Lecturer Florentin SERBAN, PhD
Professor Viorica STEFANESCU, PhD
Department of Mathematics
The Bucharest Academy of Economic Studies
Professor Massimiliano FERRARA, PhD
Department of Mathematics
University of Reggio Calabria, Italy

PORTFOLIO OPTIMIZATION IN THE FRAMEWORK MEAN –VARIANCE -VAR

Abstract. *This paper proposes a model for portfolio optimization. Firstly we compare the mean-variance method with the mean -VaR method and we search the link between the mean-variance efficient set and the mean-VaR efficient set. Then we analyze two portfolio optimization approaches. The first is a two-stage portfolio optimization approach using, in order, both mean-variance and mean-VaR approaches. The second is a general mean-variance-VaR approach using both variance and VaR as a double-risk measure simultaneously. Finally we consider the case of an equity portfolio at the Italian Stock Market. We use data analyze for portfolio selection, then estimation of risk for each stock and after we solve the portfolio optimization in the framework mean – var.*

Keywords: *portfolio optimization, risk measures, mean-var analyze, mean-variance analyze.*

JEL CLASSIFICATION : C02, C61, G11.

1. Introduction

Mean-risk models are still the most widely used approach in the practice of portfolio selection. With mean-risk models, return distributions are characterized and compared using two statistics: the expected value and the value of a risk measure. Thus, mean-risk models have a ready interpretation of results and in most cases are convenient from a computational point of view. Sceptics on the other hand may question these advantages since the practice of describing a distribution by just two parameters involves great loss of informations. It is evident that the risk measure used plays an important role in making the decisions. Variance was the first risk measure used in mean-risk models (Markowitz 1952) and, in spite of criticism and many proposals of new risk measures (Konno and Yamazaki 1991, Ogryczak and Ruszczynski1999, 2001, Rockafellar and Uryasev 2000, 2002), variance is still the most widely used measure of risk in the practice of portfolio selection.

For regulatory and reporting purposes, risk measures concerned with the left tails of distributions (extremely unfavourable outcomes) are used. The most widely used risk measure for such purposes is Value-at-Risk (VaR). However, it is known that VaR has undesirable theoretical properties (it is not subadditive) and thus fails to reward diversification). In addition, optimization of VaR leads to a non-convex NP-hard problem which is computationally intractable. In spite of a considerable amount of research, optimizing VaR is still an open problem (Larsen et al. 2002).

In this paper, we propose a two-stage portfolio optimization approach which has all the strengths of the mean-VaR and the mean-variance approaches, and overcomes their shortcomings as the two stages complement one another.
This approach also uses more information of the underlying distribution of the portfolio return. Here, variance and VaR as risk measures are used separately in two stages according to a priority order of the two risk measures.
In stage one, we use a primary risk measure to collect all efficient portfolios.
In stage two, we use a secondary risk measure to re-evaluate (optimize) these efficient portfolios from stage one. This approach provides better results than the mean-variance and the mean-VaR approaches considered separately. The mean-variance-VaR efficient portfolio may not be mean-variance efficient or mean-VaR efficient. We also show that the mean-variance and the mean-VaR approaches are special cases of the mean-variance-VaR approach.

Many papers have been published that are related to this work, the most related being works by Alexander and Baptista [2000] and Basak and Shapiro [1999].
The first study compares the mean-variance and mean-VaR approaches for two special cases: multivariate normal distribution and multivariate t-distribution.
The second study analyzes optimal policies focusing on the VaR based risk management. Our work does not only compare the mean-variance and mean-VaR approaches in a general case, but also merges the two approaches into one single approach.

The rest of this paper is organized into for sections. In Section 2, we review the mean-risk approach using both variance as well as risk measures VaR. In Section 3, we propose a more general portfolio optimization strategy: the mean-variance-VaR model. The usual mean-variance model and the mean-VaR model are special situations of this model. Both variance and VaR are used as the risk measures during the procedure of optimization. In Section 4, we consider the case of an stocks portfolio listed at the *Italian Stock Market*. We use data analyze for portfolio selection, then estimation of risk with VaR risk measures for each stock and finally we solve the portfolio optimization in the framework mean – var.

2. Mean-risk models

Mean-risk models were developed in the early 1950s for the portfolio selection problem. In his seminal work 'Portfolio selection', Markowitz (1952) proposed variance as a risk measure. Since then, many alternative risk measures have been proposed. The question of which risk measure is most appropriate is still the subject of much debate. In mean-risk models, two scalars are attached to each random variable: the expected value (mean) and the value of risk measure. Preference is then defined using a trade-off between the mean where a larger value is desirable and risk where a smaller value is desirable.

In the mean-risk approach with the risk measure denoted by ρ, random variable R_x dominates (is preferred to) random variable R_y if and only if: $E(R_x) \geq E(R_y)$ and $\rho(R_x) \leq \rho(R_y)$ with least one strict inequality. Alternative, we can say that portfolio *x* dominates portfolio *y*. In this approach, the choice *x* (or the random variable R_x) is efficient (non-dominated) if and only if there is no other choice *y* such that R_y has higher expected value and less risk than R_x. This means that, for a given level of minimum expected return, R_x has the lowest possible risk, and, for a given level of risk, it has the highest possible expected return. Plotting the efficient portfolios in a mean-risk space gives the *efficient frontier*.

Thus, the efficient solutions in a mean-risk model are Pareto efficient solutions of a multiple-objective problem, in which the expected return is maximized and the risk is minimized: $\max\{(E(R_x) - \rho(R_x)) : x \in A\}$

The problem of portfolio selection with one investment period is an example of the general problem of deciding between random variables when larger outcomes are preferred. Decisions are required on the amount (proportion) of capital to be invested in each of a number of available assets such that at the end of the investment period the return is high as possible. Consider a set of *n* assets, with asset *j* in $\{1,...,n\}$ giving a return R_j at the end of the investment period. R_j is a random variable, since the future price of the asset is not known. Let x_j be the proportion of capital invested in asset *j* ($x_j = w_j / w$ where w_j is the capital invested in asset *j* and *w* is the total amount of capital to be invested), and let $x = (x_1,..., x_n)$ represent the portfolio resulting from this choice. This portfolio's return is the *random variable*: $R_x = x_1R_1 + ... + x_nR_n$ with distribution function $F(r) = P(R_x \leq r)$ that depends on the choice $x = (x_1,..., x_n)$.

To represent a portfolio, the weights $(x_1,...,x_n)$ must satisfy a set of constrains that forms a feasible set *M* of decisions vectors. The simplest way to define a feasible set is by the requirement that the weights must sum to 1 and short selling is not allowed. For this basic version of the problem, the set of feasible decisions vectors is $M=\left\{(x_1,...,x_n)/\sum_{j=1}^{n}x_j=1, x_j\geq 0, \forall j\in\{1,...,n\}\right\}$

The next issue is to consider a practical representation for the random variables that describe asset and portfolio returns. We treat these random variables as discrete and describe by realizations under *T* states of the world, generated using scenario generation of finite sampling of historical data. Let state $i\in\{1,...,T\}$ occur with probability p_i, $\sum_{i=1}^{T}p_i=1$. Let r_{ij} be the return of asset *j* under scenario *i*, $i\in\{1,...,T\}$ $j\in\{1,...,n\}$. Thus, the random variable R_j representing the return of asset *j* is finitely distributed over $\{r_{1j},...,r_{Tj}\}$ with the probabilities $p_1,...,p_T$. The random variable R_x representing the return of portfolio $x=(x_1,...,x_n)$ is finitely distributed over $\{R_{x1},...,R_{xT}\}$, where $R_{xi}=x_1r_{i1}+...+x_nr_{in}, \forall i\in\{1,...,T\}$.

In the following, we summarize:

-Let there be *n* assets $S_j, j=1,2,...,n$ and let R_j be the random variables representing the rate of return of S_j. Let $x_j\geq 0$ be the proportion of the fund to be invested in S_j.

-The vector $x=(x_1,x_2,...,x_n)$ is called a portfolio, which has to satisfy the following condition: $\sum_{j=1}^{n}x_j=1, \quad 0\leq x_j\leq\alpha_j, \quad j=1,2,...,n$ (1)

-Let $R(x)$ be the rate of return of the portfolio: $R(x)=\sum_{j=1}^{n}x_j\cdot R_j$ (2)

and let $r(x)$ and $v(x)$ be, respectively the mean and the risk of $R(x)$.

Then the mean-variance (MV) model is represented as follows.

$$(MV_1)\left|\begin{array}{ll}\min imize & v(x)\\ subjectto & r(x)\geq\rho\\ & x\in X\end{array}\right. \qquad (3)$$

where $X\in R^n$ is an the set defined by (1). Also, it may contain additional linear constraints. And ρ is a constant to be specified by an investor.

Varying ρ and repeatedly solving the corresponding optimization problem identifies the minimum risk portfolio for each value of ρ.

Let $x(\rho)$, be an optimal solution of the problem (3). Then trajectory of $\left(r(x(\rho)), \sqrt{\upsilon(x(\rho))}\right)$ is called an efficient frontier. By plotting the corresponding values of the objectives function and of the expected return respectively in a return risk space, we trace out the efficient frontier.

There are two alternative representations of the mean variance model, namely

$$(MV_2)\left|\begin{array}{ll} \max imize & r(x) \\ subjectto & v(x) \le \sigma^2 \\ & x \in X \end{array}\right. \tag{4}$$

$$(MV_3)\left|\begin{array}{ll} \max imize & r(x) - \lambda \upsilon(x) \\ subject\ to & x \in X \end{array}\right. \tag{5}$$

All three representations are used interchangeably since they generate the same efficient frontier as we vary ρ in (MV_1), σ in (MV_2) and $\lambda \ge 0$ in (MV_3).

There are several measures of risk uses for to assess the risk such as : absolute deviation($W(x) = E\left[\left|R(x) - E[R(x)]\right|\right]$), lower semi-variance, partial moments, below-target risk, value-at-risk (VaR), conditional value-at-risk (CVaR). Most of these except VaR are convex functions of *x*.

2.1 Mean -Variance Model

The mean-variance approach is the earliest method to solve the portfolio selection problem (Markowitz [1952, 1959]). The principle of diversification is the foundation of this method and it still has wide application in risk management. However, there are some arguments against it though this approach has been accepted and appreciated by practitioners and academics for a number of years (Korn [1997]). The variance of the portfolio return is the only risk measure of this method. Controlling (minimizing) the variance does not only lead to low deviation from the expected return on the down side, but also on the up side it may bound the possible gains too.

In this section, we briefly review the mean-variance approach.

Suppose that there are *n* securities with rates of return X_i $(i = 1, \ldots, n)$.

- The means and covariances of these rates of return are:

$\mu_i = E(X_i)$ and $\sigma_{ij} = \operatorname{cov}(X_i, X_j), i, j = 1, \ldots, n$

- The portfolio vector is : $\omega = (\omega_1, \ldots, \omega_n)' \in R^n$ and $\sum_{i=1}^{n} \omega_i = 1$

-We define that set W is a collection of all possible portfolios: $W = \left\{ \omega \in R^n \middle| \sum_{i=1}^{n} \omega_i = 1 \right\}$

-The total return of portfolio ω is $R_\omega = \sum_{i=1}^{n} \omega_i X_i$

- Its mean and variance are

$\mu_\omega = E(R_\omega) = E\left(\sum_{i=1}^{n} \omega_i X_i \right) = \sum_{i=1}^{n} \omega_i \mu_i$ and $\sigma_\omega^2 = Var\left(\sum_{i=1}^{n} \omega_i X_i \right) = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij}$

There are two common models that utilize the mean-variance principle.

The idea of the first model is that for a given upper bound σ_0^2 for the variance of the portfolio return ($\sigma_\omega^2 \leq \sigma_0^2$), select a portfolio ω, such that μ_ω is maximum :

$$\begin{aligned} &\max \quad \mu_\varpi \\ &\omega \in W \\ &s.t. \quad \sigma_\omega^2 \geq \sigma_0^2 \end{aligned} \tag{6}$$

The second model states that for a given lower bound μ_0 for the mean of the portfolio return ($\mu_\omega \geq \mu_0$), select a portfolio ω, such that σ_ω^2 is minimum :

$$\begin{aligned} &\min \quad \sigma_\omega^2 \\ &\omega \in W \\ &s.t. \quad \mu_\omega \geq \mu_0 \end{aligned} \tag{7}$$

2.2 Mean-VaR Model

In recent years, VaR has become a new benchmark for managing and control risk (Dowd [1998], Jonon [1997], RiskMetrics [1995]). Unfortunately, VaR based risk management has two shortcomings. First, VaR measures have difficulties aggregating individual risks, and sometimes discourage diversification (Artzner et al [1998]). Second, the VaR based risk management is only focusing on controlling the probability of loss, rather than its magnitude (Basak and Shapiro [1999]). The expected losses, conditional on the states where there are large losses, may be higher sometimes. The mean-variance approach encourages risk diversification, but the mean-VaR approach discourages risk diversification sometimes. The mean-variance approach does not only control the risk of return on the down side, but also bounds the possible gain on the up side while the mean-VaR approach only controls risk of return on the down side. Another limitation of both approaches is that the underlying distribution of the rate of return is not well understood, and there are no higher degree information is utilized except means, covariances (variances), or values of VaR

In this section, we briefly review the concept of VaR and the mean-VaR approach.

The VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence level, and provides users a summary measure of market risk. Precisely, the VaR at the $100 \cdot \alpha\%$ confidence level of a portfolio ω for a specific time period is the rate of return q_ω such that the probability of that portfolio having a rate of return of q_ω or less is α: $P(R_\omega \le q_\omega) = \alpha$ (8). Here q_ω is also called the α^{th} quantile of the distribution of R_ω. Similar to the mean-variance method, we defined two models for the mean-VaR principle.

The first one is that for a given upper bound q_0 for the VaR of the portfolio return, select a portfolio ω, such that μ_ω is the maximum with $q_\omega \le q_0$:

$$\begin{aligned} &\max \quad \mu_\varpi \\ &\omega \in W \\ &s.t. \quad q_\omega \le q_0 \end{aligned} \tag{9}$$

The second model states that for a given lower bound μ_0 for the mean of the return , select a portfolio ω, such that its VaR (q_ω) is minimum with $\mu_\omega \ge \mu_0$:

$$\begin{aligned} &\min \quad q_\omega \\ &\omega \in W \\ &s.t. \quad \mu_\omega \ge \mu_0 \end{aligned} \tag{10}$$

2.3 Comparison of Mean -Variance and Mean -VaR Models

In this section, we compare the mean-VaR approach with the mean-variance approach. The two approaches are using completely different risk measures to optimize portfolios. The mean-variance approach only uses of the mean and variance of portfolio return. The Mean-VaR approach only uses the mean and VaR of the portfolio return. Both approaches have many advantages; however they do not sufficiently use the information from the distribution of the portfolio return. Example shows that a mean-variance efficient portfolio is not a mean-VaR efficient portfolio. Remark says that a mean-VaR efficient portfolio is not a mean-variance efficient portfolio but under the normality assumption proposition can show that a mean-VaR efficient portfolio is a mean-variance efficient portfolio.

Example : *A mean-variance efficient portfolio is not a mean-VaR efficient portfolio.*

We consider a simple two-security portfolio selection problem.

The rate of return for the first security is $X_1 = Z$, where Z is the standard normal $N(0,1)$ with mean, variance and VaR, $\mu_{X_1} = 0$, $\sigma^2_{X_1} = 1$, and $q_{X_1} = -z_\alpha$, where $1-\alpha$ is the confidence level (say $\alpha = 0{,}05$), and z_α, is the α^{th} quantile of the standard normal distribution, such that $\alpha = \int_{-\infty}^{-z_\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$

The rate of return for the second security is $X_2 = 2Z + 2z_\alpha$, with mean, variance, and VaR, $\mu_{X2} = 2z_\alpha$, $\sigma^2_{X_2} = 4$, and $q_{X_2} = 0$ ($P(X_2 < 0) = \alpha$) .

The correlation of X_1 and X_2 is $Corr(X_1, X_2) = Corr(Z, 2Z + 2z_\alpha) = 2$

For any portfolio ω, the variance of its return $R_\omega = \omega_1 X_1 + \omega_2 X_2$ is

$Var(R_\omega) = Var(\omega_1 X_1 + \omega_2 X_2) = (\omega_1 + 2\omega_2)^2 = (2-\omega_1)^2$

-This variance reaches minimum value 1 when $\omega_1 = 1$. Therefore $\omega^* = (1,0)$ is a mean-variance efficient portfolio with mean, variance, and VaR

$\mu_{\omega^*} = 0$, $\sigma^2_{\omega^*} = 1$ and $q_{\omega^*} = z_\alpha$

-But this portfolio is not mean-VaR efficient. Consider portfolio $\omega^{**} = (0,1)$. Both mean and VaR of $R_{\omega^{**}} = X_2$ are better,

$\mu_{\omega^{**}} = \mu_{X_2} = 2z_\alpha > 0 = \mu_{\omega^*}$ and $q_{\omega^{**}} = q_{X_2} = 0 < q_{\omega^*}$

Remark: A mean-VaR efficient portfolio is not a mean-variance efficient portfolio.

Proposition: Under the normality assumption, a mean-VaR efficient portfolio is mean-variance efficient.

-Under the normality assumption, the portfolio return R_ω is a $N(\mu_\omega, \sigma^2_\omega)$ random variable with VaR, $q_\omega = z_\alpha \sigma_\omega - \mu_\omega$ (11)

-If ω^* is a mean-VaR efficient portfolio, then for any portfolio, we have:

$q_\omega \geq q_{\omega^*}$ if $\mu_\omega \geq \mu_{\omega^*}$ Using result (11), we have:

$$\sigma_\omega = \frac{1}{z_\alpha}(q_\omega + \mu_\omega) \geq \frac{1}{z_\alpha}(q_{\omega^*} + \mu_{\omega^*}) = \sigma_{\omega^*}$$

3. Mean-Variance-VaR Model

In this section, we propose a general mean-variance-VaR model for portfolio optimization with two variations. We use both variance and VaR as risk control measures. Our models cover both the mean-variance model and the mean-VaR model. In other words, the two models are special cases of our models.

The first model is that for a given upper bounds σ_0^2 and q_0 for the variance and VaR of the portfolio return respectively, select a portfolio ω, such that μ_ω is the maximum with $\sigma_\omega^2 \le \sigma_0^2$ and $q_\omega \le q_0$:

$$\begin{aligned} &\max \quad \mu_\varpi \\ &\omega \in W \\ &s.t. \quad \sigma_\omega^2 \le \sigma_0^2 \\ &\qquad\quad q_\omega \le q_0 \end{aligned} \tag{12}$$

Comparing with the mean-variance model or the mean-VaR model, we use double-risk measures instead of one single risk measure. The mean-variance-VaR efficient portfolio may not be mean-variance efficient or mean-VaR efficient. Moreover, the mean-variance (6) and the mean-VaR models (9) are special cases of our model (12):

- when $q_0 = \infty$, our model (12) becomes the mean-variance model (6);
- when $\sigma_0^2 = \infty$, our model (12) becomes the mean-VaR model (9).

The second model is not that for a given lower bound μ_0^2 for the mean of the portfolio return, select a portfolio ω, such that the convex combination of variance and VaR of the portfolio return $\beta\sigma_\omega^2 + (1-\beta)q_\omega$ is the minimum with $\mu_\omega \ge \mu_0$:

$$\begin{aligned} &\min \quad \beta\sigma_\omega^2 + (1-\beta)q_\omega \\ &\varpi \in W \qquad\qquad\qquad\qquad , \beta \in [0,1] \,. \\ &s.t. \quad \mu_\varpi \ge \mu_0 \end{aligned} \tag{13}$$

For the two extreme values of β, we have

- when $\beta = 1$, our model (13) becomes the mean-variance model (7);
- when $\beta = 0$, our model (13) becomes the mean-VaR model (10).

Possible alternatives to the objectives function of model (13) are: $\beta\sigma_\omega^2 + (1-\beta)q_\omega^2$ and $\beta\sigma_\omega + (1-\beta)q_\omega$.From the computational point view, $\beta\sigma_\omega^2 + (1-\beta)q_\omega^2$ is better than $\beta\sigma_\omega + (1-\beta)q_\omega$ since square-root takes more computation time than square. We also can substitute the objectives function of model (13) by a general utility function $f(\sigma_\omega^2, q_\omega)$.

4. Case study : *Italian Stock Market*

4.1. Stage of selection of assets

In the context of nowadays financial markets it is a huge amount of available financial data. It is therefore very difficult to make use of such an amount of information and to find basic patterns, relationships or trends in data. We apply data analysis techniques in order to discover information relevant to financial data, which will be useful during the selection of assets and decision making. Consider that we have collected information on a number S of assets, each with P features, which represent various financial ratios, still called variables. Denote by y_i^j the j-th variable for stock i. Multivariate data set will be represented by a matrix $Y = \left(y_i^j\right)_{\substack{i=\overline{1,S} \\ j=\overline{1,P}}}$ and can be viewed as a set of S points in a P-dimensional space. Principal components analysis (PCA) is a useful technique for analyzing data to find patterns of data in a large-scale data space. PCA involves a mathematical procedure that transforms P variables, usually correlated in a number of $p \leq P$ uncorrelated variables called principal components. After applying the PCA, each asset *i* will be characterized by *p* variables, represented by a set of parameters $y_i^1, y_i^2, ..., y_i^p$ therefore, it is possible to form the arrays $Y_i = \left(y_i^1, y_i^2, ..., y_i^p\right),\ i = \overline{1,S}$, which correspond to a set of S assets. Suppose now that we obtained a data set $Y_i = \left(y_i^1, y_i^2, ..., y_i^p\right),\ i = \overline{1,S}$. We then use clustering techniques in order to find similarities and differences between the stocks under consideration. The idea of clustering is an assignment of the vectors $Y_1, Y_2, ..., Y_S$ in T classes $C_1, C_2, ..., C_T$. Once completed the selection of activities, we construct the initial portfolio by selecting low-risk asset in each class.

We will present some of the most important financial indicators that we will use in study:

- The P/E is calculated by dividing the current market price to the value of net profit per share for the past four consecutive quarters, net income per share is calculated by dividing the total net profit earned by the company during the reporting period (it is relevant to relate to the last 12 months) the number of shares issued and outstanding.
- *The P / BV* is calculated by dividing the current trading price to book value per share determined according to the latest financial reporting; accounting value of a share is calculated by dividing the total equity value of the company to the total of it shares issued and outstanding; equity value is determined by deducting total liabilities from total assets owned company and is "shareholder wealth", which is what remains to be recovered if the assets and liabilities would be paid.
- *Divy index* measures the performance of dividend and is calculated as the ratio between the amount of the dividend and book value or market value of the stock. and assesses the efficiency of investment in an asset.

- *Volatility* is a measure for variation of price of a financial instrument over time. It is used to quantify the risk of the financial instrument over the specified time period.
- *Evolution of price*: to observe the price level at a given time we take into acount the maximum price and minimum price achieved in the last 6 months

We used information on a total of those 40 shares representing shares of FTSE MIB Index traded on the Italian Stock Market. The aim of our study is to find similarities and differences between the current analysis and build a diversified portfolio. We take into account for each stock the 6 features described above;

We use data analysis techniques in order to process this vast amount of information. Table 1 lists, for each of the 40 stocks analyzed, the values of the six features; we used the data available on the Italian Stock Market on 27 March 2012.

Table 1: The value of the 6 features

No	Company	P / E	P / BV	DIVY %	Volatility %	P / Max	P / Min
1	A2A	11.4	0.8	8	26	0.66	1
2	Ansaldo Sts	13.9	2.8	2.5	21	0.91	1.07
3	Atlantia	8.8	5.8	6.9	25	0.81	1.34
4	Autugrill	15.8	2.7	3.7	21	0.92	1.12
5	Azimut	11.6	1.87	3.1	31	0.97	1.56
6	Banca Popolare	22.7	0.37	0.00	60	0.95	1.85
7	Bca Mps	16.5	0.18	0.00	77	0.83	1.8
8	BcaPopEmil Romagna	11.1	0.4	1.3	38	0.79	1.28
9	BcaPop Milano	38.4	0.3	0.00	64	0.87	1.92
10	Buzzi Unicem	30.7	0.6	0.00	33	1	1.61
11	Campari	18	2.58	0.07	19	0.92	1.04
12	DiaSorin	11.8	4.5	2.03	32	0.78	1.23
13	Enel	6.1	0.72	8.7	26	0.8	1

14	Enel Green Power	6.1	0.72	8.7	26	0.84	1.04
15	Eni	8.2	6.2	6.4	14	0.98	1.38
16	Exor	3.9	1.3	1.7	24	0.94	1.37
17	Fiat	23.1	0.6	0.00	36	0.89	1.38
18	Fiat Industrial	23.1	0.6	0.00	36	0.97	1.67
19	Finmeccanica	10.9	0.7	0.00	48	0.63	1.33
20	Generali	1	1.4	1.4	27	0.92	1.1
21	Impregilo	12.8	1.26	1.29	25	0.98	1.68
22	Intesa Sanpaolo	9.7	0.34	3.3	40	0.9	1.24
23	Lottomatica	10.4	7.4	5.27	23	0.98	1.38
24	Luxottica	21.8	3.0	2.3	20	0.99	1.45
25	Mediaset	9.2	1.6	5.9	34	0.98	1.16
26	Mediobanca	9.7	0.94	2.6	38	0.59	1.21
27	Mediolanum	12.9	2.9	4.4	31	0.99	1.52
28	Parmalat	16.1	0.7	3.1	32	0.98	1.39
29	Pirelli	8.9	1.3	4.3	32	0.98	1.74
30	Prysmian	10.6	1.8	2.7	30	0.97	1.48
31	Saipem	16.4	3.2	2	19	0.97	1.55
32	Salvatore Ferragamo	25.2	8.23	1.6	35	0.99	1.85
33	Snam	16	2.3	6.2	16	0.99	1.16
34	St	9.96	0.85	0.00	36	0.96	1.4

	microlectronics						6
35	Telecom Italia	6.30	0.6	5.4	26	0.97	1.17
36	Tenaris	15.6	1.9	2	27	0.65	1.65
37	Terna	16.9	1.8	7	17	1	1.24
38	Tod'S	18.8	4	3	25	0.99	1.37
39	Ubi Banca	21.4	0.43	0.00	44	0.84	1.34
40	Unicredit	12.7	0.31	0.00	79	0.57	1.73

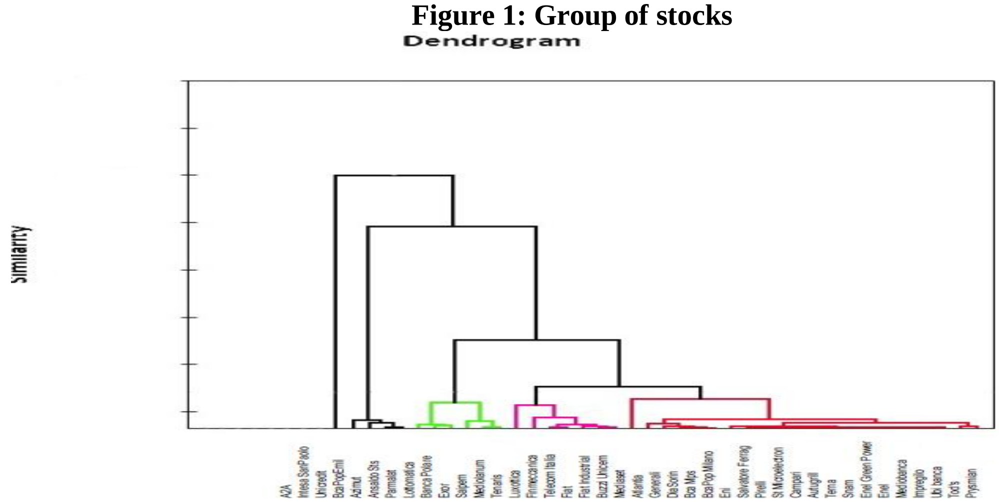
Source: www.borsaitaliana.it

We apply data analysis techniques to discover the similarities and differences between the stocks of the Bucharest Stock Exchange, using the package StatistiXL 1.8.

Figure 1 contains the tree resulted from PCA (dendrogram).

Dendrogram usually begins with all assets as separate groups and shows a combination of mergers to a single root. Stocks belonging to the same cluster are similar in terms of features taken into account. In order to build a diversified portfolio, we first choose the number of clusters, which will be taken into account. We will then choose a stock from each group and we get the initial portfolio.

Figure 1: Group of stocks



Source: package programe STATISTI XL 1.8

4.2. Phase risk estimation.

We evaluate the performance of an asset using expected future income, an indicator widely used in financial analysis. Denote by $S_j(t)$ the closing price for an asset j at time t. Expected future income attached to the time horizon $[t, t+1]$ is given by: $R_j(t) = \ln S_j(t+1) - \ln S_j(t),\ j \in \overline{1,S}$. Similary, we define the loss random variable, the variable L_j, for asset j for $[t, t+1]$ as $L_j(t) = -R_j(t) = \ln S_j(t) - \ln S_j(t+1),\quad j \in \overline{1,S}$. Using Rockafellar et al., 2000, define the risk measure VaR corresponding loss random variable L_j. Probability of L_j not to exceed a threshold $z \in \mathbf{R}$ is $G_{L_j}(z) = P(L_j \le z)$.. Value at risk of loss random variable L_j associated with the value of asset j income and corresponding probability level $\alpha \in (0,1)$ is: $VaR_\alpha(L_j) = \min\{z \in \mathbf{R} \,|\, G_{L_j}(z) \ge 1-\alpha\}$ or P (X > VaR) = α .
If G_{L_j} is strictly increasing and continuous, $VaR_\alpha(L_j)$ is the unique solution of equation $G_{L_j}(z) = 1-\alpha$ then $VaR_\alpha(L_j) = G_{L_j}^{-1}(1-\alpha)$. One of the most frequently used methods for estimating the risk is the *historical simulation method.* This risk assessment method is useful if empirical evidence indicates that the random variables in question may not be well approximated by normal distribution or if we are not able to make assumptions on the distribution. Historical simulation method calculates the value of a hypothetical changes in the current portfolio, according to historical changes in risk factors. The great advantage of this method is that it makes no assumption of probability distribution, using the empirical distribution obtained from analysis of past data. Disadvantage of this method is that it predicts the future development based on historical data, which could lead to inaccurate estimates if the trend of the past no longer corresponds. If L_j is the loss random variable and $\hat{G}_n$ is empirical distribution function of L_j and $\alpha \in (0,1)$ a fixed level of probability, then $\hat{G}_n(z) = \frac{1}{n}\sum_{i=1}^{n} I_{\{L_j \le z\}}$.

We can prove that : $VaR(L_j) = \min\left\{ z \in \mathbf{R} \,|\, \frac{1}{n}\sum_{i=1}^{n} I_{\{L_j^i \le z\}} \ge 1-\alpha \right\}$. Once completed the phase of grouping the assets in T classes by the existing similarities, we focus on the selection of the assetsş of each class to have a minimal risk. Consider the loss random variable corresponding to each asset in each obtained class $C_i,\ i = \overline{1,T}$: $\min_k VaR\,L(A_k^i)$. We used the closing price values daily for each share, corresponding to a time horizon of 50 days to measure VaR for each stock. We used the data available on the Italian Stock Market from 15 march 2012 - 30 april 2012. The following tables contain values of VaR for each stock and three levels of probability values

Class 1	0.90	0.95	0.99
Atlantia	0.0139	0.0171	0.0519
Autogrill	0.0290	0.0371	0.0653
BcaPop Milano	0.0195	0.0231	0.0361
Bca Mps	0.0134	0.0204	0.0259
Buzzi Unicem	0.0531	0.0742	0.0969
Campari	0.0333	0.0388	0.0558
DiaSorin	0.0221	0.0293	0.0330
Enel	0.0247	0.0347	0.0492
Enel Green Power	0.0248	0.0471	0.0750
Eni	0.0201	0.0248	0.0325
Generali	0.0418	0.0585	0.2624
Impregilo	0.0568	0.0807	0.1476
Mediobanca	0.0261	0.0369	0.1449
Mediaset	0.0236	0.0317	0.0529
Pirelli	0.0317	0.0485	0.0584
Prysmian	0.0282	0.0313	0.0420
Snam	0.0228	0.0422	0.0701
S.Ferragamo	0.0348	0.0383	0.0584
St microlectronics	0.0415	0.0705	0.1025
Terna	0.0534	0.0687	0.0748
Tod'S	0.0287	0.0455	0.0995
Ubi Banca	0.0465	0.0550	0.1003

Class 2	0.90	0.95	0.99
Fiat	0.0223	0.0424	0.0486
Fiat Industrial	0.0531	0.0742	0.0969
Finmeccanica	0.0512	0.0747	0.1289
Luxottica	0.0265	0.0405	0.0669
Mediolanum	0.0270	0.0299	0.0676
Telecom	0.0235	0.0372	0.0434
Tenaris	0.0328	0.0400	0.0468

Class 3	0.90	0.95	0.99
Ansaldo Sts	0.0721	0.0891	0.1325
Banca Popolare	0.0460	0.0562	0.0815
Exor	0.0237	0.0280	0.0388
Lottomatica	0.0348	0.0448	0.0566
Parmalat	0.0572	0.0650	0.1074
Saipem	0.0239	0.0250	0.0368

Class 4	0.90	0.95	0.99
Azimut	0.0225	0.0270	0.0329
BcaPopEmil Romagna	0.0276	0.0317	0.0406
Intesa Sanpaolo	0.0255	0.0305	0.0343
Unicredit	0.0242	0.0260	0.0295

Class 5	0.90	0.95	0.99
A2A	0.0167	0.0221	0.0327

4.3 Optimization portfolio phase in the framework Mean - VaR

We obtain an initial portfolio comprising a wide range of stocks with minimal risk. We will try to determine what percentage is the optimal composition of capital that needs to be invested in each of the assets under consideration, so that at the end of the investment we have a maximum return on investment.

Thus, T is a set of stocks, with stock j that leads to expected income $R_j,\ j=\overline{1,T}$;Expected income of portfolio is: $R_{\mathbf{x}}=\sum_{j=1}^{T}x_jR_j$ .The model to be solved is:

$\max\sum_{j=1}^{T}x_jR_j$, with $VaR_{\alpha}\left(L_x\right)\le v_0$, where v_0 is the model parameter.

As a consequence of applying the technique of selection, we selected a portfolio of 5 stocks, each of them representing the minimum risk stock class corresponding VaR measured probability level 0.99: Bca Mps,Telecom,Saipem,Unicredit, A2A. We will try to determine what percentage is the optimal composition of capital that needs to be invested in each of the stocks under consideration, so that at the end of the investment we have a maximum return on investment. In these conditions, the optimization problem to be solved is:

$$\max f = 0.1122\, x_1 + 0.0286\, x_2 + 0.04\, x_3 + 0.1724\, x_4 + 0.205\, x_5$$

$$\begin{cases} 0.031\, x_1 + 0.052\, x_2 + 0.044\, x_3 + 0.0354\, x_4 + 0.0392\, x_5 \le \upsilon_0 \\ \sum_{i=1}^{5} x_i = 1 \\ x_i \ge 0,\ i = \overline{1.5} \end{cases}$$

There are several methods for solving this problem (math programming or software *Scientific Work Place*). A solution to this problem remains a challenge for future.

5.Conclusion : In this paper we have discussed and compared the mean-variance approach with the mean-VaR approach. The mean-variance-VaR approach uses variance and VaR as a double-risk measure simultaneously. The mean-variance and the mean-VaR approaches are special cases of this approach. Finally we build a portfolio with stocks listed at *Italian Stock Market*.

Acknowledgment : *This work was supported from the European Social Fund through Sectoral Operational Programme Human Resources Development 2007-2013, project number POSDRU/89/1.5/S/59184 „Performance and excellence in postdoctoral research in Romanian economics science domain”.*

REFERENCES

[1] **Alexander, G., Baptista, A.(2000),** ***Economic Implications of Using a MeanVaR Model for Portfolio Selection: A Comparison with Mean-Variance Analysis***. *Working Paper,* University of Minnesota;

[2]**Basak, S., Shapiro,A.(1999),** ***Value-at-Risk Based Risk Management: Optimal Policies and Asset Prices***. *Working Paper,* University of Pennsylvania;

[3] **Dedu, S. , Fulga, C. (2011**), ***Value-at-risk. Estimation Comparative Approach with Application to Optimization Problem*** . *Economic Computation and Economic Cybernetics Studies and Research,45 (1), 127-142, ASE Publishing House, Bucharest;*

[4]**Fulga, C., Dedu, S., Şerban, F. (2009),** ***Portfolio Optimization with Prior Stock Selection***. *Economic Computation and Economic Cybernetics Studies and Research, ASE Publishing House, Bucharest, 43 (4), 157-172;*

[5] **Fulga, C., Dedu, S. (2010),** ***A New Approach in Multi-Objective Portfolio Optimization Using Value-at-Risk Based Risk Measure,*** Proceedings of The International Conference on Information and Financial Engineering ICIFE 2010, IEEE Computer Society, Computer Society Press, 765-769;

[6] **Markowitz, H.(1952),** ***Portfolio Selection*****.** *Journal of Finance*, 7, 77-91;
[7] **Rockafellar, T., Uryasev, S.(2000),** ***Optimization of Conditional Value-at-Risk***. *Journal of Risk, 2;*
[8]**Roman,D, Darby,K.,Mitra,G.(2007)**, ***Mean-risk Models Using Two Risk Measures : A Multi-objective Approach***. *Quantitative Finnance*, 7(4),443-458;
[9] **Ştefănescu, V., Ferrara, M., Dedu, S. (2008),** ***Algorithms for Hierarchical Classification with Applications in Portfolio Management*****,** *Economic Computation and Economic Cybernetics Studies and Research, ASE Publishing,* 42, 3-4, 109-122;
[10]**Ştefănescu, M.V, Şerban, F., Buşu, M., Ferrara, M. (2010),** ***Portfolio Optimization Using Classification and Functional Data Analysis Techniques*** . *Economic Computation and Economic Cybernetics Studies and Research, 44 (3), 93-108, ASE Publishing House, Bucharest;*
[11] **Wang, J.(2007)**, ***Mean-Variance-VaR Based Portfolio Optimization*** . *Working Paper*, Valdosta University.