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RISK MEASUREMENT IN POST-MODERN PORTFOLIO THEORY: DIFFERENCES FROM MODERN PORTFOLIO THEORY

Abstract. *In the context of financial crises, the risk measurement is one of the top issues debated by practitioners and financial research studies. Standard deviation represents a largely used method of measuring risk; but in the last years it was partially put under question because it does not consider the investor behavior and his expectations. The downside risk better answer to the real investment process, including investor expectation and the non-normal distributed return rates.*

We highlighted the differences between the methods of measuring risk in the post-modern and modern portfolio theory, both from a theoretical and empirical perspective. The PMPT method generates better empirical results sustained by the theoretical approach presented in the paper.

Using PMPT for quantifying risk, the investor can distinguish between the real risk of obtaining returns lower than his expectation and the premium of obtaining higher returns than expected paying for his courage of performing the investment.

Key words: *post-modern portfolio theory, downside risk, Sortino, potential return rate, premium risk.*

JEL Classification: G12

Introduction

The modern portfolio theory (MPT) represented at the middle of the past century a big step forward in the financial literature and the investment practice. The theory put a logic relation between the distribution of return rates and risk of the investment, considering that investors acts rational in taking decisions about the investment performed, that they have aversion to risk and that the distribution of return rates is following a normal distribution.

Over time, the modern portfolio theory developed, many investors and researchers intended to relax and adapt the model restrictive conditions to the market reality. Conditions such as normal distribution of return rates, stability over time of assets' correlation or iso-variance and iso-average (Beste, Leventhal, Williams, Lu, 2002) are desirable but usually do not exist.

Restrictions related to investors' rational behavior and their pure interest just in maximization of economic utility were put under question by the behavioral finance that reflects the investors' trend to obtain the emotional comfort prior to optimal financial efficiency. The prospect theory (Kahneman, Tversky, 1979) presents a new face of the investor. The investor is not just rational as the previous financial literature assumes he should be, but he is also a human person with emotions and preconceptions (Barberis, Shleifer, Vishny, 1996). The humanized investor is a person with different reactions for losses and gains resulting from his investment, depending on the individual assumption of risks.

Financial behavior presents the investor as a person that is reluctant to losses, but not to gains over the minimum expected return. Previous it was consider that an investor is interested to invest in a portfolio with return that does not vary much from the average. The research of the investor reactions shows that he is in fact interested in obtaining a minimum desired return, any result below the minimum desired return is consider a loss, while gains higher than the expected level of return do not constitute a concern, (but contrary, they are considered as premium for the courage of investing), the "good surprise" (Tsai, Wang, 2012).

The risk in the post-modern portfolio theory is consider as the possibility of return rates to be situated beneath the minimum expected return; investors are preoccupied mostly to limit this kind of variation from their investment. The post-modern portfolio theory has a wide application than the MPT and includes the expectation of investors related to a minimum desired return rate as benchmark rather than the average return rate.

Although MPT remained a significant benchmark in the portfolio theory (Elton, Gruber, 1997, Chen, Tsai, Lin, 2011), the post modern portfolio theory moves the financial theory and practice a step forward, considering the investor expectations (Nawrocki, 1999, Bawa, Lindenberg, 1977, Fishburn, 1977).

Both theories are used within the financial research but also outside this area, researchers and business people extend their application to others economic domains (such as real estate, energy portfolios, other investments except stocks) with interesting results and ways of applying the methods of quantifying risk (Madlener, Glensk, Raymond, 2009, Tsai, Wang, 2012, Hines, 2009).

Since the beginning of the present financial crises many researchers and portfolio managers revive the question regarding the MPT realism relative to market conditions. Although MPT was preferred and used for decades before financial crises in 2008, the theory was blamed for failing in those moments (Welch, 2010). Investors and researchers start to look for alternative theories that measure risk (Bertsimas, Lauprete, Samarov, 2004, Patari, 2008).

Literature review

The post modern portfolio theory (PMPT) was developed in the 1980s at the Pension Research Institute (USA) in order to better adapt the modern portfolio theory to the market reality, including the minimum return rate accepted by the investor in the measurement of risk. It includes the behavior of the investor in computation of risk measure, considering the risk as the chance that the investment return be less than the minimum return expected by the investor from his portfolio.

Until PMPT the investors were considered as having a rational behavior regarding the investment decision process, all investors having the same expectation related to market future evolution. This concept is modified in PMPT; investor is considered as having as target a minimum accepted return that insures him the emotional comfort and that the investor is concerned by the returns lower that his expected benchmark.

This is the most evident difference is the risk measurement between the two theories, but advantages of PMPT are even more important. The MPT consider that returns of portfolio are normal distributed; in real economic life, the distribution of returns in only in exceptional cases normal distributed – the restriction of normal distribution induces from start errors in analyses. PMPT does not include any restriction related to the distribution; the returns distribution may have any form. The only common restriction imposed by both theories is the continuity of the distribution function.

There is generated an area of possibilities of losses or gains, depending on the risk assumed by investors by the position of accepted return in the distribution of returns of portfolio (Sortina, Forsey, 1996). The downside risk is based on the possibility of appearance of returns inferior to the minimum accepted. The importance of measuring the chance of not obtaining the minimum return results from the investors preoccupation in taking decisions based on the balance between the downside risk and the upside risk (the potential benefit resulting from the investment made). Investors may obtain superior returns from consideration of real asymmetry of returns in comparison with the standard bell-shape of the normal distribution.

There are few studies comparing the two methods of measuring risk from the two theories, but usually revealing the superiority of the PMPT over MPT. The classic standard deviation is used on a large scale as measure of risk but has limitation in offering correct information due to consideration of returns superior to a specified return level (average return or return level established by investor) as penalty for the investment results or due to asymmetry in returns distribution. Downside risk is what investors consider to be risky and this became more “popular” among investors (Huang, 2008).

The attitude of investor regarding the returns situated over the expected return rate established by the investor in accordance with his own emotional satisfaction and interests is considered being linear, neutral or even in favor of risk (Fishburn, 1977) – this returns do not practically generate losses but determine premium gains for the investment.

The attitude of investors to risk is determined, according to several studies, to the importance of the invested sum of money – if the sum invested is important for the investor financial situation, he is more reluctant to risk, while if the sum invested has a reduce significance the investor became more risk willing and is attracted by higher returns (Kaplan, Siegel, 1994). Nawrocki (1999) completes the assumption and mentions that as the investors' expectations modify, the investment sum and the investment horizon modify; also the expected return level modifies together with changes in risk aversion.

PMPT allows models applied for portfolio management to be more adequate to reality, having higher power in representing the economic reality (Dronin, 2012, Rani, 2012). The information offered are better suited for the decisional process of managers that evaluate the investment opportunities in a competitive environment (Libby, Fishburn, 1977). Differences of decision in performing the investment are explicable due to the diverse perspective of investors regarding any specific situation and the level of risk accepted (Nawrocki, 1991).

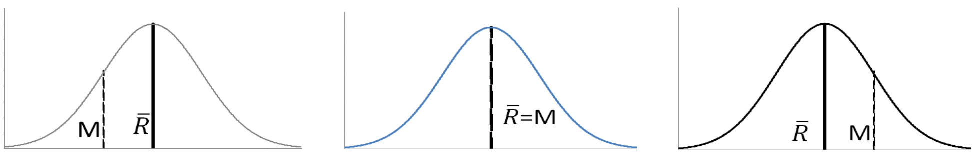
Starting from the basic elements of theory, there is a lot of developments to PMPT (Plantinga, van der Meer, Sortino, 2001, Kaplan, Knowles, 2004, Galloppo, 2010). For comparison with MPT we had to remain to main elements as probability of return, potential return rates and risk; the evolutions of theory generate steps forward in obtaining more information from applying PMPT.

Theoretical differences between the two methods of measuring risk

The attitude towards risk depends on the investor affinity to risk (Kaplan, Siegel, 1994), his wish to obtain a higher return implies accepting higher risk, so the minimum accepted return rate (M) is higher. The position of the minimum accepted return on the return rates distribution depends on the risk accepted.

There are three cases that we considered in our study for determining the differences resulting from applying the two methods of computing risk, concerning to the position of the minimum accepted return (M) in relation to average return ($\overline{R}$) – the position of minimum accepted return is the element from PMPT that determine the amplitude of risk. In order to be comparable, we shall use a normal distribution and point out the influence of M and $\overline{R}$ over the risk measurement. The three cases studied on a normal distribution are:

a) minimum accepted return rate M is lower than average return rate $\overline{R}$
b) minimum accepted return rate M equal to average return rate $\overline{R}$ – this case will be presented in paper only when it determines significant information for our study (it was computed in all analyses performed);
c) minimum accepted return rate M is higher than average return rate $\overline{R}$



a) M < $\overline{R}$ **b)** M = $\overline{R}$ **c)** $\overline{R}$ < M

Figure 1. Cases of position of M vs. $\overline{R}$ (considering a normal distribution)

Probability of return rate

The first element that we studied was the probability that portfolio return could be lower than the return considered by the investor. The topic has different significance in the two theories. For the MPT, the probability that the portfolio return to be lower than the return considered by investor is almost equal to 0.5, because the considered return of the portfolio is the average return. The average return on a normal distribution is equal to median and the module of the data series; and because the total area under the function is equal to 1, then the area from beneath the function and on the left of the average return is equal to the area beneath the function on the right of the average return, the sum of the two being equal to unit less the probability of return rate to be equal to average return.

$$P_i(r, \bar{R}) = \int_{-\infty}^{\bar{R}} f(r)dr \ ; \ P_i(r, M) = \int_{-\infty}^{M} f(r)dr$$

where $P_i(r, \bar{R})$ is the inferior (or downside) probability (that the portfolio return to be lower than the average return $\bar{R}$) and f(r) is the distribution function of the data series. For PMPT the inferior probability $P_i(r, M)$ represents the chance that the portfolio returns to be lower than the minimum return rate M expected by investor.

In case a), M being lower than $\overline{\mathrm{R}}$, the inferior probability is lower for PMPT with the area below the function and between M and $\overline{\mathrm{R}}$. In case b), when M is equal to $\overline{\mathrm{R}}$, on a normal distribution, the inferior probability computed for both theories is the same. When the investor requires a higher minimum return then the average return, the inferior probability is higher in case of PMPT. We can resume the difference between the inferior probabilities for the two theories as follow:

Difference resulting from comparison of the two inferior probability determine the area from below the distribution function and situated between the position of minimum accepted return rate M and position of average return rate $\overline{\mathrm{R}}$. This difference [noted $\mathrm{dP_i}(\mathrm{r}, \overline{\mathrm{M}, \mathrm{R}})$] reflects the investor appetite to risk and how much area potential risky exists on left of M [noted $\mathrm{d_{\%}P_i}(\mathrm{r}, \overline{\mathrm{M}, \mathrm{R}})$], risk assumed by investor in performing the investment (from establishing the minimum accepted return rate M) in compare with the area of risk involved by average return $\overline{\mathrm{R}}$. Notation $\mathrm{abs(b)}$ represents the abstract of value b.

$$\mathrm{dP_i}(\mathrm{r}, \overline{\mathrm{M}, \mathrm{R}}) = \mathrm{P_i}(\mathrm{r}, \mathrm{M}) - \mathrm{P_i}(\mathrm{r}, \overline{\mathrm{R}});\ \mathrm{d_{\%}P_i}(\mathrm{r}, \overline{\mathrm{M}, \mathrm{R}}) = \frac{\mathrm{P_i}(\mathrm{r}, \mathrm{M})}{\mathrm{P_i}(\mathrm{r}, \overline{\mathrm{R}})} \in (0.1)$$

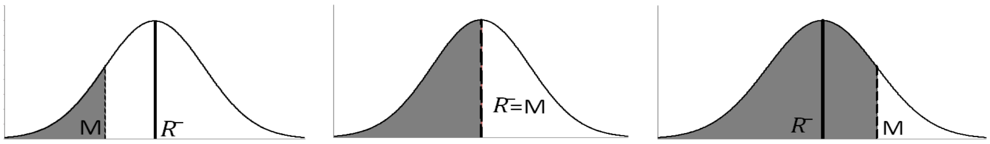
The problem is more complicated if we discuss about data series that are not normal distributed, maintaining the condition of being continuous distribution function (Sortino, Forsey, 1996). The average return $\overline{\mathrm{R}}$ is no longer in the center of the distribution area, so that the area in front of average return and below the distribution line can be less of 0.5 or can cover more of the risk. The position of M after $\overline{\mathrm{R}}$ does not necessarily mean that the area below the distribution line and in front of M is necessary more than 0.5; as well a value of M less than $\overline{\mathrm{R}}$ does not necessarily denote that the area below distribution line and in from of M is less than 0.5. The normal distribution is just a particular case of a distribution function of return rates (Madlener, Glensk, Raymond, 2009, Kolbadi, 2011).

In MPT the probability of the portfolio return to be in the area considered risky is equal to $1 - p(r = \overline{\mathrm{R}})$ (where $p(r = \overline{\mathrm{R}})$ noted as $p_{\bar{R}}$ is the probability of $\overline{\mathrm{R}}$), because only the average return is considered as acceptable and any return that deviates from it is considered as risk, no matter if the portfolio return is lower or higher than the average return.

$$P(r, \bar{R}) = P(r, p_{\bar{R}}, \bar{R}) = 1 - p(r = \overline{\mathrm{R}})$$

where $P(r, \bar{R})$ is the probability of portfolio return rate to differ from the average return rate.

The PMPT consider the investor emotional, real but rational decision to desire for a minimum acceptable return (or higher) and to accept the risk in consequence. The return higher than the minimum accepted does not imply any risk. The area considered with risk in PMPT is situated below the distribution function line and in front of M (the hashed areas in the graphics below):



a) M < $\overline{\mathrm{R}}$ **b)** M = $\overline{\mathrm{R}}$ **c)** $\overline{\mathrm{R}}$ < M

Figure 2. Risk implied by the position of M vs. $\overline{\mathbf{R}}$ (normal distribution)

In MPT the risk measured as variation of portfolio return from the average return is a characteristic of the portfolio with no relation with the investor desired return; while in PMPT the risk actually depends on the investor minimum desired return. In MPT the inferior probability is equal to superior probability, no matter the investors' wished return. While in PMPT the investors' minimum accepted return is the key for determining the probability of loss or gain.

For covering the probability that the portfolio return to be greater the minimum accepted return there is defined the superior (or upside) probability:

$$P_s(r, M) = \int_M^{\infty} f(r)dr$$

The downside and upside probability together with the probability that the portfolio return to be equal to the minimum accepted return is equal to 1.

$$P_i(r, M) + p_M(r = M) + P_s(r, M) = \int_{-\infty}^{M} f(r)dr + p_M(r = M) + \int_M^{\infty} f(r)dr = 1$$

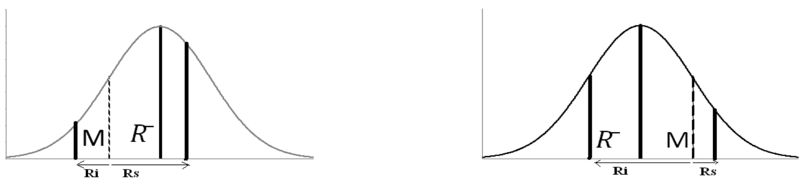
Potential return rate

The downside potential return rate is weighted distance from the return rate r to the minimum accepted return rate M. Because $r < M$, M being a positive value and weight of return rates are positive, then $R_i(r, M)$ have only negative values. As complement is the upside potential return rate, as the weighted return rate r upper to M. As r is higher than M, M being a positive values, then $R_s(r, M)$ is positive.

$$R_i(r, M) = \int_{-\infty}^{M} (r - M) * f(r)dr \ ; \ R_s(r, M) = \int_{M}^{\infty} (r - M) * f(r)dr$$

The two potential rates measure the distance from M to the potential return rate on both sides of minimum accepted return rate M; $R_i(r, M)$ to the left of M and $R_s(r, M)$ on the right side. The sum of the downside and upside potential return rates give distance from average return rate to minimum accepted return rate, and the position of the minimum accepted return to average return.

$$R_i(r, M) + R_s(r, M) = \int_{-\infty}^{\infty} rf(r)dr - M \int_{-\infty}^{\infty} f(r)dr = \bar{R} - M$$



a) $M < \bar{R}$ **b)** $\bar{R} < M$

Figure 3. Downside and upside potential return rates

Depending on the position of minimum accepted return rate M relative of the average return rate $\bar{R}$, the absolute value of the two potential return rates varies:

$$\begin{cases} abs[R_i(r, M)] < abs[R_s(r, M)], if\ M < \bar{R} \\ abs[R_i(r, M)] > abs[R_s(r, M)], if\ M > \bar{R} \end{cases}$$

For normal distribution function, in MPT the downside potential return rate is equal as absolute value with the upside potential return rate but with negative sign. We note $\bar{R}_i(r, \bar{R})$ as the downside potential return rate and $\bar{R}_s(r, \bar{R})$ as the upside potential return rate in MPT:

$$\bar{R}_i(r, \bar{R}) = \int_{-\infty}^{\bar{R}} (r - \bar{R}) * f(r)dr = -\bar{R}_s(r, \bar{R}) = -\int_{\bar{R}}^{\infty} (r - \bar{R}) * f(r)dr$$

In MPT both downside and upside potential return rate implies risk, while in PMPT only the downside potential return rate implies risk, the upside potential return rate is considered as a supplementary reward for the investor courage to put money on that portfolio.

The differences between the downside potential rate computed in the PMPT and MPT, as well as the difference between the two upside potential return rate is given by the position of minimum expected return rate M relative to average return rate $\overline{\mathrm{R}}$. We use the notation $abs[b]$ as the absolute value of number b.

For $M < \overline{\mathrm{R}}$ then $\overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}}) = \mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) + (M - \bar{R}) * P_i(r, M) + \Phi_{i<}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$ where $M < \bar{R}$ and $r < \bar{R}$, $(M - \bar{R}) < 0$, $\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) < 0$, $P_i(r, M) > 0$ and $\Phi_{i<}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) = \int_{\mathrm{M}}^{\overline{\mathrm{R}}} (\mathrm{r} - \overline{\mathrm{R}}) * \mathrm{f(r)dr} < 0$.

We can also estimate the difference between $\overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})$ and $\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M})$ if we consider the value of $\mathrm{M} \to \overline{\mathrm{R}}$, $M < \overline{\mathrm{R}}$. The difference of $\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) - \overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})$ is higher than zero, and because both downside potential rate are negative, then $\mathrm{abs}[\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M})] < abs[\overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})]$, while $\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) > \overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})$.

$$\lim_{\substack{\mathrm{M} \to \overline{\mathrm{R}} \\ M < \overline{\mathrm{R}}}} [\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) - \overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})] > 0$$

For $M > \overline{\mathrm{R}}$ then $\overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}}) = \mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) + (M - \bar{R})\bar{P}_i(r, \bar{R}) + \Phi_{i>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$ where $M > \bar{R}$ and $r > \bar{R}$, $(M - \bar{R}) > 0$, $\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M}) < 0$, $\bar{P}_i(r, \bar{R}) = \int_{-\infty}^{\bar{R}} f(r)dr$ and $\Phi_{i>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) = -\int_{\bar{R}}^{M} (r - M) * f(r)dr$ positive

The upside potential return rate computed on both MPT and PMPT has the following situation of $\mathrm{R}_{\mathrm{s}}(\mathrm{r}, \mathrm{M})$ versus $\overline{\mathrm{R}}_{\mathrm{s}}(\mathrm{r}, \overline{\mathrm{R}})$:

For $M < \overline{\mathrm{R}}$ then $R_s(r, M) = \overline{\mathrm{R}}_{\mathrm{s}}(\mathrm{r}, \overline{\mathrm{R}}) + (\overline{\mathrm{R}} - M) * \bar{P}_s(r, \overline{\mathrm{R}}) + \Phi_{s<}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$ where $\bar{P}_s(r, \overline{\mathrm{R}})$ and $\Phi_{s<}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) = \int_{M}^{\bar{R}} (r - M) * f(r)dr$ positive.

For $M > \overline{\mathrm{R}}$ then $\overline{\mathrm{R}}_{\mathrm{s}}(\mathrm{r}, \overline{\mathrm{R}}) = \mathrm{R}_{\mathrm{s}}(\mathrm{r}, \mathrm{M}) + (M - \overline{\mathrm{R}})P_s(r, M) + \Phi_{s>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$ where $\Phi_{s>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) = \int_{\overline{\mathrm{R}}}^{M} (r - \overline{\mathrm{R}}) * f(r)dr$ positive .

Resuming, Φ_i reflects the potential risk existing between the M selected by investor and the average return rate considered in MPT as benchmark. If Φ_i is negative it reflects the potential of risk reduced from benchmark $\overline{\mathrm{R}}$ in MPT to M; while a positive Φ_i reflects the potential suplimentary risk assumed by investor from choosing a M higher then $\overline{\mathrm{R}}$. Cases are presented synthetic below:

	$M < \overline{\mathrm{R}}$	$M > \overline{\mathrm{R}}$
$\mathrm{abs}[\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M})]\ vs.\ abs[\overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})]$	$\lvert R_i \rvert < \lvert \bar{R}_i \rvert$	$\lvert R_i \rvert > \lvert \bar{R}_i \rvert$
$\mathrm{R}_{\mathrm{i}}(\mathrm{r}, \mathrm{M})\ vs.\ \overline{\mathrm{R}}_{\mathrm{i}}(\mathrm{r}, \overline{\mathrm{R}})$	$\mathrm{R}_{\mathrm{i}} > \overline{\mathrm{R}}_{\mathrm{i}}$	$\mathrm{R}_{\mathrm{i}} < \overline{\mathrm{R}}_{\mathrm{i}}$
$\Phi_i(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$	-	+
$\mathrm{abs}[\mathrm{R}_s(\mathrm{r}, \mathrm{M})]\ vs.\ abs[\overline{\mathrm{R}}_s(\mathrm{r}, \overline{\mathrm{R}})]$	$\lvert \mathrm{R}_s \rvert > \lvert \overline{\mathrm{R}}_s \rvert$	$\lvert \mathrm{R}_s \rvert < \lvert \overline{\mathrm{R}}_s \rvert$
$\mathrm{R}_{\mathrm{s}}(\mathrm{r}, \mathrm{M})\ vs.\ \overline{\mathrm{R}}_{\mathrm{s}}(\mathrm{r}, \overline{\mathrm{R}})$	$\mathrm{R}_s > \overline{\mathrm{R}}_s$	$\mathrm{R}_s < \overline{\mathrm{R}}_s$
$\Phi_s(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$	+	+

Table 1. Potential inferior and superior return rate comparing position of M and $\overline{\mathbf{R}}$

It is obvious that post-modern portfolio theory, in comparison with the modern portfolio theory, covers better the risk measurement. Not only that there is no condition imposed regarding the distribution function, but the potential risk measured in PMPT distinguish between the risk of an investment and the premium for the investors' courage, taking into account the investor's preference for an expected return rate. While in MPT the risk is determine by anything that differs from a characteristic of the portfolio such as the average return and does not consider the investor return acceptance, the PMPT innovates by including the minimum accepted return rate as a characteristic of the investment policy.

Risk and variance in PMPT and MPT

The risk in PMPT is defined by the standard semi-deviation, having as benchmark the minimum accepted return rate requested by the investor in order to risk his money and perform the investment. The PMPT propose the risk as the chance of portfolio return to be lower than the minimum accepted return rate, in contrast to the MPT that consider risk every return varying from the average return rate of the portfolio.

The risk in PMPT is measured by the downside deviation, while the upside deviation gives the measure of the possibility that the portfolio return exceeds the minimum accepted return rate. While in MPT the risk is a characteristic intrinsic of the portfolio and is not influence by the investor appetite for risk, in PMPT the risk depends primarily on the investors will to accept the quantity of risk based on the portfolio return rate distribution and the minimum accepted return.

As a consequence of the fact that PMPT does not presume the bell-shape of the distribution function, the downside deviation is not necessarily equal to the upside deviation as in MPT. MPT is a particular case of PMPT imposing the normality of the distribution function, having in the middle the average return as central benchmark, then the downside deviation always equals the upside one.

The variance in PMPT is also downside and upside in relation with the investor minimum accepted return rate and is given by the following formulas:

$$V_i = \int_{-\infty}^{M} (r-M)^2 * f(r)dr \quad and \quad V_s = \int_{M}^{\infty} (r-M)^2 * f(r)dr$$

In MPT the variance reflects everything that differ from the average return rate, while the downside and upside ones are considered as semi-standard deviation, and equal one with the other. The semi-standard deviation was proposed in financial literature and is used in practice as an alternative of measuring risk of return rates to be inferior to the average expected return rate (Grootvelda, Hallerbachb, 1999).

$$\sigma^2 = \frac{1}{n-1}\sum_{r=1}^{n}(r-\bar{R})^2$$

$$s\sigma_i{}^2 = \int_{-\infty}^{\bar{R}} (r-\bar{R})^2 * f(r)dr = s\sigma_s{}^2 = \int_{\bar{R}}^{\infty} (r-\bar{R})^2 * f(r)dr$$

Further, we compared the downside deviation computed according to the PMPT in consideration of risk with the standard deviation resulted from MPT. First, for comparing the risk measured according to the MPT and that resulting from PMPT, we compared the MPT variance σ^2 with the inferior variation V_i as in PMPT; also considering the superior variance V_s from PMPT in order to be comparable measures.

$$\sigma^2 = V_i + V_s - (M-\bar{R})^2$$

The risk in PMPT is given by downside deviation; there is also an upside deviation as premium for the courage of performing the investment:

$$sV_i = \left(\int_{-\infty}^{M} (r-M)^2 * f(r)dr\right)^{\frac{1}{2}} \quad and \quad sV_s = \left(\int_{M}^{\infty} (r-M)^2 * f(r)dr\right)^{\frac{1}{2}}$$

For comparison, in MPT there is calculated the semi-standard deviation, both inferior and superior, being equal; while the risk is given by standard deviation.

$$\sigma = \left(\frac{1}{n-1}\sum_{r=1}^{n} (r-\bar{R})^2\right)^{\frac{1}{2}} = [V_i + V_s - (M-\bar{R})^2]^{\frac{1}{2}}$$

$$s\sigma_i = \left(\int_{-\infty}^{\bar{R}} (r-\bar{R})^2 * f(r)dr\right)^{\frac{1}{2}} = s\sigma_s = \left(\int_{\bar{R}}^{\infty} (r-\bar{R})^2 * f(r)dr\right)^{\frac{1}{2}}$$

For $M < \bar{R}$, the downside risk is :

$$s\sigma_i = \left[V_i + 2(M-\bar{R}) * \mathrm{R_i(r,M)} + (M-\bar{R})^2 * P_i(r,M) + \Phi^2_{\mathrm{i<}}(\mathrm{r,M,\bar{R}})\right]^{\frac{1}{2}}$$

where $s\sigma_i > sV_i$ and $\Phi^2_{\mathrm{i<}}(\mathrm{r,M,\bar{R}}) = \int_{M}^{\bar{R}} (r-\bar{R})^2 f(r)dr$ positive, representing the risk between minimum accepted return rate M and average return rate, risk reduced in PMPT method of computing risk because the investor choose M lower than the average return rate that is the benchmark in MPT, regardless of investor's will.

For case $< \bar{R}$, the upside risk represents the premium obtained by investor from performing the investment

$$sV_s = [\Phi^2_{\mathrm{s<}}(\mathrm{r,M,\bar{R}}) + \sigma_s + 2(\bar{R}-M) * \mathrm{\bar{R}_s(r,\bar{R})} + (\bar{R}-M)^2 * P(r,\bar{R})]^{\frac{1}{2}}$$

where $s\sigma_s < sV_s$ and $\Phi^2_{\mathrm{s<}}(\mathrm{r,M,\bar{R}}) = \int_{M}^{\bar{R}} (r-M)^2 * f(r)dr$ positive representing the suplimentary premium obatined by investor from the position of M inferior to the average return rate.

In case $M > \bar{R}$ the downside risk is higher than the risk involved by the average return rate:

$$sV_i = \left(\sigma_i + (\bar{R}-M)^2 * P(r,\bar{R}) + 2(\bar{R}-M) * \mathrm{\bar{R}_i(r,\bar{R})} + \Phi^2_{\mathrm{i>}}(\mathrm{r,M,\bar{R}})\right)^{\frac{1}{2}}$$

$$s\sigma_i = \left\{V_i - \left[(\bar{R}-M)^2 * P(r,\bar{R}) + 2(\bar{R}-M) * \mathrm{\bar{R}_i(r,\bar{R})} + \Phi^2_{\mathrm{i>}}(\mathrm{r,M,\bar{R}})\right]\right\}^{\frac{1}{2}}$$

where $s\sigma_i < sV_i$ and $\Phi^2_{i>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) = \int_{\bar{R}}^{M} (r - M)^2 f(r)dr$ positive representing the supplementary risk implied by choosing M higher than the average rate

The upside risk representing the premium obtained by investor for performing the investment is lower than in case of benchmark average return rate because M has a higher value the $\bar{R}$:

$$s\sigma_s = [\Phi^2_{s>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) + V_s + 2(M - \bar{R}) * \mathrm{R_s}(\mathrm{r}, \mathrm{M}) + (M - \bar{R})^2 * P_s(r, M)]^{\frac{1}{2}}$$

where $s\sigma_s > sV_s$ and $\Phi^2_{s>}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}}) = \int_{\bar{R}}^{M} (r - \bar{R})^2 * f(r)dr$ positive representing the premium lost by investor because he choose an accepted return rate higher than the average return rate.

	$M < \overline{\mathrm{R}}$	$M > \overline{\mathrm{R}}$
$s\sigma_i\ vs.\ sV_i$	$s\sigma_i > sV_i$	$s\sigma_i < sV_i$
$\Phi^2_{\mathrm{i}}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$	reducing risk	increasing risk
$s\sigma_s\ vs.\ sV_s$	$s\sigma_s < sV_s$	$s\sigma_s > sV_s$
$\Phi^2_{\mathrm{s}}(\mathrm{r}, \mathrm{M}, \overline{\mathrm{R}})$	increasing premium	reducing premium

Table 2. Downside and upside risk resulting from position of minimum accepted return rate M in compare with average return rate $\overline{\mathbf{R}}$

Research methodology for data analyses

In this chapter we used database containing trades of shares listed on Bucharest Stock Exchange (Romanian name is Bursa de Valori Bucuresti, abbreviation BVB) over the period between years 2005 and 2012. The selected period for analyze include several years before the financial crises and several years during the financial crises. This period allows us to understand also the impact of the financial crises over the evolution of measuring risk.

Data selection

The selected period starts in January 2005 and ends in June 2012, included. In this period there are 1780 days when the Bucharest Stock Exchange was opened for trading. For this period there are 80 companies that had their shares traded.

During this period not all companies were listed for 1780 days and several companies were listed during this period. We applied several criteria of filtering the database. We select companies that had at least 500 days of quotation, corresponding to more than two years of trading – considering that a year has around 250 trading days.

Because Bucharest Stock Exchange has shares with liquidity problems (several consecutive days without trading) that may influence over the correct evaluation of shares price and induce large correction and variation over shares price (Geambasu, Stancu, 2010), we put another selection criterion – we kept only shares that were trade more that 75% of the possible trading days in an year for the period the company was listed. This criterion is not applicable to the first year of listing a company on BVB; the number of trading days for the company in the first year depends on the day in the year when the company is listed.

We have to consider another criterion, in order to insure the relevance of data to present – there were selected only shares that have quotation in more than 80% of the possible trading days in the last three years.

The filtered database remained with half of the initial 80 companies; only 40 companies passed the selection criteria applied. These 40 companies were in general quoted in all the days of quotation from the selected period.

Methodology applied on the selected database

Data selected cover the period between January 2005 and June 2012. The database include daily closing quotation from the 40 companies remained after the selection criteria were applied. In order to add relevance to analyze, we included several portfolios. First we included the BVB indexes portfolios – indexes BET and BET-C (the main indexes of BVB). We also created portfolios from giving each share random weights.

$$Portfolio = \sum_{i=1}^{40} rand_i * Rshare_i \qquad where\ i\ represents\ the\ 40\ shares$$

and $rand_i$ is the random weight of share i included in the portfolio, $rand_i \in [0; 1]$ and $\sum_{i=1}^{40} rand_i = 1$. $Rshare_i$ is the return rate of share i included in the portfolio.

For the closing quotation of each share and portfolio we calculated the risk elements discussed in the previous chapter using windows of data. The windows were enlarged from 200 to 1000 daily of quotation. Each window was moved from the first quotation of the company on BVB to the last, and for each position of the window were calculated the risk elements.

We used the following notation to identify the inferior probability computed for a specific company, numbers of days included in the window and first date from the window computed:

$$P_i^{c,w,d,p}(r, M) = \int_{-\infty}^{M} f(r)dr$$

where c is the company (from the 40 companies included in the analyze), w is the window length (takes values from 200 to 1000 – number of quotation days included in the window), d – the first quotation day of the window. The windows cover from 10 month to 4 years , increasing progressively. This way ensured us that we covered the influence of two aspects with possible influence over the results of data computation: the influence of the number of days included in the window and the financial crises influence. For determining the influence of minimum expected return M, for each interval of computing data we tested various position of M. In order to determine the position of M on the distribution, we used p as percent that takes values from 1% to 99%; M is computed as:

$$M = Min + p * (Max - Min) \qquad where\ p \in [1\%; 99\%]$$

where Min is the minimum value from the interval, Max is the maximum value from the interval. Using this procedure we moved M through the distribution of returns rate from Min to Max.

For each company there were computed about 800 windows types. Depending on the date of the first quotation and on the window number of days included, each window type, for each company we had several hundred values. For example, for a company that was first quoted before January 2005 and was active over the entire analyzed period and had quotations in all days of the analyses period, the window type with 200 days included has over 1500 computed data, the first interval is from January 03rd, 2005 to October 14th, 2005, while the last interval is from September 21st, 2011 to June 29th, 2012. For the same company, for the window type with 1000 days included, the first interval is between January 03rd, 2005 and January 12th, 2009, while the last interval is between February 15th, 2008 and June 29th, 2012 – there are over 775 windows created with 1000 days included.

Results from processing data

The first analyze was performed for determining the inferior probability of returns that generate the risk: for MPT the risk is generate by every return that is different from the average return, while for PMPT the risk is generate only by those returns that are less than the minimum expected return M. The inferior probability $P_i^{c,w,d,p}$ is almost equal to 0.5 in MPT, while in PMPT it depends on the position of M (determined by p) in the distribution of returns.

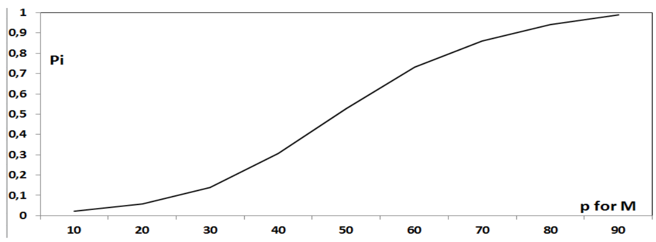


Figure 4. Inferior probability determined by position of M

The position of M directly influenced the inferior probability of returns situated on the left of the minimum accepted return – there is a direct linear relation between the two elements p and $P_i^{c,w,d,p}(r,M)$, with an R-squared of 97%,; $P_i^{c,w,d,p}(r,M)$ is highly influenced by the position of M on the distribution series. PMPT offers better results in measuring the inferior probability of returns below the minimum return expected by the investor.

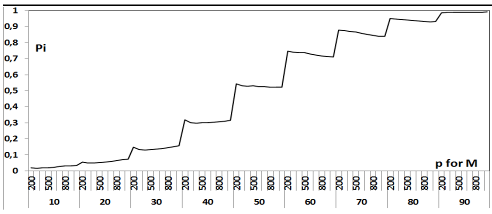


Figure 5. Inferior probability determined by position of M and window

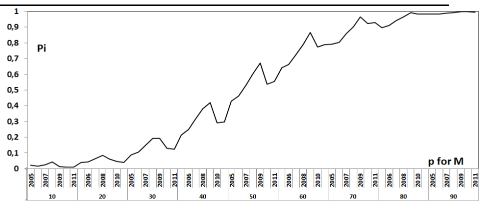


Figure 6. Inferior probability by position of M and period of window

The analyze of the windows created with various values for p, and the windows created within the same p reveals that the proportion of returns on the left of the minimum accepted return is almost the same but more stable with the increase of the number of days included in the computation windows.

In order to compare values of $P_i^{c,w,d,p}(r, M)$ with the inferior probability measured for average return, we used differential term $\Delta_i^{w,c,d,p}$. For example, in order to analyse the influence of number of days included in the windows, we noted $\Delta_i^{w(w1,w2)}$ as the differential term that measure the influence of variation of window, w taking values w1 and w2.

$$\Delta_i^{w(w1,w2),c,d,p} = \frac{P_i^{c,w1,d,p}(r,M)}{P_i^{c,w2,d,p}(r,M)} \quad where\ w1, w2\ \in W\ (possible\ values\ of\ w)$$

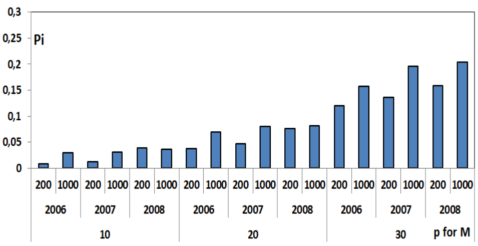


Figure 7. Inferior probability by position of M, window weight and period included in window

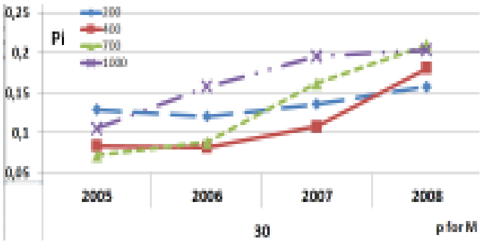


Figure 8. Inferior probability for position of M (p=30%), window weight and period included in window

In the same way we used differential terms for measure the influence of each element: Δ_i^d for measuring the movement over time (the single element varying is the first day of the window, the rest of the conditions remaining unchanged), Δ_i^c for measuring the influence of the company and Δ_i^p for measuring the influence of p – position of M. The most interesting elements are d and w, they show the influence of the number of days included in the interval and the evolution in time including the impact of financial crises.

We considered year 2005 as benchmark for the next years. In order to have data for all intervals, we use intervals that start in 2008 at most (for windows with 1000 days included in the interval analyzed, there were no intervals starting later than year 2008, the limit of database is June 2012 – the windows of 1000 days that end in June 2012 starts in 2008). We put in table below only position of M with p = 10%, 20% and 30% but the conclusion are available for all position of M – for p=90% the majority of the returns are on the left of M, while for lower position of M the influence of financial crises is more evident.

	position of M : p=10%			position of M : p=20%			position of M : p=30%		
days in window	2006	2007	2008	2006	2007	2008	2006	2007	2008
200	0.389	0.556	1.778	0.695	0.869	1.432	0.933	1.052	1.223
300	0.296	0.526	2.777	0.664	0.876	2.189	0.946	1.053	1.688
400	0.320	1.082	2.907	0.684	1.425	2.629	0.977	1.286	2.151
500	0.279	1.759	2.908	0.642	2.058	2.871	0.949	1.522	2.579
600	0.686	1.965	2.844	1.057	2.452	2.958	1.091	1.994	2.813
700	1.247	1.874	2.710	1.539	2.455	2.893	1.226	2.245	2.907
800	1.676	1.719	2.479	1.869	2.275	2.525	1.552	2.437	2.847
900	1.151	1.173	1.570	1.297	1.527	1.659	1.481	2.063	2.326
1000	0.854	0.868	1.022	1.018	1.155	1.182	1.487	1.847	1.922

Table 3. Inferior probability for p=10%, 20%, 30% and year 2005 as benchmark

Analyzing the evolution of $P_i^{c,w,d,p}(r,M)$ over time, there is evidence that the financial crises manifested starting with year 2008 affect the evolution – there is an increase in probability that the return is less than the minimum expected return M. In Figure 7 we selected only the windows of 200 and 1000 days in order to highlight the increase probability to have negative returns in years of financial crises – the data not presented in the graphic have the same evolution. This feature reflects another advantage of using the risk measurement proposed by PMPT – it clearly show the evolution of the risk based on market evolution versus the minimum accepted return, emphasizing the probability to have negative results from the investment performed. For contrary, the MPT reflects the market evolution in diminish of the average return and not on the increase of inferior probability of return rate.

As presented in the previous chapter, the risk in MPT is considered to be measured by the standard deviation of the data series relative to the average return. Because the standard deviation consider the variation of both higher and lower returns from the average return, sometimes there is used the semi-standard deviation. Semi-standard deviation measure the variation from the average return of all the returns situated on one side of the benchmark, we used it to measure the variation of returns inferior to average return. Both standard deviation and semi-standard variation depend only on the distribution of the returns and on the average return, and do not count for the minimum expected return of the investor.

We used the same methodology as for inferior probability, and note $sV_i^{c,w,d,p}(r,M)$ as the inferior risk of company c, considering a window w, the first date included in the windows as d, and p is the position of M on the distribution of return rates:

$$sV_i^{c,w,d,p}(r,M) = \left(\int_{-\infty}^{M}(r-M)^2 * f(r)dr\right)^{\frac{1}{2}}$$

In the Figure 9, the standard deviation and the semi-standard deviation are the same no matter the minimum expected return of the investor. On the other hand, the risk measured according to PMPT (continuous line on the graphic) increases as the M moves to right – movement to right of M implies that more returns rate are lower than M, so the risk is higher when p increase. There is a direct influence between increase in p, the appetite for risk (a higher return rate implies a higher risk) and the increase of risk (black line is the evolution of downside risk and red line is the evolution of semi-standard variation).

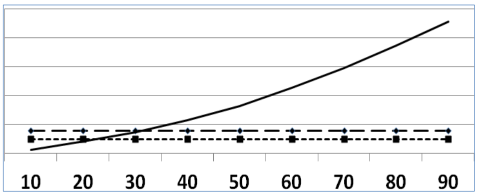


Figure 9. Evolution of downside risk standard deviation and semi-deviation

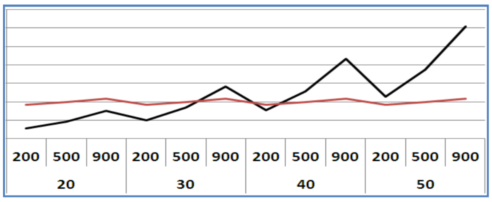


Figure 10. Evolution of downside risk considering the window wide

In order to compare values of $\mathrm{sV_i^{c,w,d,p}(r,M)}$ obtained in various combination of the criteria applied – windows wide, first day of the window and position of M on the distribution series; we used a differential term to measure each element influence. For example, in order to analyze the influence of number of days included in the windows, we noted $\Delta_i^{2\,w(w1,w2)}$ as the differential term that measure the influence of variation of element w, w taking values w1 and w2.

$$\Delta_i^{2\,w(w1,w2),c,d,p} = \frac{sV_i^{c,w1,d,p}(r,M)}{sV_i^{c,w2,d,p}(r,M)} \quad where\ w1,w2\ \in W$$

In the same way we used differential terms for measure the influence of each element: $\Delta_i^{2\,d}$ for measuring the movement of the windows, $\Delta_i^{2\,c}$ for measuring the influence of the company returns and $\Delta_i^{2\,p}$ for measuring the influence of position of M. The most interesting elements are d and w, they show the influence of the number of days included in the interval and the evolution in time considering the great impact of financial crises (black line is the evolution of downside risk and red line is the evolution of semi-standard variation).

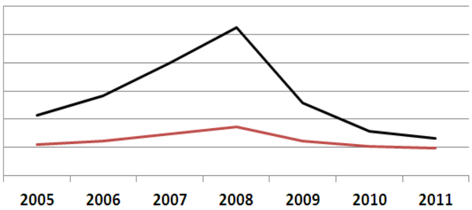


Figure 11. Evolution of downside risk over time (influence of d)

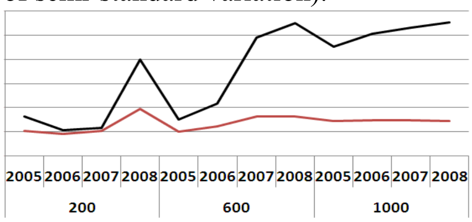


Figure 12. Evolution of downside risk considering variation of w and d

As the number of days included in the window is increasing, so is the risk, resulting that the investor is exposed to a higher risk when the period of investment is longer, including also the financial crises period.

Considering the risk measured through MPT and the risk measured through PMPT, we noted with $\Lambda_i^{w,c,d,p}$ the difference between the two values of risk:

$$\Lambda_i^{w,c,d,p} = \frac{sV_i^{c,w,d,p}(r,M)}{\sigma^{c,w,d,p}(r,\bar{R})}$$

	position of M : p=10%			position of M : p=20%			position of M : p=30%		
days in window	2006	2007	2008	2006	2007	2008	2006	2007	2008
200	0.070	0.067	0.231	0.157	0.154	0.567	0.278	0.280	0.933
300	0.064	0.067	0.298	0.143	0.193	0.751	0.252	0.358	1.242
400	0.061	0.098	0.296	0.134	0.410	0.780	0.237	0.767	1.310
500	0.060	0.122	0.302	0.133	0.586	0.815	0.237	1.100	1.387
600	0.081	0.134	0.315	0.318	0.660	0.851	0.594	1.244	1.454
700	0.111	0.137	0.326	0.529	0.680	0.884	0.992	1.292	1.517
800	0.139	0.140	0.336	0.699	0.702	0.912	1.314	1.343	1.568
900	0.140	0.143	0.323	0.710	0.721	0.904	1.343	1.388	1.574
1000	0.142	0.145	0.306	0.726	0.739	0.893	1.385	1.428	1.578

Table 4. downside risk for p=10%, 20%, 30% and year 2005 as benchmark

where c, w, d, p have the same meaning as in the previous formulas. This indicator reflects the power of $sV_i^{c,w,d,p}(r,M)$ to better measure the risk in comparison with $\sigma^{c,w,d,p}(r,\bar{R})$. For position of M to the left of the distribution seria, the risk measured by MPT is over evaluated; while on the other hand the risk is highly diminish if the minimum accepted return is moving to right of the distribution.

Discussion

As a conclusion resulting from the comparison of the two methods of measuring risk – modern portfolio theory and post modern portfolio theory – there are clear evidences that PMPT offers a better measure of risk, more flexible and adapted to the investment process reality. Investors have their own minimum accepted return limit, that constitute a benchmark in measuring the success or the failure of the investment process, while the average return rate is just a characteristic of the distribution return rates and it is usually not the investor's desirable return rate. The PMPT covers the lack of flexibility and adaptive of the MPT, by measuring the risk with consideration to the investor's requirements.

The PMPT is more flexible and has a more general application in measuring risk than MPT that limits the reality and force the distribution of return rates to be assimilated with a normal distribution. Investors usually consider risk only the return rates lower than the minimum accepted return rate they required in order to perform the investment. Risk measured according to PMPT depends on the position of the investor's minimum expected return rate (M) on the return rates distribution, with distinguish between the real risk and the premium risk of the investment performed. When the value of M required by investor increase so does the risk, because many return rates became lower than the benchmark.

Although things might seem simple with applying the PMPT, there is not the case. It is simple to compute for a share or for a portfolio already formed, for historical or predictive data, but things became more complicated if we intend to use the PMPT model in determining the portfolio assets structure. Even that this aspect does not concern directly the subject of the present article, we consider including a comment about this topic as it can be useful for further development and researches. There are attempts to include risk determined through PMPT in Markowitz model in order to determine the portfolio structure (Bawa, Lindenberg, 1977, Sing, Ong, 2000), but including the downside risk in measuring the risk of each asset composing the portfolio might be problematic. The minimum accepted return rate imposed by the investor is for the entire portfolio (the scope of investing in a portfolio is to reduce the global risk), not for each share – the shares risk included in the model must be an intrinsic characteristic of the asset. From our point of view this aspect needs further research.

Limitation

The empirical results of the present article are limited to the data and to the period included in analyses. Applying the models to other sets of data, other periods and other stock exchanges will generate different empirical results. The theoretical aspects presented in this article remain available no matter the periods and data sets involved, and are supported by both the theoretical aspects included in the models analyzed and the previous articles studied.

Another limitation of the empirical results is generated by the fact that we use historical set of data, no estimative data related to future evolution were available and certified for a pertinent analysis. The issue of data used in PMPT analyses generated a large debate in the scientific literature between two ideas. Although both parts accepted the theoretical aspects of PMPT, they do not agree over the set of data to be used. The debate has representation in the opinions of Kaplan, Siegel (1994) that sustain the use of historical data for understanding the past and prefiguring the future and Rom, Ferguson (1993), a business point of view, that intent to use predictive data about future evolution in order to determine the correct decisions to be taken for future investments. The debate did not end in a concluding and general accepted point of view and the lack of certified, general accepted predictive set of data do not allow us to apply the theories on such data.

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