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## **THE NPV CRITERION FOR VALUING INVESTMENTS UNDER UNCERTAINTY**

**Abstract.** *Corporate finance theory has established four criteria for the valuation and selection of investments: the net present value (NPV) criterion, the internal rate of return (IRR) criterion, the payback period (PP) criterion, the profitability index (PI) criterion and the excess return (ER) criterion. Each of them has its advantages and disadvantages, which we will not insist upon in this paper. Instead, we shall emphasize some interesting properties of these indicators, based on the hypothesis of the (almost) normal probability distribution of the free cash flows generated by the investment. For this purpose we have considered 15 scenarios and we have simulated possible free cash flows generated by a 10-year investment project. Our study has led to the conclusion that the indicators NPV, IRR, PP, PI and ER are approximately normally distributed, which simplifies substantially the analysis of investments under uncertainty conditions and enables us to build confidence intervals and to estimate probabilities for the lower limits of the aforementioned indicators. At the same time, our study addresses the controversial issue of computing the expected value and the standard deviation of the net present value of an investment under conditions of uncertainty.*

**Keywords:** *net present value, cash flow, discount rate, uncertainty, normal distribution, confidence interval, expected value, standard deviation of the net present value.*

### **JEL Classification G32**

#### **INTRODUCTION**

This paper aims at determining the expected value and the standard deviation of the net present value of an investment under uncertainty, the probability that investors will accept the respective project and confidence intervals for the net present value with different probabilities of occurrence. In this respect we shall consider the following investment project:

- the initial value of the investment is  $I_0 = 1.000.000$  EUR;
- the expected lifetime of the project is  $N = 10$  years;

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- the discount rate is  $k = 12\%$ ;
- the investment has no residual value ( $RV = 0$ ).

The formula of the net present value (NPV) is based on the well known principle of discounting the future free cash flows generated by the investment:

$$NPV = \sum_{i=1}^{10} \frac{FCF_i}{(1+k)^i} + \frac{RV}{(1+k)^{10}} - I_0 \quad (1)$$

where  $FCF_i$  is the free cash flow that the project generates in year  $i$  ( $i = 1, 2, 3, \dots, 10$ ).

**NPV** is the discounted value of the future free cash flows ( $FCF_i$ ) that the project will generate over its lifetime, less the initial capital invested ( $I_0$ ). The NPV criterion is based on the hypothesis of an “unsaturated money market”, according to which the cash flows can be indefinitely reinvested at a rate of  $k$  in order to generate future cash flows. Corporate finance theory is yet to identify a more robust criterion to select investments than the maximization of NPV.

We shall henceforth consider that the free cash flows are approximately normally distributed, of expected value and standard deviation given as follows:

Year $i$	1	2	3	4	5	6	7	8	9	10
$E(FCF_i)$	175,000	200,000	240,000	280,000	300,000	350,000	380,000	290,000	255,000	200,000
$\sigma(FCF_i)$	48,621	55,395	66,868	77,805	83,546	97,141	105,168	80,514	70,185	55,918

We have taken into account 15 possible scenarios whose probabilities of occurrence are presented in the following table, together with the values of FCF’s (all sums are expressed in EUR):

Scenario	$p_i$	FCF1	FCF2	FCF3	FCF4	FCF5	FCF6	FCF7	FCF8	FCF9	FCF10
1	0.02	46,333	52,952	63,542	74,133	79,428	95,408	100,609	76,780	70,233	52,952
2	0.03	80,255	91,720	110,064	128,408	137,580	160,510	174,268	132,994	123,367	91,720
3	0.04	101,343	115,820	138,984	162,148	173,730	202,685	220,058	167,939	149,775	115,820
4	0.05	110,285	126,040	151,248	176,456	189,060	220,570	239,476	182,758	161,080	126,040
5	0.06	118,684	135,639	162,767	189,894	203,458	237,368	257,714	196,676	172,940	135,639
6	0.07	125,685	145,800	172,368	201,096	215,460	251,370	277,777	208,195	183,141	143,640
7	0.09	165,393	189,020	226,824	264,628	284,064	330,785	363,104	278,300	241,001	189,020
8	0.15	175,114	200,509	239,500	283,401	301,542	353,200	380,127	291,029	255,763	199,996
9	0.12	191,200	220,112	266,485	305,920	329,998	383,257	414,978	316,846	278,606	218,514

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10	0.1	204,500	232,507	279,009	325,510	348,761	406,888	441,764	337,136	296,447	232,507
11	0.09	208,111	237,020	284,424	331,828	355,530	414,785	450,338	343,679	302,201	237,020
12	0.07	217,828	246,800	296,160	345,520	370,200	431,900	468,920	357,860	314,670	251,335
13	0.06	247,940	283,360	340,032	396,704	425,040	495,880	538,384	410,872	361,284	284,152
14	0.04	254,030	288,950	348,384	406,448	435,480	508,060	551,608	420,964	370,158	290,320
15	0.01	261,678	299,060	358,872	418,684	448,590	523,355	568,214	433,637	381,302	299,060

The discounted values of the free cash flows (considering the discount rate  $k = 12\%$ ) are given in the table:

Scenario	pi	FCF1	FCF2	FCF3	FCF4	FCF5	FCF6	FCF7	FCF8	FCF9	FCF10
1	0.02	41,369	42,213	45,228	47,113	45,070	48,337	45,510	31,010	25,327	17,049
2	0.03	71,656	73,119	78,341	81,606	78,067	81,319	78,830	53,714	44,487	29,531
3	0.04	90,484	92,331	98,926	103,048	98,579	102,687	99,543	67,828	54,010	37,291
4	0.05	98,469	100,478	107,655	112,141	107,278	111,748	108,327	73,813	58,087	40,582
5	0.06	105,968	108,130	115,854	120,681	115,448	120,258	116,577	79,434	62,364	43,672
6	0.07	112,219	116,231	122,688	127,800	122,258	127,352	125,652	84,086	66,042	46,248
7	0.09	147,672	150,686	161,449	168,176	161,186	167,586	164,250	112,401	86,907	60,859
8	0.15	156,352	159,845	170,471	180,106	171,103	178,942	171,950	117,542	92,231	64,393
9	0.12	170,714	175,472	189,679	194,418	187,250	194,170	187,715	127,969	100,468	70,356
10	0.1	182,589	185,353	198,593	206,868	197,896	206,142	199,832	136,163	106,902	74,861
11	0.09	185,813	188,951	202,447	210,883	201,737	210,143	203,710	138,806	108,977	76,314
12	0.07	194,489	196,747	210,801	219,584	210,061	218,814	212,116	144,534	113,473	80,923
13	0.06	221,375	225,893	242,028	252,113	241,179	251,228	243,538	165,944	130,283	91,489
14	0.04	226,813	230,349	247,973	258,305	247,103	257,399	249,519	170,020	133,483	93,475
15	0.01	233,641	238,409	255,438	266,081	254,542	265,148	257,031	175,139	137,501	96,289

Using the NPV formula (1) we obtain the following values for this indicator:

Scenario	$p_i$	$NPV_i$
1	0.02	-611,775
2	0.03	-329,329
3	0.04	-155,273
4	0.05	-81,423
5	0.06	-11,614

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6	0.07	50,577
7	0.09	381,171
8	0.15	462,935
9	0.12	598,210
10	0.1	695,200
11	0.09	727,782
12	0.07	801,543
13	0.06	1,065,070
14	0.04	1,114,439
15	0.01	1,179,219

We shall now calculate the expected value of the NPV using the following formula:

$$E(\text{NPV}) = \sum_{i=1}^{15} p_i \cdot \text{NPV}_i \quad (2)$$

After performing the necessary calculations we get  $E(\text{NPV}) = 457.379$  EUR. Some authors prefer the following formula for evaluating  $E(\text{NPV})$  :

$$E(\text{NPV}) = \sum_{j=1}^{10} \frac{E(\text{FCF}_j)}{(1+k)^j} - I_0 \quad (3)$$

However, the formulae (2) and (3) are equivalent and we will demonstrate this. Considering the fact that for each free cash flow  $\text{FCF}_j$  we have<sup>1</sup>:

$$E(\text{FCF}_j) = \sum_{i=1}^{15} p_i \cdot \text{FCF}_j^{(i)}$$

we get (also consider (2)):

$$E(\text{NPV}) = \sum_{j=1}^{10} \sum_{i=1}^{15} p_i \cdot \frac{\text{FCF}_j^{(i)}}{(1+k)^j} - I_0 \quad (4)$$

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<sup>1</sup>  $\text{FCF}_j^{(i)}$  is the value of the free cash flow in year j under scenario i.

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Using (4) we obtain:

$$E(\text{NPV}) = \sum_{i=1}^{15} p_i \cdot \sum_{j=1}^{10} \frac{\text{FCF}_j^{(i)}}{(1+k)^j} - I_0 = \sum_{i=1}^{15} p_i \cdot \text{NPV}_i - I_0 \quad (5)$$

as we have  $\sum_{j=1}^{10} \frac{\text{FCF}_j^{(i)}}{(1+k)^j} = \text{NPV}_i$ .

In order to evaluate the standard deviation of the NPV we will use two formulae that have raised several problems among academics and practitioners as well. The first formula is:

$$\sigma(\text{NPV}) = \sqrt{\sum_{i=1}^{15} p_i \cdot [\text{NPV}_i - E(\text{NPV})]^2} \quad (6)$$

The inputs for this formula are given in the following table:

Scenario	$p_i$	$\text{NPV}_i$	$p_i \cdot [\text{NPV}_i - E(\text{NPV})]^2$
1	0.02	-611,775	22,861,796,473
2	0.03	-329,329	18,567,313,618
3	0.04	-155,273	15,013,703,613
4	0.05	-81,423	14,515,393,602
5	0.06	-11,614	13,197,269,406
6	0.07	50,577	11,584,166,786
7	0.09	381,171	522,695,360
8	0.15	462,935	4,630,398
9	0.12	598,210	2,379,987,713
10	0.1	695,200	5,655,880,362
11	0.09	727,782	6,580,563,695
12	0.07	801,543	8,291,396,002
13	0.06	1,065,070	22,157,262,082
14	0.04	1,114,439	17,269,118,944
15	0.01	1,179,219	5,210,525,823

Summing the values in the last column of the table we get:

$$\sigma^2(\text{NPV}) = 163.811.703.879 \text{ or } \sigma(\text{NPV}) = 404.737$$

Another formula used is:

$$\sigma(\text{NPV}) = \sqrt{\sum_{i=1}^{10} \sigma_i^2 + 2 \sum_{i<j} \sigma_{ij}} = \sqrt{\sum_{i=1}^{10} \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} \sigma_i \sigma_j} \quad (7)$$

where:

- $\sigma_i^2$  = the variance of the discounted free cash flow in year  $i$ ,  $i = 1, 2, \dots, 10$ ;
- $\sigma_{ij}$  = the covariance between the discounted free cash flows of years  $i$  and  $j$ ;
- $\rho_{ij}$  = the Pearson correlation coefficient between the discounted free cash flows of years  $i$  and  $j$ .

In order to compute the covariances  $\sigma_{ij}$  we use the following formula:

$$\sigma_{ij} = \sum_{k=1}^{15} p_k \cdot \left[ \left( FCF_{ik}^{\text{disc.}} - E(FCF_i^{\text{disc.}}) \right) \left( FCF_{jk}^{\text{disc.}} - E(FCF_j^{\text{disc.}}) \right) \right] \quad (8)$$

where  $FCF_{ik}^{\text{disc.}}$  and  $FCF_{jk}^{\text{disc.}}$  represent the discounted free cash flows obtainable in scenario  $k$ ,  $k = 1, 2, \dots, 15$ , in years  $i$  and  $j$ . Of course, we have  $\sigma_{ii} = \sigma_i^2$ .

The correlation coefficient (Pearson) is computed according to the following formula:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \quad (9)$$

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Applying formula (8) we obtain the covariance matrix  $\Sigma \in M_{10}(\mathbb{R})$  whose elements are:

1,901,639,617	1,925,418,250	2,074,829,325	2,155,826,798	2,064,275,099	2,145,818,571	2,074,151,972	1,417,769,659	1,103,517,056	785,042,163
1,925,418,250	1,950,175,090	2,101,543,400	2,183,173,682	2,090,614,721	2,173,061,849	2,100,557,640	1,435,695,080	1,117,395,791	794,827,929
2,074,829,325	2,101,543,400	2,265,308,603	2,352,402,958	2,253,051,474	2,341,672,576	2,263,372,544	1,547,115,053	1,204,176,520	856,503,029
2,155,826,798	2,183,173,682	2,352,402,958	2,444,985,664	2,340,797,444	2,433,460,086	2,351,693,488	1,607,618,881	1,251,222,679	889,926,603
2,064,275,099	2,090,614,721	2,253,051,474	2,340,797,444	2,241,460,794	2,329,926,341	2,251,886,891	1,539,341,335	1,198,064,148	852,140,432
2,145,818,571	2,173,061,849	2,341,672,576	2,433,460,086	2,329,926,341	2,422,089,048	2,340,752,981	1,600,119,015	1,245,404,166	885,800,306
2,074,151,972	2,100,557,640	2,263,372,544	2,351,693,488	2,251,886,891	2,340,752,981	2,263,166,521	1,546,847,458	1,203,685,944	856,229,989
1,417,769,659	1,435,695,080	1,547,115,053	1,607,618,881	1,539,341,335	1,600,119,015	1,546,847,458	1,057,448,828	822,818,394	585,263,531
1,103,517,056	1,117,395,791	1,204,176,520	1,251,222,679	1,198,064,148	1,245,404,166	1,203,685,944	822,818,394	640,562,891	455,545,076
785,042,163	794,827,929	856,503,029	889,926,603	852,140,432	885,800,306	856,229,989	585,263,531	455,545,076	324,150,170

Formula (7) can be restated as follows:

$$\sigma(\text{NPV}) = \sqrt{\mathbf{u}^T \Sigma \mathbf{u}} \quad (10)$$

where  $\mathbf{u}$  is the vector of dimension  $10 \times 1$  with all elements equal to 1. After several calculations we get  $\sigma(\text{NPV}) = 404.737$ , which is the same result as before, when we used formula (6).

We shall now prove that the two formulae for calculating the standard deviation of NPV are equivalent. We know that:

$$\text{NPV} = \sum_{i=1}^n \text{FCF}_{\text{disc.}}^{(i)} + \text{RV}_{\text{disc.}} - I_0 \quad (11)$$

We can therefore write:

$$\sigma^2(\text{NPV}) = \sigma^2\left(\sum_{i=1}^n \text{FCF}_{\text{disc.}}^{(i)} + \text{RV}_{\text{disc.}} - I_0\right) = \sum_{i=1}^n \sigma^2(\text{FCF}_{\text{disc.}}^{(i)}) + 2 \sum_{i < j} \sigma_{ij} \quad (12)$$

On the other hand, we have:

$$\sigma^2(\text{NPV}) = \sum_{i=1}^n p_i (\text{NPV}_i - E(\text{NPV}))^2 = E(\text{NPV}^2) - E^2(\text{NPV}) \quad (13)$$

By squaring both sides of (9) we get:

$$NPV^2 = \left( \sum_{i=1}^n FCF_{disc}^{(i)} \right)^2 + RV_{disc}^2 + I_0^2 + 2RV_{disc} \cdot \sum_{i=1}^n FCF_{disc}^{(i)} - 2I_0 \cdot \sum_{i=1}^n FCF_{disc}^{(i)} - 2I_0 \cdot RV_{disc}.$$

or, equivalently,

$$NPV^2 = \sum_{i=1}^n \left( FCF_{disc}^{(i)} \right)^2 + 2 \sum_{i<j}^n FCF_{disc}^{(i)} \cdot FCF_{disc}^{(j)} + RV_{disc}^2 + I_0^2 + 2RV_{disc} \cdot \sum_{i=1}^n FCF_{disc}^{(i)} - 2I_0 \cdot \sum_{i=1}^n FCF_{disc}^{(i)} - 2I_0 \cdot RV_{disc}.$$

which leads us to:

$$\begin{aligned} E(NPV^2) &= \sum_{i=1}^n E\left(FCF_{disc}^{(i)}\right)^2 + 2 \sum_{i<j}^n E\left(FCF_{disc}^{(i)} \cdot FCF_{disc}^{(j)}\right) + RV_{disc}^2 + I_0^2 + \\ &+ 2RV_{disc} \cdot \sum_{i=1}^n E\left(FCF_{disc}^{(i)}\right) - 2I_0 \cdot \sum_{i=1}^n E\left(FCF_{disc}^{(i)}\right) - 2I_0 \cdot RV_{disc}. \end{aligned} \quad (14)$$

Considering (11), we get:

$$E(NPV) = \sum_{i=1}^n E\left(FCF_{disc}^{(i)}\right) + RV_{disc} - I_0,$$

Therefore, we have:

$$\begin{aligned} E^2(NPV) &= \sum_{i=1}^n E^2\left(FCF_{disc}^{(i)}\right) + 2 \sum_{i<j}^n E\left(FCF_{disc}^{(i)}\right) \cdot E\left(FCF_{disc}^{(j)}\right) + 2RV_{disc} \cdot \sum_{i=1}^n E\left(FCF_{disc}^{(i)}\right) - \\ &- 2I_0 \cdot \sum_{i=1}^n E\left(FCF_{disc}^{(i)}\right) - 2I_0 \cdot RV_{disc}. \end{aligned} \quad (15)$$



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Taking into account (13), (14) and (15), we find the following expression for the variance of NPV:

$$\sigma^2(\text{NPV}) = \sum_{i=1}^n \left[ E\left(\text{FCF}_{\text{disc.}}^{(i)}\right)^2 - E^2\left(\text{FCF}_{\text{disc.}}^{(i)}\right) \right] + 2 \sum_{i < j} \left[ E\left(\text{FCF}_{\text{disc.}}^{(i)} \cdot \text{FCF}_{\text{disc.}}^{(j)}\right) - E\left(\text{FCF}_{\text{disc.}}^{(i)}\right) \cdot E\left(\text{FCF}_{\text{disc.}}^{(j)}\right) \right]$$

Namely,

$$\sigma^2(\text{NPV}) = \sum_{i=1}^n \sigma^2\left(\text{FCF}_{\text{disc.}}^{(i)}\right) + 2 \sum_{i < j} \sigma_{ij} ,$$

that is exactly (12).

Because the random variables  $\text{FCF}_i$ ,  $i = 1, 2, \dots, 10$  are normally distributed, this means that the discounted free cash flows are normally distributed and therefore NPV is a normal random variable with the mean and variance given by:

$$\begin{cases} E(\text{NPV}) = 457.379 \\ \sigma(\text{NPV}) = 404.737 \end{cases}$$

Using the properties of the normal probability distribution, we can determine the probability that NPV is above a certain value  $\text{NPV}_1$ :

$$\begin{aligned} P(\text{NPV} > \text{NPV}_1) &= 1 - P(\text{NPV} \leq \text{NPV}_1) = 1 - P\left(\frac{\text{NPV} - E(\text{NPV})}{\sigma(\text{NPV})} \leq \frac{\text{NPV}_1 - E(\text{NPV})}{\sigma(\text{NPV})}\right) = \\ &= 1 - N\left(\frac{\text{NPV}_1 - E(\text{NPV})}{\sigma(\text{NPV})}\right) \end{aligned} \quad (16)$$

where  $N(\cdot)$  is the cumulative distribution function of the normal distribution. In the particular case when  $\text{NPV}_1 = 0$ , we have:

$$P(\text{NPV} > 0) = 1 - N\left(-\frac{E(\text{NPV})}{\sigma(\text{NPV})}\right) \quad (17)$$

The table below comprises some simulations for the lower bound of the NPV of our investment project:

NPV <sub>1</sub>	P(NPV > NPV <sub>1</sub> )
-500,000	98.7304%
-300,000	95.9178%
-200,000	93.2432%
-100,000	89.3815%
<b>0</b>	<b>84.1345%</b>
100,000	77.4253%
200,000	69.3520%
300,000	60.2096%
400,000	50.4669%
500,000	40.6960%
600,000	31.4745%
700,000	23.2842%
800,000	16.4385%
900,000	11.0539%
1,000,000	7.0680%

As we can see, the probability that the investment project has a positive NPV is over 84%, which is very good.

At the same time, given the fact that NPV is a normally distributed random variable, we can estimate confidence intervals with different probabilities for NPV. Thus, for  $\delta \in (0,1)$ , we shall determine NPV<sub>1</sub> and NPV<sub>2</sub> for which:

$$P(\text{NPV}_1 \leq \text{NPV} \leq \text{NPV}_2) = \delta \quad (18)$$

After some calculations we get:

$$P\left(\frac{\text{NPV}_1 - E(\text{NPV})}{\sigma(\text{NPV})} \leq \frac{\text{NPV} - E(\text{NPV})}{\sigma(\text{NPV})} \leq \frac{\text{NPV}_2 - E(\text{NPV})}{\sigma(\text{NPV})}\right) = \delta,$$

and using the notations  $z_1 = \frac{\text{NPV}_1 - E(\text{NPV})}{\sigma(\text{NPV})}$  and  $z_2 = \frac{\text{NPV}_2 - E(\text{NPV})}{\sigma(\text{NPV})}$

we obtain:

$$P\left(z_1 \leq \frac{\text{NPV} - E(\text{NPV})}{\sigma(\text{NPV})} \leq z_2\right) = \delta \quad (19)$$

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or, equivalently,

$$N(z_2) - N(z_1) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2}x^2} dx = \delta \quad (20)$$

The formula (19) leads us to the following expression for the confidence interval for NPV:

$$[E(\text{NPV}) + z_1\sigma(\text{NPV}), E(\text{NPV}) + z_2\sigma(\text{NPV})] \quad (21)$$

In order to determine  $z_1$  and  $z_2$  we shall impose the condition of minimizing the length of interval (21), because we are interested in the best estimation possible. The length of the interval will be:

$$(z_2 - z_1) \cdot \sigma(\text{NPV}) \quad (22)$$

which is the objective function for the optimization problem.

Let us construct the Lagrangean associated function:

$$L(z_1, z_2, \lambda) = (z_2 - z_1) \cdot \sigma(\text{NPV}) + \lambda \left( \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2}x^2} dx - \delta \right) \quad (23)$$

The first order conditions are:

$$\begin{cases} \frac{\partial L}{\partial z_1} = 0 \\ \frac{\partial L}{\partial z_2} = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \end{cases} \quad (24)$$

After performing certain calculations, system (24) becomes:

$$\begin{cases} \sigma(\text{NPV}) + \frac{\lambda}{\sqrt{2\pi}} e^{-\frac{z_2^2}{2}} = 0 \\ -\sigma(\text{NPV}) - \frac{\lambda}{\sqrt{2\pi}} e^{-\frac{z_1^2}{2}} = 0 \\ \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2}x^2} dx - \delta = 0 \end{cases} \quad (25)$$

The first two equations of system (25) lead us to:

$$z_1^2 = z_2^2,$$

and, as  $z_1 = z_2$  is not possible (this would mean that the confidence interval will be of length zero), we get:

$$z_1 = -z_2 \quad (26)$$

Returning to (20), we have:

$$N(z_2) - N(-z_2) = \delta \Rightarrow 2N(z_2) - 1 = \delta \Rightarrow z_2 = N^{-1}\left(\frac{1+\delta}{2}\right) = z_{1-\frac{\alpha}{2}} = -z_1 \quad (27)$$

where  $\alpha = 1 - \delta$  is the significance factor chosen.

Under these conditions, the confidence interval for NPV shall be:

$$\left[ E(\text{NPV}) - z_{1-\frac{\alpha}{2}} \cdot \sigma(\text{NPV}), E(\text{NPV}) + z_{1-\frac{\alpha}{2}} \cdot \sigma(\text{NPV}) \right] \quad (28)$$

The table below presents different confidence intervals for NPV at given values of  $\delta$  :

Probability $\delta$	Confidence interval
60%	[64,102 745,371]
65%	[26,474 782,999]
70%	[-14,746 824,219]
75%	[-60,852 870,325]
80%	[-113,954 923,427]
85%	[-177,894 987,368]

## The NPV Criterion for Valuing Investments under Uncertainty

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90%	[-260,996 1,070,469]
95%	[-388,533 1,198,006]
96%	[-426,491 1,235,964]
97%	[-473,578 1,283,582]
98%	[-536,822 1,346,295]
99%	[-637,796 1,447,269]
99.5%	[-731,373 1,540,846]

As expected, the confidence interval becomes broader as the accuracy of estimation is higher. In conclusion, we recommend that formula (6) is used for determining the standard deviation of NPV under incertitude.

When faced with a financial issue or when we try to understand the compromises implied by a business decision, there is a wide range of instruments that we can use in order to provide quantitative answers. Choosing the appropriate instrument from a multitude of available options is, without doubt, an important part of the analysis process; however, corporate finance practice has also proven that properly presenting the problem is of at least the same importance.

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