Professor Alvydas BALEŽENTIS, PhD
Mykolas Romeris University, Lithuania
Tomas BALEŽENTIS
Vilnius University, Lithuania
E-mails: a.balezentis@gmail.com; t.balezentis@gmail.com

A NOVEL METHOD FOR GROUP MULTI-ATTRIBUTE DECISION MAKING WITH TWO–TUPLE LINGUISTIC COMPUTING: SUPPLIER EVALUATION UNDER UNCERTAINTY

Abstract. *The aim of this study was to extend the MULTIMOORA–2T method for group multi–criteria decision making under linguistic environment. Whereas the previously offered MULITMOORA–2T is aimed at dealing with quantitative assessments, the new method MULTIMOORA–2T–G tackles qualitative ones. More specifically, expert evaluations expressed in 2–tuples were summarized aggregated by employing 2–tuple ordered weighted averaging (TOWA) operator and thus a new method, namely MULTIMOORA–2T–G, was offered. The method enables to summarize expert assessments when information about their significance is completely unknown. The numerical example successfully exhibited the possibilities for application of the MULTIMOORA–2T–G method. In this particular case the committee consisting of five experts ranked the potential suppliers by applying the new method with respect to different criteria identifying the potential of the considered suppliers. Such decision making method enables a company to develop an appropriate procurement policy under vague conditions.*

Keywords: *MULTIMOORA, MCDM, multi–objective optimization, group decision making, 2-tuple, TOWA operator, soft computing, linguistic reasoning, supplier selection, expert evaluation.*

JEL Classification: C44, D81, M10.

1. INTRODUCTION

The contemporary trends of the globalization of competition, diversification of customers' needs, and rapid technological advances require enterprises to make an increasing number of strategic decisions at a frenetic pace. Moreover, a firm should establish and maintain long term strategic relationships with its suppliers in order to effectively manage supply chain processes (Yucel and

Guneri, 2011). As findings of the previous studies suggest, manufacturers tend to spend some 60 per cent of total sales revenue on purchased items, namely raw materials, parts, and components (Krajewsld, Ritzman, 1996). These purchases, in turn, constitute up to 70 per cent of product cost (Ghodsypour and O'Brien, 1998). As Gencer and Gurpinar (2007) reported, over 50 per cent of all quality defects can be traced back to purchased material. The aforementioned findings, hence, suggest supplier selection to be one of the most important objectives of business activity.

On the other hand, supplier selection decision-making problem involves trade-offs among multiple criteria that involve both quantitative and qualitative factors, which may also be conflicting (Ghodsypour and O'Brien, 1998). In his reference work Dickson (1966) identified 23 commonly used criteria for supplier selection. The later studies also provided a comprehensive research of supplier selection criteria (Weber *et al.*, 1991; Evans, 1980; Shipley, 1985; Ellram, 1990). Meanwhile, some new factors, for instance, environmental requirements, are getting to be more and more important in contemporary business relations (Jabbour and Jabbour, 2009).

MCDM methods, therefore, have been widely applied in studies on supplier selection. Due to its complexity and uncertainty, the supplier selection problem might be solved by employing fuzzy logic (Zadeh, 1965). Wang *et al.* (2011) offered a supplier selection model based on fuzzy TOPSIS method. In another case, the two–tuple linguistic representation was employed to perform group decision making procedure for supplier selection (Wang, 2010). Fuzzy number and linear programming were used by Yucel and Guneri (2011), Haleh and Hamidi (2011) to solve the selection problem. Cebi and Bayraktar (2003) has structured the supplier selection problem as an integrated lexicographic goal programming and analytic hierarchy process (AHP) model including both quantitative and qualitative conflicting factors. An interrelated problem of order allocation can also be solved by using the results of MCDM analysis. The mathematical programming, hence, is a powerful tool for supplier selection exercises. Amin *et al.* (2011) combined fuzzy SWOT analysis and linear programming for that purpose. For instance, Sevkli *et al.* (2008) combined AHP and fuzzy linear programming method to handle supplier selection and order allocation problems. Liao and Kao (2011) applied multi–choice goal programming and fuzzy TOPSIS for supplier selection. Being a complex issue, nevertheless, the supplier selection problem requires new and new MCDM models for its solution.

The MULTIMOORA (Multi–Objective Optimization by Ratio Analysis plus the Full Multiplicative Form) method was introduced and developed by Brauers and Zavadskas (2006, 2010a). MULTIMOORA summarizes three methods thus offering robust ranking options. The method was applied in manufacturing and engineering environment (Kracka *et al.*, 2010; Chakraborty, 2010; Brauers *et al.*, 2008a, 2008b; Kalibatas, Turskis, 2008) as well as regional development studies (Brauers, Zavadskas, 2010b, 2011b; Brauers, Ginevičius, 2010; Brauers *et al.*, 2007, 2010; Baležentis *et al.*, 2010). Brauers and Zavadskas (2011a) introduced the Dominance theory for MULTIMOORA. In addition, MULTIMOORA has been extended to fuzzy environment (Brauers *et al.*, 2011) as well as 2–tuple data representation (Baležentis, Baležentis, 2011).

Herrera *et al.* (2000) contributed to the computing with words by introducing two–tuple linguistic representation approach. Two–tuples are used to represent, convert and map into the basic linguistic term set various crisp and fuzzy numbers. As 2–tuple linguistic is a powerful MCDM tool, many studies aimed at application and development of the method are present (Chen, Ben–Arieh, 2006; Martinez *et al.*, 2007; Tai, Chen, 2009; Halouani *et al.*, 2009; Wang, 2009; Liu, 2009; Wei, 2011; Li, 2009; Dursun, Karsak 2010; Chang, Wen, 2010; Wang, 2010; Xu and Wang, 2011; Liu, Zhang, 2011).

The aim of this study is to extend the MULTIMOORA–2T method (Baležentis, Baležentis 2011) for group decision making under linguistic environment. More specifically, expert evaluations expressed in 2–tuples will be aggregated by employing 2–tuple ordered weighted averaging (TOWA) operator and thus a new method, namely MULTIMOORA–2T–G, shall be offered. The article is organized in the following manner. Section 2 presents preliminaries of the applied methods, namely 2–tuple linguistic representation, TOWA operator, and MULTIMOORA–2T. The following Section 3 describes the proposed method MULTIMOORA–2T–G. A supplier selection according the new method is offered in Section 4.

2. PRELIMINARIES

This section consists of three parts, each describing the basic principles of 2–tuple decision–making. The first subsection presents 2–tuple linguistic representation logics. The following subsection is focused on developments of the ordered weighted averaging (OWA) operator with particular interest on TOWA operator. The third subsection provides one with MULTIMOORA–2T method.

2. 1. Two–tuple linguistic representation

After introducing fuzzy set theory, Zadeh (1975) described the fuzzy linguistic variables. The linguistic variables are very useful when describing various vague phenomena, which cannot be reasonably expressed in ordinary quantitative terms (Wang, 2009). Indeed, linguistic terms are often peculiar with finite set, odd cardinality, semantic symmetric, ordinal level, and compensative operation (Herrera-Viedma *et al.*, 2003). Consequently, Herrera and Martinez (2000a, 2000b, 2001) developed the 2–tuple linguistic representation model with various aggregation operators (Herrera *et al.*, 2000). The main advantage of such representation is the continuity of its domain. Hence any counting of information might be expressed in the universe of discourse. Moreover appropriate techniques prevents from loss of information during computing with words.

The linguistic information is expressed in a pair of values—2–tuple—consisting linguistic term and a number. Let us take, for instance, a 2–tuple $L=(s,\alpha)$, where s stands for the linguistic label of the information and α represents the symbolic translation. Actually, one can define any ordered set of linguistic terms $S_{g+1}=(s_0,s_1,...s_i,...,s_g)$, containing $g+1$ labels. As it was mentioned before, there should be odd cardinality, namely $g+1$. Let the set S_{g+1} has the following characteristics (Martinez *et al.*, 2007): 1) a negation operator $Neg(s_i)=s_j$ such that $j=g-i$; 2) a *min* and *max* operators, i. e. $s_i \le s_j \Leftrightarrow i \le j$, where $i,j \in [0,g]$. It is considered, that seven or so linguistic terms can be effectively applied (Miller, 1956). Any label $s_i=(a_i,b_i,c_i)$ can be defined in the following way (Liu, 2009):

$$\begin{cases} a_o = 0; \\ a_i = \dfrac{i-1}{g}, 1 \le i \le g; \\ b_i = \dfrac{i}{g}, 0 \le i \le g; \\ c_i = \dfrac{i+1}{g}, 0 \le i \le g-1; \\ c_g = 0. \end{cases} \tag{1}$$

However, different decision makers can use different scales (so called granularity of uncertainty), which need to be mapped onto single basic linguistic term set (BLTS) S_T. The latter set should contain the maximum number of labels if compared to scales used by different decision makers.

Definition 1. Let $\beta \in [0, g]$ be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S_{g+1}, i. e., the result of a symbolic aggregation operation. Let $i = round(\beta)$ and $\alpha = \beta - 1$ be two values, such that $i \in [0; g]$ and $\alpha \in [-0.5, 0.5)$, then α is called a symbolic translation (Herrera *et al.*, 2005; Wei, 2011).

Then linguistic representation model handles the linguistic evaluations by means of 2–tuples (s_i, α_i), where $s_i \in S_{g+1}$ and $\alpha_i \in [-0.5, 0.5)$.

Definition 2. Let $S_{g+1} = (s_0, s_1, ..., s_g)$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of symbolic aggregation operation. Then the following function returns the respective 2–tuple:

$$\Delta : [0, g] \to S_{g+1} \times [-0.5, 0.5)$$
$$\Delta(\beta) = \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where *round* is the usual rounding operation, s_i has the closest index value to β, and α is called a symbolic translation (Herrera *et al.*, 2000).

Given Definitions 1 and 2, and if S_{g+1} is the BLTS, the linguistic term of BLTS may be represented by respective 2–tuple:

$$s_i \in S_{g+1} \overset{\Delta}{\Rightarrow} (s_i, 0) \tag{3}$$

Definition 3. Let $S_{g+1} = (s_0, s_1, ..., s_g)$ be a linguistic term set and (s_i, α_i) be a 2–tuple. There exists a function Δ^{-1} which, according to 2–tuple, returns its equivalent value $\beta \in [0, g] \subset \mathbb{R}$ (Herrera *et al.*, 2000; Wei, 2011):

$$\Delta^{-1} : S_{g+1} \times [-0.5, 0.5) \to [0, g]$$
$$\Delta^{-1}(s_i, \alpha_i) = i + \alpha = \beta \tag{4}$$

Definition 4. Let (s_k, α_k) and (s_l, α_l) be two 2–tuples. Then (Herrera *et al.*, 2000):

If $k < l$, then $(s_k, \alpha_k) \prec (s_l, \alpha_l)$.

If $k = l$, then a) if $a_k = a_l$, then $(s_k, \alpha_k) \sim (s_l, \alpha_l)$;

b) if $a_k < a_l$, then $(s_k, \alpha_k) \prec (s_l, \alpha_l)$;

c) if $a_k > a_l$, then $(s_k, \alpha_k) \succ (s_l, \alpha_l)$.

Definition 5. A 2–tuple negation operator is the following (Herrera *et al.*, 2005):

$$Neg(s_i, \alpha_i) = \Delta\left(g - \left(\Delta^{-1}(s_i, \alpha_i)\right)\right) \tag{5}$$

Definition 6. Let $x = \{(r_1, \alpha_1), (r_2, \alpha_2), ..., (r_n, \alpha_n)\}$ be a set of 2–tuples from S_{g+1}. Then the 2–tuple arithmetic average is obtained as (Herrera, Martinez, 2000a, 2000b):

$$\overline{x} = (\overline{r}, \overline{a}) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(r_j, a_j)\right), \overline{r} \in S_{g+1}, \overline{a} \in [-0.5, 0.5) \tag{6}$$

Definition 7. Let $L_k = (s_k, \alpha_k)$ and $L_l = (s_l, \alpha_l)$ be two 2–tuples, then

$$d\left(L_k, L_l\right) = \Delta\,|\,\Delta^{-1}(s_k, \alpha_k) - \Delta^{-1}(s_l, \alpha_l)\,| \tag{7}$$

is called the distance between L_k and L_l (Herrera, Martinez, 2000a, 2000b).

Definition 8. Let $x = \{(r_1, \alpha_1), (r_2, \alpha_2), ..., (r_n, \alpha_n)\}$ be a set of 2–tuples from S_{g+1}. Then the 2–tuple geometric average is computed in the following way:

$$\prod_{j=1}^{n}(r_j, a_j) = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(r_j, a_j)\right)^{\frac{1}{n}}\right) \tag{8}$$

Definiton 9. Let *I* be real number, interval number, triangular fuzzy number, or trapezoidal fuzzy number etc., and $S_{g+1} = (s_0, s_1, ..., s_g)$ be linguistic term set (Gong, 2007; Gong, Liu, 2007; Liu, 2009). Thereafter *I* can be converted into 2–tuple linguistic set by the following mapping:

$$\begin{aligned} &\tau : [0,1] \to F(S_{g+1}) \\ &\tau(I) = \left\{ (s_i, \alpha_i) \mid i \in [0,1,...,g] \right\} \\ &\alpha_i = \max_y \min \left\{ \mu_I(y), \mu_{s_i}(y) \right\} \end{aligned} \tag{9}$$

where $\mu_I(y)$ and $\mu_{s_i}(y)$ are membership functions associated with *I* and s_i, respectively.

Definition 10. Let $\tau(I) = \left\{ (s_i, \alpha_i) \mid i \in [0,1,...,g] \right\}$ be 2–tuple linguistic value of the uncertain fuzzy number *I*, then 2–tuple linguistic set $\tau(I)$ can be converted into 2–tuple linguistic variable by mapping χ :

$$\begin{aligned} &\chi : F(S_{g+1}) \to [0, g] \\ &\chi\left(\tau(I)\right) = \chi\left(F(S_{g+1})\right) = \chi\left[(s_i, \alpha_i) \mid i \in [0,1,...,g] \right] = \sum_{i=0}^{g} i\alpha_i \Big/ \sum_{i=0}^{g} \alpha_i = \beta \end{aligned} \tag{10}$$

2. 2. The TOWA operator and weight determination thereof

The ordered weighted averaging (OWA) operator was defined by Yager (1988). Actually, the OWA operator is a generalization of such decision making criteria as maximax (optimistic), maximin (pessimistic), Laplace (equally likely), and Hurwicz criteria (Wang *et al.*, 2007). More specifically, different combinations of OWA weights (coefficients of significance) result in different types of data aggregation, including the aforementioned ones. The OWA operator, hence, provides a unified framework for decision making under uncertainty.

Definition 11. An OWA operator of dimension *n* is a mapping $F : \Re^n \to \Re$ with an associated weight vector $W = \left(w_1, w_2, ..., w_n\right)^T$ such that $\sum_i w_i = 1, w_i \in [0,1], i = 1, 2, ..., n$ and

$$F_W(a_1, a_2, ..., a_n) = \sum_{i=1}^{n} w_i b_i \,, \tag{11}$$

where b_i is the i^{th} largest value of a data set to be aggregated, namely $a_1, a_2, ..., a_n$ (Yager, 1988).

It is the re–ordering of the initial data set that allows applying the OWA operator and thus makes it different from traditional averaging operators. Application of different weight vectors for aggregating objects $a_1, a_2, ..., a_n$, results in the following special cases of OWA operator:

1) If $W = W^* = (1, 0, ..., 0)^T$, then $F_W(a_1, a_2, ..., a_n) = \max_i a_i$, which is purely optimistic decision (i. e. maximax criterion);
2) If $W = W_* = (0, 0, ..., 0, 1)^T$, then $F_W(a_1, a_2, ..., a_n) = \min_i a_i$, which is purely pessimistic decision (i. e. maximin criterion);
3) If $W = W_A = \left(1/n, 1/n, ..., 1/n\right)^T$, then $F_W(a_1, a_2, ..., a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i$, which is equally likely decision (i. e. Laplace criterion);
4) If $W = W_H = (\alpha, 0, ..., 0, 1-\alpha)^T$, then $F_W(a_1, a_2, ..., a_n) = \alpha \max_i a_i + (1-\alpha)\min_i a_i$, which is the Hurwicz decision model.

The OWA operator, therefore, fills the space between Min and Max operators. However, there have been many techniques developed to determine the weights for OWA operator. The most widely applied one is that proposed by Yager (1988). The weight for the i^{th} largest object in $a_1, a_2, ..., a_n$ is to be computed as:

$$w_i = Q(i/n) - Q((i-1)/n), i = 1, 2, ..., n, , \tag{12}$$

where $Q(y)$ is a non–decreasing relative quantifier (Zadeh, 1983):

$$Q(y)=\begin{cases} 0, & y<a; \\ \dfrac{y-a}{b-a}, & a\le y\le b; \\ 1, & b<y; \end{cases} \tag{13}$$

with a and b being parameters of the relative quantifier and $Q(y)$ indicating the degree to which y is compatible with the represented quantifier. Usually, the three quantifiers resembled by respective couples of parameters (a,b) might be employed: "most" – (0.3, 0.8), "at least half" – (0, 0.5), and "as many as possible" – (0.5, 1).

The weights for OWA operator, though, can be retrieved by employing mathematical programming. For instance, O'Hagan (1988) suggested a maximum entropy method; Fullér and Maljender (2003) offered a minimum variance method; Wang and Parkan (2005) presented a minimax disparity approach; Majlender (2005) developed a maximal Rényi entropy model; Wang *et al.* (2007) and Wang and Parkan (2007) offered some new methods. Application of these methods, however, certainly falls behind the scope of our paper.

The described OWA operator is designed to handle crisp information. Its extensions, therefore, are needed to deal with fuzzy data. Herrera *et al.* (1996) introduced the linguistic OWA (LOWA) operator. Merigo and Gil-Laufente (2011) combined OWA operator and Hamming distance for human resources management. Yager and Filev (1999) offered the induced OWA (IOWA) operator. Yager (2002) has also presented the heavy OWA (HOWA) operator. Xu and Da (2003) proposed the induced ordered weighted geometric averaging (IOWGA) operator, generalized induced ordered weighted averaging (GIOWA) operator, and hybrid weighted averaging (HWA) operator. Wei *et al.* (2006) further updated the HWA operator and thus introduced the two–tuple hybrid weighted arithmetic average operator (T-HWAA). Merigo and Casanovas (2011) suggested some new extensions about the OWA operator such as the induced heavy OWA (IHOWA) operator, the uncertain heavy OWA (UHOWA) operator and the uncertain induced heavy OWA (UIHOWA) operator. Merigo (2011) introduced the induced ordered weighted averaging–weighted average (IOWAWA) operator. Herrera and Martínez (2000a) developed 2-tuple arithmetic averaging (TAA) operator, 2-tuple weighted averaging (TWA) operator, 2-tuple ordered weighted averaging (TOWA) operator and extended 2-tuple weighted averaging (ET-WA) operator. Wei (2010) consequently proposed the extended 2-tuple weighted geometric (ET-WG) and the extended 2-tuple ordered weighted geometric (ET-OWG) operator. The TOWA operator will be used in our study when aggregating when aggregating 2–tuple linguistic variables.

Definition 12. Let $x=\{(r_1,a_1),(r_2,a_2),...,(r_n,a_n)\}$ be a set of 2–tuples. Then the TOWA operator of dimension *n* is a mapping $T:\Re^n \rightarrow \Re$ that has an associated vector $W=(w_1,w_2,...,w_n)^T$ such that $\sum_i w_i = 1, w_i \in [0,1], i=1,2,...,n$. Furthermore,

$$T_W(x)=\Delta\left(\sum_{j=1}^{n} w_j \Delta^{-1}\left(r_{\sigma(j)},a_{\sigma(j)}\right)\right)=(\hat{r},\hat{a}),\hat{r}\in S,a\in[-0.5,0.5), \tag{14}$$

where $(\sigma(1),\sigma(2),...,\sigma(n))$ is a permutation of $(1,2,...,n)$, such that $\left(r_{\sigma(j-1)},a_{\sigma(j-1)}\right)\geq\left(r_{\sigma(j)},a_{\sigma(j)}\right)$, $\forall j=2,3,...,n$ (Herrera and Martínez, 2000a; Wei, 2010).

The weight vector for TOWA operator might be obtained by employing Eq. 12.

2. 3. MULTIMOORA–2T

This subsection describes the MULIMOORA extended with 2–tuple linguistic representation (MULTIMOORA–2T). The method was presented by Baležentis and Baležentis (2011).

Data fusion. Let $A=(a_1,a_2,...,a_m)$ be the set of alternatives considered with respect to criteria $C=(c_1,c_2,...,c_n)$. Additionally, let $J_1 \subset C$ and $J_2 \subset C$ be subsets of benefit and cost criteria, respectively. The initial data are pooled in the decision matrix $X = x_{ij}$, where $i=1,2,...,m$ and $j=1,2,...,n$. Suppose $x_{ij} \mid j=1,2,...,r_1$ is real number; $x_{ij}=[x_{ij}^a,x_{ij}^d] \mid j=r_1+1,r_1+2,...,r_2$ is interval number; $x_{ij}=\left(x_{ij}^a,x_{ij}^b,x_{ij}^d\right) \mid j=r_2+1,r_2+2,...,r_3$ is triangular fuzzy number; $x_{ij}=s_{ij}\in S_{g+1} \mid j=r_3+1,r_3+2,...,r_4$ is linguistic variable; $x_{ij}=(x_{ij}^a,x_{ij}^b,x_{ij}^c,x_{ij}^d) \mid j=r_4+1,r_4+2,...,n$ is trapezoidal fuzzy number.

First of all, the initial data need to be normalized and summarized in the normalized decision matrix $B = b_{ij}$ (Liu, Liu, 2010):

$$b_{ij} = x_{ij} \Big/ \sqrt{\sum_{i=1}^{m} x_{ij}^2}, \forall j \in [1, 2, ..., r_1]; \tag{15}$$

$$b_{ij} = [b_{ij}^a, b_{ij}^d] = \begin{cases} b_{ij}^a = x_{ij}^a \Big/ \sqrt{\sum_{i=1}^{m} \left[(x_{ij}^a)^2 + (x_{ij}^d)^2 \right]} \\ b_{ij}^d = x_{ij}^d \Big/ \sqrt{\sum_{i=1}^{m} \left[(x_{ij}^a)^2 + (x_{ij}^d)^2 \right]} \end{cases}, \forall j \in [r_1 + 1, r_1 + 2, ..., r_2]; \tag{16}$$

$$b_{ij} =$$

$$b_{ij} = (b_{ij}^a, b_{ij}^b, b_{ij}^d) = \begin{cases} b_{ij}^a = x_{ij}^a \Big/ \sqrt{\sum_{i=1}^{m} \left[(x_{ij}^a)^2 + (x_{ij}^b)^2 + (x_{ij}^d)^2 \right]} \\ b_{ij}^d = x_{ij}^b \Big/ \sqrt{\sum_{i=1}^{m} \left[(x_{ij}^a)^2 + (x_{ij}^b)^2 + (x_{ij}^d)^2 \right]} \\ b_{ij}^d = x_{ij}^d \Big/ \sqrt{\sum_{i=1}^{m} \left[(x_{ij}^a)^2 + (x_{ij}^b)^2 + (x_{ij}^d)^2 \right]} \end{cases}, \forall j \in [r_2 + 1, r_2 + 2, ..., r_3] \tag{17}$$

The normalization of trapezoidal fuzzy number can be carried out by extending Eq. (19) with additional variable (Liu, Liu, 2010). Linguistic variables computed according to Eq. (2) do not need to be normalized.

Secondly, we have to choose the BLTS, namely $S_T = (s_0, s_1, ..., s_g)$. Usually, set with maximum granularity is chosen from the applied linguistic sets (Herrera *et al.*, 2005). Then each response b_{ij} is converted into 2–tuple $t_{ij} = (s_k, \alpha)_{ij}, s_k \in S_T, \alpha \in [-0.5, 0.5)$ by employing Eqs. 10, 9, and 2:

$$t_{ij} = \Delta\left(\chi\left(\tau_{S_{g+1}S_T}\left(b_{ij}\right)\right)\right) = (s_k, \alpha)_{ij}, \forall i, j . \tag{18}$$

Subsequently, a negation operator is used in accordance with Eq. 5 to transform cost criteria into benefit ones:

$$u_{ij} = \begin{cases} t_{ij}, \forall j \in J_1 \\ Neg(t_{ij}), \forall j \in J_2 \end{cases} \tag{19}$$

As a result, the transformed normalized decision matrix $U = u_{ij}$ is formed. Now we may proceed with aggregation of responses.

The Ratio System of MULTIMOORA–2T. The arithmetic mean will be calculated instead of simple sum of responses, for the sum could not be expressed in 2–tuples. The Eq. 6, hence, is employed:

$$y_i = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(s_k,\alpha)_{ij}\right), i = 1,2,...,m \tag{20}$$

where y_i stands for summarizing ratio of the i^{th} alternative. The alternatives with higher values of y_i are given higher ranks.

The Reference Point of MULTIMOORA–2T. The maximum for every criterion is found according to Definition 4. However, application of Δ^{-1} function would return the same results. The alternatives, therefore, are ranked by applying Min–Max metric and Eq. 7:

$$\min_i\left(\max_j d(u_{ij}, \max_i u_{ij})\right), \forall i, j. \tag{21}$$

The Full Multiplicative Form of MULTIMOORA–2T. Again, the geometric mean will be calculated instead of simple product, since the latter could not be successfully expressed in the 2–tuple form. As a result, the Eq. 8 is applied:

$$U_i = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(u_{ij})\right)^{\frac{1}{n}}\right), i = 1,2,...,m. \tag{22}$$

Alternatives with higher values of U_i are attributed with higher ranks. The final ranks for each alternative are provided according to the dominance theory (Brauers, Zavadskas, 2011).

3. THE PROPOSED METHOD MULTIMOORA–2T–G

This section presents the proposed method MULTIMOORA–2T–G. Since the new method is aimed at tackling group decision making, it consists of the following main stages of multi–criteria evaluation (Fig. 1): 1) rating (each expert rates the considered alternatives); 2) aggregation (the aforementioned rating are aggregated into single decision matrix); and 3) selection (the application of MCDM method provides ranks for each alternative and the best one is therefore chosen).

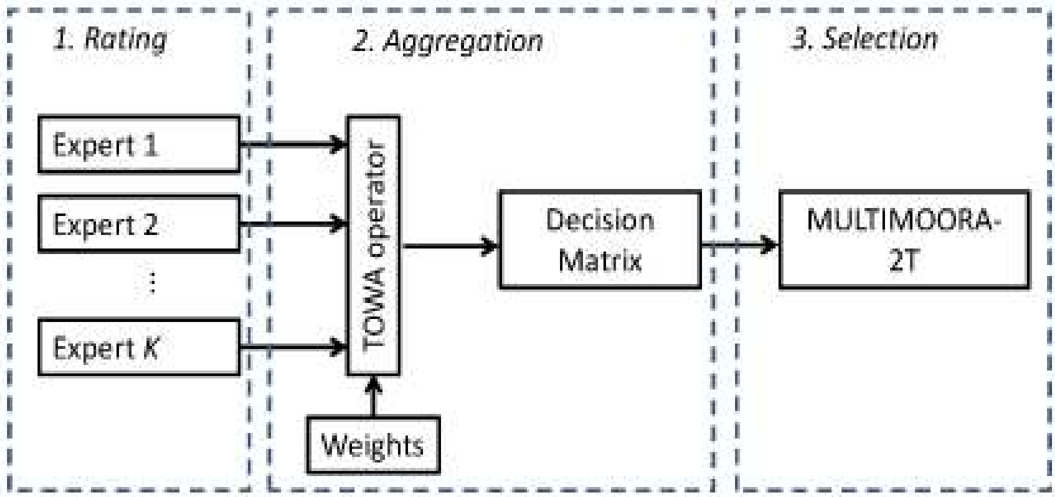

Figure 1 – The generalized model of the proposed method MULTIMOORA–2T–G.

Let $A=(a_1,a_2,...,a_m)$ be the set of alternatives considered with respect to criteria $C=(c_1,c_2,...,c_n)$. Additionally, let $J_1 \subset C$ and $J_2 \subset C$ be subsets of benefit and cost criteria, respectively. MULTIMOORA usually treats every criterion as equally important, hence no coefficients of criteria significance are used in this case. Additionally, each k^{th} expert provides ratings for each i^{th} alternative with respect to each j^{th} criterion $x_{ij}^{(k)}$, with $i=1,2,...,m$; $j=1,2,...,n$; $k=1,2,...,K$. Indeed, these ratings take form of linguistic variables from certain set $S_{g+1}=(s_0,s_1,...,s_g)$, where g is the cardinality of the linguistic term set used by certain expert. Noteworthy, if experts use different linguistic term sets they should be mapped into BLTS before aggregation by employing functions Δ_g^{-1} and Δ_T, where T is the cardinality of BLTS. The following steps describe further computations.

Step 1. By applying Eq. 3 we can rewrite the decision matrices of each expert: $X^{(k)} = \left(x_{ij}^{(k)}, 0\right)_{m \times n}$.The given expert evaluations therefore are now expressed in 2–tuples belonging to BLTS.

Step 2. The TOWA operator is employed for aggregating expert preferences into single decision matrix $X = x_{ij} = \left(x, \alpha\right)_{ij}$:

$$T_W(x_{ij}^{(k)}) = \Delta\left(\sum_{k=1}^{K} w_k \Delta^{-1}\left(x_{ij}^{\sigma(k)}, 0\right)\right) = \left(x_t, \alpha\right)_{ij}, x_t \in S_T, a \in [-0.5, 0.5), \tag{23}$$

where $\left(\sigma(1), \sigma(2), ..., \sigma(k)\right)$ is a permutation of $(1, 2, ..., K)$, such that $\left(x_{ij}^{\sigma(k-1)}, 0\right) \geq \left(x_{ij}^{\sigma(k)}, 0\right)$, $\forall k = 2, 3, ..., K$; and where $W = w_k$ is the weight vector of experts obtained by employing Eq. 12.

Step 3. Subsequently, a negation operator is used in accordance with Eq. 5 to transform cost criteria into benefit ones:

$$u_{ij} = \begin{cases} x_{ij}, \forall j \in J_1 \\ Neg(x_{ij}), \forall j \in J_2 \end{cases} \tag{24}$$

As a result, the transformed normalized decision matrix $U = u_{ij} = \left(\overline{x}_t, \overline{\alpha}\right)_{ij}$ is formed. Now we may proceed with aggregation of responses for each alternative.

Step 4. *The Ratio System of MULTIMOORA–2T.* The arithmetic mean will be calculated instead of simple sum of responses, for the sum could not be expressed in 2–tuples. The Eq. 6, hence, is employed:

$$y_i = \Delta\left(\frac{1}{n}\sum_{j=1}^{n} \Delta^{-1}(\overline{x}_t, \overline{\alpha})_{ij}\right), i = 1, 2, ..., m \tag{25}$$

where y_i stands for final rating, namely 2–tuple, of the i^{th} alternative. The alternatives with higher values of y_i are given higher ranks.

Step 5. *The Reference Point of MULTIMOORA–2T.* The maximum for every criterion is found according to Definition 4. However, application of Δ^{-1} function would return the same results. The alternatives, therefore, are ranked by applying Min–Max metric and Eq. 7:

$$\min_i \left(\max_j d(u_{ij}, \max_i u_{ij}) \right), \forall i, j \, . \tag{26}$$

Step 6. *The Full Multiplicative Form of MULTIMOORA–2T.* Again, the geometric mean will be calculated instead of simple product, since the latter could not be successfully expressed in the 2–tuple form. As a result, the Eq. 8 is applied:

$$U_i = \Delta \left(\left(\prod_{j=1}^{n} \Delta^{-1}(\overline{x}_t, \overline{\alpha})_{ij} \right)^{\frac{1}{n}} \right), i = 1, 2, ..., m \, . \tag{27}$$

Alternatives with higher values of U_i are attributed with higher ranks.

Step 7. As it was stated in Steps 4–6, each alternative now is attributed with three ranks from each part of MULTIMOORA. Thus, one needs to summarize these ranks to make find the best alternative. The final ranks for each alternative are therefore provided according to the dominance theory (Brauers, Zavadskas, 2011a).

4. EMPIRICAL APPLICATION: SUPPLIER SELECTION

In this section we employ the new method MULTIMOORA–2T–G for supplier selection. In this simulation, five experts are about to evaluate the prospective suppliers. It is considered that the required amount of objective information on considered suppliers is unavailable and the experts are therefore going to use linguist term set for evaluation. Moreover, all the experts agreed to use a seven–point scale.

In this particular case, an enterprise decides on choosing the best supplier from four candidates. The following criteria are taken into consideration: product price, product quality, time of delivery, percentage of on–time deliveries, required payment in advance, remoteness of the facilities (location), and credibility of supplier. Moreover, price, time of delivery (TOD), payment in-advance, and remoteness (location) of supplier are considered to be cost criteria and hence should be minimized. The rating of certain supplier might be provided by corresponding banks or trade insurance companies. Table 1 summarizes linguistic variables.

Table 1. Linguistic term set for qualitative evaluation.

Linguistic term	2-tuple
Very low (VL)	$(s_0, 0)$
Low (L)	$(s_1, 0)$
Medium low (ML)	$(s_2, 0)$
Moderate (M)	$(s_3, 0)$
Medium high (MH)	$(s_4, 0)$
High (H)	$(s_5, 0)$
Very high (VH)	$(s_6, 0)$

Step 1. Table 2 exhibits initial decision matrix, where expert ratings are expressed in respective 2–tuples. As we can see in Table 2, the first expert considered supplier A1 to be peculiar with "moderate" remoteness from premises of his enterprise; whereas the second expert chose "medium high" label.

Table 2. Expert ratings expressed in 2–tuples.

Expert	Supplier	Criteria: Price	Quality	TOD	On-time	Payment	Remoteness	Credibility
1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
Direction of optimization		min	max	min	max	min	min	max
DM_1	A_1	$(s_5,0)$	$(s_4,0)$	$(s_3,0)$	$(s_4,0)$	$(s_5,0)$	$(s_3,0)$	$(s_5,0)$
	A_2	$(s_3,0)$	$(s_3,0)$	$(s_4,0)$	$(s_4,0)$	$(s_5,0)$	$(s_2,0)$	$(s_5,0)$
	A_3	$(s_4,0)$	$(s_3,0)$	$(s_2,0)$	$(s_3,0)$	$(s_4,0)$	$(s_2,0)$	$(s_4,0)$
	A_4	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_5,0)$	$(s_3,0)$	$(s_3,0)$	$(s_3,0)$
DM_2	A_1	$(s_5,0)$	$(s_5,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_4,0)$	$(s_4,0)$
	A_2	$(s_4,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_3,0)$	$(s_3,0)$	$(s_4,0)$

	A3	$(s_5,0)$	$(s_4,0)$	$(s_3,0)$	$(s_3,0)$	$(s_2,0)$	$(s_4,0)$	$(s_3,0)$
	A4	$(s_4,0)$	$(s_3,0)$	$(s_2,0)$	$(s_3,0)$	$(s_4,0)$	$(s_4,0)$	$(s_5,0)$

Table 2 continued

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
DM3	A1	$(s_3,0)$	$(s_4,0)$	$(s_5,0)$	$(s_3,0)$	$(s_4,0)$	$(s_5,0)$	$(s_4,0)$
	A2	$(s_3,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$
	A3	$(s_4,0)$	$(s_2,0)$	$(s_3,0)$	$(s_5,0)$	$(s_5,0)$	$(s_3,0)$	$(s_4,0)$
	A4	$(s_3,0)$	$(s_3,0)$	$(s_4,0)$	$(s_3,0)$	$(s_4,0)$	$(s_2,0)$	$(s_1,0)$
DM4	A1	$(s_5,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_5,0)$	$(s_4,0)$	$(s_5,0)$
	A2	$(s_3,0)$	$(s_3,0)$	$(s_3,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_4,0)$
	A3	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_2,0)$	$(s_3,0)$	$(s_2,0)$	$(s_3,0)$
	A4	$(s_2,0)$	$(s_4,0)$	$(s_1,0)$	$(s_4,0)$	$(s_3,0)$	$(s_4,0)$	$(s_3,0)$
DM5	A1	$(s_4,0)$	$(s_5,0)$	$(s_5,0)$	$(s_4,0)$	$(s_4,0)$	$(s_4,0)$	$(s_5,0)$
	A2	$(s_3,0)$	$(s_4,0)$	$(s_3,0)$	$(s_5,0)$	$(s_5,0)$	$(s_4,0)$	$(s_5,0)$
	A3	$(s_5,0)$	$(s_2,0)$	$(s_2,0)$	$(s_4,0)$	$(s_4,0)$	$(s_3,0)$	$(s_3,0)$
	A4	$(s_1,0)$	$(s_3,0)$	$(s_4,0)$	$(s_5,0)$	$(s_3,0)$	$(s_5,0)$	$(s_2,0)$.

Step 2. The aggregation of expert opinions takes place by employing TOWA operator. The information on significance of experts is unknown, hence by employing Eq. 12 and quantifier "most" we obtain the following vector of weights for TOWA operator $W=(0,0.2,0.4,0.4,0)^T$, which means that the third and fourth largest ratings will have the highest impact on final rating of each alternative according to certain criterion. The extreme assessments thus are likely to be excluded from analysis. Expert ratings are reordered and thus aggregated by means of TOWA operator (Eq. 23) into single decision matrix (Table 3).

Table 3. Aggregated expert ratings (decision matrix).

Supplier	Criteria						
	Price	Quality	TOD	On-time	Payment	Remote-ness	Credibi-lity
	min	max	min	max	min	min	max
A1	$(s_5,-0.4)$	$(s_4,0.2)$	$(s_4,0.2)$	$(s_4,-0.4)$	$(s_4,0.2)$	$(s_4,0)$	$(s_5,-0.4)$
A2	$(s_3,0)$	$(s_4,-0.4)$	$(s_4,-0.4)$	$(s_4,-0.4)$	$(s_4,0.2)$	$(s_3,0.2)$	$(s_4,0.2)$
A3	$(s_4,0.2)$	$(s_3,-0.2)$	$(s_3,-0.4)$	$(s_3,0.2)$	$(s_4,-0.4)$	$(s_3,-0.4)$	$(s_3,0.2)$
A4	$(s_3,-0.2)$	$(s_3,0.2)$	$(s_3,-0.2)$	$(s_4,-0.2)$	$(s_3,0.2)$	$(s_4,-0.4)$	$(s_3,-0.4)$

Step 3. A negation operator (as defined by Eq. 24) is applied for cost criteria, namely Price, TOD, Payment, and Remoteness. Henceforth all the criteria can be considered as benefit ones and summarized into single transformed decision matrix (Table 4).

Table 4. The transformed decision matrix.

Supp-lier	Criteria						
	Price	Quality	TOD	On-time	Payment	Remote-ness	Credibi-lity
	max	max	max	max	max	max	max
A1	$(s_1,0.4)$	$(s_4,0.2)$	$(s_2,-0.2)$	$(s_4,-0.4)$	$(s_2,-0.2)$	$(s_2,0)$	$(s_5,-0.4)$
A2	$(s_3,0)$	$(s_4,-0.4)$	$(s_2,0.4)$	$(s_4,-0.4)$	$(s_2,-0.2)$	$(s_3,-0.2)$	$(s_4,0.2)$
A3	$(s_2,-0.2$	$(s_3,-0.2)$	$(s_3,0.4)$	$(s_3,0.2)$	$(s_2,0.4)$	$(s_3,0.4)$	$(s_3,0.2)$
A4	$(s_3,0.2)$	$(s_3,0.2)$	$(s_3,0.2)$	$(s_4,-0.2)$	$(s_3,-0.2)$	$(s_2,0.4)$	$(s_3,-0.4)$
$\max_i u_{ij}$	$(s_3,0.2)$	$(s_4,0.2)$	$(s_3,0.4)$	$(s_4,-0.2)$	$(s_3,-0.2)$	$(s_3,0.4)$	$(s_5,-0.4)$

Step 4. Application of Eq. 25 resulted in ranking of the suppliers according to 2–tuple reference Point approach. As one can see in Table 5, the second supplier is the most preferred one with overall rating meaning more than "moderate". Contrary, the first supplier possesses the rating which means less than "moderate". Indeed, all the suppliers are scattered around the same linguistic label "moderate".

Step 5. As Eq. 26 suggests, the Maximal Objective Reference Point was found (the last row in Table 4). Consequently, distances between each response and coordinates of the reference point were computed. As a result, the second supplier is considered as the best one.

Step 6. The suppliers are ranked by applying Eq. 27. Again, they are scattered around category "moderate". The fourth supplier, however, is the most preferred, whereas the first one has almost gone down to category "medium low".

Table 5. **Results of the MULTIMOORA–2T–G for supplier selection**.

Supplier	2–tuple Ratio System		2–tuple Reference Point		2–tuple Full Multiplicative Form		Final Rank
	y_i	Rank	$\max_j d_{ij}$	Rank	U_i	Rank	
A_1	$(s_3,-0.23)$	4	$(s_2,-0.2)$	3	$(s_3,-0.49)$	4	4
A_2	$(s_3,0.06)$	1	$(s_1,0)$	1	$(s_3,-0.04)$	2	1
A_3	$(s_3,-0.11)$	3	$(s_1,0.4)$	2	$(s_3,-0.18)$	3	3
A_4	$(s_3,0.03)$	2	$(s_2,0)$	4	$(s_3,0)$	1	2

Step 7. The theory of dominance is applied to define the final ranking of the alternatives, i. e. suppliers. (Brauers, Zavadskas, 2011a). The second supplier hence dominates over the remaining alternatives. Accordingly, the following order of preference is established: $A_2 \succ A_4 \succ A_3 \succ A_1$.

Given the results of the carried out multi–criteria assessment, the company should establish and maintain long-term strategic relationships with the second supplier and order minor quantities of the required materials from the fourth supplier.

5. CONCLUSION

The new method MULTIMOORA–2T–G is suitable for group decision making under linguistic environment. More specifically, the linguistic terms were represented by 2–tuples. Contrary to the case of fuzzy linguistic variables, the 2–tuple–based computation also results in assessment of certain alternative expressed in appropriate linguistic term as well as an additional numerical term—a linguistic translation—defining the exact position of the assessment in the linguistic scale. The decision makers can therefore better perceive and interpret the results of multi–criteria assessment. The TOWA operator was employed for aggregation of expert opinions. The further studies however might be aimed at applying different aggregation operators.

The numerical example illustrated the possibilities for application of the MULTIMOORA–2T–G method: the committee consisting of five experts ranked the potential suppliers by applying the new method. The following criteria were taken into consideration: product price, product quality, time of delivery, percentage of on–time deliveries, required payment in advance, remoteness of the facilities (location), and credibility of supplier. Indeed, the indicator system can be altered to meet requirements of different business decisions. To cap it all, the new method enables a company to develop an appropriate procurement policy under vague conditions; for it is based on linguistic assessments rather than numerical ones.

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