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ON STATISTICAL PATTERN WITH FUZZY DATA

Abstract. *In this paper, a general regression model with input-output trapezoidal fuzzy data, derived from a previous algorithm for triangular data [5], is discussed. Finally, an implementation of these results with application in estimating the efficiency of a decision making units model is presented.*

Key words: *fuzzy number, fuzzy set, estimator, forecasting.*

JEL classification: **C61, C81.**

1. THE GENERAL FRAMEWORK

It is known that for a trapezoidal fuzzy number $X = \left(x', x'', \underline{u}, \overline{u}\right)$, the membership function is given by:

$$\widetilde{X} = \begin{cases} 0, \text{ if } x \in \left(-\infty, x'-\underline{u}\right) \\ \frac{1}{\underline{u}}\left(x - x'\right) + 1, \text{ if } x \in \left[x'-\underline{u}, x'\right] \\ 1, \text{ if } x \in \left(x', x''\right] \\ -\frac{1}{\overline{u}}\left(x - x''\right) + 1, \text{ if } x \in \left(x'', x''+\overline{u}\right] \\ 0, \text{ if } x \in \left(x''+\overline{u}, +\infty\right) \end{cases}.$$

Consider the n pairs of data X_i, Y_i (where the first is independent variable, and the second is the dependent variable):

$$\left(X_i, Y_i\right),\ 1 \le i \le n$$

where X_i, Y_i trapezoidal fuzzy numbers.
It must be find the real numbers $a, b \in \mathbf{R}$ such that the relation

$$Y = a_0 + a_1 X$$

describes, in accordance with least squares approach, the best line fitting the data.

Thus, it must be minimized:

$$S(a_0,a_1)=\sum_{i=1}^{n} D_2^2(a_0+a_1X_i,Y_i) \quad (*)$$

where the distance D_2^2 (see [5]) is given, for $Z_i=\left(\underline{Z}_i(r),\overline{Z}_i(r)\right)$ and $Y_i=\left(\underline{Y}_i(r),\overline{Y}_i(r)\right)$, by

$$D_2^2(Z_i,Y_i)=\int_0^1\left[\underline{Z}_i(r)-\underline{Y}_i(r)\right]^2dr+\int_0^1\left[\overline{Z}_i(r)-\overline{Y}_i(r)\right]^2dr,$$

or, if consider the form $Z_i=\left(z'_i,z''_i,\underline{t}_i,\bar{t}_i\right)$, $Y_i=\left(y'_i,y''_i,\underline{v}_i,\bar{v}_i\right)$, with $\underline{Z}_i(r)=z'_i-\underline{t}_i+\underline{t}_ir$, $\overline{Z}_i(r)=z''_i+\bar{t}_i-\bar{t}_ir$, $\underline{Y}_i(r)=y'_i-\underline{v}_i+\underline{v}_ir$, $\overline{Y}_i(r)=y''_i+\bar{v}_i-\bar{v}_ir$:

$$D_2^2(Z_i,Y_i)=\frac{1}{2}\left[\left(z'_i-\underline{t}_i-y'_i+\underline{v}_i\right)^2+\left(z'_i-y'_i\right)^2+\left(z''_i+\bar{t}_i-y''_i-\bar{v}_i\right)^2+\left(z''_i-y''_i\right)^2\right]$$

The total error has two different forms according to the sign of a_1.

First case: $a_1>0$ (H1)

Under this hypothesis one can write

$$S(a_0,a_1)=\sum_{i=1}^{n}\left[\int_0^1\left(a_0+a_1\underline{X}_i-\underline{Y}_i\right)^2dr+\int_0^1\left(a_0+a_1\overline{X}_i-\overline{Y}_i\right)^2dr\right]$$

Second case: $a_1<0$ (H2)

In this case, the sum $S(a_0,a_1)$ is:

$$S(a_0,a_1)=\sum_{i=1}^{n}\left[\int_0^1\left(a_0+a_1\overline{X}_i-\underline{Y}_i\right)^2dr+\int_0^1\left(a_0+a_1\underline{X}_i-\overline{Y}_i\right)^2dr\right]$$

2. THEORETICAL RESULTS

First of all, the function

$$S(a_0,a_1)=\sum_{i=1}^{n} D_2^2(a_0+a_1X_i,Y_i)$$

must to be minimized, w.r.t. a_0 and a_1 (belonging to real axis).

Theorem 1.

Assume that hypothesis (H1) is true. Then the minimization problem

$$\min_{a_0,a_1\in\mathbf{R}}\sum_{i=1}^{n} D_2^2(a_0+a_1X_i,Y_i)$$

admits a single pair of real solutions.

Proof.

$$S(a,b)=\sum_{i=1}^{n}\left[\int_0^1 \left(a_0+a_1\underline{X}_i-\underline{Y}_i\right)^2 dr+\int_0^1 \left(a_0+a_1\overline{X}_i-\overline{Y}_i\right)^2 dr\right]=$$

$$=\sum_{i=1}^{n}\left[\int_0^1 \ [a_0+a_1(x'_i-\underline{u}_i+\underline{u}_i r)-y'_i+\underline{v}_i-\underline{v}_i r]^2 dr\right]+$$

$$+\sum_{i=1}^{n}\left[\int_0^1 \ [a_0+a_1(x''_i+\overline{u}_i-\overline{u}_i r)-y''_i-\overline{v}_i+\overline{v}_i r]^2 dr\right]\cong$$

$$\cong\frac{1}{2}\sum_{i=1}^{n}\{\ [a_0+a_1(x'_i-\underline{u}_i)-(y'_i-\underline{v}_i)]^2+(a_0+a_1x'_i-y'_i)^2+$$

$$+[a_0+a_1(x''_i+\overline{u}_i)-(y''_i+\overline{v}_i)]^2+(a_0+a_1x''_i-y''_i)^2\ \}$$

After establish the equality with zero for the partial derivatives of $S(a_0,a_1)$ w.r.t. a_0,a_1, one can derive:

$$4na_0+a_1\sum_{i=1}^{n}\left(2x'_i+2x''_i+\overline{u}_i-\underline{u}_i\right)=\sum_{i=1}^{n}\left(2y'_i+2y''_i+\overline{v}_i-\underline{v}_i\right)$$

$$a_0\sum_{i=1}^{n}\left(2x'_i+2x''_i+\overline{u}_i-\underline{u}_i\right)+a_1\sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)^2+(x''_i+\overline{u}_i)^2+x'^2_i+x''^2_i\right]=$$

$$=\sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)(y'_i-\underline{v}_i)+(x''_i+\overline{u}_i)(y''_i+\overline{v}_i)+x'_i\,y'_i+x''_i\,y''_i\right]$$

We have

$$\Delta(\mathrm{H}_1)=4n\ \sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)^2+(x''_i+\overline{u}_i)^2+x'^2_i+x''^2_i\right]-$$

$$-\left[\sum_{i=1}^{n}\left(2x'_i+2x''_i+\overline{u}_i-\underline{u}_i\right)\right]^2=$$

$$=\left\{\sum_{i=1}^{n}[1+1+1+1]\right\}\cdot\left\{\sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)^2+(x''_i+\overline{u}_i)^2+x'^2_i+x''^2_i\right]\right\}-$$

$$-\left\{\sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)+(x''_i+\overline{u}_i)+x'_i+x''_i\right]\right\}^2$$

Thus $\Delta(\mathrm{H1})>0$.

$$\Delta_{a_0}(\mathrm{H1})=\left\{\sum_{i=1}^{n}\left(2y'_i+2y''_i+\overline{v}_i-\underline{v}_i\right)\right\}\cdot$$

$$\cdot\left\{\sum_{i=1}^{n}\left[\left(x'_i-\underline{u}_i\right)^2+\left(x''_i+\overline{u}_i\right)^2+x'^2_i+x''^2_i\right]\right\}-$$

$$-\left\{\sum_{i=1}^{n}\left[\left(x'_i-\underline{u}_i\right)\left(y'_i-\underline{v}_i\right)+\left(x''_i+\overline{u}_i\right)\left(y''_i+\overline{v}_i\right)+x'_i\,y'_i+x''_i\,y''_i\right]\right\}\cdot$$

$$\cdot\left[\sum_{i=1}^{n}\left(2x'_i+2x''_i+\overline{u}_i-\underline{u}_i\right)\right],$$

and

$$\Delta_{a_1}(\mathrm{H1})=\left\{\sum_{i=1}^{n}\left(2y'_i+2y''_i+\overline{v}_i-\underline{v}_i\right)\right\}\cdot\left\{\sum_{i=1}^{n}\left(2x'_i+2x''_i+\overline{u}_i-\underline{u}_i\right)\right\}-$$

$$-4n\left\{\sum_{i=1}^{n}\left[\left(x'_i-\underline{u}_i\right)\left(y'_i-\underline{v}_i\right)+\left(x''_i+\overline{u}_i\right)\left(y''_i+\overline{v}_i\right)+x'_i\,y'_i+x''_i\,y''_i\right]\right\}.$$

Consequently, we obtain a unique pair which verifies the above two equations:

$a_0^{(H1)}=\left[\Delta(\mathrm{H1})^{-1}\right]\cdot\left[\Delta_{a_0}(\mathrm{H1})\right],$

$a_1^{(H1)}=\left[\Delta(\mathrm{H1})^{-1}\right]\cdot\left[\Delta_{a_1}(\mathrm{H1})\right].$ □

Theorem 2.
Assume that hypothesis (H2) is true. Then the minimization problem

$$\min_{a_0,a_1\in\mathbf{R}}\sum_{i=1}^{n}D_2^2\left(a_0+a_1X_i,Y_i\right)$$

admits a single pair of real solutions.

Proof.

$$S(a_0,a_1)=\sum_{i=1}^{n}\left[\int_0^1\left(a_0+a_1\overline{X}_i-\underline{Y}_i\right)^2dr+\int_0^1\left(a_0+a_1\underline{X}_i-\overline{Y}_i\right)^2dr\right]=$$

$$=\sum_{i=1}^{n}\left[\int_0^1\ \left[a_0+a_1\left(x''_i+\overline{u}_i-\overline{u}_ir\right)-y'_i+\underline{v}_i-\underline{v}_ir\right]^2dr\right]+$$

$$+\sum_{i=1}^{n}\left[\int_0^1\ \left[a_0+a_1\left(x'_i-\underline{u}_i+\underline{u}_ir\right)-y''_i-\overline{v}_i+\overline{v}_ir\right]^2dr\right]\cong$$

$$\cong\frac{1}{2}\sum_{i=1}^{n}\left\{\ \left[a_0+a_1\left(x''_i+\overline{u}_i\right)-\left(y'_i-\underline{v}_i\right)\right]^2+\left(a_0+a_1x''_i-y'_i\right)^2+\right.$$

$$\left.+\left[a_0+a_1\left(x'_i-\underline{u}_i\right)-\left(y''_i+\overline{v}_i\right)\right]^2+\left(a_0+a_1x'_i-y''_i\right)^2\ \right\}.$$

After writing the equality with zero for the partial derivatives of $S(a_0,a_1)$ w.r.t. a_0,a_1, one can obtain:

$$4na_0 + a_1\sum_{i=1}^{n}\left(2x'_i + 2x''_i + \overline{u}_i - \underline{u}_i\right) = \sum_{i=1}^{n}\left(2y'_i + 2y''_i + \overline{v}_i - \underline{v}_i\right)$$

$$a_0\sum_{i=1}^{n}\left(2x'_i + 2x''_i + \overline{u}_i - \underline{u}_i\right) + a_1\sum_{i=1}^{n}\left[\left(x'_i - \underline{u}_i\right)^2 + \left(x''_i + \overline{u}_i\right)^2 + x'^2_i + x''^2_i\right] =$$

$$= \sum_{i=1}^{n}\left[\left(x''_i + \overline{u}_i\right)\left(y'_i - \underline{v}_i\right) + \left(x'_i - \underline{u}_i\right)\left(y''_i + \overline{v}_i\right) + x''_i\, y'_i + x'_i\, y''_i\right]$$

We have

$$\Delta(\mathrm{H2}) = 4n \sum_{i=1}^{n}\left[\left(x'_i - \underline{u}_i\right)^2 + \left(x''_i + \overline{u}_i\right)^2 + x'^2_i + x''^2_i\right] -$$

$$-\left[\sum_{i=1}^{n}\left(2x'_i + 2x''_i + \overline{u}_i - \underline{u}_i\right)\right]^2 > 0 .$$

Moreover,

$$\Delta_{a_0}(\mathrm{H2}) = \left\{\sum_{i=1}^{n}\left(2y'_i + 2y''_i + \overline{v}_i - \underline{v}_i\right)\right\}\cdot$$

$$\cdot\left\{\sum_{i=1}^{n}\left[\left(x'_i - \underline{u}_i\right)^2 + \left(x''_i + \overline{u}_i\right)^2 + x'^2_i + x''^2_i\right]\right\} -$$

$$-\left\{\sum_{i=1}^{n}\left[\left(x''_i + \overline{u}_i\right)\left(y'_i - \underline{v}_i\right) + \left(x'_i - \underline{u}_i\right)\left(y''_i + \overline{v}_i\right) + x''_i\, y'_i + x'_i\, y''_i\right]\right\}\cdot$$

$$\cdot\left[\sum_{i=1}^{n}\left(2x'_i + 2x''_i + \overline{u}_i - \underline{u}_i\right)\right]$$

and

$$\Delta_{a_1}(\mathrm{H2}) = \left\{\sum_{i=1}^{n}\left(2y'_i + 2y''_i + \overline{v}_i - \underline{v}_i\right)\right\}\cdot\left\{\sum_{i=1}^{n}\left(2x'_i + 2x''_i + \overline{u}_i - \underline{u}_i\right)\right\} -$$

$$-4n\left\{\sum_{i=1}^{n}\left[\left(x''_i + \overline{u}_i\right)\left(y'_i - \underline{v}_i\right) + \left(x'_i - \underline{u}_i\right)\left(y''_i + \overline{v}_i\right) + x''_i\, y'_i + x'_i\, y''_i\right]\right\}$$

We get the unique real solutions for system equations:

$a_0^{(H2)} = \left\lfloor \Delta(\mathrm{H2})^{-1}\right\rfloor \cdot \left\lfloor \Delta_{a_0}(\mathrm{H2})\right\rfloor = \left\lfloor \Delta(\mathrm{H1})^{-1}\right\rfloor \cdot \left\lfloor \Delta_{a_0}(\mathrm{H2})\right\rfloor,$

$a_1^{(H2)} = \left[\Delta(\mathrm{H2})\right]^{-1} \cdot \left\lfloor \Delta_{a_1}(\mathrm{H2})\right\rfloor = \left[\Delta(\mathrm{H1})\right]^{-1} \cdot \left\lfloor \Delta_{a_1}(\mathrm{H2})\right\rfloor.$ □

Theorem 3.
The following inequality holds:

$$a_1^{(H2)} \leq a_1^{(H1)}$$

(This relation ensure us that the problem (*) always has a unique solution, if we presume $(\mathrm{H1}) \vee (\mathrm{H2})$ is true, namely a global solution).

Proof.
We have

$$\Delta(\mathrm{H1})^{-1}\Delta_{a_1}(\mathrm{H1}) - \Delta(\mathrm{H2})^{-1}\Delta_{a_1}(\mathrm{H2}) = \Delta(\mathrm{H1})^{-1}\left(\Delta_{a_1}(\mathrm{H1}) - \Delta_{a_1}(\mathrm{H2})\right) =$$

$$= \Delta(\mathrm{H1})^{-1}\, 4n\left\{\sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)(y'_i-\underline{v}_i)+(x''_i+\overline{u}_i)(y''_i+\overline{v}_i)+x'_i\, y'_i+x''_i\, y''_i\right]\right\} -$$

$$- \Delta(\mathrm{H1})^{-1} 4n\left\{\sum_{i=1}^{n}\left[(x''_i+\overline{u}_i)(y'_i-\underline{v}_i)+(x'_i-\underline{u}_i)(y''_i+\overline{v}_i)+x''_i\, y'_i+x'_i\, y''_i\right]\right\}$$

Since

$$\left\{\sum_{i=1}^{n}\left[(x'_i-\underline{u}_i)(y'_i-\underline{v}_i)+(x''_i+\overline{u}_i)(y''_i+\overline{v}_i)+x'_i\, y'_i+x''_i\, y''_i\right]\right\} -$$

$$-\left\{\sum_{i=1}^{n}\left[(x''_i+\overline{u}_i)(y'_i-\underline{v}_i)+(x'_i-\underline{u}_i)(y''_i+\overline{v}_i)+x''_i\, y'_i+x'_i\, y''_i\right]\right\} =$$

$$= \sum_{i=1}^{n}\left[\ \underline{v}_i(x''_i-x'_i) - y'_i\left(\underline{u}_i+\overline{u}_i\right) - y'_i\,(x''_i-x'_i) + \underline{v}_i\left(\underline{u}_i+\overline{u}_i\right)\ \right] +$$

$$+ \sum_{i=1}^{n}\left[\ \overline{v}_i(x''_i-x'_i) + y''_i\left(\underline{u}_i+\overline{u}_i\right) + y''_i\,(x''_i-x'_i) + \overline{v}_i\left(\underline{u}_i+\overline{u}_i\right)\ \right] +$$

$$+ \sum_{i=1}^{n}\left[\ -y'_i\,(x''_i-x'_i) + y''_i\,(x''_i-x'_i)\ \right] =$$

$$= \sum_{i=1}^{n}\left[\ \left(\underline{v}_i+\overline{v}_i\right)(x''_i-x'_i) + (y''_i-y'_i)\left(\underline{u}_i+\overline{u}_i\right) + \left(\underline{v}_i+\overline{v}_i\right)\left(\underline{u}_i+\overline{u}_i\right)\ \right] \geq 0$$

then

$$\Delta(\mathrm{H1})^{-1}\Delta_{a_1}(\mathrm{H1}) - \Delta(\mathrm{H2})^{-1}\Delta_{a_1}(\mathrm{H2}) \geq 0\,.$$

Thus

$$a_1^{(H2)} = \left[\Delta(\mathrm{H2})\right]^{-1}\cdot\left[\Delta_{a_1}(\mathrm{H2})\right] \leq \left[\Delta(\mathrm{H1})^{-1}\right]\cdot\left[\Delta_{a_1}(\mathrm{H1})\right] = a_1^{(H1)}\,. \quad \square$$

3. NUMERICAL APPLICATION

We consider the data from [2],[3] (concerning a DEA-data envelopment analysis model) with eight input-output (*inp.,outp.*) measurements: (X_i, Y_i), $i = \overline{1,8}$, where X_i , Y_i are represented as $(x'_i, x''_i, \underline{u}_i, \overline{u}_i)$ and $(y'_i, y''_i, \underline{v}_i, \overline{v}_i)$, respectively (Table 1).

Table 1: Input-output data

i	*Input* (X_i)	*Output* (Y_i)
1	(3;3;2;2)	(3;3;1;1)
2	(4;4;0.5;0.50)	(2.50;2.50;1;1)
3	(4.50;4.50;1.50;1.50)	(6;6;1;1)
4	(6.50;6.50;0.50;0.50)	(4;4;1.25;1.25)
5	(7;7;2;2)	(5;5;0.50;0.50)
6	(8;8;0.50;0.50)	(3.50;3.50;0.50;0.50)
7	(10;10;1;1)	(6;6;0.50;0.50)
8	(6;6;0.50;0.50)	(2;2;1.50;1.50)

Table 2: Extended input data

Input	$x'_i - \underline{u}_i$	$x''_i + \overline{u}_i$	$(x'_i - \underline{u}_i)^2$	$(x''_i + \overline{u}_i)^2$
(3;3;2;2)	1	5	1	25
(4;4;0.5;0.5)	3.50	4.50	12.25	20.25
(4.5;4.5;1.5;1.5)	3	6	9	36
(6.5;6.5;0.5;0.5)	6	7	36	49
(7;7;2;2)	5	9	25	81
(8;8;0.5;0.5)	7.50	8.50	56.25	72.25
(10;10;1;1)	9	11	81	121
(6;6;0.5;0.5)	5.50	6.50	30.25	42.25

Table 3: Extended output data

Output	$y'_i - \underline{v}_i$	$y''_i + \overline{v}_i$
(3;3;1;1)	2	4
(2.5;2.5;1;1)	1.50	3
(6;6;1;1)	5	7
(4;4;1.25;1.25)	2.75	2.50
(5;5;0.5;0.5)	4.50	5.50
(3.5;3.5;0.5;0.5)	3	4
(6;6;0.5;0.5)	5.50	6.50
(2;2;1.5;1.5)	0.50	3.50

After defining the auxiliary coefficients A_i B_i C_i D_i E_i, $i = \overline{1,n}$ then a new form of the theoretical equations obtained in section 2 are:

$$\underline{\text{Hypothesis 1 (H1)}} \Rightarrow \begin{cases} 4na_0 + a_1 \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} C_i \\ a_0 \sum_{i=1}^{n} A_i + a_1 \sum_{i=1}^{n} B_i = \sum_{i=1}^{n} D_i \end{cases}$$

and

$$\underline{\text{Hypothesis 2 (H2)}} \Rightarrow \begin{cases} 4na_0 + a_1 \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} C_i \\ a_0 \sum_{i=1}^{n} A_i + a_1 \sum_{i=1}^{n} B_i = \sum_{i=1}^{n} E_i \end{cases}$$

where

$$A_i = 2x'_i + 2x''_i + \overline{u}_i - \underline{u}_i \,,$$

$$B_i = \left(x'_i - \underline{u}_i\right)^2 + \left(x''_i + \overline{u}_i\right)^2 + x'^2_i + x''^2_i \,,$$

$$C_i = 2y'_i + 2y''_i + \overline{v}_i - \underline{v}_i \,,$$

$$D_i = \left(x'_i - \underline{u}_i\right)\left(y'_i - \underline{v}_i\right) + \left(x''_i + \overline{u}_i\right)\left(y''_i + \overline{v}_i\right) + x'_i\, y'_i + x''_i\, y''_i \,,$$

$$E_i = \left(x''_i + \overline{u}_i\right)\left(y'_i - \underline{v}_i\right) + \left(x'_i - \underline{u}_i\right)\left(y''_i + \overline{v}_i\right) + x''_i\, y'_i + x'_i\, y''_i \,.$$

Table 4: Auxiliary coefficients

i	A_i	B_i	C_i	D_i	E_i
1	12	44	12	40	32
2	16	64.50	10	38.75	37.25
3	18	85.50	24	111	105
4	26	169.50	16	86	86.25
5	28	204	20	142	138
6	32	256.50	14	112.50	111.50
7	40	402	24	241	239
8	24	144.50	8	49.50	46.50

Assume (H1).
Thus we have the equations (Theorem 1)

$$32a_0 + 196a_1 = 128$$

$$196a_0 + 1370.5a_1 = 820.75$$

which lead us to the first preliminary solution:

$$a_0 = 2.67\,,\ a_1 = 0.21$$

Assume (H2).
We solve the equations (Theorem 2)

$$32a_0 + 196a_1 = 128$$
$$196a_0 + 1370.50a_1 = 795.50$$

which give us

$$a_0 = 3.58\,,\ a_1 = 6.76\times10^{-2}$$

Table 5: Preliminary results

	H1	H2	
a_0	**R**	**R**	Hypothesis (membership set)
a_1	$\mathbf{R}^{*}_{+}$	$\mathbf{R}^{*}_{-}$	
a_0	2.67	3.58	Model solutions
a_1	0.21	6.76×10^{-2}	
	Y	*N*	*Eligibility*

Taking into account Table 5 and Theorem 3, we may conclude that the best forecasting line is given by mathematical relation:

$$Y = 2.67 + 0.21X \text{ (Fig. 1)}$$

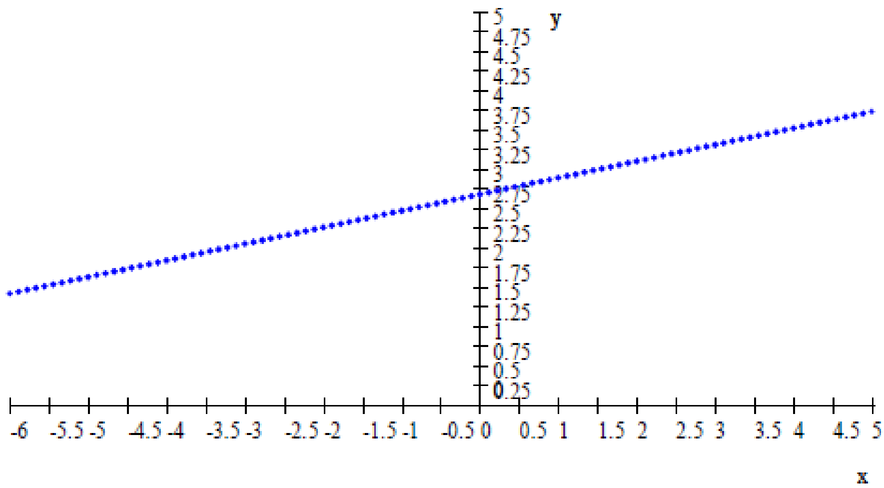

Fig. 1. Best line fitting the given eight (*inp.,outp.*) data

For $\alpha, \beta \in \mathbf{R}$ and

$$\tilde{X}(x)=\begin{cases} 0, & \text{if } x\in(-\infty, x'-\underline{u}) \\ \frac{1}{\underline{u}}(x-x')+1, & \text{if } x\in[x'-\underline{u}, x'] \\ 1, & \text{if } x\in(x', x''] \\ -\frac{1}{\overline{u}}(x-x'')+1, & \text{if } x\in(x'', x''+\overline{u}] \\ 0, & \text{if } x\in(x''+\overline{u}, +\infty) \end{cases}$$

we have

$$\alpha\tilde{X}(x)+\beta=\begin{cases} 0, & \text{if } x\in(-\infty, \alpha(x'-\underline{u})+\beta) \\ \frac{1}{\alpha\underline{u}}[x-(\alpha x'+\beta)]+1, & \text{if } x\in[\alpha(x'-\underline{u})+\beta, \alpha x'+\beta] \\ 1, & \text{if } x\in(\alpha x'+\beta, \alpha x''+\beta] \\ -\frac{1}{\alpha\overline{u}}[x-(\alpha x''+\beta)]+1, & \text{if } x\in(\alpha x''+\beta, \alpha(x''+\overline{u})+\beta] \\ 0, & \text{if } x\in(\alpha(x''+\overline{u})+\beta, +\infty) \end{cases}$$

For instance, if $X_s=(3,4,1,1)$, $s\in\mathbf{N}, s\geq 9$ then in an orthogonal system of coordinates (xOy), we may write

$$\tilde{X}_s(x)=\begin{cases} 0, & \text{if } x\in(-\infty, 2) \\ x-2, & \text{if } x\in[2,3] \\ 1, & \text{if } x\in(3,4] \\ -x+5, & \text{if } x\in(4,5] \\ 0, & \text{if } x\in(5,+\infty) \end{cases}$$

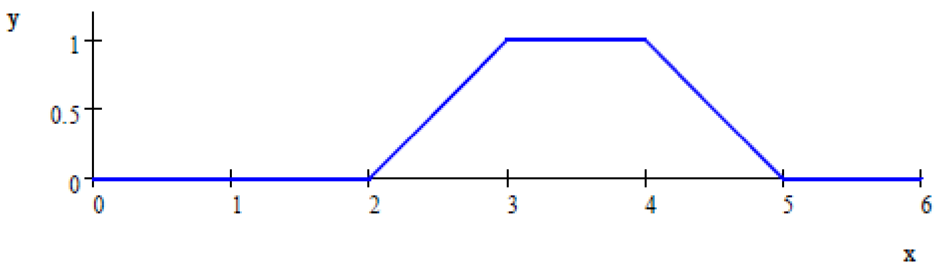

Fig. 2. Representation of input data (X_s)

Thus, estimated response is

$$\widetilde{Y}_s(x) = 2.67 + 0.21\widetilde{X}_s(x),$$

so

$$\widetilde{Y}_s(x) = \begin{cases} 0, \text{ if } x \in (-\infty;5.55) \\ \dfrac{1}{2.67}[x - 8.22] + 1, \text{ if } x \in [5.55;8.22] \\ 1, \text{ if } x \in (8.22;10.89] \\ -\dfrac{1}{2.67}[x - 10.89] + 1, \text{ if } x \in (10.89;13.56] \\ 0, \text{ if } x \in (13.56;+\infty) \end{cases}$$

In simplified form, Y_s may be represented as $Y_s = (8.22;10.89;2.67;2.67)$.

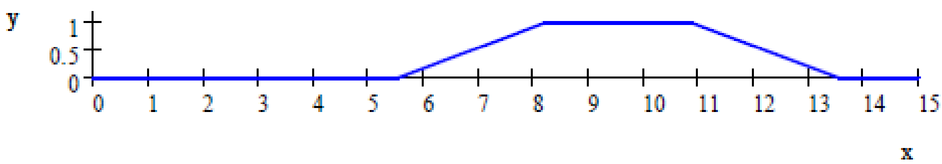

Fig. 3. Representation of estimated value (Y_s).

4. CONCLUSION

A method for estimating the linear trend of fuzzificated (in trapezoidal form, so only four cartesian coordinates are needed) statistical data was studied. The theoretical results (the existence and uniqueness of solution in the general case which are proven in second section) were applied to a set of experimental data and the forecast corresponding to a random input data was calculated.

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