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A NOVEL METHOD FOR PREDICTION OF OPTION PRICING FOR A MARKET MODEL

Abstract. *Using the principle for physical probability of price process, an actuarial approach to the option pricing problem is proposed based on the results of Mogens Bladt and Hina Hviid Rydberg. In particular, the Black-Schole (B-S) model is generalized to the case where the interest rates are stochastic and the stock prices. In such case, the accurate pricing formula and put-call parity of European option are obtained. Based on such formula, the European call-put parity relation is derived naturally. Furthermore, the new prices of European call option and the put option with continuous dividend yield are deduced from the above results. The accurate pricing formula of European option on a stock is given by actuarial approach.*

Key words: *market model; B-S model.*

JELL Classification: C12, C18, C43

1. Introduction

The pricing has been one of most important issues in the field of financial mathematics research. It is the foundation of the financial risk management to price the contingent claim correctly which is also the essential component of the modern finance. In 1973, Fisher Black and Myron Scholes published a classic paper on the theory of option pricing, where they derived the well-known Black-Scholes (B-S) formula by arbitrage reasoning and stochastic analysis and established the option pricing theory. It is a revolutionary achievement to financial theory. However, the B-S pricing model was based on many assumptions, For example, the riskless interest rate is known and constant over time, the evolution of the asset price is assumed to follow the geometric Brownian Motion, the asset pays no dividend, etc. These and other assumptions are not well compatible with the real finance market, so many scholars made a lot of improvements and modifications of the B-S model and the formula. Cox, Ross and Rubinstein (1979) put forward a binomial pricing model; Merton(1973) employed the arbitrage theory to prove the B-S model and obtained a simple solution to optimal consumption and investment decision under continuous-time model by dynamic programming methods; Duffie applied the traditional method of option pricing to deduce the B-S formula.

The pricing methods mentioned above, such as martingale method, partial differential method and discrete model approximation, are all built on the basis that the financial markets are complete and arbitrage-free, then the solutions are obtained by replication strategy. If the market is incomplete and not arbitrage-free, the performance of these methods will be suspicious, as the given contingent claim can not be replicated and hedged accurately in an incomplete market. The common practice is to search for a family of ultimate wealth by self-financing strategy to approach some claim, but, with no doubt, there exist errors. Follmer and Sondermann(1986) put forward the "mean-variance" criteria to measure this error, but the solving process was extremely complex.

In 1998, Blat and Rydberg proposed the actuarial approach to option pricing which converted the problem into determining the equivalent of the fair premium. There are no economic assumptions in the actuarial approach, so the approach is applicable not only to the arbitrage-free, equilibrium, complete markets, but also to the arbitrage, non-equilibrium, incomplete markets. Bladt and Rydberg (1998) have completely proved that the option price is consistent with the B-S formula in the continuous time. Yan and Liu(2003) got the accurate pricing formula of European option where the evolution of the stock price follows Ornstein-Uhlenback(O-U) process by actuarial approach; Zhao and He(2007) studied an option pricing model given by a fractional

O-U process in the risk-neutral market. The stock price driven by O-U process avoids that it varies in the same direction in traditional lognormal distribution, and the upward trend of the stock price is weakened. However, it is not sufficient to meet the actual background that the functions of the interest rates are determinate in the above-mentioned models. As many empirical studies have shown, in the actual financial market the interest rates are mean reverting, and the volatility of the long-term interest rates are less than the volatility of short-term interest rates; the volatility will be higher when the interest rates are in a higher level[13,14]. Liu, Deng and Yang (2007) discussed the price of the reload option in the financial market where the interest rates are driven by Hull-White model with martingale and stochastic analysis approach.

In this paper, we consider the models in the continuous time and apply the actuarial approach to price the general European option and the exchange option. To understand the role of the actuarial approach and the stochastic interest rates, we conduct a comparative analysis of numerical simulation and an empirical analysis between the B-S model and the actuarial model, using the actual data in Chinese stock market.

2. Principle

Let us consider a frictionless financial market in continuous time. Assume that there are two assets in the market. One is the riskless asset, the bond; the other is the risky asset supposed to be the stock. Give a complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ where the filtration satisfies the usual conditions.

The evolution of the stock price S(t) is assumed to follow the generalized Exp-Ornstein-Uhlenback process:

$$dS(t) = (\mu(t) - \alpha \ln S(t))S(t)dt + \sigma_s(t)S(t)dB(t), S(0) = S, \tag{1}$$

where $S > 0$, $\sigma_s(t)$ is the volatility. Both $\mu(t)$ and $\sigma_s(t)$ are assumed to be some given functions of t, and α is constant.

Assume that the short-term interest rates satisfy the Hull-White model:

$$dr(t) = (a(t) - b(t)r(t))dt + \sigma_r(t)dW(t), r(0) = r, \quad (2)$$

where $a(t)$, $b(t)$ and $\sigma_r(t)$ are some given functions of t. $a(t)$ describes the long-term average level of interest rates while $b(t)$ is the average response rate adjusting the relationship between short-term and long-term interest rates, (2) becomes Vasicek model when they are both constant.

$\{B(t): t \geq 0\}$ and $\{W(t): t \geq 0\}$ are two standard Brownian Motion defined on probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with the related coefficient ρ.

Definition 2.1[4]. The expected rate of return of the stock price process $\{S(t), 0 \leq t \leq T\}$ is defined to $\int_0^T \beta(t)dt$ as follows:

$$e^{\int_0^T \beta(t)dt} = \frac{ES(T)}{S}, \quad (3)$$

Definition 2.2[4] The value of the European option by actuarial approach is defined to the expectation of the difference between the discount value of the stock price on maturity and the strike price on probability measure of the actual distribution of the stock price if the option is exercised. The riskless asset is discounted by the riskless interest rate while the risky asset is discounted by the expected rate of return as defined in (3). The necessary and sufficient condition to execute of the European call option and the put option on the expiration date is, respectively,

$$\exp\{-\int_0^T \beta(t)dt\}S(T) > \exp\{-\int_0^T r(t)dt\}K, \exp\{-\int_0^T \beta(t)dt\}S(T) < \exp\{-\int_0^T r(t)dt\}K.$$

Let $C(K,T)$ denotes the European call option and $P(K,T)$ denotes the European put option whose stock price is $S(t)$, the strike price is K and the expiration date is T. Then the value of the European option is defined by actuarial approach as follows:

$$C(K,T)=E\left[(\exp\{-\int_0^T \beta(t)dt\}S(T)-\exp\{-\int_0^T r(t)dt\}K)1_{\{\exp\{-\int_0^T \beta(t)dt\}S(T)>\exp\{-\int_0^T r(t)dt\}K\}}\right],$$

$$P(K,T)=E\left[(\exp\{-\int_0^T r(t)dt\}K-\exp\{-\int_0^T \beta(t)dt\}S(T))1_{\{\exp\{-\int_0^T \beta(t)dt\}S(T)<\exp\{-\int_0^T r(t)dt\}K\}}\right].$$

3. Model

Lemma 3.1[12] If the random variables W_1 and W_2 are both standard normally distributed with mean 0 and variance 1, and their covariance $Cov(W_1,W_2)=\rho,$ then to any real number $a,b,c,d,k,$ we have

$$E\left[e^{cW_1+dW_2}1_{\{aW_1+bW_2\geq k\}}\right]=e^{\frac{1}{2}(c^2+d^2+2\rho cd)}N(\frac{ac+bd+\rho(ad+bc)-k}{\sqrt{a^2+b^2+2\rho ab}}).$$

Lemma 3.2 If the stock price $S(t)$ is driven by generalized Exp-O-U process(1), then

$$S(t)=S^{e^{-\alpha t}}\exp\left\{e^{-\alpha t}\int_0^t(\mu(t)-\frac{1}{2}\sigma_s^2(u))e^{\alpha u}du+e^{-\alpha t}\int_0^t\sigma_s(u)e^{\alpha u}dB(u)\right\},$$

$$ES(t)=S^{e^{-\alpha t}}\exp\left\{e^{-\alpha t}\int_0^t(\mu(t)-\frac{1}{2}\sigma_s^2(u))e^{\alpha u}du+\frac{1}{2}e^{-2\alpha t}\int_0^t\sigma_s^2(u)e^{2\alpha u}dB(u)\right\}.$$

Proof For *Itǒ* formula, we have

$$d\ln S(t)=\frac{1}{S(t)}dS(t)-\frac{1}{2S^2(t)}dS(t)$$

$$=(\mu(t)-\alpha\ln S(t)-\frac{1}{2}\sigma_s^2(t))dt+\sigma_s(t)dB(t),$$

$$d(\ln S(t)e^{\alpha t})=(\mu(t)-\frac{1}{2}\sigma_s^2(t))e^{\alpha t}dt+\sigma_s(t)e^{\alpha t}dB(t),$$

so $$\ln S(t)e^{\alpha t}=\ln S+\int_0^t(\mu(u)-\frac{1}{2}\sigma_s^2(u))e^{\alpha u}du+\int_0^t\sigma_s(u)e^{\alpha u}dB(u),$$

then $$S(t)=S^{e^{-\alpha t}}\exp\left\{e^{-\alpha t}\int_0^t(\mu(t)-\frac{1}{2}\sigma_s^2(u))e^{\alpha u}du+e^{-\alpha t}\int_0^t\sigma_s(u)e^{\alpha u}dB(u)\right\}.$$

Taking expectations on both sides, we obtain

$$ES(t)=S^{e^{-\alpha t}}\exp\left\{e^{-\alpha t}\int_0^t(\mu(t)-\frac{1}{2}\sigma_s^2(u))e^{\alpha u}du+\frac{1}{2}e^{-2\alpha t}\int_0^t\sigma_s^2(u)e^{2\alpha u}dB(u)\right\}.$$

Now, we consider two European options, the call and the put. Their underlying assets both are stock while the strike price is K, the exercise date is T.

Theorem 3.1 Suppose the stock process $\{S(t), t \geq 0\}$ is driven by generalized Exp-O-U process (1) and the short-term interest rate satisfies the Hull-White model (2）, then respectively the value of the European call and put option at the time 0 is

$$C(K,T) = SN(d_1) - K\exp\left\{\frac{1}{2}\sigma_X^2 - G(0,T)\right\}N(d_2),$$

$$P(K,T) = K\exp\left\{\frac{1}{2}\sigma_X^2 - G(0,T)\right\}N(-d_2) - SN(-d_1),$$

where

$$n(t) = \int_0^t b(t)dt, \qquad m(t,T) = \int_t^T e^{n(t)-n(s)}ds,$$

$$G(0,T) = rm(0,T) + \int_0^T a(t)m(t,T)dt,$$

$$X = \int_0^T \sigma_r(t)m(t,T)dW(t), \qquad Y = e^{-\alpha T}\int_0^T \sigma_s(t)e^{\alpha t}dB(t),$$

$$\sigma_X^2 = \int_0^T \sigma_r^2(t)m^2(t,T)dt, \qquad \sigma_Y^2 = e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt,$$

$$d_1 = \frac{\ln\frac{S}{K} + G(0,T) + \frac{1}{2}\sigma_Y^2 + \rho\sigma_X\sigma_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}}, \qquad d_2 = \frac{\ln\frac{S}{K} + G(0,T) - \sigma_X^2 - \frac{1}{2}\sigma_Y^2 + \rho\sigma_X\sigma_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}}.$$

Proof For definition 2. 2, we have

$$C(K,T) = E\left[S(T)\exp\{-\int_0^T \beta(t)dt\}\cdot 1_{\left\{S(T)\exp\{-\int_0^T \beta(t)dt\} > K\exp\{-\int_0^T r(t)dt\}\right\}}\right] -$$

$$E\left[K\exp\{-\int_0^T r(t)dt\}\cdot 1_{\left\{S(T)\exp\{-\int_0^T \beta(t)dt\} > K\exp\{-\int_0^T r(t)dt\}\right\}}\right]$$

$$= I_1 - I_2$$

For the expected rate of return in actuarial approach satisfies $e^{\int_0^T \beta(t)dt} = \frac{ES(T)}{S}$, and the lemma 3.2 we obtain:

$$\int_0^T \beta(t)dt = \ln\frac{ES(T)}{S} = (e^{-\alpha T}-1)\ln S + e^{-\alpha T}\int_0^T (\mu(t)-\frac{1}{2}\sigma_s^2(t))e^{\alpha t}dt + \frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt,$$

so

$$e^{-\int_0^T \beta(t)dt} = \exp\left\{(1-e^{-\alpha T})\ln S - e^{-\alpha T}\int_0^T (\mu(t)-\frac{1}{2}\sigma_s^2(t))e^{\alpha t}dt - \frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s(t)e^{2\alpha t}dt\right\},$$

and $e^{-\int_0^T \beta(t)dt}S(T) = \exp\left\{\ln S - \frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt + e^{-\alpha T}\int_0^T \sigma_s(t)e^{\alpha t}dB(t)\right\}.$

By the interest rate model (2) and $It\breve{o}$ formula, we have [10]

$$\int_0^T r(t)dt = G(0,T) + \int_0^T \sigma_r(t)m(t,T)dW(t).$$

If we want to meet the condition, $S(T)\exp\{-\int_0^T \beta(t)dt\} > K\exp\{-\int_0^T r(t)dt\}$, then

$$\Leftrightarrow \exp\left\{\ln S - \frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt + e^{-\alpha T}\int_0^T \sigma_s(t)e^{\alpha t}dB(t)\right\} > K\cdot$$

$$\exp\ \ G(0,T) + \int_0^T \sigma_r(t)m(t,T)dW(t)\ \ ,$$

$$\Leftrightarrow \int_0^T \sigma_r(t)m(t,T)dW(t) + e^{-\alpha T}\int_0^T \sigma_s(t)e^{\alpha t}dB(t) > \ln\frac{K}{S} - G(0,T) +$$

$$\frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt,$$

$$\Leftrightarrow X + Y > \ln\frac{K}{S} - G(0,T) + \frac{1}{2}\sigma_Y^2.$$

$\{B(t): t \geq 0\}$ and $\{W(t): t \geq 0\}$ are two standard Brownian Motion defined on probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ so according to the definition, we obtain that X, Y are two independent normal random variable to F_t, and $E^Q[X] = E^Q[Y] = 0,\ \sigma_X^2 = \int_0^T \sigma_r^2(t)m^2(t,T)dt,\ \sigma_Y^2 = e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt,$ so with lemma 3.1, we have:

$$I_1 = E\left[S\exp\left\{ -\frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt + Y \right\}\cdot 1_{\left\{X+Y>\ln\frac{K}{S}-G(0,T)+\frac{1}{2}\sigma_Y^2\right\}} \right]$$

$$= S\exp\left\{ -\frac{1}{2}e^{-2\alpha T}\int_0^T \sigma_s^2(t)e^{2\alpha t}dt \right\}\cdot E\left[e^Y \cdot 1_{\left\{X+Y>\ln\frac{K}{S}-G(0,T)+\frac{1}{2}\sigma_Y^2\right\}} \right]$$

$$= SN(d_1),$$

$$I_2 = Ke^{-G(0,T)}\cdot E\left[e^{-X} \cdot 1_{\left\{X+Y>\ln\frac{K}{S}-G(0,T)+\frac{1}{2}\sigma_Y^2\right\}} \right]$$

$$= K\exp\left\{ \frac{1}{2}\sigma_X^2 - G(0,T) \right\}N(d_2).$$

So, the pricing formula of the European call option is

$$C(K,T) = SN(d_1) - K\exp\left\{ \frac{1}{2}\sigma_X^2 - G(0,T) \right\}N(d_2),$$

In the same way, we obtain the pricing formula of European put option

$$P(K,T) = K\exp\left\{ \frac{1}{2}\sigma_X^2 - G(0,T) \right\}N(-d_2) - SN(-d_1).$$

By theorem 3.1, we can deduce the following results.

Inference 3.1 Under the model (1) and (2), the call-put parity under actuarial approach is

$$C(K,T) + K\exp\left\{ \frac{1}{2}\sigma_X^2 - G(0,T) \right\} = P(K,T) + S.$$

Inference 3.2 Consider the European call and put options with continuous dividend yield $q(t)$ under the model (1) and (2). Their respective value given by actuarial approach at time 0 is

$$C'(K,T) = S\exp\left(-\int_0^T q(t)dt \right)N(d_1') - K\exp\left\{ \frac{1}{2}\sigma_X^2 - G(0,T) \right\}N(d_2'),$$

$$P'(K,T)=K\exp\left\{\frac{1}{2}\sigma_X^2-G(0,T)\right\}N(-d_2')-S\exp\ -\int_0^T q(t)dt\ \ N(-d_1').$$

where
$$d_1'=\frac{\ln\frac{S}{K}+G(0,T)-\int_0^T q(t)dt+\frac{1}{2}\sigma_Y^2+\rho\sigma_X\sigma_Y}{\sqrt{\sigma_X^2+\sigma_Y^2+2\rho\sigma_X\sigma_Y}},$$

$$d_2'=\frac{\ln\frac{S}{K}+G(0,T)-\int_0^T q(t)dt-\sigma_X^2-\frac{1}{2}\sigma_Y^2+\rho\sigma_X\sigma_Y}{\sqrt{\sigma_X^2+\sigma_Y^2+2\rho\sigma_X\sigma_Y}}.$$

The exchange option is a new option that can be used in the file of performance pay pricing analysis. We consider the valuation of European exchange option which allows the holder the right but not the obligation to exchange one risky asset for another on the expiry date T. Under the given complete probability space $(\Omega,F,\{F_t\}_{t\geq 0},P)$, suppose that the prices of both assets satisfy the following stochastic differential equation**s**

$$dS_1(t)=(\mu_1(t)-\alpha_1\ln S_1(t))S_1(t)dt+\sigma_1(t)S_1(t)dB_1(t),S_1(0)=S_1, \quad (4)$$

$$dS_2(t)=(\mu_2(t)-\alpha_2\ln S_2(t))S_2(t)dt+\sigma_2(t)S_2(t)dB_2(t),S_2(0)=S_2. \quad (5)$$

Here $S_i(t)$ denotes the price of asset i , $\sigma_i(t)$ denotes the volatility, both $\mu_i(t)$ and $\sigma_i(t)$ are assumed to be some given functions of t , α_i is constant (i=1,2). The related coefficient of $\{B_1(t):t\geq 0\}$ and $\{B_2(t):t\geq 0\}$ is ρ .

The exchange option can be considered either as a call option on asset two with strike price $S_1(T)$ or a put option on asset one with strike price $S_2(T)$. If and only if the terminal payoff is positive, the option will be exercised. By the definition of actuarial value，the value of the exchange option $V(0,T)$ on t=0 is:

$$V(0,T)=E\left[\max\left(S_2(T)\exp\left(-\int_0^T\beta_2(t)dt\right)-S_1(T)\exp\left(-\int_0^T\beta_1(t)dt\right),0\right)\right]. \qquad (6)$$

Theorem 3.2 The value of an exchange option V(0,T) on t=0 is

$$V(0,T)=S_2N(d_3)-S_1N(d_4),$$

where

$$Y_1=e^{-\alpha_1 T}\int_0^T\sigma_1(t)e^{\alpha_1 t}dB_1(t), \qquad Y_2=e^{-\alpha_2 T}\int_0^T\sigma_2(t)e^{\alpha_2 t}dB_2(t),$$

$$\sigma_{Y_1}^2=e^{-2\alpha_1 T}\int_0^T\sigma_1^2(t)e^{2\alpha_1 t}dt, \qquad \sigma_{Y_2}^2=e^{-2\alpha_2 T}\int_0^T\sigma_2^2(t)e^{2\alpha_2 t}dt,$$

$$d_3=\frac{\ln\frac{S_2}{S_1}+\frac{1}{2}\sigma_{Y_1}^2+\frac{1}{2}\sigma_{Y_2}^2-\rho\sigma_{Y_1}\sigma_{Y_2}}{\sqrt{\sigma_{Y_1}^2+\sigma_{Y_2}^2-2\rho\sigma_{Y_1}\sigma_{Y_2}}},$$

$$d_4=\frac{\ln\frac{S_2}{S_1}-\frac{1}{2}\sigma_{Y_1}^2-\frac{1}{2}\sigma_{Y_2}^2+\rho\sigma_{Y_1}\sigma_{Y_2}}{\sqrt{\sigma_{Y_1}^2+\sigma_{Y_2}^2-2\rho\sigma_{Y_1}\sigma_{Y_2}}}.$$

Proof Just for the definition (6), we have

$$V(0,T)=E\left[S_2(T)\exp\{-\int_0^T\beta_2(t)dt\}\cdot 1_{\left\{S_2(T)\exp\{-\int_0^T\beta_2(t)dt\}>S_1(T)\exp\{-\int_0^T\beta_1(t)dt\}\right\}}\right]-$$

$$E\left[S_1(T)\exp\{-\int_0^T\beta_1(t)dt\}\cdot 1_{\left\{S_2(T)\exp\{-\int_0^T\beta_2(t)dt\}>S_1(T)\exp\{-\int_0^T\beta_1(t)dt\}\right\}}\right]=J_1-J_2.$$

If we want to satisfy the condition:

$S_2(T)\exp\{-\int_0^T\beta_2(t)dt\}>S_1(T)\exp\{-\int_0^T\beta_1(t)dt\}$, then

$$\Leftrightarrow \exp\left\{\ln S_2-\frac{1}{2}e^{-2\alpha_2 T}\int_0^T\sigma_2^2(t)e^{2\alpha_2 t}dt+e^{-\alpha_2 T}\int_0^T\sigma_2(t)e^{\alpha_2 t}dB_2(t)\right\}>$$

$$\exp\left\{\ln S_1-\frac{1}{2}e^{-2\alpha_1 T}\int_0^T\sigma_1^2(t)e^{2\alpha_1 t}dt+e^{-\alpha_1 T}\int_0^T\sigma_1(t)e^{\alpha_1 t}dB_1(t)\right\},$$

$$\Leftrightarrow Y_2-Y_1>\ln\frac{S_1}{S_2}-\frac{1}{2}\sigma_{Y_1}^2+\frac{1}{2}\sigma_{Y_2}^2.$$

By the definitions of Y_1 and Y_2, we see that they are two normal random variables independent of F_t, and

$$E^Q[Y_1] = E^Q[Y_2] = 0, \sigma_{Y_1}^2 = e^{-2\alpha_1 T} \int_0^T \sigma_1^2(t) e^{2\alpha_1 t} dt, \sigma_{Y_2}^2 = e^{-2\alpha_2 T} \int_0^T \sigma_2^2(t) e^{2\alpha_2 t} dt.$$

Then, by lemma3.1, we have

$$J_1 = E\left[\exp\left\{\ln S_2 - \frac{1}{2} e^{-2\alpha_2 T} \int_0^T \sigma_2^2(t) e^{2\alpha_2 t} dt + e^{-\alpha_2 T} \int_0^T \sigma_2^2(t) e^{\alpha_2 t} dB_2(t)\right\} \cdot 1_{\left\{Y_2 - Y_1 > \ln\frac{S_1}{S_2} - \frac{1}{2}\sigma_{Y_1}^2 + \frac{1}{2}\sigma_{Y_2}^2\right\}}\right]$$

$$= S_2 e^{-\frac{1}{2}\sigma_{Y_2}^2} E\left[e^{Y_2} \cdot 1_{\left\{Y_2 - Y_1 > \ln\frac{S_1}{S_2} - \frac{1}{2}\sigma_{Y_1}^2 + \frac{1}{2}\sigma_{Y_2}^2\right\}}\right] = S_2 N(d_3),$$

$$J_2 = E\left[\exp\left\{\ln S_1 - \frac{1}{2} e^{-2\alpha_1 T} \int_0^T \sigma_1^2(t) e^{2\alpha_1 t} dt + e^{-\alpha_1 T} \int_0^T \sigma_1^2(t) e^{\alpha_1 t} dB_1(t)\right\} \cdot 1_{\left\{Y_2 - Y_1 > \ln\frac{S_1}{S_2} - \frac{1}{2}\sigma_{Y_1}^2 + \frac{1}{2}\sigma_{Y_2}^2\right\}}\right]$$

$$= S_1 e^{-\frac{1}{2}\sigma_{Y_1}^2} E\left[e^{Y_1} \cdot 1_{\left\{Y_2 - Y_1 > \ln\frac{S_1}{S_2} - \frac{1}{2}\sigma_{Y_1}^2 + \frac{1}{2}\sigma_{Y_2}^2\right\}}\right] = S_1 N(d_4),$$

So we obtain the pricing formula of the exchange option

$V(0,T) = S_2 N(d_3) - S_1 N(d_4).$

4. Case study

We analyze the differences of the two models, B-S model and actuarial model, using the actual data in Chinese stock market. Even though, there are only three warrants that are similar to the call option. The only difference between the warrant and the option is that the option is a standardized contract but the warrant is not. However, it would not affect the results. So we choose the data of Changhong CWB1 which is equivalent to a European call option, as its exercise ratio is 1:1 and the underlying stock has not paid dividend during the chosen period, then we need not to adjust the prices of the stock and the warrant. The data covers the period from August 19, 2009 when the Changhong CWB1 was listed to May 20, 2010 and the time interval between each observation is one day. This means that we end up with 181 observations, and this is the longest sample available to us. The data are taken from the Shanghai Stock Exchange and are collected through the Dazhihui quote software.

Next, we need to estimate the parameters in the models. Some parameters we can get from public information: the strike price of the Changhong CWB1 is $K = 5.23$, the exercise date $T = 2$ years, the interval is $\Delta t = 1/250$, the interest rate at 0 time $r = 0.0225$ which is the official interest rate for one-year deposits during the period 2009-2010. As for the parameters in the interest rate model, we use the daily data of 7-day repo rates in the Shanghai Stock Exchange from August 19, 2009 to May 20, 2010 and the MLE method to estimate the parameters by Matlab7.0. The result shows that the estimated parameters of the short-term interest rates driven by the Hull-White model in China market are $a = 0.0322$, $b = 0.1937$, $\sigma_r = 0.0863$. In order to estimate the parameters of the underlying stock, we use the historical data of Changhong shares during the same time to compute the results: $\sigma_s = 0.6968$ and $\alpha = 0.1359$ by the MLE method. A large number of the literature has revealed that the correlation of the stock price and the interest rate is weak negative, and we indeed get the parameter $\rho = -0.284$ during the chosen time.

Having specified the data and the parameters used in the simulation procedure, we now turn to the empirical results obtained from Matlab7.0. Figure 3 shows the comparison results among the real values, actuarial model prices and B-S model prices. The main features are that the actuarial prices are lower than the B-S prices mostly and the actual price points are much closer to the points computed by the actuarial model than the B-S model. Moreover, we analyze the goodness of fitting by the correlation coefficients. The results shows that the correlation coefficient of the actual prices and the actuarial model prices is 0.9652, and the correlation coefficient of the actual prices and the B-S model prices is 0.9607 which is lower than the former. It is an explicit evidence that the actuarial model is superior to the B-S model. The analysis also shows that the correlation of the actuarial prices and the B-S prices is strong and positive as their correlation coefficient is 0.9996.

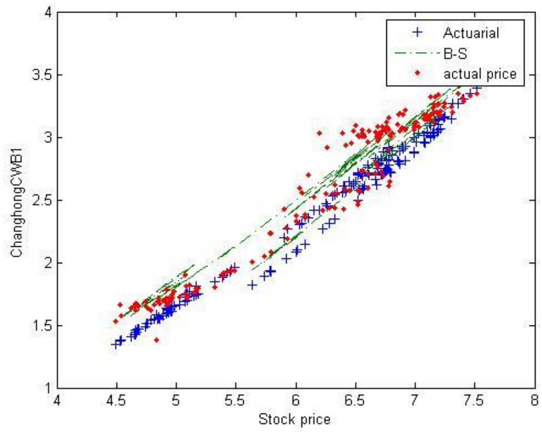

Figure 1. Comparison of the values among the actual data, actuarial model and B-S model

5. Conclusions

On the basis of the general actuarial approach, this paper assumes that the stock price process is driven by generalized Exp-Ornstein-Uhlenback process and the interest rates are assumed as the Hull-White model did to meet the actual situation, then the exact solutions of the general European option and the exchange option are obtained with the help of the related theorem of stochastic differential equation. Furthermore, the European call-put parity relation is derived naturally and the new European call and put prices with continuous dividend yield are deduced from the above results.

Ackonwledgements: ***The research is sponsored by Chinese National Educational Science Foundation(Project Numbers: DJA100325) and by Chinese National Natural Science Foundation (Project Numbers: 71072129;71132005).***

REFERENCES

[1] **Black F., Scholes M. (1973),** ***The Pricing of Options and Corporate Liabilities***; *Journal of Political Economy* 81, pp. 637-654;

[2] **Merton R.C.(1973),** ***Theory of Rational Option Pricing****; Bell Journal of Economics and Management Science* 4, pp. 141-183;

[3] **John, C. C., Stephen A. R., Mark R. (1979),** ***Option Pricing: A Simplified Approach****; Journal of Financial Economics 7*, pp. 229-263;

[4] **Bladt M., Rydberg H. T.(1998),** ***An Actuarial Approach to Option Pricing under the Physical Measure and without Market Assumptions****; Insurance: Mathematics and Economics* 22, pp. 65-73;

[5] **Cox J., Ingersoll J. Ross S. (1985),** ***A Theory of the Term Structure of Interest Rates****; Econometrica* 58, pp. 385-407;

[6] **Follmer H. Sondermann D. (1986),** ***Hedging of Non-redundant Contingent Claims****; in: W. Hildenbrand and A. Mas-Colell (eds.);* Contributions to Mathematical Economics, pp. 205-223;

[7] **Li J. L., Clemons C. B., Young G. W. (2008),** ***Solutions of Two-factor Models with Variable Interestrates****; Journal of Computational and Applied Mathematics* 222, pp. 30-41;

[8] **Johnson R., Stafford Z., Richard A. (2006),** ***Binomial Pricing of Fixed-income Securities for Increasing and Decreasing Interest Rate Cases***; *Applied Financial Economics,* 16, pp. 1029-1046;

[9] **Yan H. F. Liu S. Y.(2003),** ***New Method to Option Pricing for the General Black-Scholes Model ：An Actuarial Approach;*** *Applied Mathematics and Mechanics*, 24, pp. 730-738;

[10] **Liu J., Deng G. H., Yang X. Q. (2007),** ***Pricing Reload Option with Stochastic Interest Rate under Ornstein-Uhlenback Process****; Journal of Engineering Mathematics*, 24, pp. 237-241;

[11] **Zhao W. He T. M. (2007),** ***Model of Option Pricing Driven by Fractional Ornstein-Uhlenback Process****; Chinese Journal of Management Science*, 15, pp. 1-5;

[12] **Chen S. N. (2002),** ***Financial Engineering***. Fudan University Press, Shanghai, pp. 130-133;

[13] **Dong L. (2006),** ***An Empirical Test of Mean-reversion Hypothesis of Short-term Interest Rate in China****; The Journal of Quantitative and Technical Economics*, 11, pp. 151-160;

[14] **Christiansen C. (2010),** ***Mean Reversion in US and International Short Rates****; North American Journal of Economics and Finance,* 21, pp. 286-296.