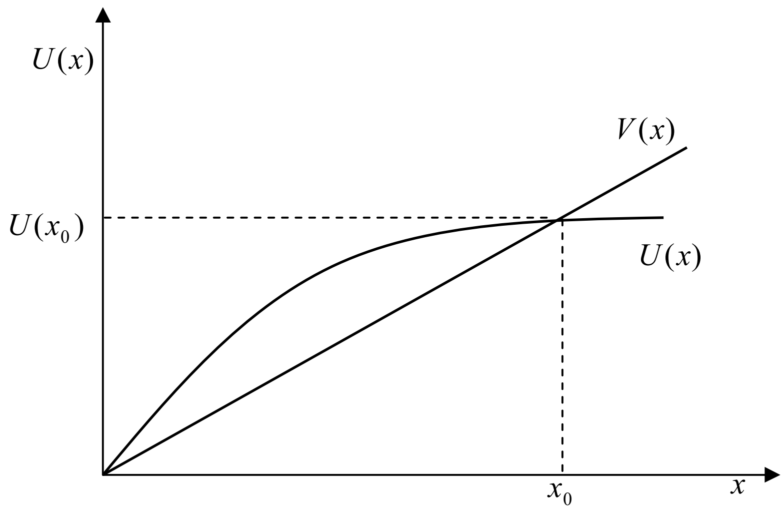

Figure 1. ***The two utility functions and the distance from neutrality***

The smaller the area between the graphs of the functions $U(x)$ and $V(x)$ – over the interval $\left[0, x_0\right]$ - is, the agent A has a smaller distance toward neutrality. We can express this using the ratio of the area between the graph of $U(x)$ and *'x'* axis and the respective area of $V(x)$, both calculated over the interval $\left[0, x_0\right]$. Mathematically, it is represented by:

$$r_g(x_0) = \frac{\int_0^{x_0} U(x)dx}{\frac{1}{2}U(x_0)x_0} \tag{10}$$

Note that the values of this ratio satisfy:

$$1 \le r_g(x_0) \le 2$$

The closer is this ratio to 1, the more risk neutral is the agent on the interval $\left[0, x_0\right]$.

Next, we will derive the global distance from risk neutrality as being the following limit:

$$\overline{r}_g = \lim_{x_0 \to \infty} r_g(x_0)$$

Using relation (10), this limit is determined as:

$$\overline{r}_g = \lim_{x_0 \to \infty} \frac{\int_0^{x_0} U(x)dx}{\frac{1}{2}U(x_0)x_0} = \lim_{x_0 \to \infty} \frac{U(x_0)}{\frac{1}{2}\left(U'(x_0)x_0 + U(x_0)\right)}$$

or

$$\overline{r}_g = \frac{1}{\frac{1}{2} + \frac{1}{2}\lim_{x_0 \to \infty}\left(U'(x_0) : \frac{U(x_0)}{x_0}\right)}$$

We will use the following definition of the utility function elasticity:

$$E_{U(x)/x}\Big|_{x=x_0} = U'(x_0) : \frac{U(x_0)}{x_0}$$

Therefore, the limit can be expressed as:

$$\overline{r}_g = \frac{1}{\frac{1}{2} + \frac{1}{2}\lim_{x_0 \to \infty} E_{U(x)/x}\Big|_{x=x_0}}$$

But any concave function $U(\cdot)$, with $U(\cdot 0)=0$, verifies the assumptions in Proposition 3, and hence this function satisfies:

$$U'(x_0) < \frac{U(x_0)}{x_0},\ \forall x_0 > 0$$

Then, we obtain:

$$0 < E_{U(x)/x}\Big|_{x=x_0} < 1, \forall x_0 > 0$$

And thus the limit $\lim\limits_{x\to x_0} E_{U(x)/x}\Big|_{x=x_0}$ exists and it is finite.

It is worth to briefly comment this result: the closer to 1 is the value of the global distance from neutrality, the more risk neutral is the agent having preferences represented by the concave utility function $U(\cdot)$.

5. Conclusions

We considered a basic model of two-asset portfolio, with one risky asset and one safe asset. We were interested in determining the optimal proportions of income or wealth invested in these two actives by an expected utility maximizing risk averse economic agent. Using some local measures of risk aversion, we derived the necessary and sufficient conditions for the problem of choosing the optimal portfolio. Supposing that the amount invested in the risky active is strictly positive, the optimization problem has an interior solution, whose features are summarized in the paper. We derived necessary and sufficient conditions for the problem.

We investigated the relationship between the coefficient of absolute risk aversion and the return of the safe asset. We found that, if the coefficient of absolute risk aversion is an increasing function, then the return of the safe asset is bounded below.

We also derived a relationship between the amount invested in risky active and the rate of return for the safe asset and we analyzed the sign of the corresponding derivative, using the monotonicity property of the absolute risk aversion coefficient. We showed that, if the absolute risk aversion coefficient is an increasing function of income, then the return of the safe asset and the amount invested in the risky asset change in opposite directions (the derivative is negative). Finally, we presented an alternative way of analyzing agent's behavior toward risk, the non-neutrality measure of risk aversion and we derived a measure of the global approach to the neutrality.

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