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PORTFOLIO SELECTION USING MINIMUM PSEUDODISTANCE ESTIMATORS

Abstract. *Mean-variance portfolios constructed using maximum likelihood estimates of the mean and covariance matrix perform poorly in practice due to estimation errors. For this reason, researchers have recently focused on robust mean-variance portfolios which rely on stable estimates of the mean and covariance matrix. In this paper we consider robust minimum pseudodistance estimators in the framework of Markowitz's mean-variance portfolio selection model. The use of these estimators leads to robust weights for the optimal portfolio. Our numerical results on empirical data sets confirm that the proposed portfolios are more stable than the traditional mean-variance portfolios.*

Keywords: portfolio selection, financial returns, pseudodistance, robust estimation.

JEL Classification: C02, C13, G10.

1 Introduction

The mean-variance portfolio selection model proposed by Markowitz (1952) is one of the most known models for asset allocation. Practitioners often use this model although, in recent literature, it was underlined that the traditional approach can lead to financially irrelevant sub-optimal portfolios. Among the reasons of this drawback, an important role is played by the unbounded influence of the extreme returns on the procedure. The mean-variance optimization method aims to find the optimal portfolio which maximizes some objective function whose input parameters are the mean and the covariance matrix of the asset returns. The optimal portfolio is very sensitive to inputs, these inputs being subject to estimation errors of the expected return and covariance. Small changes in these estimates can induce significant changes in the portfolio composition. Therefore, diverse attempts were made to improve the performance of portfolio optimization. Some authors suggested portfolio optimization using robust estimators. Among the papers that consider this approach, we recall Perret-Gentil and Victoria-Feser (2005), Welsch and Zhou (2007), DeMiguel and Nogales (2009), Fabozzi et al. (2010), Grossi and Laurini (2011).

Our approach in the present paper is based on using robust minimum pseudodistance estimators of location and covariance. Pseudodistances are generalizations of distances between probability measures usually not satisfying the triangle inequality. By its definition, a pseudodistance satisfies two properties, namely the nonnegativity and the fact that the pseudodistance between two probability measures P and Q equals to zero, if and only if the two measures are equal. Note that the divergences are moreover characterized by the information processing property, i.e. by the complete invariance with respect to the statistically sufficient transformations of the observation space. In general, a pseudodistance may not satisfy this property. The use of divergence measures in statistical inference is an important issue. We refer to the books of Pardo (2006) and Basu et al. (2011) for a variety of statistical methods based on divergences, as well as to some recent papers such as Toma (2008), Broniatowski and Keziou (2009), Toma (2009), Toma and Leoni-Aubin (2010), Bouzebda and Keziou (2010), Karagrigoriou and Mattheou (2010), Panayiotis and Karagrigoriou (2011), Toma and Broniatowski (2011) and Vonta et al. (2012) which develop divergence based estimation and test procedures. The minimum pseudodistance estimators are defined by minimizing an empirical version of pseudodistance between the assumed parametric model and the true model underlying the data, by using the empirical measure pertaining to the sample. The pseudodistances that we consider, as well as the corresponding minimum pseudodistance estimators, are indexed by a positive escort parameter α which controls the equilibrium between robustness and efficiency. We consider these estimators in the case of the multivariate normal model and apply them in the framework of Markowitz mean-variance portfolio selection model.

The content of this paper is organized as follows. In Section 2, we recall the Markowitz mean-variance portfolio selection model. In Section 3 we present minimum pseudodistance estimators of location and covariance together with their robustness properties. These estimators are used in the framework of the Markowitz model in order to construct robust and efficient portfolios. In Section 4 some examples based on real data are given to illustrate the performance of this approach for robust portfolio optimization.

2 The Markowitz mean-variance portfolio selection model

The Markowitz mean-variance portfolio selection model is one of the most known and studied models for asset allocation. There is an extensive literature related to this model and its applications (see for example Markowitz (1959), Fabozzi et al. (2007), as well as some recent papers such as Dedu and Fulga (2011) and Şerban et al. (2011)).

Consider N financial assets and denote by $X := (X_1, \dots, X_N)^t$ the random vector of the asset returns, X_i being the random variable associated to the return of the i^{th} asset, $i = 1, \dots, N$. Let $\mu = (\mu_1, \dots, \mu_N)^t := \mathrm{E}(X)$ be the vector of expected returns and $\Sigma := \mathrm{cov}(X)$ be the covariance matrix of the returns of the assets. Denote by p_i the weight of the i^{th} asset in the portfolio and let $p := (p_1, \dots, p_N)^t$. Then, the total return of the portfolio is given by the random variable $\sum_{i=1}^N p_i X_i$. The expected return of the portfolio is $R(p) := \mathrm{E}(\sum_{i=1}^N p_i X_i) = p^t \mu$ and the variance of the portfolio return is $S(p) := \mathrm{Var}(\sum_{i=1}^N p_i X_i) = p^t \Sigma p$.

The Markowitz's approach for optimal portfolio selection consists of determining the portfolio p^* solution to the following optimization problem. For a given positive value of the parameter λ, representing the investor's risk aversion, the portfolio p^* is solution of

$$\arg\max_p \{R(p) - \frac{\lambda}{2} S(p)\} \tag{2.1}$$

with the constraint $p^t e_N = 1$, where $e_N := (1, \dots, 1)$ is the N-dimensional vector of ones. The solution to the optimization problem (2.1) is explicit and is given by

$$p^* = \frac{1}{\lambda} \Sigma^{-1} (\mu - \eta e_N) \tag{2.2}$$

where

$$\eta = \frac{e_N^t \sum^{-1} \mu - \lambda}{e_N^t \sum^{-1} e_N}.$$

Different positive values of λ give different investment strategies and determine the so-called mean-variance efficient frontier. The greater the value of λ, the more risk averse the investor is. This is the case when short selling is allowed. When short selling is not allowed, all the portfolio weights p_i, $i = 1, \dots, N$, have positive values.

Traditionally, the unknown parameters μ and Σ are estimated using their sample counterparts, namely the maximum likelihood estimators under multivariate normal distribution. It is known that the portfolio optimization based on sample mean and covariance performs poorly in practice. Since the maximum likelihood estimators of μ and Σ, which are inputs in the optimization procedure, are very sensitive to outlying observations, the weights of the resulted portfolio, which are outputs of this procedure, may be substantially affected by such observations.

Our approach for robust portfolio optimization is based on using robust minimum pseudodistance estimators of location and covariance.

3 Minimum pseudodistance estimators

Recently, Broniatowski et al. (2012) introduced a class of parametric estimators called minimum pseudodistance estimators.

The considered family of pseudodistances is indexed by a positive tuning parameter α and is defined as follows. Let P and Q two probability measures with densities p, respectively q, with respect to the Lebesgue measure. The pseudodistance between P and Q is defined by

$$R_\alpha(P,Q) := \frac{1}{\alpha+1}\ln\int p^\alpha \mathrm{d}P + \frac{1}{\alpha(\alpha+1)}\ln\int q^\alpha \mathrm{d}Q - \frac{1}{\alpha}\ln\int p^\alpha \mathrm{d}Q$$

for $\alpha > 0$ and satisfy the limit relation

$$R_\alpha(P,Q) \to R_0(P,Q) := \int \ln\frac{q}{p}\mathrm{d}Q \ \text{ for } \ \alpha \downarrow 0.$$

We note that the pseudodistances $R_\alpha(P,Q)$ are related to the Renyi entropy functionals as studied by Källberg et al. (2012).

Let $(P_\theta)_{\theta\in\Theta}$ be a parametric model, with the parameter space $\Theta \subset \mathbb{R}^d$, and assume that each probability measure P_θ has a density p_θ with respect to the Lebesgue measure. Let $X^1,\ldots,X^T$ be a sample on P_θ. A minimum pseudodistance estimator $\widehat{\theta}_n$ of the parameter θ is defined by

$$\widehat{\theta}_n := \arg\inf_\theta R_\alpha(P_\theta, P_n),$$

where P_n is the empirical measure pertaining to the sample, or equivalently as

$$\widehat{\theta}_n = \begin{cases} \arg\sup_\theta \frac{1}{TC_\alpha(\theta)}\sum_{i=1}^T p_\theta^\alpha(X^i) & \text{if } \alpha > 0 \\ \arg\sup_\theta \frac{1}{T}\sum_{i=1}^T \ln p_\theta(X^i) & \text{if } \alpha = 0 \end{cases}$$

where $C_\alpha(\theta) = \left(\int p_\theta^{\alpha+1} d\lambda\right)^{\frac{\alpha}{\alpha+1}}$. Note that the case $\alpha = 0$ leads to the definition of the maximum likelihood estimator of the parameter θ.

When p_θ is the N-variate normal density with $\theta = (\mu, \Sigma)$

$$p_\theta(x) = \left(\frac{1}{2\pi}\right)^{N/2}\sqrt{\det\Sigma^{-1}}\exp\left(-\frac{1}{2}(x-\mu)^t\Sigma^{-1}(x-\mu)\right)$$

it holds

$$C_\alpha(\theta) = \frac{\left(\frac{1}{2\pi}\right)^{\frac{N\alpha^2}{2(\alpha+1)}}\left(\sqrt{\det\Sigma^{-1}}\right)^{\frac{\alpha^2}{\alpha+1}}}{\left(\sqrt{\alpha+1}\right)^{\frac{N\alpha}{\alpha+1}}}$$

and then the minimum pseudodistance estimators corresponding to positive α may be written as

$$\widehat{\theta}_n = \arg\sup_{\theta} (\sqrt{\det \Sigma^{-1}})^{\frac{\alpha}{\alpha+1}} \sum_{i=1}^{T} \exp\left(-\frac{\alpha}{2}\|X^i - \mu\|^2_{\Sigma^{-1}}\right),$$

where $\|X^i - \mu\|^2_{\Sigma^{-1}} = (x-\mu)^t \Sigma^{-1}(x-\mu)$.

By direct differentiation with respect to μ and Σ, we see that the minimum pseudodistance estimators of these parameters are solutions of the system

$$\begin{aligned}
\mu &= \sum_{i=1}^{T} \frac{\exp(-\frac{\alpha}{2}\|X^i - \mu\|^2_{\Sigma^{-1}})}{\sum_{i=1}^{T}\exp(-\frac{\alpha}{2}\|X^i - \mu\|^2_{\Sigma^{-1}})} X^i \\
\Sigma &= \sum_{i=1}^{T} \frac{(\alpha+1)\exp\left(-\frac{\alpha}{2}\|X^i - \mu\|^2_{\Sigma^{-1}}\right)}{\sum_{i=1}^{T}\exp(-\frac{\alpha}{2}\|X^i - \mu\|^2_{\Sigma^{-1}})} (X^i - \mu)(X^i - \mu)^t.
\end{aligned}$$

The definitions of the minimum pseudodistance estimators in the unidimensional normal case, as well as some empirical results in this case are given in Toma (2012).

For studying the robustness of an estimator the influence function is often used. Recall that, a map T defined on a set of probability measures and parameter space valued is a statistical functional corresponding to an estimator $\widehat{\theta}_n$ of the parameter θ, if $\widehat{\theta}_n = T(P_n)$. As it is known, the influence function of T at P_θ is defined by

$$\mathrm{IF}(x; T, P_\theta) := \left.\frac{\partial T(\widetilde{P}_{\varepsilon x})}{\partial \varepsilon}\right|_{\varepsilon=0}$$

where $\widetilde{P}_{\varepsilon x} := (1-\varepsilon)P_\theta + \varepsilon\delta_x$, $\varepsilon > 0$, δ_x being the Dirac measure putting all mass at x (see Hampel et al. (1986)). The influence function measures the standardized effect of an infinitezimal contamination in a point x on the asymptotic value of the estimator. Whenever the influence function is bounded with respect to x, the corresponding estimator is called robust.

Broniatowski et al. (2012) derived the influence function of the statistical functional T_α corresponding to a minimum pseudodistance estimator

$$\mathrm{IF}(x; T_\alpha, P_\theta) = M_\alpha(\theta)^{-1}[p_\theta^{\alpha-1}(x)\dot{p}_\theta(x) - c_\alpha(\theta)p_\theta^\alpha(x)],$$

where

$$M_\alpha(\theta) = \int p_\theta^{\alpha-1}\dot{p}_\theta \dot{p}_\theta^t d\lambda - \frac{\int p_\theta^\alpha \dot{p}_\theta d\lambda (\int p_\theta^\alpha \dot{p}_\theta d\lambda)^t}{\int p_\theta^{\alpha+1} d\lambda}$$

and $c_\alpha(\theta) := \frac{\int p_\theta^\alpha \dot{p}_\theta d\lambda}{\int p_\theta^{\alpha+1} d\lambda}$, $\dot{p}_\theta$ being the derivative of p_θ with respect to θ.

When P_θ is the N-variate normal model and $\theta = (\mu, \Sigma)$ is the parameter of interest, the influence functions of the corresponding minimum pseudodistance estimators are

$$\mathrm{IF}(x; \mu, P_{\mu,\Sigma}) = (\sqrt{\alpha+1})^{N+2}(x-\mu)\exp\left(-\frac{\alpha}{2}\|x-\mu\|^2_{\Sigma^{-1}}\right) \tag{3.1}$$

$$\mathrm{IF}(x; \Sigma, P_{\mu,\Sigma}) = (\sqrt{\alpha+1})^{N+4}\left[(x-\mu)(x-\mu)^t - \frac{1}{\alpha+1}\Sigma\right]\exp\left(-\frac{\alpha}{2}\|x-\mu\|^2_{\Sigma^{-1}}\right) \tag{3.2}$$

In the particular case $\alpha = 0$, we find the influence functions of the maximum likelihood estimators of location and covariance

$$\begin{aligned} \mathrm{IF}(x; \mu, P_{\mu,\Sigma}) &= x - \mu \\ \mathrm{IF}(x; \Sigma, P_{\mu,\Sigma}) &= (x-\mu)(x-\mu)^t - \Sigma. \end{aligned}$$

These influence functions are unbounded and consequently the maximum likelihood estimators are not robust, as it is well known.

When $\alpha > 0$, the estimators gain robustness, in this case the influence functions (3.1) and (3.2) being bounded with respect to x. By using robust minimum pseudodistance estimators of μ and Σ in formula (2.2), we obtain robust estimates of the portfolio weights.

4 Application to real financial data

In this section we use empirical data sets to illustrate the stability and performance of the proposed portfolios.

In a first example, we consider monthly log-returns of 3 indexes (BET, BET-C, BET-FI) from the Bucharest Stock Exchange for the period January 2001 to May 2012 included. Data come from Bucharest Stock Exchange (www.bvb.ro) and are displayed in Figure 1. Figure 2 contains normal quantile plots, while Figure 3 presents boxplots for these data. All these graphical representations highlight the presence of outliers among data. The normal quantile plots show that the bulk of the data follows normality fairly closely, but values in the tails depart from normality in various degrees. In this situation, we choose to use robust minimum pseudodistance estimators for the vector of the expected returns and for the covariance matrix of the returns. These estimators will be inserted in the optimized portfolio according to the formula (2.2). For comparison with the classical Markowitz's approach, we also consider maximum likelihood estimates in the optimization procedure. Previous studies (Broniatowski et al. (2012)) showed that a good choice of the tuning parameter associated to the pseudodistance is one not far to zero, in order to achieve robustness and high efficiency of the estimation procedure. We choosed $\alpha = 0.2$ in our study. As noted before, the choice $\alpha = 0$ in pseudodistance corresponds to the maximum likelihood estimates.

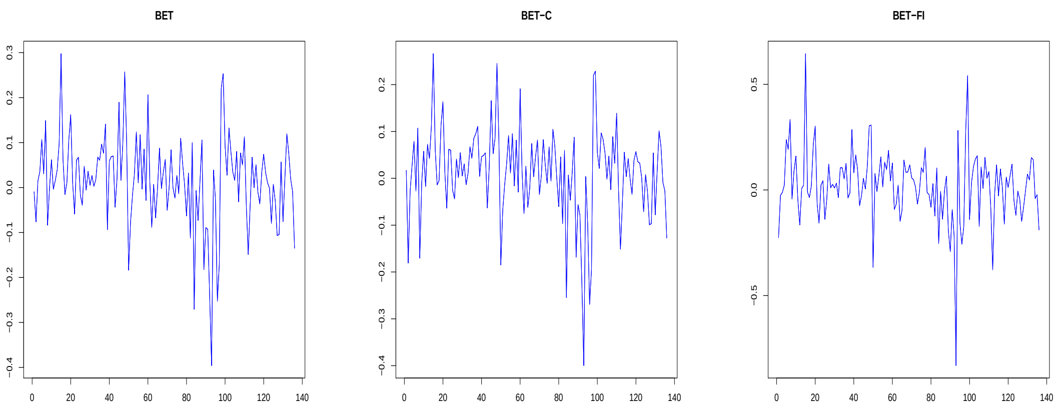

Figure 1. Monthly log-returns of the indexes BET, BET-C, BET-FI, for the period January 2001 - May 2012

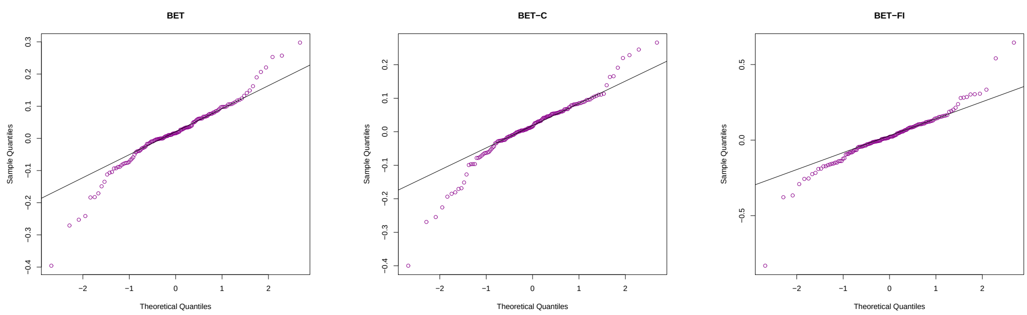

Figure 2. Normal QQ-plots for monthly log-returns of the indexes BET, BET-C, BET-FI

In Figure 4 expected returns estimates and variance estimates of the 3 indexes are presented. The expected returns estimates obtained with robust minimum pseudodistance estimators are superior to the ones provided by the classical maximum likelihood estimators, while the robust estimates of variances obtained with minimum pseudodistance estimators are lower than the classical ones.

In Figure 5 mean-variance efficient frontiers are represented. First, short selling is allowed, case in which we use the formula (2.2) for computing portfolio weights. In this case, a comparison between the efficient frontier based on robust minimum pseudodistance estimators and the efficient frontier based on classical maximum likelihood estimators can be seen in the left hand side of Figure 5. Then, we considered that short selling is not allowed. For this case, the two efficient frontiers,

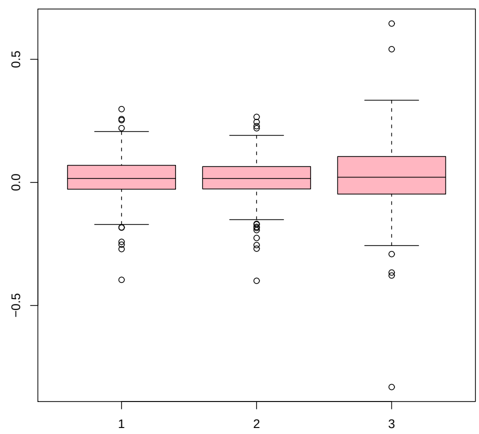

Figure 3. Boxplots for monthly log-returns (1: BET, 2: BET-C, 3: BET-FI)

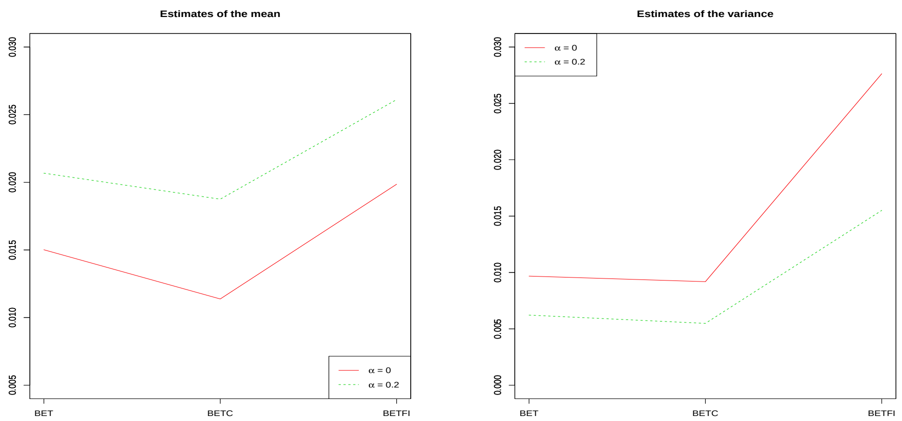

Figure 4. Expected returns estimates (left) and variance estimates (right) for the indexes BET, BET-C, BET-FI

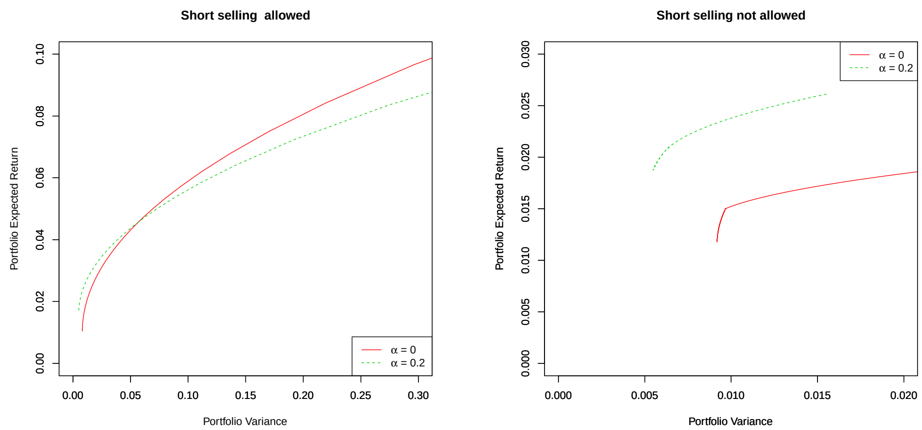

Figure 5. Mean-variance efficient frontiers for the indexes BET, BET-C, BET-FI

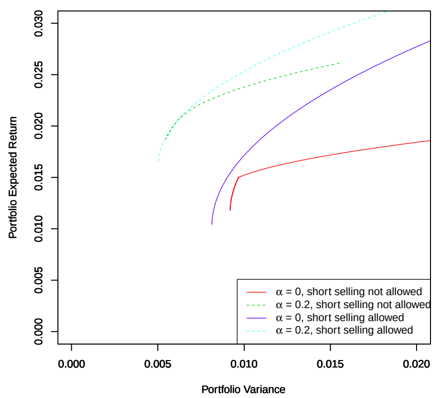

Figure 6. Mean-variance efficient frontiers for the indexes BET, BET-C, BET-FI

namely the robust one based on robust minimum pseudodistance estimators and the classical one based on maximum likelihood estimators are given in the right hand side of Figure 5. Moreover, we provide an overview of all these efficient frontiers in Figure 6. Note that, in both cases, the robust efficient frontier is situated higher and more to the left than that based on classical maximum likelihood estimates. This indicates that, for a given level of the portfolio variance, the total return of the portfolio is higher when using robust estimates.

In a second example, we use an empirical data set representing monthly log-returns of 8 assets from Bucharest Stock Exchange (ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP). The data span from January 2004 to May 2012 and are provided by www.tranzactiibursiere.ro. Time series of asset log-returns are given in Figure 7. In the case of SIF3, we remark the presence of large outliers in the sample. Indeed, the price of this asset in October 2008 was 0.0008 RON and consequently the log-returns constructed with this data are very distant with respect to the other log-returns in the sample. But outliers in the data set come from the other assets, too. This can be seen in Figure 7 and also in Figure 9 where boxplots of the data are represented. Moreover, the normal QQ-plots from Figure 8 indicate the deviation of the distribution of the data from the normal distribution. Thus, we are motivated to apply a robust method in order to construct optimal portfolios.

Figure 10 (right hand side) shows how much the presence of extreme outliers (especially determined by values of log-returns of SIF3) influenced the estimation of variance when using the classical maximum likelihood estimators. In contrast, the robust estimators reduced significantly the effect of large outliers. Also, the robust estimates of the expected returns are in general higher that those obtained with classical estimators. Efficient frontiers obtained with robust minimum pseudodistance estimations, respectively with classical maximum likelihood estimations, are represented in Figure 11 and Figure 12. The two cases, "short selling allowed" and "short selling not allowed" are considered. As in the previous example, the robust efficient frontier is located higher than and to the left of the classical one. For a given value of the portfolio variance, the portfolio expected return is higher in the case "short selling allowed" than in the case "short selling not allowed". This is valid both when using the classical procedure and when using the robust procedure based on minimum pseudodistance estimators. These results are illustrated in Figure 12.

The obtained numerical results show that the proposed portfolios are less-sensitive to deviations of the returns distribution from normality than those provided by the traditional approach. The stability of the portfolios makes them a viable alternative to the traditional portfolios, because investors are usually reticent to use methods for which the portfolio weights change drastically over the time.

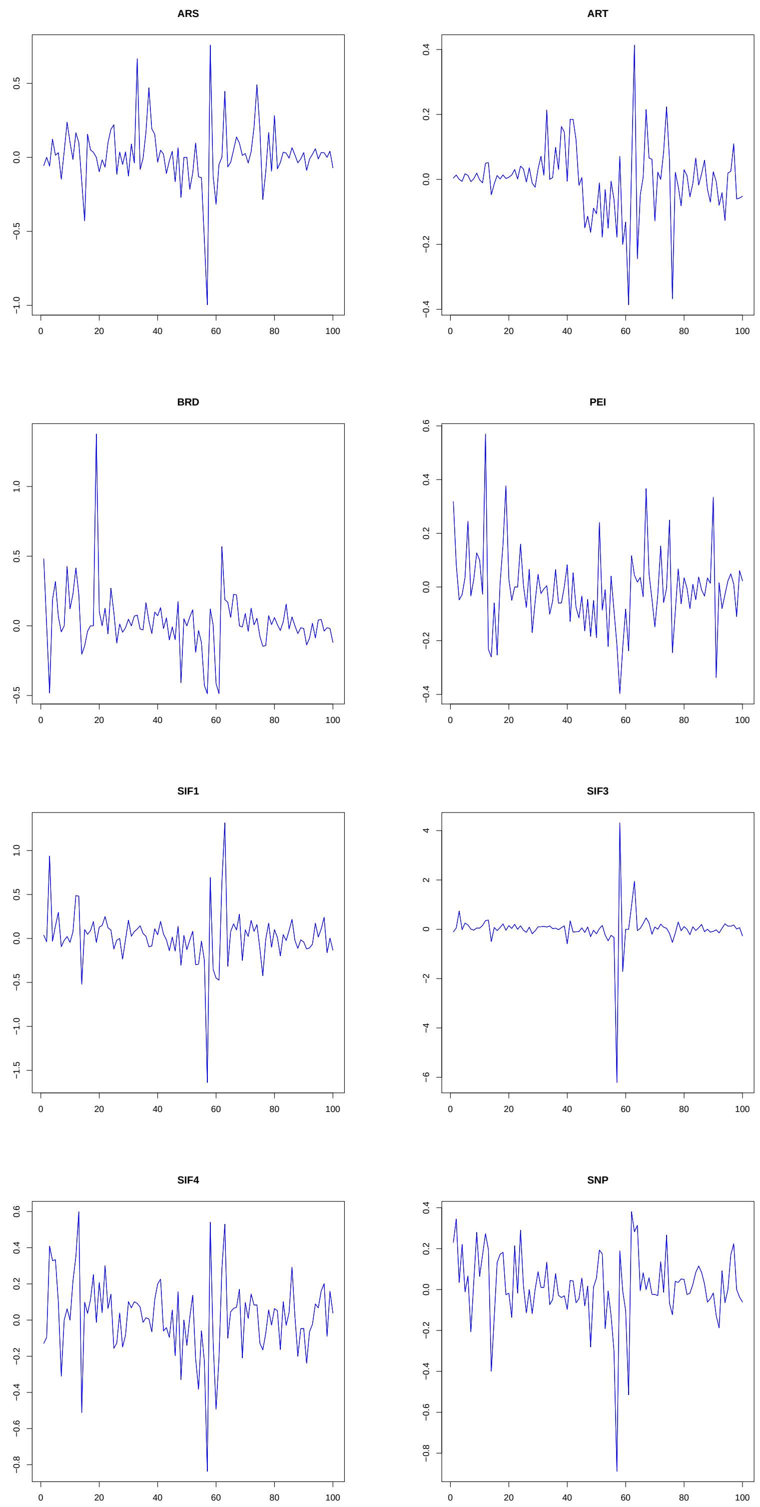

Figure 7. Monthly log-returns of the assets ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP, for the period January 2004 - May 2012

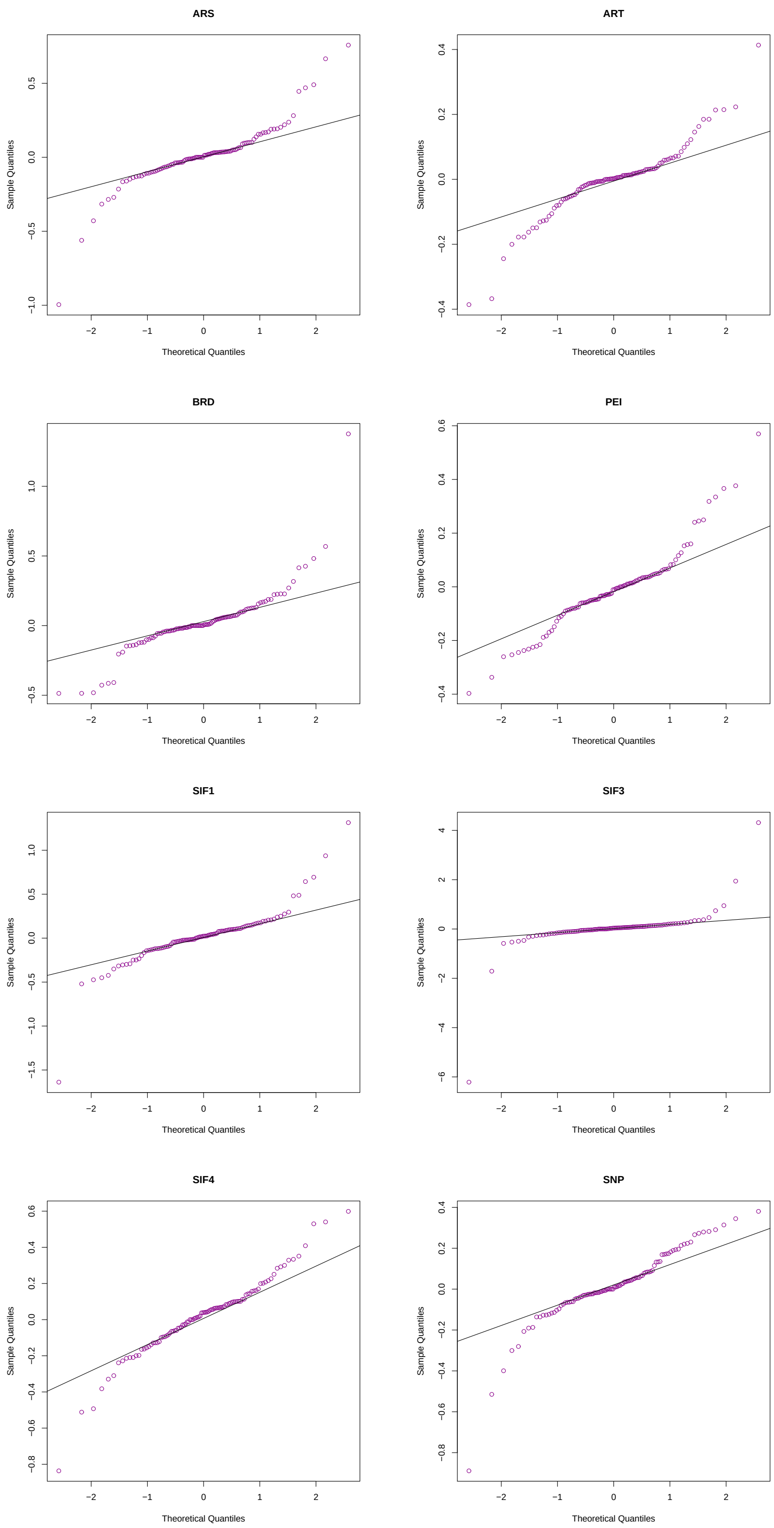

Figure 8. Normal QQ-plots for monthly log-returns of the assets ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP

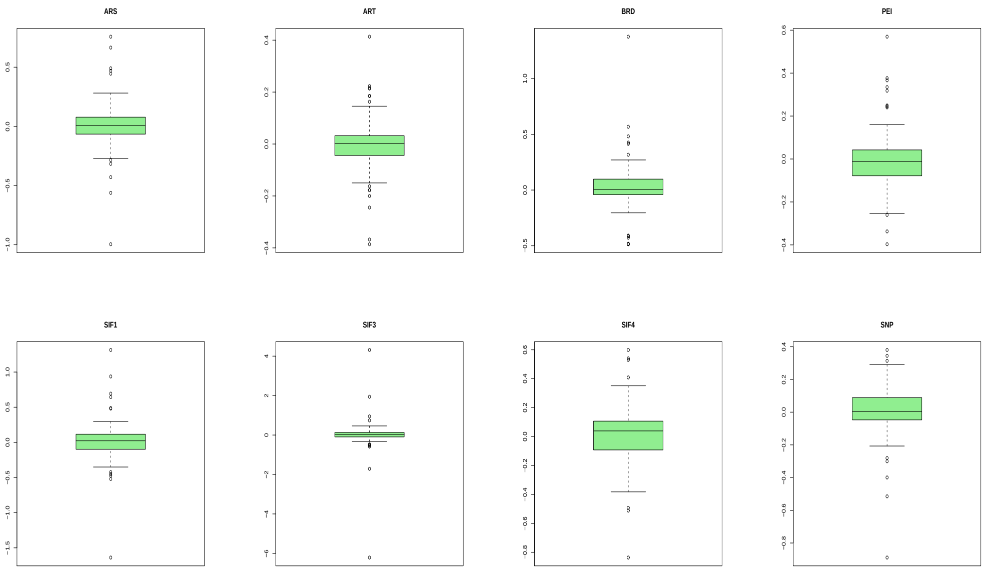

Figure 9. Boxplots for monthly log-returns corresponding to ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP

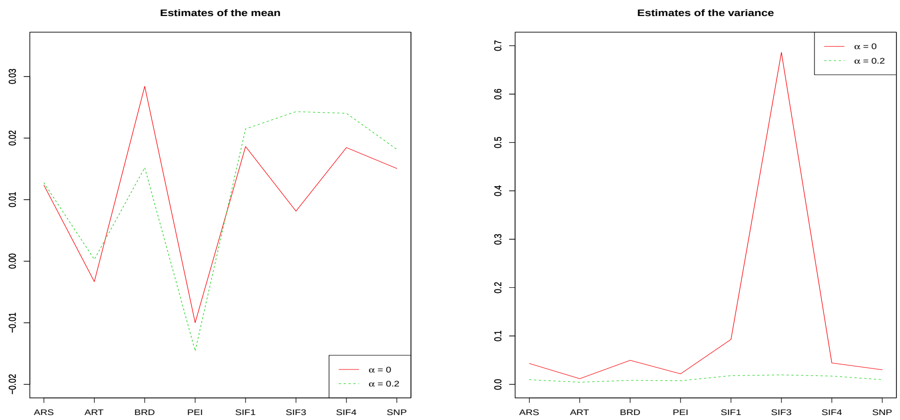

Figure 10. Expected returns estimates (left) and variance estimates (right) for the assets ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP

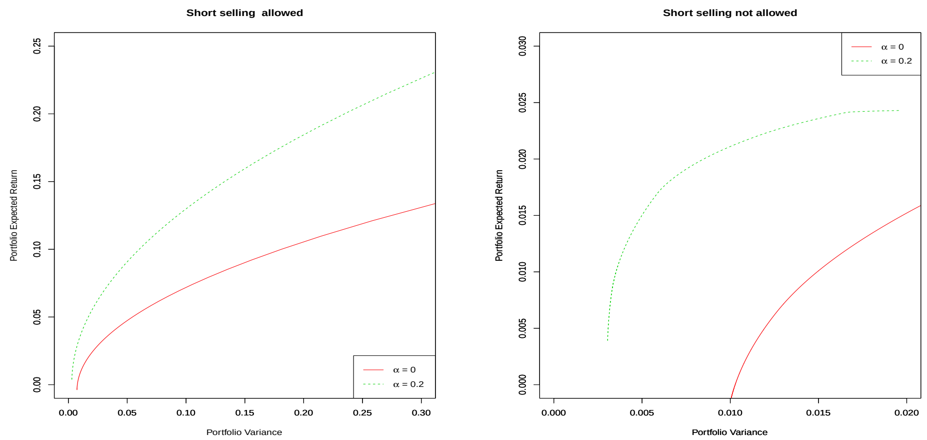

Figure 11. Mean-variance efficient frontiers for ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP

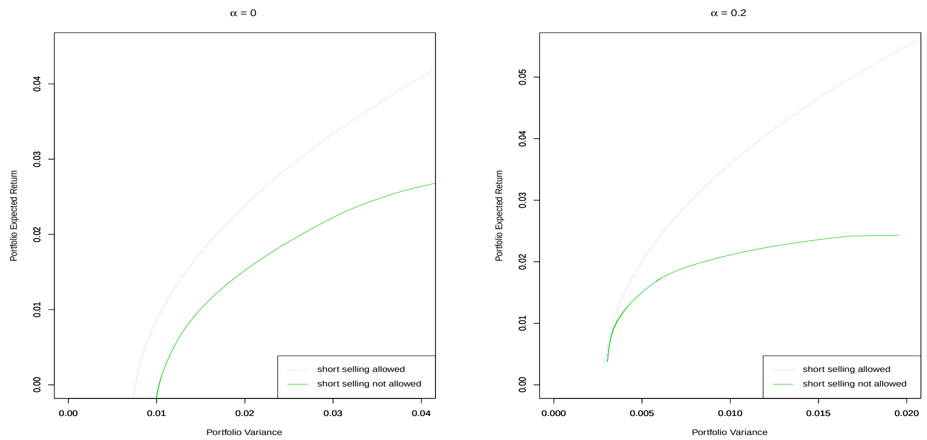

Figure 12. Mean-variance efficient frontiers for ARS, ART, BRD, PEI, SIF1, SIF3, SIF4, SNP

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