Tatiana Corina DOSESCU, PhD
"Dimitrie Cantemir"Christian University, Bucharest
Constantin RAISCHI, PhD
Department of Mathematics
The Bucharest Academy of Economic Studies

THE ECONOMETRIC MODELING OF A SYSTEM OF THREE RANDOM VARIABLES WITH THE β DEPENDENCE

Abstract. *Within classical econometric modelling, because of the complexity of „data – generating process" we have chosen it to be presented with the help of a set of assumptions, where the random is as controlled as possible, so that the activity should be monitored and even dosed.*

This paper has chosen a systemic and cybernetic approach of the display of the „data – generating process". The first stage of the „data – generating process" decoding, the authors study a system made of three random variables defined on different probability space between which there is a special a priori dependence, called β dependence. The research includes setting a probabilistic model of such a system and making a representative example. The research ends with an analysis of the systemic and cybernetic repercussions corresponding to the β dependence.

Key words: *probability spaces, product of probability spaces, stochastic processes, time series, economic variables, econometric model.*

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1. INTRODUCTION

In a system of economic variables, the classical regression analysis consists mainly in [1], [3], [7], [8], [9], [10], [13], in considering one of the system variables as dependent on the rest of the variables and all the economic variables values produced by an underling „data – generating process", on which we make a set of assumptions.

For example, we may consider the system made of the following three macroeconomic indexes (variables): GDP, consumption and investments. This system represents a subsystem of the whole system of indexes which are used for the entire economy that are part of the National Accounts System.

Making use of the index system evolution during 1990-1995, where the indexes are:

Index	Measurement unit	1990	1991	1992	1993	1994	1995
GDP	Mild. lei	857,9	2203,9	6029,2	20035,7	49767,6	72559,7
Consumption (C)	Mild. lei	679,5	1672,5	4642,5	15235,8	37417,5	56315,4
Investments (I)	Mild. lei	168,410	314,014	888,566	2821,819	8004,621	12995,49

Source: The Romanian Statistics Yearbook, 1996

We shall research what happens when we try a regression analysis based on the economic links between the three indexes.

In case of the classical regression analysis, we first presuppose that each index is a non-random variable. Then we presuppose that one of the indexes, for example investments, depends on the other indexes and we make an econometric model of linear regression, such as $I = a \cdot GNP + b \cdot C + \varepsilon$. Here we have some parameters, such as *a* and *b*, which will be estimated according to the data in the table, and the random variable ε, called disturbance, which cumulates the effect of other unknown or neglected factors, on which we make certain assumptions.

The entire outcome depends on the data existing in the table. It is obvious that the data are the result of all economic processes within the national economy, which have been mixed with other processes involving the population behaviour at a national level, the international economic relations as well as the world wide status of the economy.

Within the classical econometric modelling, because of the complex „data – generating process" we have chosen that it will be presented according to a set of assumptions, where the random is as controlled as possible, so that the activity be monitored and even dosed.

The present paper chooses a systemic and cybernetic approach based on „data – generating process".

Such an approach leads the system econometric modelling made of three indexes to a system of three random variables defined on different probability spaces, for which there is no need to calculate the covariance among the pairs of random variables or to determine the regression function in relation to the others. In conclusion, there is no use for the econometric models of regression within this approach. Moreover, an important section of the classical econometric modelling is not adequate for such an economic index system.

The present paper proposes the research of a system made of three random variables between which there is a special a priori dependence, called β dependence, as the first stage of decoding the „data – generating process". The paper includes setting a probabilistic model of such a system and the construction of a representative example. It ends with an analysis of the systemic and cybernetic repercussions corresponding to the β dependence.

2. THE RANDOM SYSTEM WITH 3 CONSTITUENTS THAT GENERATES TIME SERIES DATA

We consider a random system *S*(*t*), which develops in discrete time and is defined by the ordered triplet $(X_1(t), X_2(t)), X_3(t)), \quad t \in \mathbf{N}^*$. The order refers to the fact that, each *t* time, we first consider $X_1(t)$, then $X_2(t)$ and then $X_3(t)$. $X_i(t)$ is a random variable which is defined on the probability space $(\Omega_i(t), \mathcal{K}_i(t), \mathrm{P}^i(t))$, having $\Omega_i(t)$ a set that is at the most numerable, $\forall\, i = \overline{1,3}$. Let us consider $x_i(t) = X_i(t)(\omega)$, with $\omega \in \Omega_i(t)$ and $i = \overline{1,3}$ where $x_i(t)$ means the value of the random variable X_i at *t* time, $\forall\, i = \overline{1,3}$.

The values of the three random variables make three time series data that correspond to the indicators marked I_1 , I_2 and I_3.

There is a reciprocal dependence between the three random variables that is defined as follows:

The β dependence

$\overline{X_2}$'s dependence on X_1

At $t \in \mathbf{N}^* \setminus \{1\}$ time, the probability space on which $X_2(t)$ random variable is defined depends on the value of $X_1(t)$ as follows:

If $x_1(t)$ meets the condition imposed by C_1, then

$\Omega_2(t) \neq \Omega_2(t-1), \Omega_2(t) \supset \Omega_2(t-1)$, $\mathcal{K}_2(t)=\mathcal{B}\ (\Omega_2(t))$, $P^2(t) \neq P^2(t-1)$, $P^2(t)$ being part of the same class of probability laws or probability distributions, such as $P^2(t-1)$ and $X_2(t) \neq X_2(t-1), X_2(t): \Omega_2(t) \to \mathbf{R}$, where

$$X_2(t)(\omega) = \begin{cases} X_2(t-1)(\omega), & \omega \in \mathbf{\Omega}_2(t-1) \\ k_2(t), & \omega \in \mathbf{\Omega}_2(t) \setminus \mathbf{\Omega}_2(t-1) \end{cases},$$

$k_2(t) \in \mathbf{R} \setminus X_2(t-1)(\mathbf{\Omega}_2(t-1))$.

If $x_1(t)$ does not meet the condition imposed by C_1, then $\Omega_2(t) = \Omega_2(t-1)$, $\mathcal{K}_2(t)= \mathcal{K}_2(t-1)$, $P^2(t) = P^2(t-1)$ and $X_2(t) = X_2(t-1)$.

b) X_3's dependence on X_2

At $t \in \mathbf{N}^* \setminus \{1\}$ time, the probability space on which $X_3(t)$ random variable is defined depends on the value of $X_2(t)$ as follows:

If $x_2(t)$ meets the condition imposed by C_2, then $\mathbf{\Omega}_3(t) \neq \mathbf{\Omega}_3(t-1)$,

$\mathbf{\Omega}_3(t) \supset \mathbf{\Omega}_3(t-1)$, $\mathcal{K}_3(t)=\mathcal{B}(\mathbf{\Omega}_3(t))$, $P^3(t) \neq P^3(t-1)$, $P^3(t)$ being part of the same class of probability laws or probability distributions, such as $P^3(t-1)$ and $X_3(t) \neq X_3(t-1)$, $X_3(t):\mathbf{\Omega}_3(t) \to \mathbf{R}$, where

$$X_3(t)(\omega) = \begin{cases} X_3(t-1)(\omega), & \omega \in \mathbf{\Omega}_3(t-1) \\ k_3(t), & \omega \in \mathbf{\Omega}_3(t) \setminus \mathbf{\Omega}_3(t-1) \end{cases},$$

$k_3(t) \in \mathbf{R} \setminus X_3(t-1)(\mathbf{\Omega}_3(t-1))$.

If $x_2(t)$ does not meet the condition imposed by C_2, then $\mathbf{\Omega}_3(t) = \mathbf{\Omega}_3(t-1)$, $\mathcal{K}_3(t)= \mathcal{K}_3(t\text{-}1)$, $P^3(t) = P^3(t-1)$ and $X_3(t) = X_3(t-1)$.

c) <u>X_1's dependence on X_3</u>

At t+1 time, $t \in \mathbf{N}^*$, the probability space on which $X_1(t+1)$ random variable is defined, depends on the value of $X_3(t)$ as follows:

If $x_3(t)$ meets the condition imposed by C_3, then

$\Omega_1(t+1) \neq \Omega_1(t), \Omega_1(t+1) \supset \Omega_1(t)$, $\mathcal{K}_1(t+1)= \mathcal{B}(\Omega_1(t+1))$,

$P^1(t+1) \neq P^1(t)$, $P^1(t+1)$ being part of the same class of probability laws or probability distributions, such as $P^1(t)$ and

$X_1(t+1) \neq X_1(t)$, $X_1(t+1):\Omega_1(t+1) \to \mathbf{R}$, where

$$X_1(t+1)(\omega) = \begin{cases} X_1(t)(\omega), & \omega \in \mathbf{\Omega}_1(t) \\ k_1(t+1), & \omega \in \mathbf{\Omega}_1(t+1) \setminus \mathbf{\Omega}_1(t) \end{cases},$$

$k_1(t+1) \in \mathbf{R} \setminus X_1(t)(\Omega_1(t))$.

If $x_3(t)$ does not meet the condition imposed by C_3, then $\Omega_1(t+1) = \Omega_1(t)$, $\mathcal{K}_1(t+1)= \mathcal{K}_1(t)$, $P^1(t+1) = P^1(t)$ and $X_1(t+1) = X_1(t)$.

The relation $S(t) \overset{\sim}{\mapsto} (X_1(t), X_2(t)\ X_3(t))$ is meaningful on the basis of the $\underline{\beta(3)}$ dependence, in order to designate the fact that the state of the random system at t time is characterized by the trinity (X_1(t), $X_2(t)$, $X_3(t)$).

Remark 1. At $t+1 \in \mathbf{N}^*$ time, the system has the state $S(t+1) \overset{\sim}{\mapsto} (X_1(t+1), X_2(t+1)\ X_3(t+1))$, which depends on the state $S(t)$, because $X_1(t+1)$ depends on $X_3(t)$, according to the c) procedure from the $\underline{\beta(3)}$ dependence. On the other hand, $X_2(t+1)$ depends on $X_1(t+1)$ according to a) procedure and $X_3(t+1)$ depends on $X_2(t+1)$ according to b) procedure.

Remark 2. At t time the effect of $x_1(t)$'s value upon the random variable $X_2(t)$ and the effect of $x_2(t)$'s value upon the random variable $X_3(t)$ are instantaneous. In this way, the simultaneity relations between the three random variables are postulate.

Remark 3. The procedure c) is a variety of feedback between constituents of system S(t).

Theorem T. The probability model of the state $S(t+1)$ depends on the values of $x_3(t)$ which appear in the $S(t)$ state as follows:

If $x_3(t)$ meets the condition imposed by C_3, then the probability model of $S(t+1)$ is represented by the three-dimensional random variable $X(t+1) = (X_1(t+1), X_2(t+1), X_3(t+1))$ which is defined on the probability space $(\Omega(t+1), \mathcal{K}(t+1), P(t+1))$, $\mathbf{\Omega}(t+1) = \mathbf{\Omega}_1(t+1) \times \mathbf{\Omega}_2(t+1) \times \mathbf{\Omega}_3(t+1)$,
$\mathcal{K}(t+1) = \mathcal{B}(\Omega(t+1))$, $P(t+1): \mathcal{K}(t+1) \to [0,1]$,
$P(t+1)(\{e\} \times \{f\} \times \{g\}) = P^1(t+1)(\{e\}) \cdot P^{2,e}(t+1)(\{f\}) \cdot P^{3,e,f}(t+1)(\{g\})$
$\forall e \in \mathbf{\Omega}_1(t+1)$, $\forall f \in \mathbf{\Omega}_2(t+1), \forall g \in \mathbf{\Omega}_3(t+1)$,

$$P^{2,e}(t+1) = \begin{cases} P^{2,a}(t+1), & e \in \Omega_1^a(t+1) \\ P^2(t), & e \in \Omega_1(t+1) \setminus \Omega_1^a(t+1) \end{cases},$$

$\Omega_1^a(t+1) = \{\omega \in \Omega_1(t+1) | X_1(t+1)(\omega) = x_1(t+1)$ meets the condition imposed by the $C_1\}$,
$P^{2,a}(t+1): \mathcal{K}_2(t+1) \to [0,1]$, $P^{2,a}(t+1) \neq P^2(t)$, $\mathcal{K}_2(t+1) = \mathcal{B}(\Omega_2(t+1))$,
$\Omega_2(t+1) \neq \Omega_2(t)$, $\Omega_2(t+1) \supset \Omega_2(t)$,

$$P^{3,e,f}(t+1) = \begin{cases} P^{3,e,a}(t+1), & f \in \Omega_2^a(t+1) \\ P^3(t), & f \in \Omega_2(t+1) \setminus \Omega_2^a(t+1) \end{cases},$$

$\mathbf{\Omega}_2^a(t+1) = \{\omega \in \mathbf{\Omega}_2(t+1) | X_2(t+1)(\omega) = x_2(t+1)$ meets the condition imposed by the $C_2\}$,
$P^{3,e,a}(t+1): \mathcal{K}_3(t+1) \to [0,1]$, $P^{2,e,a}(t+1) \neq P^3(t)$, $\mathcal{K}_3(t+1) = \mathcal{B}(\Omega_3(t+1))$,
$\mathbf{\Omega}_3(t+1) \neq \mathbf{\Omega}_3(t)$, $\mathbf{\Omega}_3(t+1) \supset \mathbf{\Omega}_3(t)$.

Also happens

$\forall A \in \mathcal{K}(t+1)$ with $A_e = \{f \in \Omega_2(t+1) | (e,f) \in pr_{\Omega_1(t+1)}A \times pr_{\Omega_2(t+1)}A\}$ and
$A_{e,f} = \{ g \in \mathbf{\Omega}_3(t+1) | (e,f,g) \in \mathbf{\Omega}(t+1) \}$,

$$P(A) = \sum_{e \in \Omega_1(t+1)} \sum_{f \in A_e} P^1(t+1)(\{e\}) \cdot P^{2,e}(t+1)(A_e) \cdot P^{3,e,f}(t+1)(A_{e,f}) \text{ or}$$

$$P(A) = \int\int P^{3,e,f}(t+1)(A_{e,f}) dP^1(t+1)(e) dP^{2,e}(t+1)(f).$$

If $x_3(t)$ does not meet the condition imposed by C_3, then the probability model of $S(t+1)$ is represented by the three-dimensional random variable $X(t+1) = (X_1(t+1), X_2(t+1), X_3(t+1))$ which is defined on the probability

space $(\Omega(t+1)$, $\mathcal{K}(t+1)$, $\mathrm{P}(t+1))$, $\Omega(t+1)=\Omega_1(t)\times\Omega_2(t+1)\times\Omega_3(t+1)$, $\mathcal{K}(t+1) = \mathcal{B}(\Omega(t+1))$, $P(t+1)$: $\mathcal{K}(t+1) \to [0,1]$,
$P(t+1)(\{e\}\times\{f\}\times\{g\}) = P^1(t)(\{e\})\cdot P^{2,e}(t+1)(\{f\})\cdot P^{3,e,f}(t)$,
$\forall e\in\Omega_1(t),\ f\in\Omega_2(t+1), g\in\Omega_3(t+1)$,

$$P^{2,e}(t+1)=\begin{cases} P^{2,a}(t+1), & e\in\Omega_1^a(t+1) \\ P^2(t), & e\in\Omega_1(t+1)\setminus\Omega_1^a(t+1)\end{cases},$$

$\Omega_1^a(t+1)=\{\omega\in\Omega_1(t+1) | X_1(t+1)(\omega)=x_1(t+1)$ meets the condition imposed by the $C_1\}$,
$P^{2,a}(t+1):\mathcal{K}_2(t+1)\to[0,1]$, $P^{2,a}(t+1)\neq P^2(t)$, $\mathcal{K}_2(t+1)= \mathcal{B}(\Omega_2(t+1))$,
$\Omega_2(t+1)\neq\Omega_2(t)$, $\Omega_2(t+1)\supset\Omega_2(t)$,

$$P^{3,e,f}(t+1)=\begin{cases} P^{3,e,a}(t+1), & f\in\Omega_2^a(t+1) \\ P^3(t), & f\in\Omega_2(t+1)\setminus\Omega_2^a(t+1)\end{cases},$$

$\mathbf{\Omega}_2^a(t+1)=\{\omega\in\mathbf{\Omega}_2(t+1) | X_2(t+1)(\omega)=x_2(t+1)$ meets the condition imposed by the $C_2\}$,
$P^{3,e,a}(t+1):\mathcal{K}_3(t+1)\to[0,1]$, $P^{2,e,a}(t+1)\neq P^3(t)$, $\mathcal{K}_3(t+1)= \mathcal{B}(\Omega_3(t+1))$,
$\mathbf{\Omega}_3(t+1)\neq\mathbf{\Omega}_3(t)$, $\mathbf{\Omega}_3(t+1)\supset\mathbf{\Omega}_3(t)$.

Also happens
$\forall\ A\in\mathcal{K}(t+1)$, $A_e=\{f\in\Omega_2(t+1) | (e,f)\in pr_{\Omega_1(t)}A\times pr_{\Omega_2(t+1)}A\}$ and $A_{e,f}$ = $=\{\ g\in\mathbf{\Omega}_3(t+1) | (e,f,g)\in\mathbf{\Omega}(t+1)\ \}$,

$$P(A)=\sum_{e\in\Omega_1(t)}\sum_{f\in A_e}P^1(t)(\{e\})\cdot\ P^{2,e}(t+1)(A_e)\cdot P^{3,e,f}(t+1)(A_{e,f})\ \text{or}$$

$$P(A)=\int\int P^{3,e,f}(t+1)(A_{e,f})dP^1(t)(e)dP^{2,e}(t+1)(f).$$

Proof.

Suppose that $x_3(t)$ meets the C_3 condition. Then, according to the c) procedure from the β (3) dependence, we have $\Omega_1(t+1)\supset\Omega_1(t)$, $\Omega_1(t+1)\neq\Omega_1(t)$, $\mathcal{K}_1(t+1)= \mathcal{B}(\Omega_1(t+1))$, that is $\mathcal{K}_1(t+1)$ is the Borelian generated field of $\Omega_1(t+1)$, $P^1(t+1)\neq P^1(t)$, but $P^1(t+1)$ is part of the same class of probability laws, as well as $P^1(t)$.

Following the statement above, at t+1 time the random variable $X_1(t+1)$ is defined on the probability space $(\Omega_1(t+1),\ \mathcal{K}_1(t+1), P^1(t+1))$.

According to a) procedure from the $\beta(3)$ dependence, we may only have two situations:

1) $x_1(t+1)$ meets the C_1 condition. In this case, $\Omega_2(t+1) \neq \Omega_2(t)$, $\Omega_2(t+1) \supset \Omega_2(t)$, $\mathcal{K}_2(t+1)= \mathcal{B}\,(\Omega_2(t+1))$, $P^2(t+1) \neq P^2(t)$ and $P^2(t+1)$ is part of the same class of probability laws, as well as $P^2(t)$. We term $P^{2,a}(t+1)$ the probability law $P^2(t+1)$. Following, at $t+1$ time the random variable $X_2(t+1)$ is defined on the probability space $(\Omega_2(t+1),\ \mathcal{K}_2(t+1), P^{2,a}(t+1))$.

2) $x_1(t+1)$ does not meet the C_1 condition. In this case $\Omega_2(t+1) = \Omega_2(t)$, $\mathcal{K}_2(t+1)=\mathcal{K}_2(t)$, $P^2(t+1) = P^2(t)$ and at $t+1$ time the random variable $X_2(t+1)$ is defined on the probability space $(\Omega_2(t), \mathcal{K}_2(t), P^2(t))$.

Out of the two situations, we may deduce that there are defined on the measured space $(\Omega_2(t+1), \mathcal{K}_2(t+1))$ two probabilities: $P^{2,a}(t+1)$ and $P^2(t)$, where $P^2(t)(\omega)=0$ for $\omega \in \mathbf{\Omega}_2(t+1) \setminus \mathbf{\Omega}_2(t)$.

According to b) procedure from the $\beta(3)$ dependence, we may only have two situations:

1) $x_2(t+1)$ meets the C_2 condition. In this case, $\mathbf{\Omega}_3(t+1) \neq \mathbf{\Omega}_3(t)$, $\mathbf{\Omega}_3(t+1) \supset \mathbf{\Omega}_3(t)$, $\mathcal{K}_3(t+1)= \mathcal{B}\,(\mathbf{\Omega}_3(t+1))$, $P^3(t+1) \neq P^3(t)$ and $P^3(t+1)$ is part of the same class of probability laws, as well as $P^3(t)$. We term $P^{3,a}(t+1)$ the probability law $P^3(t+1)$. Following, at $t+1$ time the random variable $X_3(t+1)$ is defined on the probability space $(\mathbf{\Omega}_3(t+1),\ \mathcal{K}_3(t+1), P^{3,a}(t+1))$.

2) $x_2(t+1)$ does not meet the C_2 condition. In this case $\mathbf{\Omega}_3(t+1) = \mathbf{\Omega}_3(t)$, $\mathcal{K}_3(t+1)=\mathcal{K}_3(t)$, $P^3(t+1) = P^3(t)$ and at $t+1$ time the random variable $X_3(t+1)$ is defined on the probability space $(\mathbf{\Omega}_3(t), \mathcal{K}_3(t), P^3(t))$.

Out of the two situations, we may deduce that there are defined on the measured space $(\Omega_3(t+1), \mathcal{K}_3(t+1))$ two probabilities: $P^{3,a}(t+1)$ and $P^3(t)$, where $P^3(t)(\omega)=0$ for $\omega \in \Omega_2(t+1) \setminus \Omega_2(t)$.

In conclusion, we have the probability space $(\Omega_1(t+1), \mathcal{K}_1(t+1), P^1(t+1))$, the measurable space $(\Omega_2(t+1), \mathcal{K}_2(\mathrm{t}+1))$, where two probabilities $P^{2,a}(t+1)$ and $P^2(t)$ are defined and the measurable space $(\Omega_3(t+1), \mathcal{K}_3(t+1))$, where two probabilities $P^{3,a}(t+1)$ and $P^3(t)$ are defined.

According to the proposition P from the Annex [4], [14], it follows that the probability model of the state $S(t+1)$ is given by the three-dimensional random variable $X(t+1) = (X_1(t+1), X_2(t+1), X_3(t+1))$, which is defined on the probability space $(\Omega(t+1), \mathcal{K}(t+1), \mathrm{P}(t+1))$,

where:

$\Omega(t+1)=\Omega_1(t+1)\times\Omega_2(t+1)\times\Omega_3(t+1)$, $\mathcal{K}(t+1)=$
$=\mathcal{K}_1(t+1)\times\mathcal{K}_2(t+1)\times\mathcal{K}_3(t+1)$ and
$P(t+1):\mathcal{K}(t+1)\to[0,1]$, $\forall(e,f,g)\in\Omega_1(t+1)\times\Omega_2(t+1)\times\Omega_3(t+1)$,

$$P(t+1)(\{e\}\times\{f\}\times\{g\})=P^1(t+1)(\{e\})\cdot P^{2,e}(t+1)(\{f\})\cdot P^{3,e,f}(t+1)(\{g\}),$$

where: $P^{2,e}(t+1)=\begin{cases}P^{2,a}(t+1), & e\in\Omega_1^a(t+1)\\ P^2(t), & e\in\Omega_1(t+1)\setminus\Omega_1^a(t+1)\end{cases}$,

and $\Omega_1^a(t+1)$ is the set defined in the theorem content and

$$P^{3,e,f}(t+1)=\begin{cases}P^{3,e,a}(t+1), & f\in\Omega_2^a(t+1)\\ P^3(t), & f\in\Omega_2(t+1)\setminus\Omega_2^a(t+1)\end{cases},$$

and $\Omega_2^a(t+1)$ is the set defined in the theorem content.

Out of above results, $\forall\ A\in\mathcal{K}(t+1)$

with $A_e=\{f\in\Omega_2(t+1)\,|\,(e,f)\in pr_{\Omega_1(t+1)}A\times pr_{\Omega_2(t+1)}A\}$ and $A_{e,f}=$
$=\{\ g\in\mathbf{\Omega}_3(t+1)\,|\,(e,f,g)\in\mathbf{\Omega}\ (t+1)\ \}$ and by the relation:

$$\sum_{(e,f,g)\in A}(\cdot)=\sum_{e\in\Omega_1(t+1)}\ \sum_{f\in A_e}\ \sum_{g\in A_{e,f}}(.),$$

it follows:

$$P(A)=\sum_{e\in\Omega_1(t+1)}\ \sum_{f\in A_e}P^1(t+1)(\{e\})\cdot\ P^{2,e}(t+1)(A_e)\cdot P^{3,e,f}(t+1)(A_{e,f})\ \text{or}$$

$$P(A)=\int\int P^{3,e,f}(t+1)(A_{e,f})dP^1(t+1)(e)dP^{2,e}(t+1)(f).$$

Suppose that $x_3(t)$ does not meet the C_2 condition. Then according to c) from the $\beta\ (3)$ dependence we have: $\Omega_1(t+1)=\Omega_1(t)$, $\mathcal{K}_1(t+1)=\mathcal{K}_1(t)$, $P^1(t+1)=P^1(t)$ and $X_1(t+1)=X_1(t)$.

According to a) from the $\beta(3)$ dependence there are two situations that we have described in 1.

In conclusion, we have the probability space $(\Omega_1(t),\ \mathcal{K}_1(t),P^1(t))$, the measurable space $(\Omega_2(t+1),\ \mathcal{K}_2(t+1))$ on which the two probabilities $P^{2,a}(t+1)$ and $P^2(t)$ are defined and presented in 1 and the measurable space $(\Omega_3(t+1),\mathcal{K}_3(t+1))$, where two probabilities $P^{3,a}(t+1)$ and $P^3(t)$ are defined and presented in 1.

According to the proposition P from the Annex, it follows that the probability model of the state $S(t+1)$ is given by the three-dimensional random variable $X(t+1)=(X_1(t),X_2(t+1),X_3(t+1))$ which is defined on the probability ($\Omega \left(t+1\right)$,$\mathcal{K} \left(t+1\right)$, P $\left(t+1\right)$), where: $\Omega\left(t+1\right)=\Omega_1(t)\times\Omega_2(t+1)\times\Omega_3\left(t+1\right)$, $\mathcal{K}\left(t+1\right)$ = $\mathcal{K}_1(t)\times\mathcal{K}_2(t+1)\times\mathcal{K}_3(t+1)$ and $P\left(t+1\right)$: $\mathcal{K}\left(t+1\right)\to[0,1]$, $\forall(e,f,g)\in\Omega_1(t)\times\Omega_2(t+1)\times\Omega_3(t+1)$,

$P(\{e\}\times\{f\}\times\{g\})=P^1(t)(\{e\})\cdot P^{2,e}(t+1)(\{f\})\cdot P^{3,e,f}\left(t+1\right)\left(\{g\}\right)$ where $P^{2,e}(t+1)$ and $P^{3,e,f}$ probabilities being defined in 1.

Out of above results, $\forall\ A\in\ \mathcal{K}\left(t+1\right)$

with $A_e=\{f\in\Omega_2(t+1)\big|(e,f)\in pr_{\Omega_1(t)}A\times pr_{\Omega_2(t+1)}A\}$ and $A_{e,f}$ =

={ $g\in\mathbf{\Omega}_3\left(t+1\right)\big|\left(e,f,g\right)\in\mathbf{\Omega}\left(t+1\right)$ } and by the relation:

$$\sum_{(e,f,g)\in A}(\cdot)=\sum_{e\in\Omega_1(t)}\sum_{f\in A_e}\sum_{g\in A_{e,f}}(.),$$

it follows:

$$P\left(A\right)=\sum_{e\in\Omega_1(t)}\sum_{f\in A_e}P^1\left(t\right)\left(\{e\}\right)\cdot\ P^{2,e}\left(t+1\right)\left(A_e\right)\cdot P^{3,e,f}\left(t+1\right)\left(A_{e,f}\right)\text{ or}$$

$$P(A)=\int\int P^{3,e,f}(t+1)(A_{e,f})dP^1(t)\left(e\right)dP^{2,e}\left(t+1\right)\left(f\right).$$

3. EXAMPLE

We consider a system S made up of three urns U_1 , U_2 and U_3. Urn U_1 contains n_1 balls of the same size, numbered from 1 to n_1, $n_1\geq 10$, from which a are white and the rest black, $0<a<n_1$. Urn U_2 contains n_2 balls of the same size, numbered from 1 to n_2 and urn U_3 contains n_3 balls of the same size, numbered from 1 to n_3 . Extractions are done from system S. An extraction from system S consists in a random extraction from urn U_1 followed by an extraction of the same type from urn U_2 and next extraction from urn U_3.

We consider three conditions C_1, C_2 and C_3, which refer to the results of the extractions.

If the extracted ball from urn U_1 is white, i.e. it meets the condition C_1, then we introduce a same type ball into urn U_2 assigning it the number n_2+1. On a contrary situation, the structure of urn U_2 does not change. Then we do the extraction from urn U_2.

If the result of the extraction from urn U_2 meets the condition C_2, then we introduce a same ball into urn U_3, assigning it the number n_3+1, on a contrary situation the structure of urn does not change. Then we do the extraction from urn U_3.

If the result of the extraction from urn U_3 is an even number, i.e. it meets the condition C_3 ,then we introduce a white ball into urn U_1 assigning it the number n_1+1, on a contrary situation the structure of urn U_1 does not change. Then we do the following extraction from the system.

We will attach to each urn a probability space and a random variable defined on this space. In order to do this we will consider the following notations.

Let us take t as the number of extractions from system S, $t \in \mathbf{N}^*$, $n_i(t)$ the number of balls from urn U_i , before the extraction of the t category, $i = \overline{1,2}$. We notice that $n_i(t) \le n_i(t+1), \forall t \in \mathbf{N}^*$.

Let us take $a(t)$ as the number of white balls from the urn U_1 before the extraction of the t category. We notice that $a(t) \le a(t+1), \forall t \in \mathbf{N}^*$.

The probability space attached to urn U_1 before the extraction of the category t is $(\Omega_1(t), \mathcal{P}(\Omega_1(t)) = \mathcal{K}_1(t), P^1(t))$, where

$$\Omega_1(t) = \{\omega_1, \ldots, \omega_{n_1(t)}\},\ P_1(t)(\omega) = \begin{cases} \dfrac{a(t)}{n_1(t)}, & \omega \in \Omega_1^a(t) \\ 1 - \dfrac{a(t)}{n_1(t)}, & \omega \in \Omega_1(t) \setminus \Omega_1^a(t) \end{cases}.$$

$\Omega_1^a(t) = \{\omega \in \Omega_1(t) \,|\, \omega \text{ the event of obtaining a white ball}\}$.

Let us take $X_1(t) : \Omega_1(t) \to \mathbf{R}$, $X_1(t)$ is the random variable that represents the result of the extraction of the category t from urn U_1 and let us take $x_1(t) = X_1(t)(\omega), \omega \in \Omega_1(t)$.

The probability space attached to urn U_2 before the extraction of the category t is $(\Omega_2(t), \mathcal{K}_2(t), P^2(t))$ where $\Omega_2(t) = \{\omega_1, \ldots, \omega_{n_2(t)}\}$, $\mathcal{K}_2(t) = \mathcal{P}(\Omega_2(t))$, $P^2(t)(\omega) = \dfrac{1}{n_2(t)}, \forall \omega \in \Omega_2(t)$.

Let us take $X_2(t) : \Omega_2(t) \to \mathbf{R}$, $X_2(t)$ is the random variable that represents the result of the extraction of the category t from urn U_2 and let us take $x_2(t) = X_2(t)(\omega), \omega \in \Omega_2(t)$.

The probability space attached to urn U_3 before the extraction of the category t is $(\Omega_3(t), \mathcal{K}_3(t), P^3(t))$ where $\Omega_3(t) = \{\omega_1, \ldots, \omega_{n_3(t)}\}$, $\mathcal{K}_3(t) = \mathcal{P}(\Omega_3(t))$, $P^3(t)(\omega) = \dfrac{1}{n_3(t)}, \forall \omega \in \Omega_3(t)$.

Let us take $X_3(t):\Omega_3(t)\to\mathbf{R}$, $X_3(t)$ is the random variable that represents the result of the extraction of the category t from urn U_3 and let us take $x_3(t)=X_3(t)(\omega),\ \omega\in\Omega_3(t)$.

Following the significance of the three random variables we can $S(t)\mapsto(X_1(t),\ X_2(t),\ X_3(t))$ write.

According to the extraction mode from the system S defined above, there is a dependence $\beta\,(3)$ between the three random variables.We must analyze successively the three dependencies from the definition of $\beta(3)$.

a) X_2's dependence on X_1.

In the extraction of the category $t\in\mathbf{N}^*$ the probability space on which the random variable $X_2(t)$ is defined depends on the value of $X_1(t)$, as follows:

- if $x_1(t)$ is a white ball, i.e. it meets the condition imposed by C_1, then $\Omega_2(t)\neq\Omega_2(t-1)$, $\Omega_2(t)=\{\omega_1,\ldots,\omega_{n_2(t)}\}\supset\Omega_2(t-1)=\{\omega_1,\ldots,\omega_{n_2(t-1)}\}$ because $n_2(t)>n_2(t\text{-}1)$,

$\mathcal{K}_2(t)=\mathcal{P}(\Omega_2(t))$, $P^2(t)\neq P^2(t-1)$, $P^2(t)(\omega)=\dfrac{1}{n_2(t)}$, $\forall\omega\in\Omega_2(t)$.

It follows that $P^2(t)(\omega)\neq P^2(t-1)(\omega),\ \forall\omega\in\Omega_2(t-1)$.

$X_2(t):\Omega_2(t)\to\mathbf{R}$, where

$$X_2(t)(\omega)=\begin{cases}X_2(t-1)(\omega), & \omega\in\mathbf{\Omega}_2(t-1)\\ n_2(t-1)+1=n_2(t), & \omega\in\mathbf{\Omega}_2(t)\setminus\mathbf{\Omega}_2(t-1)\end{cases}.$$

-if $x_1(t)$ is a black ball, i.e. it does not meet the condition imposed by C_1, then $\Omega_2(t)=\Omega_2(t-1)$, $\mathcal{K}_2(t)=\mathcal{K}_2(t\text{-}1)$, $P^2(t)=P^2(t-1)$ and $X_2(t)=X_2(t-1)$.

b) X_3's dependence on X_2.

In the extraction of the category $t\in\mathbf{N}^*$ the probability space on which the random variable $X_3(t)$ is defined depends on the value of $X_2(t)$, as follows:

- if $x_2(t)$ meets the condition imposed by C_2, then $\Omega_3(t)\neq\Omega_3(t-1)$, $\Omega_3(t)=\{\omega_1,\ldots,\omega_{n_3(t)}\}\supset\Omega_3(t-1)=\{\omega_1,\ldots,\omega_{n_3(t-1)}\}$ because $n_3(t)>n_3(t\text{-}1)$,

$\mathcal{K}_3(t)=\mathcal{P}(\Omega_3(t))$, $P^3(t)\neq P^3(t-1)$, $P^3(t)(\omega)=\dfrac{1}{n_3(t)}$, $\forall\omega\in\Omega_3(t)$.

It follows that $P^3(t)(\omega)\neq P^3(t-1)(\omega),\ \forall\omega\in\Omega_3(t-1)$.

$X_3(t):\Omega_3(t)\to\mathbf{R}$, where

$$X_3(t)(\omega)=\begin{cases}X_3(t-1)(\omega), & \omega\in\Omega_3(t-1)\\ n_3(t-1)+1=n_3(t), & \omega\in\Omega_3(t)\setminus\Omega_3(t-1)\end{cases}.$$

- if $x_2(t)$ does not meet the condition imposed by C_2, then $\Omega_2(t)=\Omega_2(t-1)$, $\mathcal{K}_2(t)=\mathcal{K}_2(t\text{-}1)$, $P^2(t)=P^2(t-1)$ and $X_2(t)=X_2(t-1)$.

c) X_1's dependence on X_3.

The extraction of the category $t+1, t \in \mathbf{N}^*$. Happened, the probability space on which the random variable $X_1(t+1)$ is defined depends on the value of $X_3(t)$, as follows:

- if $x_3(t)$ is "a ball with an even number", i.e. it meets the condition imposed by C_3, then $\Omega_1(t+1) = \{\omega_1, \dots, \omega_{n_1(t+1)}\} \supset \Omega_1(t)$ because $n_1(t) < n_1(t+1)$;

$\mathcal{K}_1(t+1)=\mathcal{P}(\Omega_1(t+1))$, $P^1(t+1)(\omega) = \dfrac{a(t+1)}{n_1(t+1)}$, $\forall \omega \in \Omega_1(t+1)$, $a(t) < \; < a(t+1)$.

It follows that $P^1(t+1)(\omega) \neq P^1(t)(\omega), \forall \omega \in \Omega_1(t)$.

$X_1(t+1) : \Omega_1(t+1) \to \mathbf{R}$, where

$$X_1(t+1)(\omega) = \begin{cases} X_1(t)(\omega), & \omega \in \mathbf{\Omega}_1(t) \\ n_1(t)+1, & \omega \in \mathbf{\Omega}_1(t+1) \setminus \mathbf{\Omega}_1(t) \end{cases}.$$

- if $x_3(t)$ is "a ball with an odd number", i.e. it does not meet the condition imposed by C_2, then $\Omega_1(t+1) = \Omega_1(t)$, $\mathcal{K}_1(t+1)=\mathcal{K}_1(t)$, $P^1(t+1) = P^1(t)$ and $X_1(t+1) = X_1(t)$.

Let us apply the Theorem T to the system. It follows that the probability model of the state $S(t+1)$ depends on the value of $x_3(t)$ which appears with the state $S(t)$ when the t extraction happens.

1. If $x_3(t)$ is a "a ball with an even number", then the probability model is represented by the three-dimensional random variable $X(t+1) = (X_1(t+1), X_2(t+1), X_3(t+1))$, defined on the probability space $(\Omega(t+1), \mathcal{K}(t+1), \mathrm{P}(t+1))$, where $\Omega(t+1) = \Omega_1(t+1) \times \Omega_2(t+1) \times \Omega_3(t+1)$, $\mathcal{K}(t+1)=\mathcal{K}_1(t+1) \times \mathcal{K}_2(t+1) \times \mathcal{K}_3(t+1)$, $\mathrm{P} : \mathcal{K}(t+1) \to [0,1]$,

$\forall (e, f, g) \in \Omega_1(t+1) \times \Omega_2(t+1) \times \Omega_3(t+1)$,

$$P(\{e\} \times \{f\} \times \{g\}) = P^1(t+1)(\{e\}) \cdot P^{2,e}(t+1)(\{f\}) \cdot P^{3,e,f}(t+1)(\{g\}),$$

$$P^{2,e}(t+1) = \begin{cases} P^{2,a}(t+1), & e \in \mathbf{\Omega}_1^a(t+1) \\ P^2(t), & e \in \mathbf{\Omega}_1(t+1) \setminus \mathbf{\Omega}_1^a(t+1) \end{cases},$$

$\Omega_1^a(t+1) = \{\omega \in \Omega_1(t+1) \mid x_1(t+1)$ meets the condition imposed by the $C_1\}$,

$P^{2,a}(t+1) : \mathcal{K}_2(t+1) \to [0,1]$, $P^{2,a}(t+1)(\omega) = \dfrac{1}{n_2(t+1)}$, $\forall \omega \in \Omega_1(t+1)$

$P^{2,a}(t+1) \neq P^2(t)$, $\mathcal{K}_2(t+1)= \mathcal{B}(\Omega_2(t+1))$, $\Omega_2(t+1) \neq \Omega_2(t)$,

$\Omega_2(t+1) \supset \Omega_2(t)$,

$$P^{3,e,f}(t+1)=\begin{cases} P^{3,e,a}(t+1), & f\in\Omega_2^a(t+1) \\ P^3(t), & f\in\Omega_2(t+1)\setminus\Omega_2^a(t+1)\end{cases},$$

$\mathbf{\Omega}_2^a(t+1)=\{\omega\in\mathbf{\Omega}_2(t+1)\,|\,X_2(t+1)(\omega)=x_2(t+1)$ meets the condition imposed by the $C_2\}$,

$P^{3,e,a}(t+1):\mathcal{K}_3(t+1)\to[0,1]$, $P^{3,e,a}(t)(\omega)=\dfrac{1}{n_3(t+1)}$, $\forall\,\omega\in\Omega_3(t+1)$

$P^{2,e,a}(t+1)\neq P^3(t)$, $\mathcal{K}_3(t+1)=\mathcal{B}(\Omega_3(t+1))$, $\mathbf{\Omega}_3(t+1)\neq\mathbf{\Omega}_3(t)$, $\mathbf{\Omega}_3(t+1)\supset\mathbf{\Omega}_3(t)$.

Also happens

$\forall\ A\in\mathcal{K}(t+1)$ with $A_e=\{f\in\Omega_2(t+1)\,|\,(e,f)\in pr_{\Omega_1(t+1)}A\times pr_{\Omega_2(t+1)}A\}$ and

$A_{e,f}\ =\{\ g\in\mathbf{\Omega}_3(t+1)\,|\,(e,f,g)\in\mathbf{\Omega}\,(t+1)\ \}$,

$$P(A)=\sum_{e\in\mathbf{\Omega}_1(t+1)}\sum_{f\in\mathbf{\Omega}_2(t+1)}P^1(t+1)(\{e\})\cdot P^{2,e}(t+1)(A_e)\cdot P^{3,e,f}(t+1)(A_{e,f})$$

2. If $x_3(t)$ does not meet the condition imposed by C_3, then the probability model of $S(t+1)$ is represented by the three-dimensional random variable $X(t+1)=(X_1(t),X_2(t+1),X_3(t+1))$, defined on the probability space $(\Omega(t+1),\mathcal{K}(t+1),\mathrm{P}(t+1))$, where:

$\Omega(t+1)=\Omega_1(t)\times\Omega_2(t+1)\times\Omega_3(t+1)$, $\mathcal{K}(t+1)=\mathcal{K}_1(t)\times\mathcal{K}_2(t+1)\times\mathcal{K}_3(t+1)$ and $P(t+1):\mathcal{K}(t+1)\to[0,1]$,

$\forall(e,f,g)\in\Omega_1(t)\times\Omega_2(t+1)\times\Omega_3(t+1)$,

$$P(\{e\}\times\{f\}\times\{g\})=P^1(t)(\{e\})\cdot P^{2,e}(t+1)(\{f\})\cdot P^{3,e,f}(t+1)(\{g\}),$$

$$P^{2,e}(t+1)=\begin{cases} P^{2,a}(t+1), & e\in\Omega_1^a(t+1) \\ P^2(t), & e\in\Omega_1(t+1)\setminus\Omega_1^a(t+1)\end{cases},$$

$$P^{3,e,f}(t+1)=\begin{cases} P^{3,e,a}(t+1), & f\in\Omega_2^a(t+1) \\ P^3(t), & f\in\Omega_2(t+1)\setminus\Omega_2^a(t+1)\end{cases}$$

It is the same case for $\forall A\in\mathcal{K}$, having A_e and $A_{e,f}$ from above,

$$P(A)=\sum_{e\in\mathbf{\Omega}_1(t+1)}\sum_{f\in\mathbf{\Omega}_2(t+1)}P^1(t+1)(\{e\})\cdot P^{2,e}(t+1)(A_e)\cdot P^{3,e,f}(t+1)(A_{e,f}).$$

4. CONCLUSIONS

These are the conclusions on the β dependence.

1. The conditions $C_i, i=\overline{1,3}$ are of a deterministic nature and they form an interface between the S system and the outer environment. This way the system is open and they allow a cybernetic approach to it, because there is the possibility to regulate through economic or other measures.
2. The conditions $C_i, i=\overline{1,3}$ are bivalent: accomplished/non-accomplished. It is possible to formulate multivalent conditions, which would refine communication with the outer environment.
3. The nature of the three components belonging to the β dependence is different. Thus, the a) and b) dependences refer to the influence of a random variable on the following one, while the c) dependence is a feedback type, and it introduces a circulation relation between X_1 and X_3.
4. If at t and $t+1$ moments those three conditions are fulfilled, then the random variables $X_i(t+1), i=\overline{1,3}$ are defined on the various probability spaces, which differ from those at $t+1$ and $t+2$ moments.
5. If there is a time span $T \subseteq \mathbf{N}^*$ so that all conditions are not fulfilled, then $X_i(t+1) = X_i(t), \forall t \in T, i=\overline{1,3}$. In this instance, between those three random variables of S system, the β dependence is not displayed and the classical regression study makes sense. Also, in the hypothesis that S system is a subsystem of the $\mathcal{S}$ system, between those three random variables there may exist another dependence, which is displayed at the level of the $\mathcal{S}$ system.

ANNEX

PROPOSITION **P**. Let E, F, G be sets that are at the most numerable. Then the probabilities P on the measured space $(E \times F \times G, P(E \times F \times G))$, where $P(M)$ is the set of sides of the set M, are in one-to-one and onto correspondence with the systems $\left(Q, \left(Q^e\right)_{e\in E, Q(e)>0}, \left(Q^{e,f}\right)_{f\in F, Q^e(\{f\})>0}\right)$, where Q is a probability on the measured space $(E, P(E))$, $Q^e, e \in E$ are the probabilities on the measured space $(F, P(F))$, but $Q^{e,f}$ are the probabilities on the measured space $(G, P(G))$. The correspondence is thus: to one probability P on the measured space $(E \times F \times G, P(E \times F \times G))$ attaches probabilities:

$Q = P \circ pr_E^{-1}$, with $Q(\{e\}) = P\left(pr_E^{-1}(\{e\})\right) = P(\{e\} \times F \times G)$,

$Q^e = P_{pr_E^{-1}(\{e\})} \circ pr_F^{-1}$, with $Q^e(\{f\}) = P_{pr_E^{-1}(\{e\})}\left(pr_F^{-1}(\{f\})\right) =$

$= \dfrac{P(\{e\} \times \{f\} \times G)}{P(\{e\} \times F \times G)}$,

$Q^{e,f} = P_{pr_E^{-1}(\{e\}) \cap pr_F^{-1}(\{f\})} \circ pr_G^{-1}$, with $Q^{e,f}(\{g\}) = P_{pr_E^{-1}(\{e\}) \cap pr_F^{-1}(\{f\})}\left(pr_G^{-1}(\{g\})\right)$

$= \dfrac{P(\{e\} \times \{f\} \times \{g\})}{P(\{e\} \times \{f\} \times G)} = P_{\{e\} \times \{f\} \times G}(E \times F \times \{g\})$, where Q^e and $Q^{e,f}$ are conditioned probabilities;

to one system $\left(Q, \left(Q^e\right)_{e \in E, Q(e) > 0}, \left(Q^{e,f}\right)_{f \in F, Q^e(\{f\}) > 0}\right)$ attaches the probability P on the measured space $(E \times F \times G, P(E \times F \times G))$, with

$$P(\{e\} \times \{f\} \times \{g\}) = Q(\{e\}) Q^e(\{f\}) Q^{e,f}(\{g\}),$$

and for $A \in P(E \times F \times G)$ with $A_e = \{f \in F | (e,f) \in pr_E(A) \times pr_F(A)\}$ and

$A_{e,f} = \{g \in G | (e,f,g) \in A\}$, we have

$$P(A) = \sum \sum Q(\{e\}) Q^e(A_e) Q^{e,f}(A_{e,f})$$

or $P(A) = \iint Q^{e,f}(A_{e,f}) dQ(e) dQ^e(f)$.

REFERENCES

[1] **Andrei, T. (2003),** ***Statistică şi econometrie.*** Economica Publishing House, Bucharest;

[2] **Agapie, Adriana, Bădin, L. (2009),** ***Confidence Intervals for the Guesstimation Algorithm: A Bootstrap Approach***, Economic Computation and Economic Cybernetics Studies and Research, Vol. 43, no. 3, ASE Publishing House, Bucharest;

[3] **Bourbonnais, R. (2002),** ***Econometrie - manuel et exercises corriges.*** Dunod, Paris;

[4] **Cuculescu, I. (1988),** ***Teoria probabilităților***, ALL Publishing House, Bucharest **;**

[5] **Dosescu, T. C. and Raischi, C. (2008),** ***The Econometric Modeling of a Random System that Generates Time Series Data***, Economic Computation and Economic Cybernetics Studies and Research, Vol. 42, n0. 3-4, ASE Publishing House, Bucharest;

[6] **Fuller, A. W. (1996),** ***Introduction to Statistical Time Series.*** John Wiley and Sons Inc., New York;

[7] **Green, W.(2008),** ***Econometric Analysis, Sixth Edition,*** Prentice Hall Publishing;

[8] **Groebner, D. F., Shannon, P. W., Fry, C. Ph. and Smith, K. D. (2005),** ***Business Statistics***. Updated sixth edition, Pearson Education International;
[9] **Iosifescu, M., Mihoc, Gh. and Theodorescu, R. (1966):** ***Teoria probabilităților și statistică matematică.*** Tehnica Publishing House, Bucharest;
[10] **Kerry, Patterson (2000)**, ***An Introduction to Applied Econometrics: A Time Series Approach***. Published by PALGRAVE;
[11] Ruxanda, Gh., Stoian, Adr. (2008), ***Modelling of the Relationship between Foreign Direct Investment and Economic Growth***, Economic Computation and Economic Cybernetics Studies and Research, Vol. 42, no. 3-4, ASE Publishing House, Bucharest;
[12] **Spătaru, S. (2007),** ***Modele şi metode econometrice.*** A.S.E. Publishing House, Bucharest;
[13] **Stock, H. James (2003),** ***Introduction to Econometrics***. Addison Wesley, Boston, San Francisco, New York, Paris, Montreal;
[14] **Tudor, C. (2004):** ***Teoria Probabilităților***. University of Bucharest Publishing House.