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MAXIMIZING THE NET PRESENT VALUE OF A PROJECT UNDER INFLATIONARY CONDITIONS

Abstract. *Project scheduling to maximize the net present value of the cash flows has been a topic of recent research. These researches assume that activities cost or price for materials and service in the marketplace remain relatively unchanged over the lifecycle of the project. Unfortunately, due to the increasing prices of goods in most countries during the project, it is not generally a realistic assumption. In this paper, we consider project scheduling problem with discounted cash flows under inflationary conditions. We propose two different situations due to the type of contract between contractor and client as fixed-price contract and cost-reimbursement contract. To interpret the situation, we use a lemma which shows the importance of entering inflation for both sides of a project clients and contractor. Finally, an example is solved and the proposed procedure is applied to interpret the solution.*

Keywords: *Net Present Value; Scheduling; Project; Inflation; Discounted Cash Flows.*

JEL Classification: C44, E31

1. Introduction and Literature Review

The management of a project requires the scheduling of a set of activities. Construction of a feasible project schedule is an easy task, but the difficulty arises when improved schedules are desired to optimize a stated objective function. Past research has employed various objectives, for example minimization of the project duration, maximization of Net Present Value (NPV) of the project cash flows, maximization of the project resource utilization, or minimization of the project total costs.

As it can be deduced from the literature, the minimization of the project duration and the maximization of NPV are the two most commonly emphasized objectives in the field. Models incorporating NPV criteria usually assume a project due date since otherwise activities involving negative cash flows would be delayed indefinitely to maximize NPV.

Since the project manager's main objective is to maximize financial aspect of the project, it comes into picture through cash in- and out-flows associated with the activities and/or events. The cash flows may be either the project costs or the payments made for the project during its life cycle. It is important for the manager to develop a schedule that balances the early receipt of progress payment with the delay of particularly large expenditures. Time value of money is taken into consideration through discounting the cash flows. The most commonly employed financial objective is the maximization of NPV of the cash flows where cash inflows are treated as positive (receipts) and cash outflows as negative (expenditures). A project scheduling problem in which the objective is to maximize NPV of the project cash flows is called Project Scheduling Problems with Discounted Cash Flows (PSPDCF). Russell (1970) introduced the problem of the maximizing NPV in project scheduling problem. He proposed a successive approximation approach to solve the problem. Grinold (1972) added a project deadline to the model, formulated the problem as a linear programming problem, and proposed a method to solve it. Doersch and Patterson (1977) presented a zero-one integer-programming model for the NPV problem. Their model included a constraint on capital expenditure of the activities in the project, while the available capital increased as progress payments were made. Bey et al. (1981) considered the implications of a bonus/penalty structure on optimal project schedules for the NPV problem. Russell (1986) considered the resource-constrained NPV maximization problem. He introduced priority rules of selecting activities for resource assignment based upon information derived from the optimal solution to the unconstrained problem. Smith-Daniels and Smith-Daniels (1987) extended the Doersch and Patterson Zero-one formulation to accommodate material management costs. Icmeli and Erengus (1996) introduced a branch and bound procedure to solve the resource constrained project-scheduling problem with discounted cash flows. Najafi and Niaki (2006) introduced a new resource investment project scheduling problem in which the goal was to maximize the discounted cash flows of the project payments, and called it as a Resource Investment Problem with Discounted Cash Flows.

In all of the PSDCF researches, it is assumed that activities costs or prices for materials and services in the marketplace remain relatively unchanged over the life cycle of the project. Unfortunately, this is not generally a realistic assumption. A study of the past performance of most economies throughout the world reveals that the prices for goods and services are continually increasing. Inflation is the term used to describe the rate of change in the prices of goods and services over a given period of time. Inflation may arise from a variety of factors but the basic reason for inflation is having too much money competing to buy the available goods at their existing prices, allowing those prices to rise. In most countries, particularly the developing countries, inflation rises daily. Inflation is, therefore, an important determinant of project management and scheduling.

This paper is the first to consider the condition of inflation and the objective is to maximize the net present value of all cash flows of the project. This research is new in the area of project scheduling and the only work which has considered the inflation in the project scheduling problem was proposed by Jolayemi and Oluleye (1993), they developed a linear programming model for project scheduling problem. One can categorize the characteristics of the Jolayemi-Oluleye model as follows:

- The objective function is to minimize the project total costs.
- No payments made for the project during its life cycle.
- They do not involve the concept of time-value-of-money.

Considering the fact the in real-world projects, the time-value-of-money of not only the project cash flows, but also the payments made for the project is very important for a project manager, in this research, we consider a project scheduling problem under inflation in which the goal is to maximize NPV of the project cash flows, the cash flows being the project costs and the payments made for the project during the life cycle of the project. We call this problem a Project Scheduling Problem with Discounted Cash Flows under Inflation (PSDCFI).

The contents of this paper is outlined as follows. Section 2 gives the importance of the problem. In section 3, we formulate the problem mathematically then we propose a procedure to solve the model. In section 5, an example of the application of the algorithm will be solved and section 6 contains conclusions.

2. Problem Importance

When the PSDCF models are applied, it assumes that costs of the project activities remain unchanged over the project duration. If the inflation exists and is high, then this assumption is not reasonable. Nevertheless, why is inflation important and should be considered?

Inflation is really important, because it will reduce the purchasing power of the resident agent (Bacescu, 2009). Inflation was considered as an important factor of Macroeconomic indicators in the third wave of financial crisis (Dardac and Moinesco, 2009). Zaman and Goschin (2007) was considered Romania Macroeconmoic indicators such as inflation. Based on October 2008 World Economic Outlook report, prepared by International Monetary Fund, despite the deceleration of global growth, inflation has risen around the world, especially in emerging and developing economies, to the highest rates since the late 1990s, pushed up by the surge in fuel and food prices; where the role of food prices is particularly significant and the contribution of energy prices is smaller in comparison, with stronger effects in advanced economies. In the advanced economies, 12-month inflation registered 4¼ percent in August 2008. The resurgence in inflation has been more marked in the emerging and developing economies, with headline inflation reaching 8¼ percent in the aggregate in August and with a wide swath of countries now experiencing double digit inflation. Inflation expectations have also begun to mount, especially in emerging economies, where wages have been on the rise amid generally tight labor markets, see Figure 1.

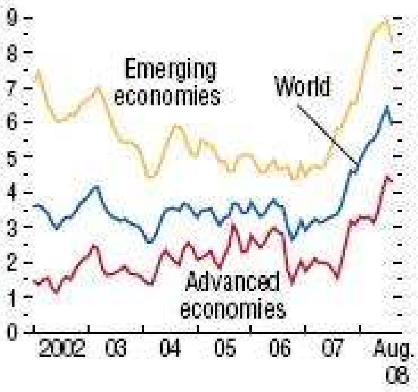

Figure 1. Comparison between Inflation in Advanced and Emerging Economies

Based on the October 2008 World Economic Outlook report, inflation Crisis seems to be more important in the Middle East countries, which are profited from fuel price, some countries like Iran, Saudi Arabia, United Arab Emirates, Egypt, Jordan, and Lebanon suffers from two digits inflation rate with 26%, 11.5%, 12.9%, 11.7%, 15.8%, and 11%. Average inflation rate in Emerging Asia countries is 7.3% and Vietnam with 24% has the highest inflation rate. In African counties, which are suffering from food and fuel price, this problem seems to be more severe than Middle Eastern countries, these countries have two-digit inflation: Ethiopia, Sudan, Congo, Kenya, Angola, Nigeria, Ghana with inflation rates as follows: 25.3%, 15%, 17.5%, 25%, 12.1%, 11% and 16.8% . In countries like Zimbabwe, inflation rate has dramatically increased with 2200000% daily. It seems that Commonwealth of Independent states is the place with the harshest problem; Ukraine, Azerbaijan, Kyrgyz Republic and Tajikistan with 25.3%, 22.4%, 24.5% and 21.6% have the most dominant inflation rate this region , although other countries of this region have high inflations Kazakhstan with 17.6%, Belarus with 15.3%, Russia with 14%, Moldova with 13.7%, Turkmenistan with 13%, Uzbekistan with 11.1% and Georgia with 10 percent are all with two-digit inflation and experienced this problem. Average inflation in Emerging European Countries including Turkey is 7.8% Latvia with 15.9% has the most Inflation rate, and in Western Hemisphere countries is 7.9%; Venezuela with 27.2% has the most inflation rate. In addition, the inflation is one of the most world economic threats as shown in Figure 2.

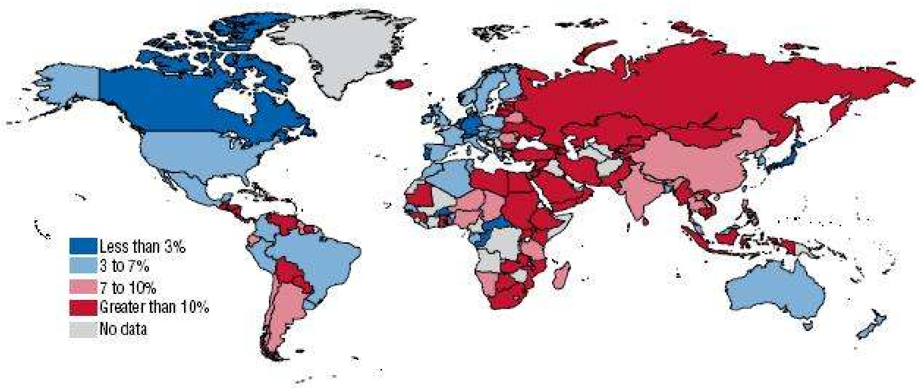

Figure 2. Inflation as a Global Economic Problem

Galloping inflation concerns have brought increasing attention to the issue of the importance of the calculating inflation in economic decisions as project management and scheduling.

3. Problem Formulation

An exact definition of the problem investigated in this paper is as follows: A project is given with a set of *n* activities indexed from *1* to *n*. Activities *1* and *n* are dummies that represent the start and completion of the project, respectively. Precedence relations of activities are shown by an activity on node network with no loops. Each activity *i* has a set of predecessor activities *p(i)*. We define NCF_i is the negative cash flow associated with activity *i*. It states the outward cash flow include resources costs, material costs and other costs of activity *i*. Also, assume PCF_i is the positive cash flow associated with activity *i* . It states the inward cash flows include payments from client (who is the owner of the project) to contractor (whose job is to execute the project). Note, NCF_i and PCF_i are discounted at starting time of activity *i*. The activities are to be scheduled such that the makespan of the project does not exceed a given due date (*DD*). Also, α and *f* are the discount rate and the inflation rate, respectively. To formulate the problem, let us define S_i as start of activity *i*.

In the classical models, there is no inflation ($f \approx 0$), as a result, inflation rate is not considered. In this case, they define cash flow (CF) of an activity equals to the sum of the positive and the negative cash flows of an activity. In other words, we have:

$$CF_i = PCF_i + NCF_i \tag{1}$$

The problem can be formulated as follows:

$$Max\, Z_0 = \sum_{i=1}^{n} CF_i\, e^{-\alpha S_i} \tag{2}$$

Subject to

$$S_j + d_j \leq S_i \quad ; \forall\, j \in p(i) \quad ; i = 1,2, \dots, n \tag{3}$$

$$S_n \leq DD \tag{4}$$

$$S_i \geq 0 \quad , i = 1,2, \dots, n \tag{5}$$

The objective function (2) maximizes the net present value of the project. Equation (3) enforces the precedent relations between activities. Constraint (4) ensures that the project ends by the latest allowable completion time. Finally, equation (5) denotes the domain of the variables.
If there is the inflation (f >0), the above model is invalid and therefore inflation rate should be considered. Due to type of contract between contractor and client, inflation affects the project scheduling. In real life situation, Contract types are grouped into two broad categories: fixed-price contract and cost-reimbursement contract. In fixed-price contract, payments from client to contractor are fixed amounts and increasing at prices of materials and expenditures over the life cycle of the project, do not change the contract price of the project. A fixed-price contract provides for a price that is not subject to any adjustment on the basis of the contractor's cost experience in performing the contract. This contract type places upon the contractor maximum risk and full responsibility for all costs and resulting profit or loss. It provides maximum incentive for the contractor to control costs and perform effectively and imposes a minimum administrative burden upon contracting parties. In the project cash flows point of view, inflation has effects on negative cash flows, but it does not have effect on positive cash flows in fixed-price contract. Cost-reimbursement contract provides for payment of allowable incurred costs, to the extent prescribed in the contract. Cost-reimbursement contracts are suitable for use only when uncertainties involved in contract performance do not permit costs to be estimated with sufficient accuracy to use any type of fixed-price contract. In the project cash flows point of view, inflation influences each of two

types of project cash flows (negative cash flows and positive cash flows) in cost-reimbursement contract.
Consequently, considering the contract type, two models may happen as follows:

Model 1: Fixed-price contract
In this case, inflation affects negative cash flows, but it does not have any effect on positive cash flows. The mathematical formulation of the problem is as follows:

$$Max\, Z_1 = \sum_{i=1}^{n} PCF_i\, e^{-\alpha S_i} + \sum_{i=1}^{n} NCF_i\, e^{-(\alpha-f)S_i} \tag{6}$$

Subject to Equations (3), (4) and (5).

Model 2: Cost-reimbursement contract
In this case, inflation affects negative cash flows and positive cash flows, too. The mathematical formulation of the problem is as follows:

$$Max\, Z_2 - \sum_{i=1}^{n} PCF_i\, e^{-(\alpha-f)S_i} + \sum_{i=1}^{n} NCF_i\, e^{-(\alpha-f)S_i} = \sum_{i=1}^{n} (PCF_i + NCF_i)\, e^{-(\alpha-f)S_i} \tag{7}$$

Subject to Equations (3), (4) and (5).

It seems that optimum value of net present value of the project in second model (cost-reimbursement contract) is greater than or equal to the same in the first model (fixed-price contract), we prove this claim this way.

Lemma. The optimum value of the objective function in cost- reimbursement contract model is greater than or equal to the optimum value of the objective function in the fixed-price contract model.

Proof: We prove the lemma with the following stages:

Stage 1: Let *S* be a solution for the problem and $Z_1(S)$ and $Z_2(S)$ denote the values of the objective functions of fixed-price contract model and cost-reimbursement contract model, respectively. It is proved that $Z_2(S) \geq Z_1(S)$ as follows.

$$Z_2(S) - Z_1(S) = \sum_{i=1}^{n} PCF_i\, e^{-(\alpha-f)S_i} + \sum_{i=1}^{n} NCF_i\, e^{-(\alpha-f)S_i} - \left(\sum_{i=1}^{n} PCF_i\, e^{-\alpha S_i} + \sum_{i=1}^{n} NCF_i\, e^{-(\alpha-f)S_i}\right) = \sum_{i=1}^{n} PCF_i\, e^{-\alpha S_i}\left(e^{f S_i} - 1\right) \tag{8}$$

Because $e^{fS_i} \geq 1, PCF_i \geq 0$ and $e^{-\alpha S_i} \geq 0$ then $Z_2(S) \geq Z_1(S)$.
Stage 2: Let S^{*1} be optimal scheduling for model 1. According to stage 1, we have $Z_2(S^{*1}) \geq Z_1(S^{*1})$.

Stage 3: Let us define S^{*2} as optimal scheduling of model 2, because it is the optimal solution of model 2, then $Z_2(S^{*2}) \geq Z_2(S^{*1})$.

Stage 4: According to resulting of stages 2 and 3, we have $Z_2(S^{*2}) \geq Z_1(S^{*1})$. It means that the optimum value of the objective function in the cost-reimbursement contract model is greater than or equal to the optimum value of the objective function in the fixed-price contract model.

Now we can easily conclude from the above concepts that if we are solving the problem for contractor, it would be suitable for him to choose cost-reimbursement contract and if we are solving on client behalf, it would be suitable for him to choose fixed-price contract.

4. A Solution Procedure

In order to solve the derived models in section three, we try to convert them to the linear models. The derived models are non-linear programming models, but it can be solved iteratively by successive approximations using Taylor expansion approximation. The Taylor expansion transforms a function *f* around a point (solution) x_0 as:

$$f(x) = f(x_0) + \frac{f^{(1)}(x_0)}{1!}(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \quad (9)$$

To apply this manner, a feasible solution is needed that we assume it as $S^0 = (S_1^0, S_2^0, \ldots, S_n^0)$, which can be obtained by using critical path method. If we consider the first two terms of the Taylor expansion, the objective functions are transformed to linear forms and be maximized very easily by classic methods for instance the Simplex algorithm. Therefore, the objective function (6) for the fixed-price contract model is transformed as follows:

$$Max\, Z_1 = \sum_{i=1}^{n}(F_i^+ + F_i^- e^{fS_i^0}) - \sum_{i=1}^{n}(\alpha F_i^+ + (\alpha - f)F_i^- e^{fS_i^0})(S_i - S_i^0) \quad (10)$$

where:

$$F_i^+ = PCF_i e^{-\alpha S_i^0} \tag{11}$$

$$F_i^- = NCF_i e^{-\alpha S_i^0} \tag{12}$$

The objective function (7) for the cost-reimbursement contract model is transformed as follows:

$$Max\, Z_2 = \sum_{i=1}^{n}(F_i^+ + F_i^-)e^{fS_i^0} - (\alpha - f)\sum_{i=1}^{n}(F_i^+ + F_i^-)e^{fS_i^0}(S_i - S_i^0) \tag{13}$$

The solving method starts an initial solution S^0 and then the optimal solution derived by approximated function is replaced by x_0 and this process is iterated until the difference between the objective function values reaches to a threshold (e.g. 0.01). By this process, the optimal or a very close to optimal solution is determined. Russell (1970) proved that the process converges in a finite number of iterations.

5. A Numerical Example

In order to illustrate the proposed method, consider a project network with eight activities. Figure 3 shows the activity-on-node representation of the network with the node numbers denoting the activity numbers. Activities 1 and 8 represent the start and completion of the project, respectively.

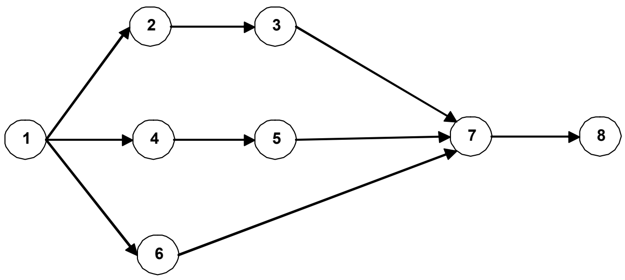

Figure 3: The Project Network of the Example Problem

The deadline is 10 and the discount rate is taken to be 1% per period (e.g. month). Table 1 presents the durations, the negative cash flow and the positive cash flow of the activities.

Table 1: Activity Data of the Example Problem

Activity (i)	Duration (d_i)	Negative cash flow (NCF_i)	Positive cash flow (PCF_i)
1	0	0	2000
2	5	-3000	0
3	2	-2000	0
4	2	-1000	0
5	3	-3000	4000
6	2	-1000	1000
7	1	-1500	3000
8	0	0	3700

To illustrate the impact of inflation on the project scheduling, the problem is solved in three cases: (a) there is no inflation, (b) there is inflation and the contract type is fixed-price and (c) the contract type is cost-reimbursement while there is inflation.

(a) The inflation rate is expected to be 0%. In this situation, the problem is formulated as equations (2), (3), (4) and (5). The related mathematical modeling is solved; the value of the objective function of the optimum solution, *NPV* is 1894. Figure 4 shows the time schedule of the solution. We call this solution as inflation-free solution.

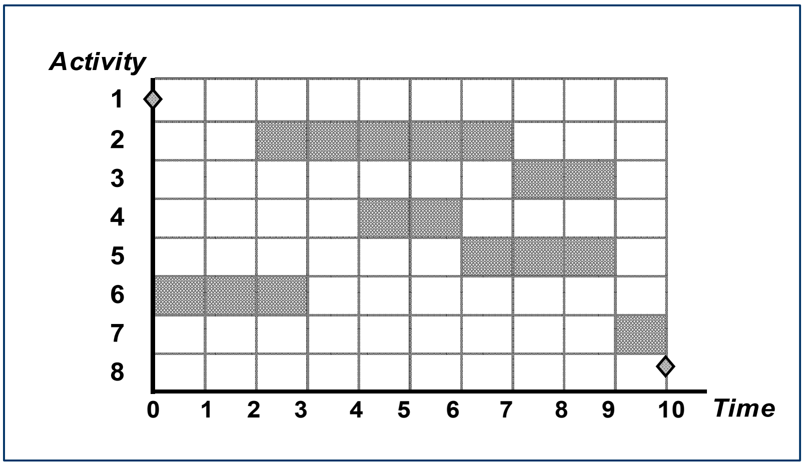

Figure 4: The inflation-free schedule

(b) It is estimated that the future rate of inflation will be ½% per period and the contract type is fixed-price. Therefore, the model 1 is formulated by using the example data and solved according to the proposed procedure. The time schedule is shown in Figure 5. For this solution, NPV is 1764. This solution is called inflation-fixed-price solution.

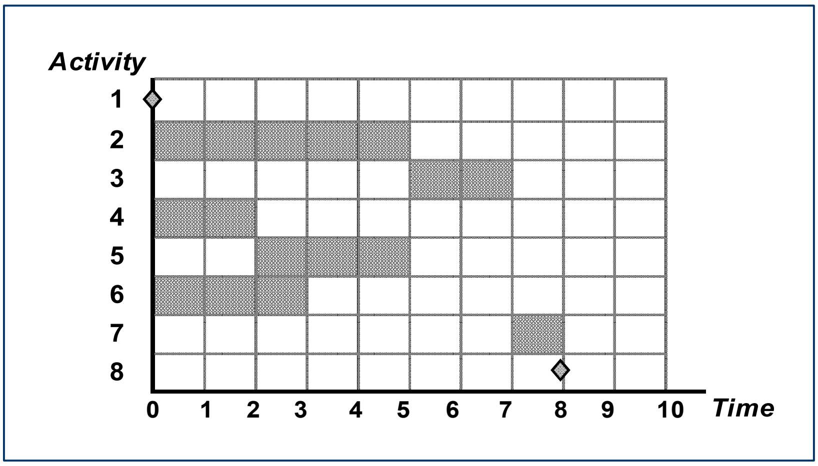

Figure 5: The inflation-fixed-price schedule

(c) The contract type is cost-reimbursement and the inflation rate is expected to be ½% per period. In this situation, the model 2 must be solved according to the solution procedure. For this model, Figure 6 shows the time schedule of the solution. We call this solution as inflation-cost-reimbursement schedule. The NPV of the solution is 2043.

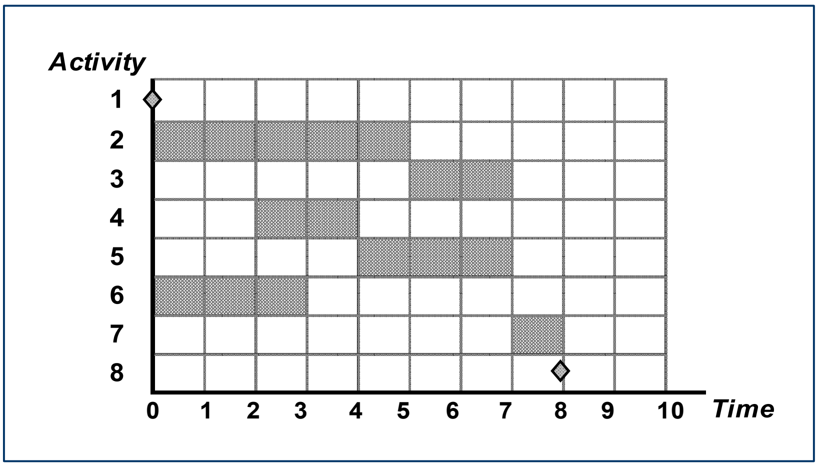

Figure 6: The inflation-cost-reimbursement schedule

For analysis of the inflation effect on project scheduling in this example, the value of the objective functions (NPV) of three above cases are obtained for the final solutions of each cases as shown in Table 2.

Table 2: Analysis of the inflation effect on project scheduling in the example

Solution Name	Solution (S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8)	The value of objective function for solution		
		Z_0 (.)	Z_1(.)	Z_2(.)
inflation-free	(0, 2, 7, 4, 6, 0, 9 ,10)	1894	1630	2041
inflation-fixed-price	(0, 0, 5, 0, 2, 0,7 ,8)	1891	1764	2042
inflation-cost-reimbursement	(0, 0, 5, 2, 4, 0,7 ,8)	1892	1726	2043

The analysis of table 2 shows that:

- Optimal solution of each case is not optimal solution for other cases. It means that each model of solving is unable to obtain optimal solution for other models.
- Type of the contract will be changing the activities scheduling.
- If we are solving the problem for contractor, it would be suitable to choose cost-reimbursement contract and if we are solving on client behalf, it would be suitable to choose fixed-price contract.
- According to the lemma, it shows $Z_2 > Z_1$ for each solution.

6. Conclusions

In this paper, we developed the project scheduling problem with discounted cash flow under inflation environment. We proposed two different situations due to type of contract between contractor and client as fixed-price contract and cost-reimbursement contract and concluded contractor might choose cost-reimbursement contract and client might choose fixed-price contract. Some extensions of this research might be of interest. While in this paper, we only considered the "zero-lag finish-to-start precedent constraints", some other precedence relation such as generalized precedence may be considered in the project. The other extension of this research would be to investigate a resource constrained project scheduling under inflation environment is to maximize the NPV of the project.

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