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A COMPARATIVE DCC-GARCH AND ROLLING WAVELET CORRELATION ANALYSIS OF INTERDEPENDENCE BETWEEN THE SLOVENIAN AND EUROPEAN STOCK MARKETS

Abstract. *This paper examines the comovement and spillover dynamics between returns of the Slovenian and some European (the UK, German, French, Austrian, Hungarian and the Czech) stock markets. It aims to answer these question: i) Is correlation (comovement) between the Slovenian and European stock markets time-varying and scale dependent; ii) What effect did financial crises in the period from April 1997 to May 2010 have on the comovement between the Slovenian and European stock markets; iii) Are there return and volatility spillovers between European and Slovenian stock markets; iv) Do DCC-GARCH and wavelet correlation correlation estimates differ and which one should international investor resort to when making international stock market investments?*

A DCC-GARCH and maximal overlap discrete wavelet transform analysis is applied to returns series of representative national stock indices for the period April 1997- May 2010. The main findings of the paper are: i) Comovement between Slovenian and European stock markets is time-varying; ii) There are significant return spillovers between the Slovenian and European stock markets; iii) Return spillovers are not just time-varying, but also scale dependent; iv) The global financial crisis of 2007-2008 has increased comovement between the Slovenian and European stock markets; v) As the scale (frequency) increases, we can observe larger discrepancies between DCC-GARCH and wavelet correlation estimates suggesting one should resort to rolling wavelet correlation estimates when making longer horizon international portfolio decisions.

Keywords: DCC-GARCH, wavelet analysis, stock markets, Slovenia, return comovement.

JEL Classification: G15, G11, F36

1 Introduction

International stock market linkages are of great importance for the financial decisions of international investors. Since the seminal works of Markowitz (1958) and the empirical evidence of Grubel (1968), it has been widely accepted that international diversification reduces the total risk of a portfolio. This is due to non-perfect positive comovement between the returns of portfolio assets. Increased comovement between asset returns can therefore diminish the advantage of internationally diversified investment portfolios (Ling and Dhesi, 2010).

Modeling the comovement of stock market returns is a challenging task. The conventional measure of market interdependence, known as the Pearson correlation coefficient, is a symmetric, linear dependence metric (Ling and Dhesi, 2010) suitable for measuring dependence in multivariate normal distributions (Embrechts et al., 1999). However, correlations may be nonlinear and time-varying (Égert and Kočenda, 2010). Also, the dependence between two stock markets as the market rises may be different than the dependence as the market falls (Necula, 2010). It only represents an average of deviations from the mean without making any distinction between large and small returns, or between negative and positive returns (Poon et al., 2004). A better understanding of stock market interdependencies may be achieved by applying econometric methods: Vector Autoregressive (VAR) models (Malliaris and Urrutia, 1992; Gilmore and McManus, 2002), cointegration analysis (Gerrits and Yuce, 1999; Patev et al., 2006), GARCH models (Tse and Tsui, 2002; Égert and Kočenda, 2010; Cho and Parhizgari, 2008) and regime switching models (Schwender, 2010). A novel approach and promising approach is based on wavelet analysis (Ranta, 2010; Zhou, 2011).

The GARCH models are used to analyze the volatility of individual assets (Bollerslev et al.; 1994; Shephard, 1996), while international investors are more interested in comovement and spillovers between the assets (or markets). Comovement between assets (or markets) may be time-varying (Tse and Tsui, 2002; Bae et al., 2003; Égert and Kočenda, 2010; Cho and Parhizgari, 2008; Égert and Kočenda, 2010) and can be analyzed by multivariate GARCH models (MGARCH – Multivariate Generalized Autoregressive Conditional Heteroskedasticity).

There are several MGARCH models, of which the DCC-GARCH (Dynamic Conditional Correlation GARCH) models have greatly increased in popularity. They offer both the flexibility of univariate GARCH models and the simplicity of parametric correlation in the model and are an extension of CCC-GARCH (Constant Conditional Correlation GARCH) models. More DCC-GARCH models have been developed: the version by Engle (2002), the version by Engle and Sheppard (2001), the model by Tse and Tsu (2002), a model by Christodoulakis and Satchell (2002), a model by Lee et al. (2006).

Interdependencies between stock markets may be not be just time, but also scale dependent (Ranta, 2010; Zhou, 2011). Candelon et al. (2008) argue that the stock market comovement analysis should consider the distinction between short and long-term investors. From a portfolio diversification point of view, the short term investors are more interested in the stock market interdependencies at shorter time horizons (that is at higher frequencies or short term movements), and the long term investors focus on the lower frequencies interdependencies. As such, one has to resort to the scale (frequency) domain analysis to obtain insights about the international interdependencies of stock markets at the scale level (Pakko, 2004; Sharkasi et al., 2005). In such a context, with both the time horizon of economic decisions and the strength and direction of economic relationships between variables that may differ according to the time scale of the analysis, a useful analytical tool may be represented by wavelet analysis.

Wavelets in finance are primarily used as a signal decomposition tool (e.g. Mallat and Zhang, 1993; Gençay et al., 2001a; Gençay et al., 2003; Vuorenmaa, 2006), or a tool to detect interdependence between variables (In & Kim, 2006; In et al., 2008; Kim and In, 2007). There are several studies using MODWT (Maximal Overlap Discrete Wavelet Transform) variance, wavelet correlation and wavelet cross-correlation to investigate interdependence between economic (or financial) variables at different time scales (In and Kim, 2006; Kim and In, 2007; Gençay et al., 2001a; Gallegati, 2008; Conlon et al., 2009; Ranta, 2010; Zhou, 2011). These studies confirm that interdependence between financial (or economic) variables is scale dependent, exhibiting different correlation structure at different time scales. Ranta (2010) and Zhou (2011), using MODWT rolling correlation technique, show also, that return linkage between stock indices is time varying and its dynamics varies across scales.

This paper aims to answer these question: i) Is correlation (comovement) between the Slovenian and European stock markets time-varying and scale dependent; ii) What effect did financial crises in the period from April 1997 to May 2010 have on the comovement between the Slovenian and European stock markets; iii) Are there return and volatility spillovers between European and Slovenian stock markets. These questions will be answered by applying two modern techniques: a DCC-GARCH model of Engle and Sheppard (2001) and the maximal overlap discrete wavelet transform (MODWT) rolling correlation analysis.

2 Methodology

2.1 The DCC-GARCH model

The DCC-GARCH model of Engle and Sheppard (2001) assumes that returns from k assets are conditionally multivariate normal with zero expected value (r_t) and covariance matrix H_t. Returns of the asset (stocks, stock indices), given the information set available at time $t-1$, have the following distribution[1]:

$r_t \mid \Im_{t-1} \sim N(0, H_t)$, and $H_t \equiv D_t R_t D_t$,

[1] The description of the DCC-GARCH models is from Engle and Sheppard (2001). The same notations as by the authors are used.

where D_t is the $k \times k$ diagonal matrix of time varying standard deviations from univariate GARCH models with $\sqrt{h_{it}}$ on the ith diagonal, and R_t is the time varying correlation matrix.

The loglikelihood of this estimator is written as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}(k\, log(2\pi) + 2\log(|D_t|) + \log(|\, R_t|) + \epsilon_t' R_t^{-1}\epsilon_t),$$

where $\epsilon_t \sim N(0, R_t)$ are the residuals standardized by their conditional standard deviation. Elements of the matrix D_t are given by a univariate GARCH model (Engle and Sheppard 2001):

$$h_{it} = \omega_i + \sum_{p=1}^{P_i}\alpha_{ip} r_{it-p}^2 + \sum_{q=1}^{Q_i}\beta_{iq} h_{it-q},$$

for $i = 1,2, \ldots, k$ (variables, in our case stock indices), with the usual GARCH restrictions (for non-negativity and stationarity $\sum_{p=1}^{P_i}\alpha_{ip} + \sum_{q=1}^{Q_i}\beta_{iq} < 1$). Dynamic correlation structure is defined by the following equations:

$$Q_t = (1 - \sum_{m=1}^{M}\alpha_m - \sum_{n=1}^{N}\beta_n)\bar{Q} + \sum_{m=1}^{M}\alpha_m(\epsilon_{t-m}\epsilon'_{t-m}) + \sum_{n=1}^{N}\beta_n Q_{t-n},$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1},$$

where M is the length of the innovation term in the DCC estimator, and N is the length of the lagged correlation matrices in the DCC estimator $(\alpha_m \geq 0$, $\beta_n \geq 0$, $\sum_{m=1}^{M}\alpha_m + \sum_{n=1}^{N}\beta_n < 1)$.

$\bar{Q}$ is the unconditional covariance of the standardized residuals resulting from the first stage estimation and Q_t^* *is* a diagonal matrix composed of the square root of the diagonal elements of Q_t:

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & \ldots & 0 \\ 0 & \sqrt{q_{22}} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \sqrt{q_{kk}} \end{bmatrix}.$$

The elements of the matrix R_t are:

$$\rho_{ijt} = \frac{q_{ijt}}{\sqrt{q_{ii} q_{jj}}}.$$

The DCC-GARCH model is estimated in two stages. In the first stage univariate GARCH models are estimated for each residual series, and in the second stage, residuals, transformed by their standard deviation estimated during the first stage, are used to estimate the parameters of the dynamic correlation. More specific, the parameters of the DCC-GARCH model, θ, are written in two groups: $(\phi_1, \phi_2, \dots, \phi_1, \psi) = (\phi, \psi)$, where the elements of ϕ_i correspond to the parameters of the univariate GARCH model for the ith asset series, $(\phi_i = \omega_i, \alpha_{1i}, \dots, \alpha_{P_i i}, \beta_{1i}, \dots, \beta_{Q_i i})$.

In empirical applications, normally a bivariate DCC(1,1)-GARCH(1,1) model is estimated, with two financial assets, $r_{1,t}$ and $r_{2,t}$ (Engle, 2002; Lebo and Box-Steffensmeier, 2008; Égert and Kočenda, 2010).

To estimate a DCC(1,1)-GARCH(1,1) model of stock indices return comovements, we first estimate a VAR (Vector Autoregressive) model:

$$r_{1,t} = \mu_1 + \sum_{i=1}^{p} a_{1,i} r_{1,t-i} + \sum_{i=1}^{p} b_{1,i} r_{2,t-i} + \varepsilon_{1,t}$$
$$r_{2,t} = \mu_2 + \sum_{i=1}^{p} a_{2,i} r_{2,t-i} + \sum_{i=1}^{p} b_{2,i} r_{1,t-i} + \varepsilon_{2,t},$$

and then, using residuals of the VAR model, estimate a DCC(1,1)-GARCH(1,1) model:

$$h_{it} = \omega_i + \alpha_{i1} r_{it-1}^2 + \beta_{i1} h_{it-1}$$
$$Q_t = (1 - \alpha_1 - \beta_1)\bar{Q} + \alpha_1 \left(\epsilon_{t-1} \epsilon_{t-1}^{'}\right) + \beta_1 Q_{t-1}.$$

2.2 The Maximal overlap discrete wavelet transform method

Similar to Fourier analysis, wavelet analysis involves the projection of the original series onto a sequence of basis functions, which are known as wavelets. There are two basic wavelet functions: the father wavelet (also known as a scaling function), ϕ, and the mother wavelet (also known as a wavelet function), ψ , which can be scaled and translated to form a basis for the Hilbert space $L^2(\mathbb{R})$ of square integrable functions. The father and mother wavelets are defined by the functions:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi(2^{-j} t - k), \ \psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi(2^{-j} t - k),$$

where $j = 1, \dots, J$ is the scaling parameter in a J-level decomposition and k is a translation parameter ($j, k \in \mathbb{Z}$). The long term trend of the time series is captured by the father wavelet, which integrates to 1, while the mother wavelet, which integrates to 0, describes fluctuations from the trend. The continuous wavelet transform of a square integrable time series $X(t)$ consists of the scaling, $\alpha_{J,k}$, and wavelet coefficients, $\beta_{j,k}$, (Craigmile and Percival, 2002):

$\alpha_{J,k} = \int \phi_{J,k}(t)X(t)$ and $\beta_{j,k} = \int \psi_{j,k}(t)X(t)$.

It is possible to reconstruct $X(t)$ from these transform coefficients using:

$$X(t) = \sum_k \alpha_{J,k}\phi_{J,k}(t) + \sum_k \beta_{J,k}\psi_{J,k}(t) + \cdots + \sum_k \beta_{j,k}\psi_{j,k}(t) + \cdots + \sum_k \beta_{1,k}\psi_{j,k}(t).$$

In practice, we observe a time series for a finite number of regularly spaced times, so we can make use of a maximal overlap discrete wavelet transform (MODWT). The MODWT is a linear filtering operation that transforms a series into coefficients related to variations over a set of scales. It is similar to the discrete wavelet transform (DWT), but it gives up the orthogonality property of the DWT to gain other features that render MODWT more suitable for the aims of our study. As noted by Percival and Mojfeld (1997) this includes: i) the ability to handle any sample size regardless of whether the series is dyadic (that is of size 2^{J_0}), or not; ii) increased resolution at coarser scales as the MODWT oversamples the data; iii) translation-invariance, which ensures that MODWT wavelet coefficients do not change if the time series is shifted in a "circular" fashion; and iv) the MODWT produces a more asymptotically efficient wavelet variance estimator than the DWT.

Let[2] $\boldsymbol{X}$ be an N dimensional vector whose elements represent the real-valued time series $\{X_t: t = 0, \dots, N-1\}$.. For any positive integer, J_0, the level J_0 MODWT of $\boldsymbol{X}$ is a transform consisting of the $J_0 + 1$ vectors $\widetilde{\boldsymbol{W}}_1, \dots, \widetilde{\boldsymbol{W}}_{J_0}$ and $\widetilde{\boldsymbol{V}}_{J_0}$, all of which have dimension N. The vector $\widetilde{\boldsymbol{W}}_j$ contains the MODWT wavelet coefficients associated with changes on scale $\tau_j = 2^{j-1}$ (for $j = 1, \dots, J_0$), while $\widetilde{\boldsymbol{V}}_{J_0}$ contains MODWT scaling coefficients associated with averages on scale $\lambda_{J_0} = 2^{J_0}$[3]. Based upon definition of MODWT coefficients we can write (Percival and Walden, 2000, 200):

$$\widetilde{\boldsymbol{W}}_j = \widetilde{\mathcal{W}}_j \boldsymbol{X} \text{ and } \widetilde{\boldsymbol{V}}_{J_0} = \widetilde{\mathcal{V}}_{J_0} \boldsymbol{X},$$

where $\widetilde{\mathcal{W}}_j$ and $\widetilde{\boldsymbol{V}}_{J_0}$ are $N \times N$ matrices. Vectors are denoted by bold.

By definition, the elements of $\widetilde{\boldsymbol{W}}_j$ and $\widetilde{\boldsymbol{V}}_{J_0}$ are outputs obtained by filtering $\boldsymbol{X}$, namely:

$$\widetilde{W}_{j,t} = \sum_{l=o}^{L_{j-1}} \tilde{h}_{j,l} X_{t-l \bmod N} \text{ and } \widetilde{V}_{j,t} = \sum_{l=o}^{L_{j-1}} \tilde{g}_{j,l} X_{t-l \bmod N},$$

[2] Concepts and notations as in Percival and Walden (2000) are used. Another thorough description of MODWT using matrix algebra is found in Gençay et al. (2002).

[3] Percival and Walden (2000) denote scales for wavelet coefficient as τ and scales for scaling coefficients as λ. We use these notations as well.

for $t = 0, \ldots, N-1$, where $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are jth MODWT wavelet and scaling filters.

The MODWT treats the series as if it were periodic, whereby the unobserved samples of the real-valued time series $X_{-1}, X_{-2}, \ldots X_{-N}$ are assigned the observed values at $X_{N-1}, X_{N-2}, \ldots X_0$.

The MODWT coefficients are thus given by:

$\widetilde{W}_{j,t} = \sum_{l=o}^{N-1} \tilde{h}^{\circ}_{j,l} X_{t-l \, mod N}$ and $\widetilde{V}_{j,t} = \sum_{l=o}^{N-1} \tilde{g}^{\circ}_{j,l} X_{t-l \, mod N}$ (for $t = 0, \ldots, N-1$).

This periodic extension of the time series is known as analyzing $\{X_t\}$ using ″circular boundary conditions″ (Percival and Walden, 2000; Cornish et al., 2006). There are $L_j - 1$ wavelet and scaling coefficients that are influenced by the extension (″the boundary coefficients″). Exclusion of boundary coefficients in the wavelet variance, wavelet correlation and covariance provides unbiased estimates (Cornish et al., 2006). One of the important uses of the MODWT is to decompose the sample variance of a time series on a scale-by-scale basis. Since the MODWT is energy conserving (Percival and Mojfeld, 1997):

$$\|\boldsymbol{X}\|^2 = \sum_{j=1}^{J_0} \left\|\widetilde{\boldsymbol{W}}_j\right\|^2 + \left\|\widetilde{\boldsymbol{V}}_{J_0}\right\|^2,$$

a scale-dependent analysis of variance from the wavelet and scaling coefficients can be derived (Cornish et al., 2006):

$$\hat{\upsilon}_X^2 = \frac{1}{N}\|\boldsymbol{X}\|^2 - \bar{X}^2 = \frac{1}{N}\sum_{j=1}^{J_0}\left\|\widetilde{\boldsymbol{W}}_j\right\|^2 + \frac{1}{N}\left\|\widetilde{\boldsymbol{V}}_{J_0}\right\|^2 - \bar{X}^2.$$

Wavelet variance is defined for stationary and nonstationary processes with stationary backward differences. Considering only the non-boundary wavelet coefficient, obtained by filtering stationary series with MODWT, the wavelet variance $\upsilon_X^2(\tau_j)$ is defined as expected value of $\widetilde{W}_{j,t}^2$.

In this case $\upsilon_X^2(\tau_j)$ represents the contribution to the (possibly infinite) variance of $\{X_t\}$ at the scale $\tau_j = 2^{j-1}$and can be estimated by the unbiased estimator (Percival and Walden 2000, 306):

$$\hat{\upsilon}_X^2(\tau_j) = \frac{1}{M_j}\sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^2,$$

where $M_j \equiv N - L_j + 1 > 0$ is the number of non-boundary coefficients at the *j*th level.

Given two stationary processes $\{X_t\}$ and $\{Y_t\}$, an unbiased covariance estimator $\hat{\upsilon}_{XY}(\tau_j)$ is given by (Percival, 1995):

$$\hat{\upsilon}_{X,Y}(\tau_j) = \frac{1}{M_j}\sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^{(X)}\,\widetilde{W}_{j,t}^{(Y)},$$

where $M_j \equiv N - L_j + 1 > 0$ is the number of non-boundary coefficients at the jth level.

The MODWT correlation estimator for scale τ_j is obtained by making use of the wavelet cross-covariance and the square root of wavelet variances:

$$\hat{\rho}_{X,Y}(\tau_j) = \frac{\hat{\upsilon}_{X,Y}(\tau_j)}{\hat{\upsilon}_X(\tau_j)\hat{\upsilon}_Y(\tau_j)},$$

where $|\hat{\rho}_{X,Y}(\tau_j)| \leq 1$. The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gençay et al., 2002, 258).

3 Empirical results

3.1 Data

Stock indices returns are calculated as differences of logarithmic daily closing prices of indices ($\ln(P_t) - \ln(P_{t-1})$, where P is an index price). The following indices are considered: LJSEX (for Slovenia), ATX (for Austria), CAC40 (for France), DAX (for Germany), FTSE100 (for the UK), BUX (for the Hungary) and PX (for the Czech Republic). The period of observation is April 1, 1997 – May 12, 2010. Days of no trading on any of the observed stock market were left out. Total number of observations amounts to 3060 days. Data sources of LJSEX, PX and BUX indices are their respective stock exchanges, data source of ATX, CAC40, DAX and FTSE100 indices is Yahoo Finance. Table 1 presents some descriptive statistics of the data.

Table 1: Descriptive statistics of indices return series

	Min	Max	Mean	Std. deviation	Skewness	Kurtosis
BUX	-0.1803	0.2202	0.0004859	0.02021	-0.30	15.90
ATX	-0.1637	0.1304	0.0002515	0.01558	-0.40	14.91
CAC40	-0.0947	0.1059	0.0001206	0.01628	0.09	7.83
DAX	-0.0850	0.1080	0.0002071	0.01756	-0.06	6.58
FTSE100	-0.0927	0.1079	0.0000774	0.01361	0.09	9.30
PX	-0.199	0.2114	0.0002595	0.01667	-0.29	24.62
LJSEX	-0.1285	0.0768	0.0003521	0.01062	-0.87	20.19

To test stationarity of stock index return time series Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test are applied. Results of stationarity tests are presented in table 2.

Table 2: Results of stationarity tests

	KPPS test (a constant + trend)	KPSS test (a constant)	PP test (a constant + trend)	PP test (a constant)	ADF test (a constant + trend)	ADF test (a constant)
PX	0.158* (10)	0.170 (10)	-55.022*** (10)	-55.029*** (10)	-16.676*** (L=8)	- 16.676*** (L=8)
ATX	0.186** (12)	0.191 (13)	-53.586*** (15)	-53.594*** (15)	- 40.604** (L=1)	- 40.608*** (L=1)
CAC40	0.110 (15)	0.250 (15)	-57.840*** (14)	-57.787*** (14)	- 36.142*** (L=2)	- 36.108*** (L=2)
DAX	0.099 (1)	0.105 (1)	-57.805*** (3)	-57.812*** (3)	- 57.692*** (L=0))	- 57.698*** (L=0)
FTSE100	0.089 (9)	0.101 (9)	-58.284*** (7)	-58.287*** (7)	-29.112*** (L=3)	- 29.111*** (L=3)
BUX	0.065 (6)	0.065 (6)	-54.295*** (6)	-54.304*** (6)	-54.301*** (L=0)	- 54.310*** (L=0)
LJSEX	0.249*** (11)	0.591** (12)	-44.099*** (0)	-43.795*** (3)	-37.229*** (L=1)	-37.128*** (L=1)

*Notes: All tests were performed for two models: for a model with a constant and for the model with a constant plus trend. For KPSS and PP test Bartlet Kernel estimation method was used with Newey-West automatic bandwidth selection. Optimal bandwidth is indicated in parenthesis under the statistics. The number of lags to be included (L) for ADF test were selected by SIC criteria (30 was a maximum lag). Exceeded critical values for rejection of null hypothesis are marked by *** (1% significance level), ** (5% significance level) and * (10% significance level).*

The null hypothesis of KPSS test (i.e. the time series is stationary) for a model with a constant plus trend can be rejected at the 5% significance level for the return series of LJSEX and ATX. Since trend is not significantly different from zero, we give advantage to KPSS model results with no trend. For that model we cannot reject the null hypothesis of stationary process for any stock index return series (expect for LJSEX) at the 1% significance level. The null hypothesis of PP and ADF tests is rejected for all stock indices. On the basis of the stationarity tests we conclude that all indices return time series are stationary.

3.2 DCC-GARCH conditional correlation analysis results

Before estimating a DCC(1,1)-GARCH(1,1) model, time series have to be filtered to assure zero expected (mean) value of the time series. A bivariate Vector Autoregressive (VAR) model for the return series was used to initially remove potential linear structure between pairs of stock index returns. Then the residuals of the VAR model were used as inputs for the DCC-GARCH model.

The results for the DCC(1,1)-GARCH(1,1) model are presented in table 3. All estimated GARCH model parameters ($\omega_{\text{LJSEX - other index}}$, $\omega_{\text{other index – LJSEX}}$, $\alpha_{\text{LJSEX - other index}}$, $\alpha_{\text{other index - LJSEX}}$, $\beta_{\text{LJSEX - other index}}$ and $\beta_{\text{other index - LJSEX}}$) are statistically significant. Conditional variance of LJSEX returns is influenced by past return innovations in the foreign index in the pair ($\alpha_{\text{LJSEX - other index}}$ and $\alpha_{\text{other index - LJSEX}}$) and by its lagged variances ($\beta_{\text{LJSEX - other index}}$ and $\beta_{\text{drugi indeks – LJSEX}}$). Statistically significant parameters $\beta_{\text{LJSEX - other index}}$ and $\beta_{\text{other index - LJSEX}}$ indicate, that volatility transmission is bi-directional between the indices in pairs (so they are transmitted to Slovenian stock market and, vice versa, from the Slovenian stock market to the other markets). The DCC parameter β is statistically significant in all cases, while α is significant only for stock indices pairs LJSEX-PX, LJSEX-BUX and LJSEX-ATX. If we also consider that $\beta > \alpha$ for all indices pairs, we can argue, that behaviour of current variances is more affected by magnitude of past variances as by past return innovations. Having value β close to 1 indicates high persistance in the series of correlations R_t. The sum of the DCC parameters ($\alpha + \beta$) is larger than zero (meaning that conditional correlation between the pairs of indices returns is not constant); actually, values close to 1 are observed, indicating that conditional variances are highly persistent and only slowly mean-reverting (Lebo and Box-Steffensmeier, 2008). Results of the Ljung-Box statistics do not reject the null hypothesis of no serial correlation in squared residuals of estimated DCC-GARCH model, suggesting a DCC(1,1)-GARCH(1,1) model is appropriately specified.

Table 3: Results of the DCC(1,1)-GARCH (1,1) model for indices in pair with LJSEX

Parameter	LJSEX-PX	LJSEX-BUX	LJSEX-ATX	LJSEX-CAC40	LJSEX-DAX	LJSEX-FTSE100
$\omega_{LJSEX\text{-}other\ index}$	4.369636e-06 (3.4461)***	4.496051e-06 (3.5365)***	4.535121e-06*** (3.1793)	4.399285e-06*** (2.7560)	4.365765e-06*** (3.2594)	4.430459e-06*** (2.8542)
$\alpha_{LJSEX\text{-}other\ index}$	0.357100*** (6.1872)	0.3531817 (5.8959)***	0.3541330*** (5.4017)	0.336292*** (4.4445)	0.342869*** (5.2884)	0.336162*** (4.5178)
$\beta_{LJSEX\text{-}other\ index}$	0.642898*** (12.3724)	0.6468163*** (12.4241)	0.645865*** (10.8304)	0.663706*** (9.5277)	0.657129*** (11.3731)	0.663836*** (9.8781)
Ljung-Box $Q^2(10)$ statistics	12.81	14.57	16.25*	13.91	13.88	13.66
$\omega_{other\ index\text{-}LJSEX}$	7.546223e-06*** (4.3904)	1.55260e-05** (2.0502)	3.494421e-06*** (3.7631)	2.387093e-06*** (2.7644)	3.319945e-06*** (3.0601)	1.319709e-06*** (3.1459)
$\alpha_{other\ index\text{-}LJSEX}$	0.138853*** (8.5966)	0.1550270*** (2.6635)	0.120194*** (5.7198)	0.093023*** (7.0157)	0.114016*** (6.8342)	0.094799*** (8.0858)
$\beta_{other\ index\text{-}LJSEX}$	0.836669*** (57.4213)	0.8117493*** (12.4677)	0.866595*** (42.3760)	0.902160*** (67.1917)	0.880248*** (55.2197)	0.901835*** (78.9879)
Ljung-Box $Q^2(10)$ statistics	11.42	6.26	13.61	8.74	11.12*	9.77
α	0.023455*** (2.5499)	0.0304473*** (2.4516)	0.003861** (1.6986)	0.002871* (1.4543)	0.014298* (1.5558)	0.016878 (0.6658)
β	0.9181218*** (25.5845)	0.8686862*** (14.2335)	0.992664*** (172.8324)	0.994840*** (211.2200)	0.954110*** (25.7305)	0.927541*** (5.8284)

*Notes: Parameters $\omega_{LJSEX\text{-}other\ index}$, $\alpha_{LJSEX\text{-}other\ index}$, $\beta_{LJSEX\text{-}other\ index}$ are estimated parameters of a univariate GARCH (1,1) model, with residuals input from the estimated bivariate Vector Autoregressive (VAR) model with LJSEX returns as dependent variable and the other index returns as explanatory variable. $\omega_{other\ index\text{–}LJSEX}$, $\alpha_{other\ index\ \text{–}LJSEX}$, $\beta_{other\ index\text{–}LJSEX}$ are estimated parameters of a univariate GARCH (1,1) model, with residuals input from the estimated bivariate Vector Autoregressive (VAR) model with LJSEX returns as explanatory variable and the other index returns as dependent variable. In parenthesis under the parameter estimation, t-statistics are given: *** (**/*) denote rejection of the null hypothesis that parameter is equal zero at 1% (5%/10%) significance level. Ljung-Box $Q^2(10)$ statistics reports the value of the statistics at lag 10: ***(**/*) indicate that the null hypothesis of no serial correlation in squared residuals of estimated DCC-GARCH model can be rejected at 1% (5%/10%) significance level.*

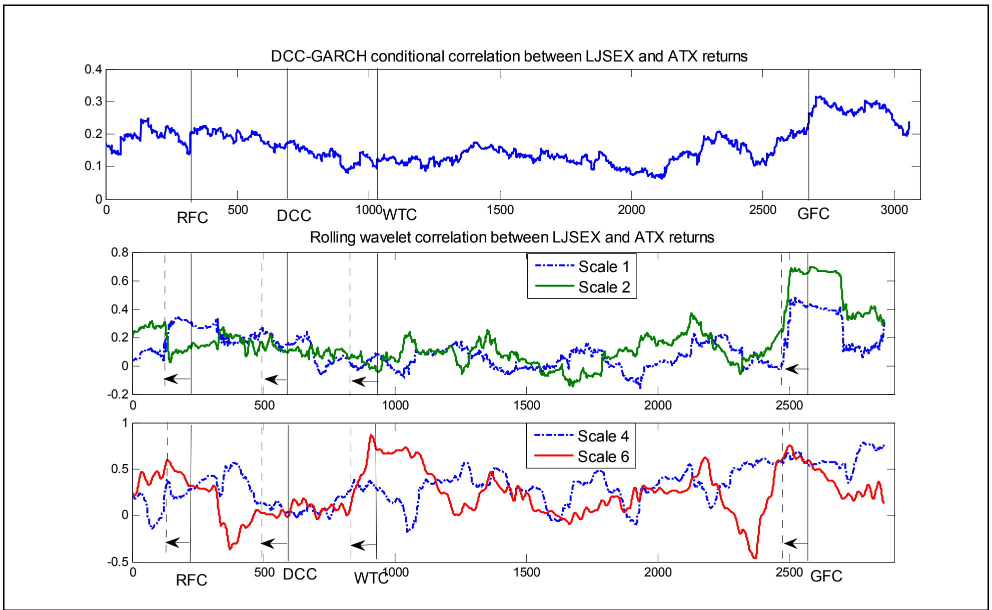
3.3 MODWT results

MODWT transformation of the indices returns series is performed by using a Daubechies least asymmetric filter with a wavelet filter length of 8 (LA8). This is a common wavelet filter applied in empirical studies on financial market interdependence (Gençay et al., 2001b; Ranta, 2010). Wavelet coefficients W_1 to W_6 correspond to changes in averages over physical scales of $\tau_j = 2^{j-1}$ days, scaling coefficients V_6 corresponds to averages of the index return series over a scale $\lambda_j = 2^j$ (Percival and Walden, 2000). To achieve an optimal level balance between sample size and the length of the filter, the maximum number of levels that we use in the decomposition is 6 ($J_0 = 6$). Scale 1 measures the dynamics of returns over 2-4 days, scale 2 over 4-8 days, scale 3 over 8-16 days, scale 4 over 16-32 days, scale 5 over 32-64 days and scale 6 over 64-128 days. Only scales 1,2 (representing low scales, high frequency returns dynamics), scale 4 (mid-frequency returns dynamics) and scale 6 (low frequency returns dynamics) are analyzed in detail.

To examine if wavelet correlation is time-varying, rolling correlations (that is correlations computed in moving windows) are calculated. Using this approach, correlation between the two stock indices return series at time t is calculated from w observations (where w is size of the window), centered around time t. The window is rolled forward one day at a time, resulting in a time series of wavelet correlation. This way we obtain $N - w$ correlation coefficients. The window size has to capture enough data points to obtain reasonable estimates for higher scales. We choose $w = 200$ days, as in Ranta (2010). Experimenting with larger window (400 day) sizes only led to slight changes in the time-varying wavelet correlation.

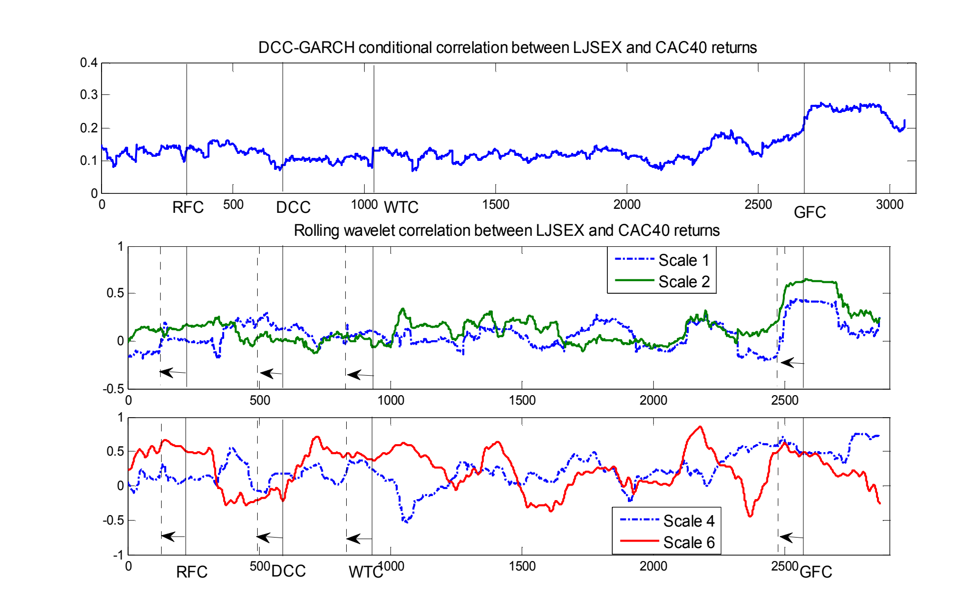
The DCC(1,1)-GARCH(1,1) conditional correlation and rolling wavelet correlation graphs are presented in figures 1 to 6.

Figure 1: DCC-GARCH conditional correlation and rolling wavelet correlation between LJSEX and ATX returns



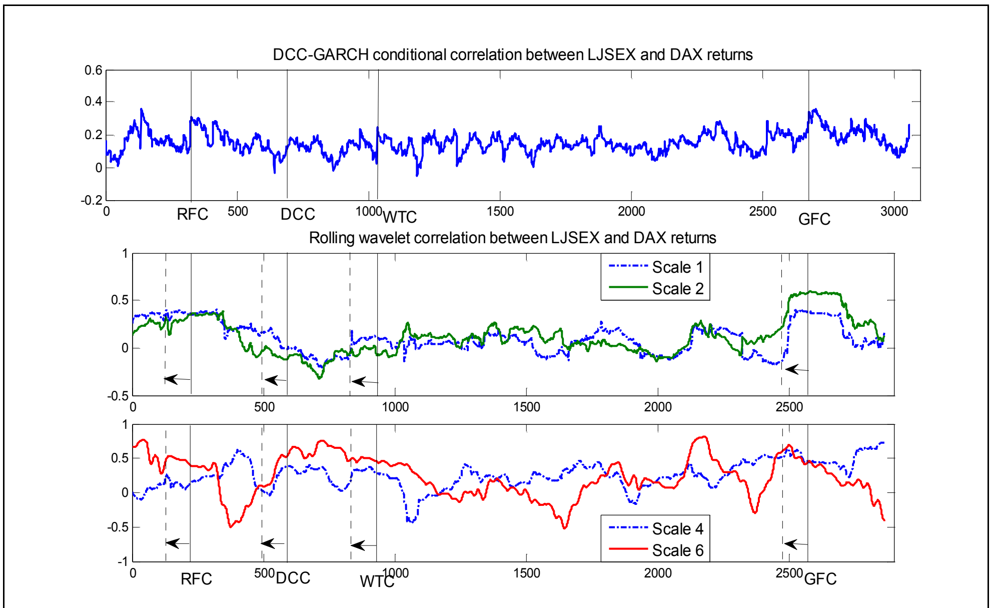
Notes: As calculating rolling correlation on sample size N gives only $N-w$ rolling correlation coefficients, whereas the DCC-GARCH conditional correlation N conditional correlation coefficients, the graph of the later is longer for 100 time units (days) at the start and 100 time units (days) at the end, to achieve that the graphs are time aligned. On the time axis the financial crises are denoted: RFC = Russian financial crisis (outbreak on August 13, 1998), DCC = Dot-Com crisis (the date, March 24, 2000, is taken, when the peak of S&P500 was reached, before the dot-com crisis began), WTC = attack on WTC in New York (September 11, 2001), GFC = Global financial crisis (September 16, 2008). The rolling window of 200 days is taken. The vertical lines indicate these events. The dotted lines in the rolling correlation graphs are drawn 100 days (half the window length) before the actual date of the event, as due to the construction characteristics of rolling correlation coefficient the effect of the event should start to show up in the graph 100 days before the actual time of event.

Figure 2: DCC-GARCH conditional correlation and rolling wavelet correlation between LJSEX and CAC40 returns



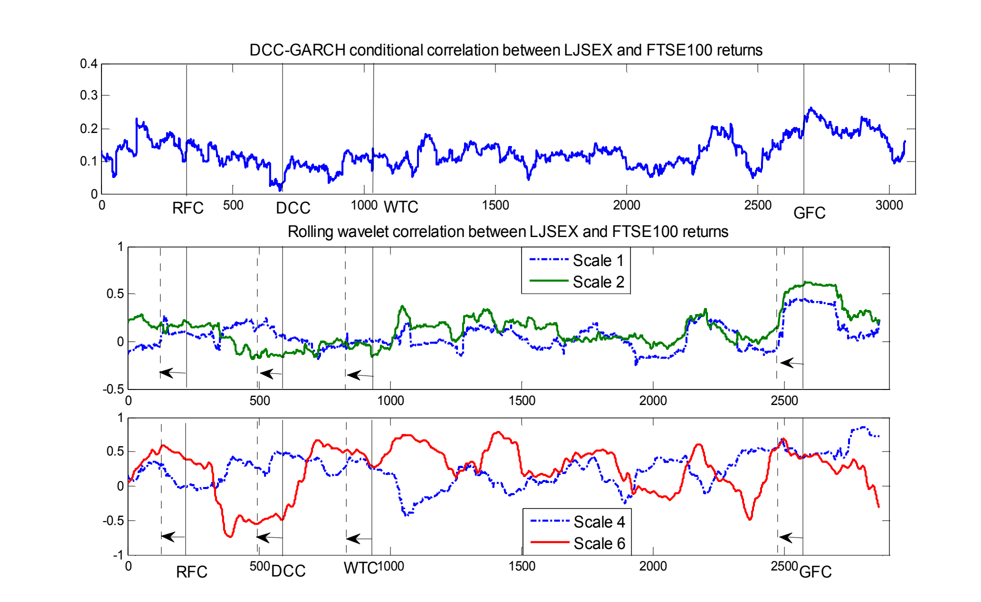
Notes: See figure 1 notes.

Figure 3: DCC-GARCH conditional correlation and rolling wavelet correlation between LJSEX and DAX returns



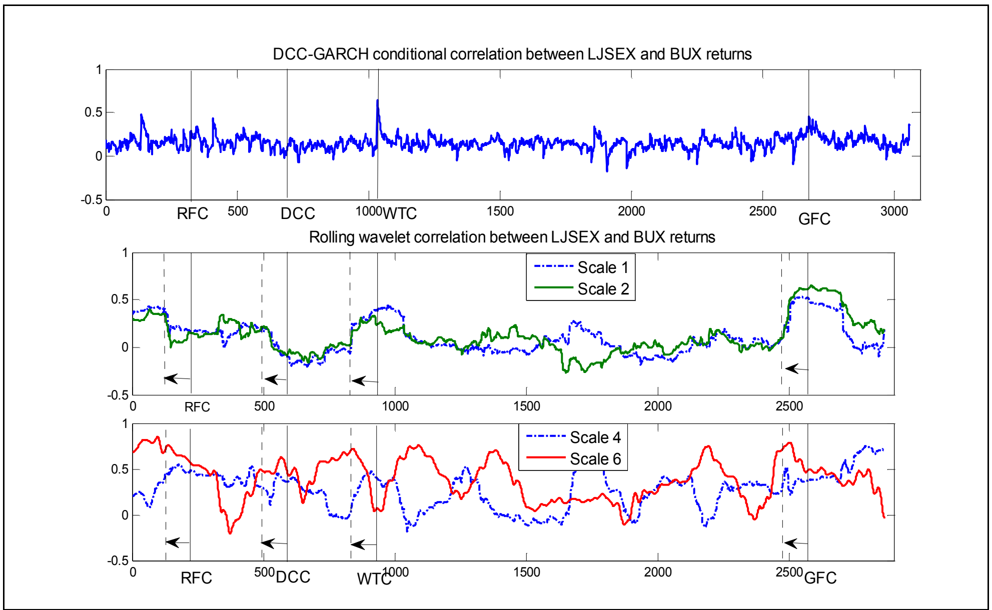
Notes: See figure 1 notes.

Figure 4: DCC-GARCH conditional correlation and rolling wavelet correlation between LJSEX and FTSE100 returns



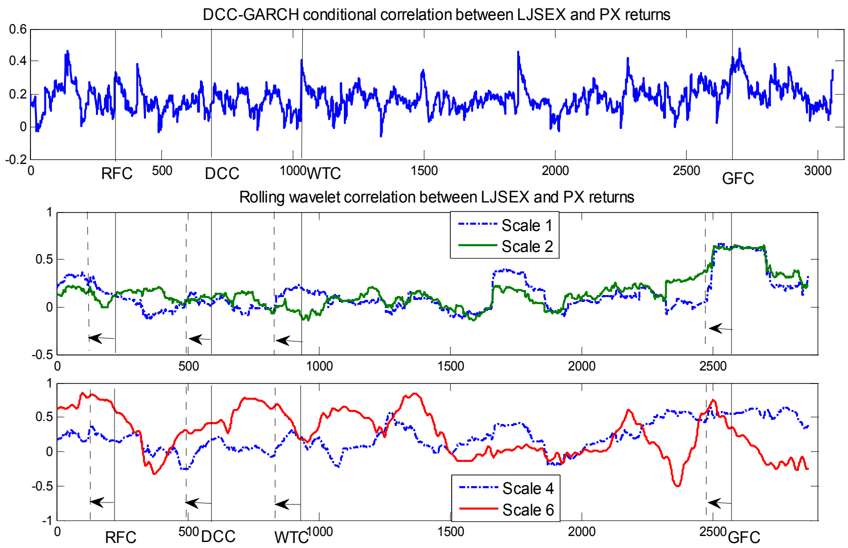
Notes: See figure 1 notes.

Figure 5: DCC-GARCH conditional correlation and rolling wavelet correlation between LJSEX and BUX returns



Notes: See figure 1 notes.

Figure 6: DCC-GARCH conditional correlation and rolling wavelet correlation between LJSEX and PX returns



Notes: See figure 1 notes.

Analyzing figures 1 to 6, more findings can be noted: (1) First of all, a high volatility of conditional correlations between LJSEX and European stock indices returns can be observed, meaning correlation (comovement) between Slovenian and European stock market returns is time-varying. This finding is in accordance with the empirical literature on measuring international stock market comovements (Forbes and Rigobon, 2002; Phylaktis and Ravazzolo, 2005; Gilmore et al., 2008; Kizys and Pierdzioch, 2009). (2) Next, differences in rolling wavelet correlation levels and their time paths suggest that stock market comovement is not just time-varying, but also scale dependent. Similar results, but for other stock markets and time periods, were obtained by Ranta (2010) and Zhou (2011). (3) The trend of correlation between Slovenian and European stock markets in observed period was rising, indicating that Slovenian stock market became more interdependent with these stock markets. This can be confirmed by observing DCC-GARCH conditional correlation and scale 1, 2 and 4 rolling wavelet correlations. However, the highest scale (scale 6), representing long investment horizon, does not confirm this. (4) Financial crises, especially the global financial crisis of 2007-2008 had a major impact on increased comovement of Slovenian stock market with European stock markets. There is mounting evidence that correlations among international markets tend to increase when stock returns fall precipitously (Karolyi and Stulz, 1996; Chesnay and Jondeau, 2001; Ang and Bekaert, 2002; Baele, 2005). However, we also notice, that after the crises (100-400 days after the start of the crisis), comovement between stock markets falls.

Differences between DCC-GARCH and rolling window correlation estimates point to the differences between time-domain and time-frequency domain analysis. DCC-GARCH dynamic correlation is conceptually a time-domain measure, whereas wavelet-based measure allows one to assess simultaneously the comovement at the scale (frequency) level and over time (Rua, 2009). The financial market consists of a variety of agents with different time horizons, and therefore it is postulated that market linkage could differ across time scales. Our findings confirm this – the comovement of returns between stock markets is a scale phenomena. As the scale (frequency) increases, we can observe larger discrepancies between DCC-GARCH and wavelet correlation estimates suggesting one should resort to rolling wavelet correlation estimates when making longer horizon international portfolio decisions.

4 Conclusion

In this paper the comovement and spillover dynamics between the Slovenian and six European stock markets returns (the United Kingdom, German, French, Austrian, Hungarian and the Czech stock market) were studied. Key findings of the paper are the following: (1) Conditional correlations between LJSEX and European stock indices returns in the observed period were highly volatile; (2) Differences in rolling wavelet correlation levels and their time paths suggest that stock market comovement between Slovenian and European stock markets is not just time-varying, but also scale dependent; (3) Financial crises, especially the global financial crisis of 2007-2008, had a major impact on comovement of Slovenian stock with European stock markets; (4)The comovement of returns between stock markets is a scale phenomenon. As the scale (frequency) increases, larger discrepancies between DCC-GARCH and wavelet correlation estimates show up, suggesting one should resort to rolling wavelet correlation estimates when making longer horizon international portfolio decisions.

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