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TOWARDS A PROABILITY TRANSITION MATRIX MODEL OF THE TAX COMPETITION

Abstract. *The tax competition remains one of the most and widely studied problems in macroeconomics. We are looking for empirical evidence which supports our preliminary theoretical conceptual model intended for the adequate description of the economic systems experiencing the tax competition. The model is intended not only for the utilization within the EU-27, but its wider use is assumed which combines the effect of the tax revenue function gradients with the harmonization tendency as an alternative. The analytical part of the study underlines the potential of the perturbative calculations which might be relevant for further macroeconomic research. The theoretical research represents ground for the empirical research searching for the causal relationship between the change of the revenue and time shifted tax burden.*

Keywords: *tax competition, inter-country transfer of GDP, tax revenue trend.*

JEL Classification: C63, C65, H21

1.Introduction

The European Union single market initiative named EC 92 has led to the removal of the barriers and stimulation of the production factors mobility. Consequently, relatively independent tax systems have started to develop in the European economic community. The gap between a complete monetary union and incomplete fiscal and political integration has stimulated governments of the EU members to encourage the inflow of productive resources via the mechanism of tax competition (TC). In general, the TC process between localities may be stimulated by the different tax systems with distinct tax rates. High tax rates mostly encourage mobile capital to move to another, less taxed country. The actual situation with regards tax benefits can be considerably blurred by numerous policy and non-policy factors such as relative country size (Haufler et al., 1991, Kanbur et al., 1993), sparsity/density of the populations (Trandel, 1994).

Despite considerable scientific activity aimed at TC, the identifying of the effect of TC itself and its implications is not clearly understood yet. The distortions among the tax systems may affect the savings, investments and labor (Mendoza et al., 2005, Razin et al., 2006). The undesirable and negative impacts of TC are the basic features of the plausible *race to the bottom hypothesis*, where some competing local governments attract the mobile capital by tax rates reduction. This consequently induces a reaction of the others resulting in an overall cut not only of the tax rates but also of the tax revenues at all. This fact leads the economists to ask for alternative tax policy - tax harmonization, which, on the other hand, does not take into account short-term priorities of the participating national economies. The TC (in)efficiency is still under ongoing research. Tax compentence respecting the free population migration analyzed in (Razin et al., 2011, Wildasin, 1989) presents harmful TC consequences with the attraction of the mobile capital from the high tax regions triggered by the strategic lowering of the domestic tax rates. This concept is akin to the scope of (Tanzi, 1996), according to which TC typically leads to the lower tax revenues, reduces progressiveness and makes the structure of a tax system less neutral. The study of (Razin et al., 2006) shows that rich countries as sources of FDI, tend to the equilibrium with a high corporate tax rates and high provision of the public goods. The scenario goes opposite way for the poorer countries.

Against the arguments emphasizing negative consequences of TC stands an opposite view expressed by the hypothesis (Tiebout, 1956) based on the role of the non-central auto-regulatory adjustment of the taxes among the localities (Razin et al., 1991). Undoubtedly, the option to harmonize tax rates via the collective agreements of European governments or via the responsible central authorities (Sinn, 1990) is often dismissed since it may be perceived as the accession to the sovereignty of the EU countries.

In order to shed light on the issue of TC we have suggested a preliminary model interlinking the revenue, GDP, GNP and tax burden. The main purpose of our study is to construct the hypothetical and logically consistent relations for GDP transfer. We believe that the resulting model may serve as a basis for better assessing of the systematic discrepancies between shortsighted tax policy of the EU countries and long-term prospects across the EU community.

2. The static relationship between GDP and GNP

The problem may be seen as a variant of n-country game, where countries - their governments interact in a setting of the strategic interdependence in order to decide upon the level of tax rate for the forthcoming tax season. We consider the set of n countries -

players enumerated as $1,2,\ldots,n$ forming the community. Thus under very simplifying conditions the strategy of i th country may involve a choice of single *tax rate* parameter $t_i \in (0,1)$. If we are concentrated just on the short-term optimality, i.e. instant effect, the objective function of i th country government (without making coalitions with rivals) may be associated with the actual *tax revenue*

$$T_i(t_1,t_2,\ldots,t_n) = t_i Y_i^{GDP}(t_1,t_2,\ldots,t_n), \tag{1}$$

where Y_i^{GDP} be Gross domestic product (GDP) produced within the borders of i th country. In trying to deduce trends towards optimality in n dimensional landscape, it may be useful to express GDP in terms of its Gross National Income (GNI) contributions denoted as Y_i. It includes income obtained from other countries. We suppose Y_i^{GDP} originates from the mixing of the n terms Y_i, $i = 1,2,\leq,n$. The situation is expressed by the equation

$$Y_i^{GDP} = Y_i + \sum_{j=1,\, j\neq i}^{n} (Y_j p_{ji} - Y_i p_{ij}), \tag{2}$$

where $p_{ij} \in [0,1]$ is the transition probability which describes the the transfer of GNI from the country i to the country j during the 1 year period. Thus $Y_i^{GDP} - Y_i$ has the meaning of the transfer rate per country per year. Speaking illustratively, in the extreme case $p_{ij} = 1$ the whole Y_i is moved from i to j. (Then it becomes subordinated to a taxation with the resulting revenue $t_j Y_i$). The model structure implies an important conservation property to hold

$$\sum_{j=1}^{n} Y_j^{GDP} = \sum_{j=1}^{n} Y_j = invariant \tag{3}$$

Of course, it is clear that there remains a problem whether the selected economic system meets the conditions of the closeness (or at least approximate closeness). (The existence of tax havens may be also placed into the model by supposing an extra country with extraordinary low tax). As usual, the total probability of a transition including transition to itself is 1: $\sum_{j=1}^{n} p_{ij} = 1$.

The normalization combined with Eq.(2) simply gives

$$Y_i^{GDP} = \sum_{j=1}^{n} Y_j p_{ji}. \tag{4}$$

It should be noted that in practice, we have a direct access to data, however data on

GNI is not available in general, in further we consider them as latent. In order to uncover hidden value Y_j we need to perform the matrix inversion of p (where $[p]_{ij} \equiv p_{ij}$) as it follows

$$Y_i = \sum_{j=1}^{n} Y_j^{GDP} \left[\mathbf{p}^{-1} \right]_{ji}. \tag{5}$$

Let's add here the comment on the algorithm that in the case of large p matrices, the algorithmic inversion may be carried out as an iterative refinement $Y_i^{(iterat+1),\ preliminary} \leftarrow Y_i^{(iterat)} + \gamma \sum_{k=1}^{n} (Y_k^{GDP} - \sum_{j=1}^{n} Y_j^{(iterat)} p_{jk}) p_{ik}$ designed to tend to minimal square $\sum_{k=1}^{n} (Y_k^{GDP} - \sum_{j=1}^{n} Y_j^{(iterat)} p_{jk})^2$ of the Eq.(4). The convergence rate is controlled by the parameter $\gamma > 0$. The additional procedure against the conventional matrix inversion should also guarantee the Eq.(3). It means that some preliminary (rough) estimate $Y_i^{(iterat+1),\ preliminary}$ (where the superscript $iterat$ refers to the number of iterations) may be improved by the normalization $Y_i^{(iterat+1)} \leftarrow Y_i^{(iterat+1),\ preliminary}$ $[\sum_{j=1}^{n} Y_j^{GDP} / \sum_{k=1}^{n} Y_k^{(iterat+1),\ preliminary}]$. The final GNI estimate will be: $Y_i = Y_i^{(iterat)} |_{iterat \to \infty}$. We checked numerically that it is reasonable to initialize the iterations from $Y_j^{(iter=0)} = Y_j^{GDP}$. Clearly, the calculus of $\mathbf{p}^{-1}$ starts to be difficult for highly interconnected $p_{ij} : 1/n$ systems.

3. The parametrization of the transfer matrix p_{ij}

In what follows we focus on the model joining p_{ij} with the tax rates. To achieve the normalization of p_{ij} we introduce auxiliary link variables

$$p_{ij} = \frac{\phi_{ij}}{\Phi_i}, \qquad \Phi_i = \sum_{s=1}^{n} \phi_{is}. \tag{6}$$

Now we turn attention to the specific parameterization of the link variables. We assume

$$\phi_{ij} = \delta_{ij} + \mu(1-\delta_{ij}) f_{ij} \qquad which implies\ \Phi_i = 1 + \mu \sum_{s=1, s\neq i}^{n} f_{is}. \tag{7}$$

Here δ_{ij} is the Kronecker symbol, $\mu \in [0, \infty)$ is the parameter of the propensity to contribute to the foreign economic activity we call *pressure to external mobility* or shortly *mobility*, i.e. $\mu = 0$ with consequence $p_{ij} = \delta_{ij}$ belongs to the complete domestic taxation of GDP, the f_{ij} term will be explained later. The initial approximation of $\mathbf{p}^{-1}$ may be obtained by means of the analytical perturbative technique. By the virtue of the smallness of μ we derived

$$[\mathbf{p}^{-1}]_{ji} = \delta_{ji} + \mu\left(\delta_{ji}\sum_{s\neq j}^{n} f_{js} - (1-\delta_{ji}) f_{ji}\right) + O(\mu^2). \quad (8)$$

An intermediate auxiliary step in determination of the f_{ij} is to introduce a parameter attractiveness a_i. Let us suppose that GNI is routed by the competitive pair-wise effects driven by the pair a_i, a_j as it follows

$$f_{ij} = F(a_i, a_j). \quad (9)$$

Clearly, the concept of attractiveness (or repulsiveness) involves the attraction due to lower tax. The links between the concepts of attraction of country and tax are known in the economic literature. The study of the attractiveness considered from the point of view corporate taxation, tax policy and foreign direct investment has been discussed in Simmons(2003). At the simplest level, we may think of the basic function relations between a_i and t_i : $a_i = A(t_i)$, where one may suppose monotonous decrease $\mathrm{d}a_i/\mathrm{d}t_i = A'(t_i) < 0$ and concavity of $A(.)$ in accordance with the well-known *Laffer-Kaldun* model. We shall assume that for $a_i \neq a_j$ we have $F(a_i, a_j) \neq F(a_j, a_i)$, which yields asymmetry $f_{ij} \neq f_{ji}$. Three basically different situations may be distinguished

$$F(a_i, a_j) = \begin{cases} <1 & a_j < a_i, \quad (\mathit{foreign\,repeller\,or\,domestic\,attractor - recepient}), \\ =1 & a_j = a_i, \quad (\mathit{neutral\,case}), \\ >1 & a_j > a_i, \quad (\mathit{foreign\,attractor\,or\,domestic\,repeller}). \end{cases} \quad (10)$$

In the last case ($a_j > a_i$) the GDP flux is preferentially pushed from the source country i towards the target, host country j (when $\phi_{ij} > \phi_{ji}$). One may see now a clear analogy of the attractor and repeller concepts with the more specific concepts including pull and push factors which are known from the theory of the migration or class of the well known gravity models (see e.g. their application in (Lewer, 2008)).

3.1 The example of *F(.,.)* function

As an example of F can serve us the homogeneous function of degree zero: $F(a_i, a_j) = a_j/a_i$. To illustrate above introduced model, let us suppose the effect of different tax rates and mobility in the case of the minimalist system of two coupled countries with

$$Y_1^{\mathrm{GDP}} = a_1\left(\frac{Y_1}{a_1+\mu a_2} + \frac{\mu Y_2}{a_2+\mu a_1}\right), \qquad Y_2^{\mathrm{GDP}} = a_2\left(\frac{\mu Y_1}{a_1+\mu a_2} + \frac{Y_2}{a_2+\mu a_1}\right). \quad (11)$$

The application of basic limits clarifies the obtained result. In the case $\mu = 0$ we have $Y_1^{\mathrm{GDP}} = Y_1$, $Y_2^{\mathrm{GDP}} = Y_2$. As $a_1 \to \infty$, $a_2 > 0$, the 1-st country drains both contributions so as $Y_1^{\mathrm{GDP}} \to Y_1 + Y_2$ and $Y_1^{\mathrm{GDP}} \to 0$. For the extremal mobility $\mu \to \infty$ we get the exchange $Y_1^{\mathrm{GDP}} = Y_2$, $Y_2^{\mathrm{GDP}} = Y_1$. From the inverse

$$p^{-1} = \frac{1}{\mu^2 - 1}\begin{pmatrix} -(1/a_1)(a_1 + a_2\mu) & (\mu/a_2)(a_1 + a_2\mu) \\ (\mu/a_1)(a_2 + a_1\mu) & -(1/a_2)(a_2 + a_1\mu) \end{pmatrix} \quad (12)$$

it follows that p becomes singular as $\mu = 1$ (un-identifiable GNI) which is equivalent to $Y_1^{\mathrm{GDP}}/Y_2^{\mathrm{GDP}} = a_1/a_2$.

3.2 The variability of *T_i* along *t_j* dimension, the role of parameter *μ*

We assume that the processes of change in tax rates are primarily driven by the gradients of the tax revenues $\partial T_i/\partial t_k$, which reflecmotivemporal greedy motives of the governments. Let us analyze the structure of $\partial T_i/\partial t_k$ afore stems from the afore-introduced model. With the help of Eq.(4) and Eq.(5) we obtained the linear in GDP form

$$\frac{\partial T_i}{\partial t_k} = \delta_{ki} Y_i^{\mathrm{GDP}} + t_i \sum_{s=1}^{n}\sum_{j=1}^{n} [\mathbf{p}^{-1}]_{sj} \frac{-\partial p_{ji}}{\partial t_k} Y_s^{\mathrm{GDP}} . \quad (13)$$

It eliminates Y_i , however, the problem of the inversion of $\mathbf{p}$ remains open in higher dimensions. Despite of this, we proceed with the analysis of the structure of $\partial p_{ji}/\partial t_i$ from the perspective of the Eqs.(6), (7) and (9). The differentiation of ϕ_{ji}/Φ_j applied to the Eq.(6) yields

$$\frac{\partial p_{ji}}{\partial t_k} = \frac{1}{\Phi_j^2}\left(\Phi_j \frac{\partial \phi_{ji}}{\partial t_k} - \phi_{ji}\frac{\partial \Phi_j}{\partial t_k}\right) \tag{14}$$

including

$$\frac{\partial \Phi_j}{\partial t_k} = \mu \sum_{s=1,s\neq j}^{n} \frac{\partial f_{js}}{\partial t_k}, \quad \frac{\partial \phi_{ji}}{\partial t_k} = \mu(1-\delta_{ji})\frac{\partial f_{ji}}{\partial t_k}, \quad \frac{\partial f_{js}}{\partial t_k} = A'(t_k)\left(\frac{\partial F}{\partial a_j}\delta_{jk} + \frac{\partial F}{\partial a_s}\delta_{sk}\right). \tag{15}$$

A valuable insight about the behavior of the model can be obtained through the perturbative analysis within the $0 < \mu = 1$ region. The Taylor expansion applied to the Eqs.(7) and (9) with $\partial p_{ji}/\partial t_k ; O(\mu)$, and even rough approximation $[p^{-1}]_{kj} = \delta_{kj} + O(\mu)$ [see Eq.(8)] suffices to get $O(\mu)$ order approximation of Eq.(13) in the (Taylor expansion) form

$$\frac{\partial T_i}{\partial t_k} = Y_i^{GDP}\delta_{ki} + \mu t_i \sum_{j=1,j\neq i}^{n}\left[Y_j^{GDP}\frac{\partial f_{ji}}{\partial t_k} - Y_i^{GDP}\frac{\partial f_{ij}}{\partial t_k}\right] + O(\mu^2). \tag{16}$$

As we see from this explicit example that $\partial T_i/\partial t_k$ is formed by the aggregates of the link derivatives [see also Eq.(15)]. We also see that the term $\propto \mu$ is especially typical consequence of the TC presence.

4. Empirical identification of the gradient driven tax dynamics

Our empirical investigation has been directed to demonstrate the importance of the tax revenue gradients in the studies of the TC changes. We focus here on the data from countries presently forming EU-27. For this purpose the quarterly data of 27 countries has been extracted from the *Eurostat database* for the time span 1999 - 2012. We decided to monitor the time ($\tau = 1,2,\ldots$) variations of $T_i^{(\tau)}$, $Y_i^{(\tau)}$, $t_i^{(\tau)}$ within EU-27 countries even before their integration into the EU-27.

The quantity we analyze is the tax burden, which we call here also tax quota. This term more suitably expresses the fact that tax is an instrument of the government to affect the future tax revenues. To explore given data from the point of view of TC we have used two tax causal indicators - systemic averages $I_{(h),+}^{(\tau)}$, $I_{(h),-}^{(\tau)}$ localized at the time stamp τ and characterized by the lag h. The purpose is to characterize in a cumulative manner the instantaneous consequences of revenue and the variations of the tax quota on the tax revenues in future periods. The indicators

$$I_{(h),+}^{(\tau)} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(t_i^{(\tau-3)} < t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} < T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2+h)} < t_i^{(\tau-1+h)}) \tag{17}$$
$$+\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(t_i^{(\tau-3)} > t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} > T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2+h)} < t_i^{(\tau-1+h)}),$$

$$I_{(h),-}^{(\tau)} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(t_i^{(\tau-3)} < t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} > T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2+h)} > t_i^{(\tau-1+h)}) \tag{18}$$
$$+\frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(t_i^{(\tau-3)} > t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} < T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2+h)} > t_i^{(\tau-1+h)})$$

have been constructed (for $h = 0,1,2$) by means of the indicator functions $\mathbf{1}(\ldots)$ with standard properties

$$\mathbf{1}(\ statement\, is\, true) = 1, \qquad \mathbf{1}(\ statement\, is\ false) = 0. \tag{19}$$

Let us remind that the product of the indicator functions belongs to the intersection of the sets specified by the consecutive events. Both indicators estimate the ratio of the countries participating in the *gradient driven process*. The subscripts $+$ and $-$ are used to highlight the different nature of the positive and negative terms

$$\frac{\Delta T_i^{(\tau-2)}}{\Delta t_i^{(\tau-2)}} = \frac{T_i^{(\tau-2)} - T_i^{(\tau-3)}}{t_i^{(\tau-2)} - t_i^{(\tau-3)}} \cong \left(\frac{\partial T_i}{\partial t_i}\right)^{(\tau-2)}, \tag{20}$$

which approximate $(\partial T_i/\partial t_i)^{(\tau-2)}$ we prefer to call gradient in analogy to gradient descent local optimization. Clearly, the view becomes more thoroughly substantiated when using the fact that the sign of the estimated gradients is determined by the signs of the corresponding differences, which form the numerator and denominator of Eq.(20). From the construction of Eq.(17) it follows that the positive gradients contribute to $I_{(h),+}^{(\tau)}$ only with the simultaneous consecutive tax increase $t_i^{(\tau-2+h)} < t_i^{(\tau-1+h)}$, whereas the negative gradients followed by the tax drop $t_i^{(\tau-2+h)} > t_i^{(\tau-1+h)}$ constitute $I_{(h),-}^{(\tau)}$. Now we can make the assignment of economic meaning to the concept of $I_{(h),-}^{(\tau)}$ indicator. It may be interpreted as a share of typical tax competition events. Here, any change in the tax quota is associated with the opposite change in the tax revenues (increase of the tax quotas causes a reduction of the tax revenues and vice versa). Then, in the following periods, the government is willing to decrease the tax quotas in order to increase the tax revenues. The indicator $I_{(h),+}^{(\tau)}$ describes an inverse situation, where the increase (the decrease) of the tax quotas is associated with the increase (the decrease) of the tax revenues, which consequently encourages the government to raise the tax quotas. Here the government tax policy seems to be robust against the tax quota movements in other

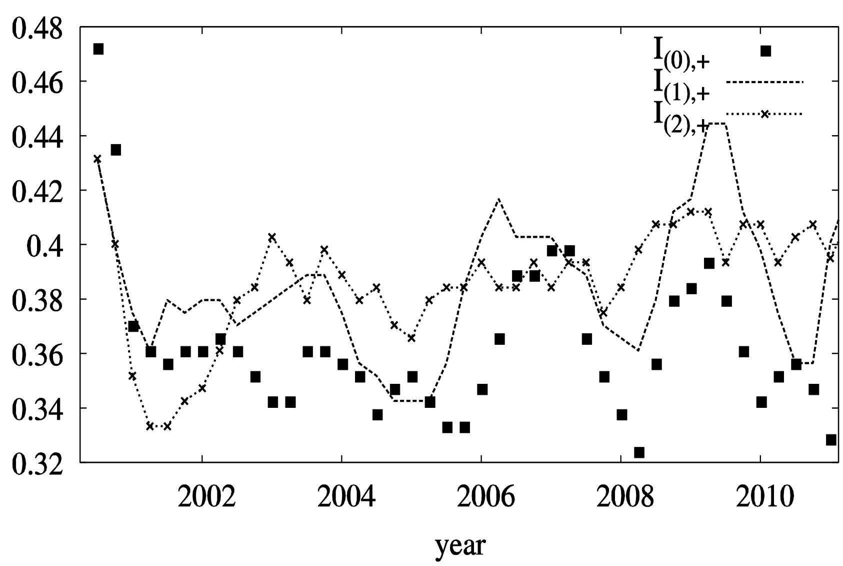
countries, which we decided to call “tax quota utilization strategy”.
Indicator $I_{(h),-}^{(\tau)}$ involves also the so called "race to the bottom" situation, described by the specific indicator $I_{(h),\mathrm{R}B}^{(\tau)}$ where the decrease of the tax quotas is interlinked to an increase of the tax revenues, which leads the governments to lower the tax quotas. The intensity of the "race to the bottom" process is then indicated by

$$I_{(h),\mathrm{R}B}^{(\tau)} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{1}(t_i^{(\tau-3)} > t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} < T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2+h)} > t_i^{(\tau-1+h)}). \quad (21)$$

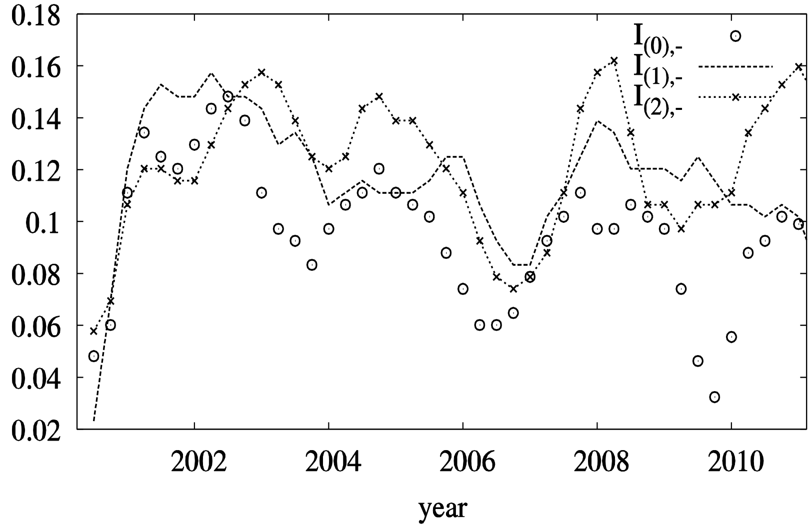
In an effort to level out seasonal variations the calculated indicators have been seasonally adjusted by means of the classical seasonal decomposition by moving averages (additive method). The results of data processing are presented in Fig.1.

Figure 1
The impact of TC represented by the indicators.

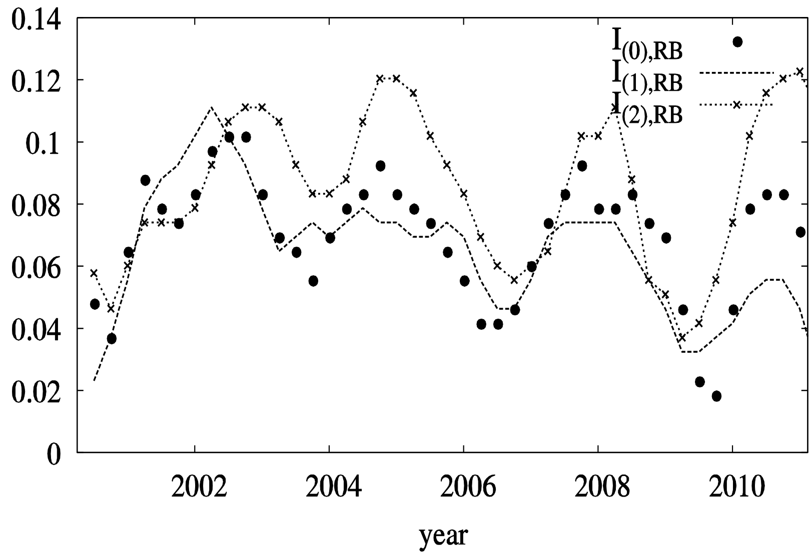
(a)



(b)



(c)



Note: Calculated for different time lags $h = 0,1,2$. The TC is associated with the participation of countries in the process driven by the gradient of T_i. The increased variability of the indicators near to the emergence of global financial crisis (year 2007, 2008) are visible.

We see that the competition case indicator reveals rather regular oscillations (see Fig. 1b). Since the indicator involves also the “race to the bottom” pattern, by comparing the both graphical patterns (Fig. 1b vs. Fig. 1c) we conclude that the oscillations are enforced by the “race to the bottom” motives. Here, the regular movements up and down can be explained quite intuitively. A growing number of the race to the bottom trials is reversed because of the natural lower limits of the tax systems, since the tax revenues are not long-term sustainable anymore. That is why these attractive “race to the bottom strategies” must be replaced by more preferable tax utilization strategies. However, in case of the tax utilization strategies the following finding may be seen as interesting: First, proportions of the tax utilization strategies is much higher (about 0.38), than the tax competing ones (about 0.07). Second, the regular oscillations in tax utilisation cases exhibit obviously distinct amplitudes if compared to the tax competition ones. Phase shift of both time diagrams is ambiguous, however, in some periods the movements of the tax competition cases and tax utilisation ones is developing in the opposite direction (see Fig.1 a, Fig. 1 b; period until the end of 2004). Years of financial crisis are accompanied by the increase of the tax competition, which is consistent with the finding (Giuli, 2010) about the TC and the financial crisis relationship.

Table 1 summarizes the result of the classification of countries, where the value of the tax change $t_i^{(\tau-1)} - t_i^{(\tau-2)}$ is comprised in the set of supplementary indicators

$$\begin{aligned}
\Delta_{+,1}t_i &= (t_i^{(\tau-1)} - t_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-3)} < t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} < T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2)} < t_i^{(\tau-1)}) \\
\Delta_{+,2}t_i &+ (t_i^{(\tau-1)} - t_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-3)} > t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} > T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2)} < t_i^{(\tau-1)}) \\
\Delta_{-,1}t_i &= (t_i^{(\tau-1)} - t_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-3)} < t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} > T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2)} > t_i^{(\tau-1)}) \\
\Delta_{-,2}t_i &= (t_i^{(\tau-1)} - t_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-3)} > t_i^{(\tau-2)})\mathbf{1}(T_i^{(\tau-3)} < T_i^{(\tau-2)})\mathbf{1}(t_i^{(\tau-2)} > t_i^{(\tau-1)}).
\end{aligned} \tag{22}$$

In Table 1, the first column indicates a continuation of the patterns of “the tax quota utilization strategy”. It is represented by the policy of the new EU member states until 2008. After that, Ireland, Estonia and Greece damaged by the economic and financial crisis, seems to decide to use this strategy. The second column represents a tax quota reversal pattern, when after negative experiencing with lowering tax quotas (unsuccessful race to the bottom attempt) the reversal in tax quotas followed. Here, the United Kingdom, Finland dominate in the list of the countries. During the pre-crisis period Finland dominates. This dominance can be explained by the tax reform implemented in the year 2005. In the crisis period, the UK position is seemingly determined by the public finance problem at all. The last two columns reflect the variants of the race to the bottom strategies. In the list of the countries, the new EU members dominate, which we explain by the rapid development of these countries restructuring their public sector. This situation creates a space for the incentives to continue the tax competition strategies. It may be also an argument, why the new EU members dominate in the list of the countries with the lowest tax quotas (see the last column).

Table 1

The selection of country selected according to above threshold value $\Delta_{thr} = 0.06$.

Year	Tax quota utilizations		Tax competitions		Lowest tax quota countries (V.)
	$\Delta_{+,1} > \Delta_{thr}$ (I.)	$\Delta_{+,2} > \Delta_{thr}$ (II.)	$\Delta_{-,1} < -\Delta_{thr}$ (III.)	$\Delta_{-,2} < -\Delta_{thr}$ (IV.)	
2001	$RO_{16}^{(I)}$, CY_1			$RO_{16}^{(III)}$	CY, IR, EE
2002	$RO_5^{(I)}$, CY_1, LU_{18}	PL_{11}, FI_{25}	$RO_3^{(II)}$	$RO_2^{(III)}$, BG_2	CY, BG, IR
2003	BG_9, CY_1				LV, CY, EE
2004	BG_{10}				LV, LT, RO
2005	$CY_6^{(IV)}$	FI_{25}	$CY_5^{(II)}$	BG_5	LV, LT, IR
2006		FI_{25}, UK_{15}			LT, RO, LV
2007	$BG_8^{(I)}$	FI_{25}, CY_7		$BG_9^{(III)}$	RO, LT, LV
2008	IR_5	UK_{15}		ES_{14}	LT, RO, LV
2009	EE_5	UK_{17}	RO_2		LT, RO, SK
2010		RO_4			LT, RO, SK
2011	GR_9	RO_2, UK_{17}			RO, LV, SK

Note: *The information about the country is supplemented by one or two indices. The lower index informs about the rank of the country when countries are ordered in ascending order. If the situation changed during a year we add an upper index to emphasize the dependence on the year's quarter. The fourth column provides information about the actual three countries having the lowest tax burden.*

We see that in many points the results correlate well with the general opinion on the location of the main tax competitors. The next section briefly closes our study by presenting a hypothetical tax dynamic model which will be used in our further studies.

4.1 Gradient driven adaptive tax dynamics and related equilibrium

The gradient concept may be interpreted in the sense that governments make their competitive and greedy rational decisions, which are beneficial for them from the short term perspective. Instead of the using overparametrized modeling by means of the well-known *Laffer-Kaldun* model, we make here simplified hypothetical proposal which combines the idea of the gradient-driven TC effect with the tax harmonization adoption as the alternative objective. For i-th country at the time $(\tau+h+1)$ the tax quota may be written as

$$t_i^{(\tau+h+1)} = t_i^{(\tau+h)} + \alpha\left(t^{harm} - t_i^{(\tau+h)}\right) + \frac{\beta}{T_i^{(\tau)}}\left(\frac{\partial T_i}{\partial t_i}\right)^{(\tau)} + \varepsilon_i^{(\tau)}, \qquad (23)$$

where $\beta > 0$ be the TC related constant of the gradient driving, t^{harm} is the term, which describes the uncompetitive limit of t_i (state of the absence of TC), $\alpha > 0$ is the positive adoption quota constant (the inverse characterstic is the adoption time) while $\varepsilon_i^{(\tau)}$ is the stochastic term. Under the assumption $E(\varepsilon_i) = 0$ the model structure implies the presence of the stochastic equilibrium with the properties $E(t_i) = t^{harm}$, $E[\frac{1}{T_i}(\partial T_i/\partial t_i)] = 0$.

5. Conclusions

The model of TC based on the transition probability matrix has been introduced. It offers a fundamental combination of the tenets of the TC process, but its nonlinear structure with unknown parameters will require an advanced analysis and application of the special purpose methods. We hope that continued research will shed light on the methodology of the assignment of the empirical values to p matrix elements. An interesting and constructive issue for further research may represent an evaluation of the residui $\varepsilon_i^{(\tau)}$ from the Eq.(23) of the former section. It may assist in the proposal of more advanced models of TC. The authors would like to express their thanks to the grant VEGA no. 1/1195/12.

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