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A DESIGN AND VALUATION OF AN INDEXED EXECUTIVE STOCK OPTION

Abstract. *Executive stock options indexed to the market are a useful compensation scheme in that they protect shareholders from excessively rewarding executives during bull markets. However, despite their usefulness, most large firms in the US have not granted such indexed options. According to academic research (Hall and Murphy (2003)) on executive stock options, the probability of expiring in the money is too small to be an incentive for risk-averse executives to work more efficiently. In addition, in bear markets even a typical indexed option model (Johnson and Tian (2000b)) can provide payoffs to executive that are too high with regard to their performance. This paper develops a new indexed option model with different implications from those of existing indexed option models. First, this paper shows that the new indexed option has higher probability of expiring in the money than existing indexed options. Second, it finds that the existing indexed option can generate excess rewards to even poorly performing executives in bear markets, and that averaging indexed options can remedy this problem.*

Keywords*: Executive stock options; Option valuation; Indexed options; Average performance.*

JEL Classification: G13, J33

1. Introduction

A major issue in corporate finance is corporate compensation that provides incentives to align the interests of executives with those of shareholders. Among various corporate compensation schemes, executive stock options have drawn the attention of many corporations. Typically, executive stock options provide executives with the right to buy shares of stock at a price set to be the stock's market price at the grant date. To the extent of an executive's efforts, he or she can be rewarded so long as performance is better than the firm's expected performance on the grant date.

Although they provide a vehicle for increasing executive incentives and reducing agency costs, executive stock options can also be an inefficient tool. For example, the financial press reports that a certain CEO "received a $2.1 million bonus and 200,000 new options last year even though net income fell by half and shareholder returns were flat at his company."[1] Moreover, one of the problems with executive stock options is that they can provide much greater rewards to executives relative to their efforts. During market upturns, traditional executive stock options can generate enormous rewards, even when firm performance is poor compared to the market or a comparable company. This happens because the stock price can ride on market upturns and, as a result, be greater than the exercise price fixed at the grant date, regardless of the executive's efforts.[2]

To make executive stock options more efficient at preventing excessive rewarding, many studies suggest indexing stock option exercise prices to the market.[3] Various elements of the financial press also propose that executive stock options be indexed to the market.[4] Compared to traditional stock options (in general, using the Black-Scholes option pricing model), indexed options can provide more efficient ways to strengthen executives' incentives, in that indexed options protect executives from uncertainties that are beyond their control.

Despite the usefulness of indexed executive stock options, no large firms in the United States have an indexed option plan (see Hall and Murphy (2003)). Indexed options almost never exist in practice for the following reasons. First, indexed option grants are treated as expenses in firms' accounting statements. Second,

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1 "An Answer to the Options Mess," *BusinessWeek*, April 19, 2002.

2 For example, see "Executive pay: Stock options plus a bull market made a mockery of many attempts to like pay to performance," *BusinessWeek*, April 20, 1998.

3 For example, Rapport (1999), Meulbroek (2001a, b) and Johnson and Tian (2000b).

4 For example, see 'How to ensure that top execs aren't rewarded even when their outfits perform poorly? Index stock options to a benchmark like the S&P 500', BusinessWeek, April 19, 2002.

according to Hall and Murphy (2003), the values of indexed executive stock options are much lower than corresponding traditional option values. Because stock returns are skewed to the right… half of the firms in an index will have returns that exceed the average. Thus, indexing reduces the company's cost of granting an option, but it reduces the executive's value even more because risk-averse executives attach very low values to options likely to expire worthless. Therefore, to deliver the same value to the executive, it costs the company more to grant indexed options rather than traditional options.[5]

In addition, this paper shows that even the typical indexed model (Johnson and Tian (2000b)) can provide payoffs that are too high at maturity for poorly performing executives in bear markets (see Panel B of Table 3). The indexed model generates excessive exercise values even when firm performance is poorer than market performance.

For these reasons, this paper aims to design a new indexed executive stock option whose payoff structure is different from that of the existing indexed option of Johnson and Tian (2000b). In Johnson and Tian (2000b), the indexed option pays off only if the firm's stock price exceeds the benchmark price indexed to the market at the option's maturity date.[6] This implies that the firm evaluates a manager's performance only on the option's payment date. This paper considers a new indexed option (hereafter, called the averaging indexed option) that has a strike price that is the expected average of the stock price conditional on the average of the index. For example, a firm's solid performance can worsen as the option's maturity approaches. In this case, it is very difficult for an executive to be rewarded through the indexed option. Compensation using an averaging indexed option creates fewer burdens on the executive. This is because an executive holding an averaging indexed option can be rewarded on the company's average performance.[7]

[5] One reason why indexed options are not popular is due to the accounting rules proposed by the Financial Accounting Standards Board (FASB). This paper studies indexed options from the perspective of option design.
[6] Both Jørgensen (2002) and Duan and Wei (2005) adopt the indexed exercise price proposed by Johnson and Tian (2000b).
[7] Obviously, there is also a similarity between the indexed option and the averaging

After designing and pricing the averaging indexed option, this study investigates its properties. It shows that, in contrast to the indexed option, the averaging indexed option has a significantly higher probability of expiring in the money. Since the possibility of the stock price at the option's maturity exceeding the indexed exercise price after many years (typically 10 years) is very low, the value incentive of the indexed option is small for the option holder. This paper also shows that the difference in value between traditional and averaging indexed options is less than that between traditional and indexed options. For example, with reasonable parameters, the prices of indexed and averaging indexed options are 34% and 70% of the prices of traditional option counterparts, respectively.

The organization of this paper is as follows. Section 2 reviews studies related to this paper. Section 3 sets up the benchmark exercise price of the averaging indexed executive stock option and derives a closed-form formula for the averaging indexed executive stock option. Section 4 provides some numerical analysis. Section 5 presents the paper's conclusions.

2. Related Work

Numerous papers focus on the specific features of executive stock options including reload options, options with reset provisions, and indexed option. An option that resets its exercise price is such that the firm has the right to reduce the exercise price relative to the corresponding at the grant date. As for a reset feature, Brenner, Sundaram, and Yermack (2000) derive a closed-form solution for a European executive stock option with a reset provision. Acharya, John, and Sundaram (2000) also investigate the incentive effects of options when the strike price is reset. On the other hand, reload options are such that executives can reload their stock options if options are exercised prior to maturity. Dybig and Lowenstein (2003) obtain the values of reload options under a continuous-time framework, while previous works (e.g., Saly, Jagannathan and Huddart (1999)) use a binomial framework.

indexed option. Similar to the indexed option of Johnson and Tian (2000b), the averaging indexed option links option values with only the firm performance after filtering out the market performance. This aspect does not allow executives to be rewarded excessively.

This paper concentrates on an indexed option model. Johnson and Tian (2000b) formally design such a model and derive an indexed option formula. Several other papers research the indexed executive stock option. Jørgensen (2002) derives the American indexed executive stock option formula and analyzes the characteristics of such options when there is a vesting period. Duan and Wei (2005) investigate the effects of systematic risk on the values and incentives of the indexed option as well as the non-indexed option under a GARCH option pricing framework. This paper proposes a new indexation for executive stock option.

3. Design and Pricing an Averaging Indexed Executive Stock Option

This section presents a simple executive stock option model where the option's strike price is indexed to a benchmark. Assume that there are no arbitrage opportunities in the economy. Suppose that under the physical probability measure *P*, the dynamics of the firm's stock price and the index are governed by

$$dS_t = (\mu_S - \delta_S)S_t dt + \sigma_S S_t dW_t \tag{1}$$

and

$$dI_t = (\mu_I - \delta_I)I_t dt + \sigma_I I_t dZ_t, \tag{2}$$

where both W_t and Z_t are standard Brownian motions and ρ is the correlation coefficient of W_t and Z_t. Here, the expected rates of return of the stock and the index are denoted by μ_S and μ_I, respectively, the volatilities by σ_S and σ_I, and continuous dividend yields by δ_S and δ_I, and it is assumed that these are all constants. In addition, there is a riskless asset, that is, a money market account that has dynamics:

$$dM_t = rM_t dt, \quad M_0 = 1, \tag{3}$$

where the riskless interest rate r is assumed to be a constant.

As in Johnson and Tian (2000b), we consider the following relation between stock and index returns:

$$\mu_S = r + \beta(\mu_I - r), \tag{4}$$

where β is the same as that in the capital asset pricing model (CAPM):

$$\beta = \rho\frac{\sigma_S}{\sigma_I} \tag{5}$$

If the index is the market portfolio, equation (4) is identical to the CAPM. Equation

(4) also states that the excess return on the firm's stock is zero. Since the excess performance $(\mu_S - r) - \beta(\mu_I - r)$ measures the executive's performance (e.g., Holmstrom and Milgrom (1987)), a zero excess return implies that the firm assesses executive performance based on firm-specific performance.

To set up a benchmark exercise price that reflects only the firm's performance after filtering out market performance, Johnson and Tian (2000b) suggest that the benchmark exercise price H_t be set as the conditional expectation of the stock price S_t on the market index I_t; that is, $H_t = E[S_t \mid I_t]$. Johnson and Tian (2000b) derive the benchmark exercise price:

$$H_t = S_0 \left(\frac{I_t}{I_0} \right)^{\beta} e^{\eta t}, \tag{6}$$

where

$$\eta = r - \delta_S - \beta(r - \delta_I) + \frac{1}{2} \rho \sigma_S \sigma_I (1 - \beta). \tag{7}$$

Under this assumption, the payoff to a firm executive at the option's maturity date T is

$$\max(S_T - H_T, 0). \tag{8}$$

This implies that the possibility of the firm’s performance beating the market performance at the option's maturity is an important factor in determining the option’s value.

However, even when the firm’s performance is better than the market’s performance before the option’s maturity, the option’s payoffs can be very small if the firm’s performance is poor near the option’s maturity. Contrary to the Johnson and Tian’s assumption, we consider the benchmark exercise price as the average of executive performance, filtering out average market performance. The averages of the executive and market performance from the grant date to the option's maturity date are calculated by

$$\exp\left(\frac{1}{T} \int_0^T \ln S_u du \right) \tag{9}$$

and

$$\exp\left(\frac{1}{T}\int_0^T \ln I_u du\right). \tag{10}$$

We define the new benchmark exercise price as

$$B_T = E[A_T^S \mid A_T^I]. \tag{11}$$

where A_T^S and A_T^I denote $\exp\left(\frac{1}{T}\int_0^T \ln S_u du\right)$ and $\exp\left(\frac{1}{T}\int_0^T \ln I_u du\right)$, respectively.

Equation (11) represents the firm's average performance against the market average performance if the index is the market portfolio. From equations (4) and (11), the firm rewards executives based on the idiosyncratic portion of the average performance after filtering out the market average performance. Then the payoff at maturity is

$$\max(S_T - B_T, 0). \tag{12}$$

Lemma 1. *Since* $\ln A_t^S$ *and* $\ln A_t^I$ *follow a bivariate normal distribution,*[8] *the conditional expectation of* A_t^S *on* A_t^I, $E[A_t^S \mid A_t^I]$ *is*

$$S_0\left(\frac{A_t^I}{I_0}\right)^{\beta} e^{\phi t}, \tag{13}$$

where

$$\phi = \frac{1}{2}(r - \delta_S - \beta(r - \delta_I)) - \frac{1}{6}(\sigma_S^2 - (3 - 2\beta)\beta\sigma_I^2)). \tag{14}$$

Proof. The derivation of the benchmark exercise price is provided in Appendix A.

To make the design of an option grant flexible, we introduce an additional parameter, λ into the exercise price:

$$\lambda S_0\left(\frac{A_t^I}{I_0}\right)^{\beta} e^{\phi t}. \tag{15}$$

8 See the Lemma A1 of Hansen and Jørgensen (2000).

Then the payoff of the averaging indexed executive stock option at maturity T is

$$\max\left(S_T - \lambda S_0\left(\frac{A_T^I}{I_0}\right)^{\beta} e^{\phi T}, 0\right). \tag{16}$$

Generally, in practice, the executive stock option is at-the-money on the date of grant. If λ is set to one, then one can check that the option defined in equation (16) is, by construction, at-the-money on the grant date.[9] If λ is greater (smaller) than one, then the option is initially out-of-the-money (in-the-money).

Now we proceed to the valuation of the averaging indexed executive stock option with payoffs of equation (16) at maturity. Because of no arbitrage opportunities, there exists the equivalent martingale measure Q with respect to the probability measure P. Under the measure Q, the firm's stock and index processes are

$$dS_t = (r - \delta_S)S_t dt + \sigma_S S_t dW_t^Q \tag{17}$$

and

$$dI_t = (r - \delta_I)I_t dt + \sigma_I I_t dZ_t^Q \tag{18}$$

where W_t^Q and Z_t^Q are standard Brownian motions under Q.

To determine the value of the indexed executive stock option at time t, we need to calculate the equation

$$c_t = e^{-r(T-t)} E_t^Q\left[\max\left(S_T - \lambda S_0\left(\frac{A_T^I}{I_0}\right)^{\beta} e^{\phi T}, 0\right)\right] \tag{19}$$

where $E_t^Q[\cdot]$ is the expectation operator at time t under Q. As in Johnson and Tian (2000b), this averaging indexed option can be considered as an option to exchange B_T for executive's stock S_T. Thus, we can apply the exchange option formula of Margrabe (1978) to equation (19).

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[9] This is because by the L'Hopital's rule, A_T^I approaches I_0 as T approaches zero.

Theorem 1. *The value of the averaging indexed option at time* t *is given by*

$$c_t = S_t e^{-\delta_S (T-t)} N(d_1) - \lambda \left(\frac{S_0}{I_0^{\beta}} \right) e^{-r(T-t)+\gamma} N(d_2), \tag{20}$$

where

$$\gamma = \beta m + \frac{(T-t)^3}{6T^2} \rho^2 \sigma_S^2 + \phi T \tag{21}$$

$$m = \frac{t}{T} \ln A_t^I + \frac{T-t}{T} \ln I_t + \frac{(T-t)^2}{2T} (r - \delta_I - \sigma_I^2/2) \tag{22}$$

$$d_1 = \frac{\ln\left(\frac{S_t I_0^{\beta}}{S_0} \right) + (r - \delta_S + \sigma_S^2/2)(T-t) - \phi T - \beta m - \frac{(T-t)^2}{2T} \rho^2 \sigma_S^2}{\sigma_S \sqrt{(T-t) + \rho^2 \left((T-t)/3T - 1 \right) (T-t)^2 / T}} \tag{23}$$

$$d_2 = d_1 - \sigma_S \sqrt{(T-t) + \rho^2 \left((T-t)/3T - 1 \right) (T-t)^2 / T} \tag{24}$$

and $N(\cdot)$ *is a cumulative standard normal distribution.*

Proof. The proof is presented in Appendix B.

4. Numerical Analysis

4.1 Probability of Expiring In-The-Money

This subsection examines the probability of expiring in-the-money at the option's maturity (the firm's payout probability). To obtain payout probabilities, we choose the parameter values used in Hall and Murphy (2002): no dividends ($\delta_S = \delta_I = 0$), $\beta = 1$, $\sigma_S = 0.2$, the risk-free rate of 6%, and an equity premium of 6.5%.[10] In addition, the volatility of the market index σ_I is assumed to be 0.15. This implies that the correlation coefficient between the firm and the market index returns is 0.5. Before investigating the in-the-money probabilities for option's

[10] Here, the above risk-free rate and an equity premium values are (simply) compounded annual returns. Therefore, we calculate continuous compounded annual returns and then use these values to be applied in this model.

moneyness, as in Hall and Murphy (2002), the differences between traditional and indexed (averaging indexed) options need to be considered. While the in-the-money probabilities can be calculated for each moneyness in the case of traditional options, indexed (averaging indexed) options, by construction, do not allow one to consider their moneyness. Therefore, using the above manipulation of the exercise price of averaging indexed options λ is added to equation (6) as follows:

$$\lambda S_0 \left(\frac{I_t}{I_0} \right)^{\beta} e^{\eta t} . \tag{25}$$

This is the same exercise price as given by the equation (21) of Johnson and Tian (2000b), that is, $\lambda S_0 (I_t / I_0)^{\beta} e^{(g+\eta)t}$, when g is equal to zero.

We examine six values of the current stock price (\$10, \$20, \$30, \$40, \$50, \$60). The exercise price λS_0 is set at \$30. Given the current stock prices and the fixed exercise price at the grant date, λ takes on the values: 3, 1.5, 1, 0.75, 0.6 and 0.5.

Table 1: **Probabilities that will be in-the-money at the option's maturity**

Stock price	T	I	A
10	0.3480	0.0402	0.1582
20	0.6330	0.1829	0.4277
30	0.7785	0.3406	0.6169
40	0.8578	0.4758	0.7381
50	0.9042	0.5835	0.8163
60	0.9329	0.6674	0.8680

Table 1 provides probabilities of expiring in the money at the option's maturity with respect to stock price. We examine payout probabilities using the following parameter values: $\delta_S = \delta_I = 0$, $\beta = 1$, $\sigma_S = 0.3$, *and* $\sigma_I = 0.15$. *The time to maturity is ten years. The risk-free rate is 6% and an equity premium is 6.5%. T, I, and A denote traditional, indexed and averaging indexed options, respectively.*

Table 1 illustrates the probabilities that options with a 10-year maturity will expire in-the-money. As stated in Johnson and Tian (2000b), their model's payout probabilities are much lower than those implied in the Black-Scholes model (traditional option).[11] The probability that an at-the-money indexed option ($S_0 = \$30$) will expire in-the-money does not exceed 50%. Since many companies these days tend to grant out-of-the-money executive stock options on the grant date (Johnson and Tian (2000b)), it is interesting to examine the probabilities that the options are in-the-money at the option's maturity when S_0 is less than \$30. Table 1 reveals that the lower the initial stock price, the greater the differences of the in-the-money probabilities.

Now we examine the probabilities that an averaging indexed option will expire in- the-money. For averaging indexed options, the probability of expiring in-the-money is

$$N\left(\frac{\left\{6\beta(\delta_I - r) + 6(r - \delta_S) - (5 - 2\rho^2)\sigma_S^2 - \ln\lambda\right\}\sqrt{T}}{12\sqrt{\dfrac{\beta^2\sigma_I^2 + 3\sigma_S^2}{3} - \dfrac{2\rho^2\sigma_S^2}{\sqrt{3}}}}\right). \qquad (26)$$

This equation leads to the following results. First, the probabilities that an averaging indexed option will be in-the-money after 10 years are much higher than corresponding Johnson and Tian's corresponding probabilities. When S_0 is \$30, that is, the option is initially set at the money, the payout probability of the averaging indexed option is around 62%, while the payout probability of the indexed option is around 34%. The averaging indexed option's payout probability is twice as high as the indexed option's. The lower S_0 is, the more significant the differences are. Second, the differences between the payout probabilities of the traditional option and those of the averaging indexed option are not relatively large. For at-the-money options, the difference in payout probabilities between traditional and averaging indexed options is around 16% point, while the difference in payout probabilities between traditional and indexed options is around 44%.

[11] Although Hall and Murphy (2002) do not use the Johnson and Tian model but use a certainty equivalence approach to calculate the payout probability, they also obtain the similar results.

In sum, we show that the payout probability of the indexed option is very low. In addition, the payout probabilities of averaging indexed options are higher than those of indexed options irrespective of moneyness. Since the difference in payout probabilities between traditional and averaging indexed options is not relatively large, granting traditional options can be replaced by granting averaging indexed options without incurring additional costs.

4.2 Implications of Option Prices

This subsection investigates the option prices of traditional, indexed and averaging indexed options on the grant date ($t=0$). The base parameter values are as follows: Both the stock and index continuous dividend yields are 2%. The index volatility is 15%. The correlation between the stock and index returns is 0.75. The option values are examined at three stock prices ($90, $100, and $110) and for three values of stock return volatility (0.1, 0.2, and 0.3). For traditional options the strike price is $100. The risk-free interest rate is 8%. Note that on the grant date, the stock price is the same as the strike price for indexed and averaging indexed options.

Table 2: Prices of traditional, indexed and averaging indexed options on grant date

Stock price	Volatility	T	I	A
90	0.1	29.2086	3.1859	15.4861
90	0.2	33.1372	9.1931	21.8746
90	0.3	38.6094	15.3228	28.8533
100	0.1	37.1516	6.8194	22.6373
100	0.2	40.3530	13.5648	28.3549
100	0.3	45.5975	20.1643	35.3187
110	0.1	45.2252	11.9815	30.3215
110	0.2	47.8027	18.6991	35.2668
110	0.3	52.7731	25.5018	42.0824

Table 2 describes values for traditional, indexed and averaging indexed options on grant date. To calculate option prices, the values of parameters are as follows: Both the stock and *index continuous dividend yield are 2%. The index volatility is 15%. The correlation between stock and index returns is 0.75. For traditional options the strike price is $100. T, I, and A denote traditional, indexed and averaging indexed options, respectively.*

Traditional options have higher values than their indexed and averaging indexed counterparts, which is consistent with Hall and Murphy (2002), where the traditional option values are too high. Also, the differences in value between traditional and indexed options are greater than between traditional and averaging indexed options. For example, for the case where $S_0 = \$100$ and $\sigma_S = 0.2$, the averaging indexed option value is $28.35, or 70.3% of the traditional option counterpart ($40.35), while the indexed option value is $13.56, or 33.6% of the traditional option counterpart ($40.35). The results show that the lower the stock price and return volatility, the greater the differences in value.

Table 3: Exercise values of traditional, indexed and averaging indexed options at the maturity of the options

Panel A

$\beta = 0.75$					$\beta = 1.0$		$\beta = 2.0$	
μ_S	μ_I	T	I	A	I	A	I	A
0.25	0.20	215.82	104.12	182.09	69.86	171.68	0.00	89.76
0.20	0.20	145.96	34.26	112.23	0.00	101.82	0.00	19.90
0.15	0.20	91.55	0.00	57.83	0.00	47.41	0.00	0.00
0.20	0.15	145.96	70.45	124.20	54.41	118.76	0.00	80.57
0.10	0.10	49.18	3.68	38.32	0.00	36.93	0.00	28.18
0.07	0.10	28.40	0.00	17.54	0.00	16.15	0.00	7.40
0.10	0.07	49.18	19.16	44.38	20.78	45.03	30.06	48.87

Panel B

		$\beta = 0.75$			$\beta = 1.0$		$\beta = 2.0$	
μ_S	μ_I	T	I	A	I	A	I	A
-0.05	-0.05	0.00	0.00	0.00	0.00	0.00	33.68	13.73
-0.03	-0.05	0.00	0.00	0.00	7.41	0.00	41.09	21.14
-0.05	-0.03	0.00	0.00	0.00	0.00	0.00	25.54	7.76
-0.15	-0.15	0.00	0.00	0.00	0.00	0.00	29.21	8.33
-0.08	-0.15	0.00	3.67	0.00	17.91	0.00	47.12	26.24
-0.15	-0.08	0.00	0.00	0.00	0.00	0.00	15.49	0.00
-0.20	-0.10	0.00	0.00	0.00	0.00	0.00	10.97	0.00

Table 3 describes the exercise values for traditional, indexed and averaging indexed options at the maturity of the options. Parameter values are as follows: Both the stock yield and the index continuous dividend yield are 2%. The stock volatility is 45% and index volatility is 15%. For traditional options the strike price is $100. T, I, and A denote traditional, indexed and averaging indexed options, respectively.

Table 3 shows the option's exercise values that are calculated by the expected stock prices at the option's maturity less the expected benchmark exercise prices. To examine the exercise values under different values of β (systematic risk), the parameter values used in Table 2 are selected, except that $T = 5$. Also, $\ln A_t^S$ follows a normal distribution with mean $\ln S_0 + (\mu_S - \delta_S - \sigma_S^2/2)t/2$ and variance $\sigma_S^2 t/3$. Panels A and B of Table 3 illustrate the exercise values when the expected returns of the stock and index are positive (bull markets) and negative (bear markets), respectively. One can observe several results. First, in Panel A of Table 3, whenever the firm's expected return is lower than (or the same as) the index counterpart, the exercise values of the indexed option are zero. On the contrary, if the firm's expected return is higher than the index counterpart, then the indexed option has nonzero exercise values only when β is smaller than one, that is, when the systematic risk is low. Despite the firm's expected return being higher than the index return, the exercise values of the indexed option are very small or zero. In contrast to the indexed option, most exercise values of the averaging indexed option are nonzero. Although the firm's expected return is lower than the index counterpart, the exercise values of the averaging indexed option are still nonzero (except for the case where $\mu_S = 0.15$, $\mu_I = 0.15$ and $\beta = 2.0$). This may be caused by averaging effects when calculating the benchmark exercise price. Second, Panel B of Table 3 shows that when β is greater than one, the averaging option can protect shareholders from excessively rewarding executives. In bear markets, the exercise values of the averaging indexed option are zero whenever the firm's expected return is lower than the index counterpart. However, indexed options provide higher payoffs even though the firm's expected return is lower than the index counterpart. This contrasts with the zero exercise values of the indexed option as long as firm performance is poorer than the index counterpart in Panel A of Table 3.

In sum, indexed options are useful for protecting shareholders from excessively compensating poorly performing executives during market upturns. While this effect appears in Panel A of Table 3, indexed options can excessively reward executives with poor performance in bear markets. In contrast, averaging indexed options can be a more efficient scheme to tie the benefits from options to executive performance.

5. Conclusions

The executive stock option plan is an important and good vehicle for alleviating agency costs between shareholders and executives. Although many firms have granted executive stock options until now, problems associated with them are brought up in academic studies. One of the problems is that during 1990s' bull market many executives were rewarded excessively relative to their performance. To settle this problem, several studies propose indexing the strike price of executive stock options to the market. This indexed option plan protects shareholders from compensating executives too much because the option has value at maturity only when firm performance is better than the index performance. However, contrary to expectations, practically no large US firms have granted such indexed options. This is because indexed option values are much lower than those of traditional option counterparts, and because indexed options may not be attractive to risk-averse executives.

This paper designs a new indexed option model. The strike price of this indexed option is set to be the firm's average performance relative to the market's average performance. Using this indexation scheme, this paper compares the values of averaging indexed options with those of traditional and existing indexed options. As stated in Hall and Murphy (2000), the payout probabilities of indexed options are too small to act as incentives to risk-averse executives; in contrast, averaging indexed options have significantly higher payout probabilities. In line with higher probabilities, the differences in value between traditional and averaging indexed options are smaller than between traditional and indexed options. This study also shows that in bear markets indexed options can generate excess rewards, even to a poorly performing executive, and that averaging indexed options can remedy this problem.

While this paper focuses on the design of a new indexed stock option, the investigation of the incentive effects of various indexed option models is an important issue for future research.

Appendix A. Proof of Lemma 1

By the Lemma A1 of Hansen and Jørgensen (2000), $\ln A_t^S$ follows a normal distribution with mean $\ln S_0 + \frac{\mu_S - \delta_S - \sigma_S^2/2}{2}t$ and variance $\frac{\sigma_S^2}{3}t$. Similarly, $\ln A_t^I$ follows a normal distribution with mean $\ln I_0 + \frac{\mu_I - \delta_I - \sigma_I^2/2}{2}t$ and variance $\frac{\sigma_I^2}{3}t$. Also, covariance between $\ln A_t^S$ and $\ln A_t^I$ is $\frac{\rho\sigma_S\sigma_I}{3}t$.

Let m_S and m_I be $\ln S_0 + \frac{\mu_S - \delta_S - \sigma_S^2/2}{2}t$ and $\ln I_0 + \frac{\mu_I - \delta_I - \sigma_I^2/2}{2}t$, respectively. From the well known conditional distribution on joint bivariate normal distribution (see Kotz, Balakrishnan, and Johnson (2000)), the distribution of $\ln A_t^S$ conditional on $\ln A_t^S$ is a normal distribution with mean $m_S t + \rho \frac{\sigma_S}{\sigma_I}(a - m_I)$ and variance $\frac{\sigma_S^2(1-\rho^2)}{3}t$.

Then the conditional expectation can be calculated as follows:

$$E\left[A_t^S \middle| A_t^I\right] = E\left[\exp\left(\ln A_t^S\right) \middle| \exp\left(\ln A_t^I\right)\right]$$

$$= \exp\left(m_S + \rho\frac{\sigma_S}{\sigma_I}(\ln A_t^I - m_I) + \frac{\sigma_S^2(1-\rho^2)}{6}t\right)$$

$$= \left(A_t^I\right)^\beta \exp\left(m_S + \beta m_I + \frac{\sigma_S^2(1-\rho^2)}{6}t\right)$$

Inserting the values of m_S and m_I into the above equation delivers

$$S_0\left(\frac{A_t^I}{I_0}\right)^\beta \exp\left(\frac{1}{2}(r - \delta_S - \beta(r - \delta_I) - \frac{1}{6}(\sigma_S^2 - (3-2\beta)\beta\sigma_I^2))t\right).$$

Let ϕ be $\frac{1}{2}(r - \delta_S - \beta(r - \delta_I) - \frac{1}{6}(\sigma_S^2 - (3-2\beta)\beta\sigma_I^2))$. Then we obtain a benchmark exercise price of equation (13).

Appendix B. Proof of Theorem 1

When pricing averaging indexed stock options, the following Lemma is useful.

Lemma A. *Let X and Y be joint normal random variables with mean vector M_{XY} and covariance matrix Σ_{XY} as follows:*

$$M_{XY} = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \ \Sigma_{XY} = \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix} \tag{27}$$

Then

$$E\left[e^X \middle| X \geq Y + k\right] = e^{\mu_X + \sigma_X^2/2} N\left(\frac{-k + \mu_X - \mu_Y - \rho\sigma_X\sigma_Y + \sigma_X^2}{\sqrt{\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y}}\right) \tag{28}$$

and

$$E\left[e^{Y}\middle|X \ge Y+k\right] = e^{\mu_Y+\sigma_Y^2/2}N\left(\frac{-k+\mu_X-\mu_Y+\rho\sigma_X\sigma_Y-\sigma_Y^2}{\sqrt{\sigma_X^2+\sigma_Y^2-2\rho\sigma_X\sigma_Y}}\right), \tag{29}$$

where k *is a constant.*

Proof of Lemma A. See Lemma 5, Appendix A in Amin and Jarrow (1991).

The price of the averaging indexed option can be obtained from the calculation of the following expectation.

$$c_t = e^{-r(T-t)}E_t^Q\left[\max\left(S_T-\lambda S_0\left(\frac{A_T^I}{I_0}\right)^{\beta}e^{\phi T},0\right)\right]$$

The above equation is rewritten as

$$\begin{aligned} &e^{-r(T-t)}E_t^Q\left[e^{\ln S_T}\middle|\ln S_T \ge \beta\ln A_T^I+\ln S_0-\beta\ln I_0+\phi T\right] \\ &-e^{-r(T-t)}\left(\frac{S_0}{I_0^{\beta}}\right)e^{\phi T}E_t^Q\left[e^{\beta\ln A_T^I}\middle|\ln S_T \ge \beta\ln A_T^I+\ln S_0-\beta\ln I_0+\phi T\right] \end{aligned} \tag{30}$$

Since $\ln S_T$ and $\beta\ln A_T^I$ are normally distributed, one can apply these into the above Lemma by denoting $\ln S_T$ and $\beta\ln A_T^I$ by X and Y, respectively. Then the means and variances of X and Y are

$$E_t^Q\left[X\right] = \ln S_t + \left(r-\delta_S-\frac{\sigma_S^2}{2}\right)(T-t) \tag{31}$$

$$E_t^Q\left[Y\right] = \beta\frac{t}{T}\ln A_t^I+\beta\frac{T-t}{T}\ln I_t+\beta\frac{\left(r-\delta_I-\frac{\sigma_I^2}{2}\right)}{2T}(T-t)^2 \tag{32}$$

$$Var_t^Q\left[X\right] = \sigma_S^2(T-t) \tag{33}$$

$$Var_t^Q\left[Y\right] = \beta^2\frac{\sigma_I^2}{3T^2}(T-t)^3 \tag{34}$$

Also, the covariance between X and Y is

$$\operatorname{cov}_t^Q\left(X,Y\right)=\beta\frac{\rho\sigma_S\sigma_I}{2T}(T-t)^2 \tag{35}$$

A straightforward calculation delivers the equation (20).

Acknowledgement: I am very grateful to Joonho Hwang, Young Ho Oh, and seminar participants at Hallym University, Korea University and 2008 Joint Conference with Allied Korea Finance Associations for valuable comments. An earlier version of this paper was circulated under the title "An Indexed Executive Stock Option: Design, Pricing and Incentive Effects."

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