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FUZZY MULTI-ATTRIBUTE DECISION MAKING METHOD BASED ON NEW SIMILARITY MEASUREMENT UNDER SINGLE-VALUED NEUTROSOPHIC SETS

Abstract. *Single-valued neutrosophic number (SVNN) is widely used in multiple attribute decision making especially when the judgment information is not accurate. The similarity measurement of single-valued neutrosophic sets (SVNSs) is an important step involved in SVNN. In this paper, the grey system theory is used first to determine the objective weight, and then the objective weight is combined with the subjective weight to form the combined weight. Then the information of alternatives with different attributes is described in the form of SVNN. In the following process of decision-making, a new similarity measurement between SVNSs based on geometric mean minimization operator is proposed to compare each alternative with the ideal optimal solution and the similarity between them is calculated which will result in the best alternative. Finally, an example is applied to illustrate the effectiveness and practicability of the proposed method*

Keywords*: Fuzzy multi-attribute decision making, Similarity measurement, Geometric mean minimization operator, Single-valued neutrosophic sets .*

JEL Classification: C02, C11, C45, C46, C63

1. Introduction

Multiple-attribute decision making (MADM) is one of the most popular research topics in the subject of group decision making at present. In a multiple-attribute decision making problem the optimal alternative is selected or the alternatives are ranked according to multiple attributes. MADM is an important part of the modern decision-making science and its theory and methods are widely used in engineering, technology, economy, management and military field. Experts need to provide judgment information for all alternatives when making the decisions.

DOI: 10.24818/18423264/56.2.22.14

In decision-making, because of the fuzziness of people’s thinking and the complexity of objective things, it is difficult for decision-makers to provide the accurate judgment information. In 1965, an American scholar L.A.Zadeh (1965) established fuzzy sets to describe fuzzy phenomena. In this method, the object to be investigated and the fuzzy concept are regarded as a fuzzy set, and the membership function is established. Through the relevant operation and transformation of the fuzzy set, the fuzzy object is analyzed. Based on fuzzy mathematics, fuzzy set theory studies the phenomenon of imprecision.

Atanassov K.T. (1986) put forward the concept of intuitionistic fuzzy sets. Because intuitionistic fuzzy sets consider both the information of membership and non-membership, it has stronger ability to express uncertainty than Zadeh′s fuzzy sets, and it can depict the fuzzy essence of the realistic world.

Smarandache(1998) introduced uncertainty degree into intuitionistic fuzzy set and put forward the concept of the neutrosophic set, which considers truth-membership, indeterminacy-membership and falsity-membership. Since then, single-valued neutrosophic number (SVNN) has been widely used to help experts to make the accurate judgment for alternatives. In order to choose the best alternative, how to measure the similarity of single-valued neutrosophic sets (SVNSs) has attracted many experts' attentions. Some algorithms are therefore put forward to calculate the similarity between SVNSs.

Jun Ye and Qiansheng Zhang(2014) studied single-valued neutrosoghic similarity measurement for multiple attribute decision-making. A similarity measurement between SVNSs based on the minimum and maximum operators has been suggested and a new multiple attribute decision-making method based on the weighted similarity measurement of SVNSs has been proposed.

Kalyan Mondal and Surapati Pramanik(2015) studied neutrosophic tangent similarity measurement and its application in multiple attribute decision making. The tangent measure of neutrosophic sets has been proposed and its properties have been explored.

Luo Minxia and Wu Lixian etc (2019) studied a new tangent similarity between single valued neutrosophic sets. A best-worst multi-criteria decision making method based on the single valued neutrosophic sets is proposed. To achieve this goal, an algorithm to identify the best and worst criteria is designed through computing the outdegrees and in-degrees of the collective single valued neutrosophic preference relation directed network, and then the optimal weight vector of attributes is calculated.

Jun Ye(2014) studied multiple attribute group decision-making method with completely unknown weights based on similarity measurements under single valued neutrosophic environment. A distance-based similarity measurements of single valued neutrosophic sets is proposed and then extended to group decision making.

Zhi Kang Lu and Jun Ye(2017) studied cosine measures of neutrosophic cubic sets for multiple attribute decision-making. In this paper, three cosine measures between neutrosophic cubic sets based on the included angle cosine of two vectors, distance, and cosine functions have been proposed and their properties have been investigated.

Maji (2013) proposed the single-valued neutrosophic soft set which combined the neutrosophic set with the soft set. On the basis of this theory, Xindong Peng and Chong Liu(2017) proposed three novel single-valued neutrosophic soft set (SVNSS) methods to solve a single-valued neutrosophic soft decision making problem by evaluation based on distance from average solution (EDAS), similarity measurement and level soft set with a new axiomatic definition for single-valued neutrosophic similarity measurement.

Smarandache(2017) introduced the neutrosophic multiset and the neutrosophic multiset algebraic structures, in which one or more elements are repeated for some times, keeping the same or different neutrosophic components.

For the decision-making problem, the influence of each attribute on the decision-making result is not equal. The weight of attribute plays an important role in the decision-making process. It is particularly important to determine the attribute weight reasonably.

Deng Julong, a Chinese scholar, developed the grey system theory, which is a new method to study the uncertainty system with small data sample and less information, in which part of information is known and part is unknown. The theory can extract valuable information from the generation and development of known information so as to monitor the system effectively and describe the systems' operation behavior and evolution law correctly.

In this paper, the objective weight is first calculated by using the grey correlation degree and then the objective weight is combined with the subjective weight to generate the combined weight by the weighted summation. The weight adjustment coefficient is taken into account as well, which has not been reported in the existing literatures.

The remainder of this paper is organized as follows: Section 2 introduces the basic concept about SVNS, its properties and similarity measurement for SVNSs. Section 3 demonstrates how to calculate the combined weight. Section 4 presents the single valued neutrosophic decision making based on similarity measurement. In section 5, an example is illustrated and a comparison is analyzed to show the effectiveness and practicability of this method. Finally, section 6 concludes the paper.

2. Basic concepts

In this part, some basic concepts and definitions about SVNS and the similarity measurement are introduced. And then an ideal SVNS is proposed.

2.1. Single-valued neutrosophic set(SVNS)

Definition 1. Let U be a universe of discourse, then a SVNS A in U is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, $F_A(x)$ $\in [0,1]$, $0 \le T_A(x)+I_A(x)+F_A(x) \le 3$. A can be expressed as follows(Changxing Fan, En Fan ,Jun Ye,2018):

$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in U\}$

Property 1.

If there are two SVNSs A and B, their relations can be defined as follows:

① $A = B$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$ and $F_A(x) = F_B(x)$ for any x in U;

② $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$ and $F_A(x) \ge F_B(x)$ for any x in U;

③ $A^c = \{< x, F_A(x), 1 - I_A(x), T_A(x) > | x \in U\}$

④ $\lambda A = \{< x, 1-(1-T_A(x))^\lambda, (I_A(x))^\lambda, (F_A(x))^\lambda > | x \in U\}$

Definition 2. Let $A = (T_A, I_A, F_A)$ be a SVNN, then the score function $s(A)$ can be defined as follows (Xindong Peng, Chong Liu, 2017):

$$s(A) = \frac{2}{3} + \frac{T_A}{3} - \frac{I_A}{3} - \frac{F_A}{3} \tag{1}$$

Equation (1) measures the hamming similarity between $A = (T_A, I_A, F_A)$ and the ideal solution (1,0,0). Obviously, $0 \le s(A) \le 1$. The larger the value of $s(A)$ is, the better the scheme is.

2.2. Similarity measurement for SVNSs

There are many methods to measure the similarity of SVNSs, such as Tangent similarity measurement, Jaccard similarity measurement, Dice similarity measurement and so on.

If A and B are two SVNNs , A and B can be expressed as follows:

$A = (T_A, I_A, F_A)$

$B = (T_B, I_B, F_B)$

Based on the geometric mean minimization operator, the similarity measurement between A and B can be defined as follows:

$$S(A,B) = \frac{\min(T_A, T_B) + \min(I_A, I_B) + \min(F_A, F_B)}{\sqrt{T_A \cdot T_B} + \sqrt{I_A \cdot I_B} + \sqrt{F_A \cdot F_B}} \tag{2}$$

Obviously, the similarity measurement $S(A,B)$ between A and B satisfies the following properties:

Property 2.

① $0 \le S(A,B) \le 1$

② $S(A,B) = 1$ if and only if $A = B$

③ $S(A,B) = S(B,A)$

In a universe of discourse $U = \{x_1, x_2, \cdots, x_n\}$, let A and B be two SVNSs, then A and B can be expressed as follows:

$A = \{< x_i, T_A(x_i), I_A(x_i), F_A(x_i) > | x_i \in U\}$

$B = \{< x_i, T_B(x_i), I_B(x_i), F_B(x_i) > | x_i \in U\}$

The similarity measurement between A and B can be defined as follows:

$$S(A,B) = \sum_{i=1}^{n} w_i \frac{\min(T_A(x_i), T_B(x_i)) + \min(I_A(x_i), I_B(x_i)) + \min(F_A(x_i), F_B(x_i))}{\sqrt{T_A(x_i) \cdot T_B(x_i)} + \sqrt{I_A(x_i) \cdot I_B(x_i)} + \sqrt{F_A(x_i) \cdot F_B(x_i)}} \tag{3}$$

Where w_i means the weight of each element $x_i\, (i = 1, 2, \cdots, n)$. $0 \le w_i \le 1$, $\sum_{i=1}^{n} w_i = 1$.

In the same way, the similarity measurement $S(A,B)$ between A and B also has the following properties:

① $0 \le S(A,B) \le 1$

② $S(A,B) = 1$ if and only if $A = B$

③ $S(A,B) = S(B,A)$

2.3. An ideal SVNS

In the decision-making process, the decision-maker will consider a number of influencing factors, which are referred to as indicators. Indicators are divided into benefit indicators and cost indicators. For benefit indicators, such as profit and rate of capital return, the greater the value is, the better the scheme is. For cost indicators, such as investment risk, investment amount, and maintenance cost, the smaller the value is, the better the scheme is. Suppose the set of benefit indicators is represented by B, and the set of cost indicators is represented by C.

In the process of decision-making, an ideal optimal solution has to be identified. The value of each alternative is expressed by single valued neutrosophic value (SVNV). For benefit indicators, the ideal value can be expressed as follows:

$r_j^* =< T_j^*, I_j^*, F_j^* >=< \max_i(T_{ij}), \min_i(I_{ij}), \min_i(F_{ij}) >$, $0 \le j \le m$ if $j \in B$;

For cost indicators, the ideal SVNV can be expressed as follows:

$r_j^* =< T_j^*, I_j^*, F_j^* >=< \min_i(T_{ij}), \max_i(I_{ij}), \max_i(F_{ij}) >$, $0 \le j \le m$ if $j \in C$.

The score function of the ideal SVNN can be described by $s(r_j^*)$.

$$s(r_j^*) = \frac{2}{3} + \frac{T_j^*}{3} - \frac{I_j^*}{3} - \frac{F_j^*}{3} \qquad 0 \le j \le m$$

3. Computing the combined weight

For the decision-making problem, the influence of each attribute on the decision-making result is not equal. The relative importance of attributes can be described by the weight of them. For example, for the problem of deciding the marketing strategy for consumer goods in different regions, the income level and consumption level of the target region have a greater impact on the choice of marketing strategy, while other factors have a smaller impact. In other words, the weight of the income level and consumption level is greater.

Obviously, the decision-making results highly depend on the value of each attribute and its weight. Therefore, it is very important to decide a reasonable weight for each attribute which is directly related to the correctness and credibility of the decision-making results.

At present, there are many methods to determine the weight of attributes. According to the different sources of the original data, these methods can be divided into the following three categories. The first one is the subjective weighting method in which the weights are given by experts according to their experience. There are expert survey method, analytic hierarchy process (AHP) method and so on in this category; The second one is the objective weighting method in which the weights are calculated automatically according to certain rules other than experts' subjective judgment; The third one is the subjective and objective combination weighting method in which the weights are calculated by combining the first two methods together. In the following section, the grey system method which is an objective weighting method will be used and then it will be combined with the subjective weighting method to calculate the combined weight.

3.1. Determining the objective weight: the grey system method

The basic idea of grey correlation analysis is to judge the correlation degree between factors according to the geometric relationship of sequence. If the two curves are similar in shape, the correlation degree is high; otherwise the correlation degree is low. Grey correlation analysis can be used to determine the objective weight. If the value of an alternative scheme under a certain index is closer to the value of the ideal scheme, the index will be given a greater weight. On the contrary, if the value of an alternative scheme under a certain index is far away from the value of the ideal scheme, the index will be given a smaller weight.

Since each SVNN A has a corresponding score function $s(A)$, for the same index, the grey correlation coefficient between each scheme and the ideal scheme can be expressed as Δ_{ij} .

$$\Delta_{ij} = \frac{\min\limits_{1\le i\le n}\left|s_{ij}-s_j^*\right| + k\max\limits_{1\le i\le n}\left|s_{ij}-s_j^*\right|}{\left|s_{ij}-s_j^*\right| + k\max\limits_{1\le i\le n}\left|s_{ij}-s_j^*\right|}$$

where $1 \le i \le n$ and $0 \le j \le m$ (4)

In equation (4) $k \in [0,1]$ and k is a predetermined constant which stands for the resolution coefficient. In general k is equal to $\frac{1}{2}$.

Thus, the grey correlation degree of each index Δ_j can be calculated as follows:

$$\Delta_j = \frac{1}{n}\sum_{i=1}^{n}\Delta_{ij} \quad \text{where } j = 1, 2, \cdots, m \tag{5}$$

Finally, the weight of each index μ_j can be calculated by normalizing the grey correlation degree as follows:

$$\mu_j = \frac{\Delta_j}{\sum_{j=1}^{m}\Delta_j} \qquad \text{where } j = 1, 2, \cdots, m \tag{6}$$

3.2. Combining the grey system method and the subjective method

In this part, the subjective and objective combination weighting method will be used to calculate the combined weight. This method will combine the objective weight obtained by the grey system method in the previous part with the subjective weight obtained from experts' experience and knowledge judgment by the weighted summation method.

Let $\lambda_j (j = 1, 2, \cdots, m)$ be the subjective weight of each index and $\mu_j (j = 1, 2, \cdots, m)$ be the objective weight of each index. The combined weight w_j can be calculated by the weighted summation method as follows:

$$w_j = \rho \cdot \lambda_j + (1-\rho)\mu_j \qquad \text{where } j = 1, 2, \cdots, m \tag{7}$$

In equation (7), ρ is the weight adjustment factor and $0 \le \rho \le 1$. The larger ρ will give more emphasis on the subjective weight and the smaller ρ will give more emphasis on the objective weight. If $\rho = \frac{1}{2}$, then the subjective term and the objective term are equally important while calculating the combined weight.

4. Single valued neutrosophic decision making based on similarity measurement

In this part, how to find the best solution among multiple alternatives under the single valued neutrosophic environment is studied.

Let $A=\{A_1,A_2,\cdots,A_n\}$ be a set of alternatives, and $C=\{C_1,C_2,\cdots,C_m\}$ be a set of indicators. The original data matrix can be expressed as follows:

$$R=\begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{pmatrix}$$

Where, $r_{ij}=<T_{ij},I_{ij},F_{ij}>$, $0\le T_{ij},I_{ij},F_{ij}\le 1$ and $0\le i\le n, 0\le j\le m$.

There are m indexes in each alternative to represent its characteristics, which is:

$$A_i=(r_{i1},r_{i2},\cdots,r_{im})=(<T_{i1},I_{i1},F_{i1}>,<T_{i2},I_{i2},F_{i2}>,\cdots,<T_{im},I_{im},F_{im}>)$$

where $0\le i\le n$

In the process of decision making, an ideal optimal scheme needs to be decided first. The ideal optimal scheme is a combination of the optimal value in each index.

For benefit indicators, the ideal SVNV can be expressed as follows:

$$r_j^*=<T_j^*,I_j^*,F_j^*>=<\max_i(T_{ij}),\min_i(I_{ij}),\min_i(F_{ij})>$$

where $0\le j\le m$ if $j\in B$

For cost indicators, the ideal SVNV can be expressed as follows:

$$r_j^*=<T_j^*,I_j^*,F_j^*>=<\min_i(T_{ij}),\max_i(I_{ij}),\max_i(F_{ij})>$$

where $0\le j\le m$ if $j\in C$

The ideal optimal scheme can be expressed by A^*.

$$A^*=(r_1^*,r_2^*,\cdots,r_m^*)=(<T_1^*,I_1^*,F_1^*>,<T_2^*,I_2^*,F_2^*>,\cdots,<T_m^*,I_m^*,F_m^*>)$$

Next the objective weight can be calculated by using the grey system method which will be combined with the subjective weight later to calculate the combined weight.

Then, each scheme can be compared with the ideal optimal one, and their closeness can be calculated. The closer the degree is, the better the scheme is.

The similarity measurement $S(A_i,A^*)$ between A_i and A^* can be calculated as follows:

$$S(A_i,A^*)=\sum_{j=1}^{m}w_j\frac{\min(T_{ij},T_j^*)+\min(I_{ij},I_j^*)+\min(F_{ij},F_j^*)}{\sqrt{T_{ij}\cdot T_j^*}+\sqrt{I_{ij}\cdot I_j^*}+\sqrt{F_{ij}\cdot F_j^*}} \tag{8}$$

where $0\le i\le n$

Finally, the alternatives will be ranked according to the similarity degree. The greater the degree of similarity is, the better the corresponding scheme is.

5. A numerical example and comparative analysis

5.1. Numerical example

In this part, a numerical example will be used to verify the effectiveness of the above method. An example in the literature [14] is used with a few changes.
Family A recently plans to purchase a car, and intends to choose one from the following four models: (1) A_1 (2) A_2 (3) A_3 (4) A_4. In order to decide which car to buy, family A needs to consider the following five indicators: (1) C_1: price; (2) C_2: cost per service; (3) C_3: fuel consumption per 100 km; (4) C_4: comfort; (5) C_5: safety. The first three indicators are cost indicators. The last two indicators are benefit indicators. For family A, these five indicators are not equally important, so each indicator is given a different weight. The weight vector of the five indicators is as follows:

$$W = (w_1, w_2, w_3, w_4, w_5)^T = (0.3, 0.15, 0.15, 0.2, 0.2)^T$$

Family A will evaluate them separately after investigating the four types of cars. The evaluation values will be given in the form of SVNV.
Step 1: Establish the decision matrix.

Family A evaluates the different indicators of each car, and the evaluation values form the decision matrix in table 1.

Table 1. Decision Matrix

Car Type	C_1	C_2	C_3	C_4	C_5
A_1	<0.7,0.3,0.6>	<0.4,0.4,0.5>	<0.8,0.7,0.6>	<0.5,0.2,0.8>	<0.7,0.7,0.5>
A_2	<0.7,0.7,0.1>	<0.7,0.6,0.8>	<0.9,0.4,0.6>	<0.5,0.1,0.9>	<0.5,0.2,0.7>
A_3	<0.6,0.3,0.7>	<0.2,0.2,0.2>	<0.6,0.5,0.2>	<0.4,0.2,0.2>	<0.9,0.5,0.5>
A_4	<0.8,0.6,0.1>	<0.3,0.5,0.2>	<0.1,0.7,0.2>	<0.4,0.2,0.8>	<0.5,0.4,0.5>

Step 2: Decide the ideal optimal scheme.

$$A^* = (<0.6, 0.7, 0.7>, <0.2, 0.6, 0.8>, <0.1, 0.7, 0.6>, <0.5, 0.1, 0.2>, <0.9, 0.2, 0.5>)$$

Step 3: Calculate the objective weight.
At first, the score function for each pair of A_i and C_j is calculated as follows in table 2.

Table 2. The Score Function

Car Type	C_1	C_2	C_3	C_4	C_5
A_1	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
A_2	$\frac{19}{30}$	$\frac{13}{30}$	$\frac{19}{30}$	$\frac{1}{2}$	$\frac{8}{15}$

A_3	$\frac{8}{15}$	$\frac{3}{5}$	$\frac{19}{30}$	$\frac{2}{3}$	$\frac{19}{30}$
A_4	$\frac{7}{10}$	$\frac{8}{15}$	$\frac{2}{5}$	$\frac{7}{15}$	$\frac{8}{15}$
A^*	$\frac{2}{5}$	$\frac{4}{15}$	$\frac{4}{15}$	$\frac{11}{15}$	$\frac{11}{15}$

Then, the grey correlation coefficient between every scheme and the ideal scheme is calculated as follows in table 3.

Table 3 The Grey Correlation Coefficient

Car Type	C_1	C_2	C_3	C_4	C_5
A_1	1	$\frac{5}{8}$	$\frac{11}{19}$	$\frac{4}{11}$	$\frac{1}{3}$
A_2	$\frac{15}{31}$	$\frac{1}{2}$	1	$\frac{4}{11}$	$\frac{7}{19}$
A_3	$\frac{15}{37}$	1	1	$\frac{2}{3}$	$\frac{7}{13}$
A_4	$\frac{5}{9}$	$\frac{5}{7}$	$\frac{11}{25}$	$\frac{1}{3}$	$\frac{7}{19}$

Next the total correlation degree of each index Δ_j and the objective weight μ_j can be calculated as follows.

$\Delta_1 = 0.6112$ $\Delta_2 = 0.7098$ $\Delta_3 = 0.7547$ $\Delta_4 = 0.4318$ $\Delta_5 = 0.4022$

$\mu_1 = 0.2101$ $\mu_2 = 0.2439$ $\mu_3 = 0.2594$ $\mu_4 = 0.1484$ $\mu_5 = 0.1382$

Step 4: Calculate the combined weight.

Suppose the weight adjustment factor $\rho = \frac{1}{2}$ which means the subjective weight and objective weight are equally important. Then the combined weight w_j can be calculated as follows.

$w_1 = 0.2551$ $w_2 = 0.1969$ $w_3 = 0.2047$ $w_4 = 0.1742$ $w_5 = 0.1691$

Step 5: Calculate the similarity degree.

According to equation (8), the similarity measurement $S(A_i, A^*)$ between A_i and A^* can be calculated as follows.

$S(A_1, A^*) = 0.4548$

$S(A_2, A^*) = 0.7601$

$S(A_3, A^*) = 0.7777$

$S(A_4, A^*) = 0.7226$

Step 6: Choose the best alternative.

According to the degree of similarity, the alternatives can be sorted as follows. The closer the alternative is to the ideal optimal scheme, the better the alternative is.

$S(A_2, A^*) > S(A_3, A^*) > S(A_1, A^*) > S(A_4, A^*)$

The priority of each alternative can be decided as follows:

$A_2 \succ A_3 \succ A_1 \succ A_4$

Therefore, the best alternative should be A_2.

5.2. Comparative analysis with different methods

In this part, the proposed method is compared with other methods in literature [4] and [10] with the same weights as follows and the comparison results are shown in Table 4.

$w_1 = 0.3$ $w_2 = 0.15$ $w_3 = 0.15$ $w_4 = 0.2$ $w_5 = 0.2$

Table 4. Comparison Results of Different Methods

Method	Result	Ranking	The Best Alternative
Method 1 based on similarity measurement in[4]	$S(A_1, A^*) = 0.6560$ $S(A_2, A^*) = 0.6979$ $S(A_3, A^*) = 0.6827$ $S(A_4, A^*) = 0.6195$	$A_2 \succ A_3 \succ A_1 \succ A_4$	A_2
Method 2 based on EDAS in[10]	$AS_1 = 0.2145$ $AS_2 = 0.4805$ $AS_3 = 0.85445$ $AS_4 = 0.45445$	$A_3 \succ A_2 \succ A_4 \succ A_1$	A_3
Method 3 based on cosine measurement in[14]	$S(A_1, A^*) = 0.9389$ $S(A_2, A^*) = 0.9782$ $S(A_3, A^*) = 0.9456$ $S(A_4, A^*) = 0.9240$	$A_2 \succ A_3 \succ A_1 \succ A_4$	A_2
The method proposed in this paper	$S(A_1, A^*) = 0.7925$ $S(A_2, A^*) = 0.8314$ $S(A_3, A^*) = 0.8335$ $S(A_4, A^*) = 0.7936$	$A_3 \succ A_2 \succ A_4 \succ A_1$	A_3

From table 4, the results of method 2 and the proposed method have the same best alternative A_3 and the results of method 1 and method 3 have the best alternative A_2. Method 1 uses the minimum maximum operator in which some information will be lost and which may lead to the change of the priority sequence, as shown in Table 4. In method 2, evaluating distance from average solution (EDAS) is used to identify the best alternative which needs a lot of calculation with eight steps. However, the method proposed in this paper only needs four steps which has greatly reduced the calculation work.

5.3. Comparative analysis with different weight adjustment factor

The combined weight in the proposed method can be calculated by equation (7) which depends on the objective weight, the subjective weight and the weight adjustment factor. In this part, three different value of $\rho(\rho = 0, \rho = \frac{1}{2}, \rho = 1)$ will be applied in the proposed method to show the impact of weights on the decision making results. And the comparison results of the proposed method with different weight adjustment factor are shown in Table 5.

The computation results in Table 5 show that different weights will lead to different decision making results. Therefore it is very important to determine a reasonable weight.

Table 5. Comparison Results of the Proposed Method with Different Weight Adjustment Factor

ρ	Weights	Results	Ranking	The Best Alternative
$\rho = 0$	$W = \begin{pmatrix} 0.2101 \\ 0.2439 \\ 0.2594 \\ 0.1484 \\ 0.1382 \end{pmatrix}$	$S(A_1, A^*) = 0.8050$ $S(A_2, A^*) = 0.8313$ $S(A_3, A^*) = 0.7817$ $S(A_4, A^*) = 0.7996$	$A_2 \succ A_1 \succ A_4 \succ A_3$	A_2
$\rho = \frac{1}{2}$	$W = \begin{pmatrix} 0.2551 \\ 0.1969 \\ 0.2047 \\ 0.1742 \\ 0.1961 \end{pmatrix}$	$S(A_1, A^*) = 0.7987$ $S(A_2, A^*) = 0.8315$ $S(A_3, A^*) = 0.8076$ $S(A_4, A^*) = 0.7967$	$A_2 \succ A_3 \succ A_1 \succ A_4$	A_2
$\rho = 1$	$W = \begin{pmatrix} 0.3 \\ 0.15 \\ 0.15 \\ 0.2 \\ 0.2 \end{pmatrix}$	$S(A_1, A^*) = 0.7925$ $S(A_2, A^*) = 0.8314$ $S(A_3, A^*) = 0.8335$ $S(A_4, A^*) = 0.7936$	$A_3 \succ A_2 \succ A_4 \succ A_1$	A_3

6. Conclusion

This paper first introduces the SVNS characterized by truth-membership function, indeterminacy-membership function and falsity-membership function. And then it defines a novel similarity measurement of two SVNNs based on the geometric mean minimization operator, and describes the properties of this similarity measurement. This similarity formula is further extended to two SVNSs.

For the multi-attribute decision-making problem, the influence of each attribute on the decision-making result is not equal. The relative importance of attributes can be described by the weight of them. How to formulate the weight scientifically is vital as well. In this paper, a combined weight is considered by combining the objective weight and the subjective one with the weighted summation method. The objective weight is determined by using the grey system theory. And the subjective weight is obtained from experts' experience and knowledge judgment. In calculating the combined weight, the weight adjustment coefficient is taken into account as well, which has not been reported in the existing literatures. The weight adjustment coefficient is determined by the importance of subjective weight. The combined weight considers not only the knowledge and experience of experts, but also the information contained in the digital characteristics of evaluation indicators.

Finally, an example in the literature [14] with a few changes is used to illustrate the calculation of the proposed method. And then the result of the proposed method is compared with other three methods in literature [4] [10] and [14]. The comparison shows that the proposed method is effective and can identify the optimal scheme quickly.

The method proposed in this paper can be applied not only to SVNSs, but also to interval-valued neutrosophic sets. In the future, more other methods which can be combined into SVNSs and other neutrosophic sets will be explored to solve the decision making problems.

ACKNOWLEDGEMENTS

This research is supported by the National Social Science Fund of China (No. 17BGL176).

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