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## **CORRELATION ANALYSIS AND PREDICTION OF STOCK BASED ON VMD-LASSO MODEL**

***Abstract.** For nonlinear non-stationary sequences, variational mode decomposition (VMD) is a novel, efficient, adaptive, quasi-orthogonal, completely non-recursive data decomposition method, which still has a solid theoretical basis. It iteratively searches for the optimal solution of the variational model to determine the frequency center and bandwidth of each component, so that the frequency domain segmentation of the signal and the effective separation of components can be adaptively realized. At the same time, the Lasso method is an effective method for performing variable screening. Therefore, this paper proposes a least absolute shrinkage and selection operator (LASSO) regression method based on the effective variable selection of components derived from VMD decomposition. The VMD-LASSO model is established for stock data. It is found that there is a strong interaction between the two stocks, and the influence of each component is one-to-one. VMD-LASSO model is used to predict stock series, and the results are compared with those of three traditional methods. The results show that the proposed model has higher prediction accuracy.*

***Keyword:** VMD, LASSO, Variable selection, Stock forecasting.*

**JEL Classification: C32, C53**

### **1. Introduction**

A time series is a collection of values that are arranged at the same or different intervals in order of appearance. Time series data is widely used in various fields of natural sciences and social sciences, such as stock prices(Xiaoli

Zhou & Zhixiong Wu ,2016), airline passenger flow(Mao et al.,2015), electricity demand(Wang et al.,2014), crude oil prices (Jianwei et al.,2017) and so on.

The actual time series are generally non-linear and non-stationary and multivariate. However, the current frequency domain analysis method for nonlinear and non-stationary data is not sufficient, and the selection of effective variables for multiple time series is also a difficult point. Therefore, the exploration of effective variable selection and frequency domain analysis methods is urgently needed to be solved.

In fact, the essence of time series analysis is still to establish a regression model. For regression models, the accuracy of the models depends mainly on the choice of variables and the value of regression parameters. Considering the general linear regression model, there are two shortcomings. The first is the problem of prediction accuracy, followed by the interpretability of the model. Robert Tibshirani put forward a new variable selection method in 1996——Least Absolute Shrinkage and Selection Operator (LASSO) (Tibshirani R,1996). Based on the sparsity of Lasso solution and the stability of variable selection, it has been widely applied in biomedicine, optics and other fields (Krissinel E & Henrick K, 2004).

Most of the traditional time-frequency analysis methods have limitations, which makes the traditional methods have a limited effect on the local feature analysis of nonlinear non-stationary signals, such as local time features or local frequency features. In 1987, wavelet first appeared as an analytical basis in multiresolution theory (Jin Yuzhu, 2013). Since the wavelet analysis was put forward, it has made outstanding achievements in the perfection of mathematical theory and the extensiveness of mathematical applications. Wavelet analysis is also widely used for nonlinear non-stationary time series. A wavelet-based approach to functional linear regression has received some attention in recent literature. In this context, a wavelet-based LASSO regression method is proposed (Zhao et al., 2012). Wavelets have some drawbacks, such as lack of adaptability, and wavelet basis functions need to be pre-selected. Aiming at the shortage of wavelets, Huang et al. proposed the Empirical Mode Decomposition (EMD) method. It can adaptively decompose complex signals into some intrinsic mode functions (IMF). Then EMD became a popular method in signal processing. EMD regression method is proposed, which shows better prediction performance (Huang et al., 1998; Wu & Huang, 2004). Combining the advantages of EMD and LASSO, a LASSO regression method based on EMD is generated (Lei et al., 2016). The method uses the time-frequency structure in the data to reveal the interactions between the two variables, so that the future events can be predicted more accurately. However, there are limitations in the EMD method, such as modal mixing and endpoint effects, noise sensitivity and selection of interpolation methods. Based on empirical mode decomposition, Wu and Huang proposed the Ensemble Empirical mode decomposition (EEMD) method. This method applies noise-assisted analysis to empirical mode decomposition to promote anti-aliasing decomposition. It solves an important defect mode aliasing phenomenon in EMD and embodies the superiority

of EEMD (Wu & Huang, 2009). A regularized EEMD for reducing the decomposition error is proposed, called LASSO EEMD. This method has better performance than cubic regression in estimating blood flow velocity (Shen & Lee, 2012). To a certain extent, EEMD overcomes the modal aliasing phenomenon of EMD, but the analysis results still have modal aliasing (Zhou et al., 2013).

A new signal adaptive decomposition method proposed by Dragomiretskiy and Zosso in 2014: Variational Mode Decomposition (VMD). The method is based on Wiener filtering, one-dimensional Hilbert transform and analytical signals, heterodyne demodulation and other theories. Its greatest advantage is that it overcomes the lack of mathematical theory support for empirical mode decomposition methods, and is not as sensitive to noise as empirical mode decomposition. VMD can simultaneously estimate the modalities of different center frequencies. The essence is a set of adaptive Wiener filter banks, which is different from EMD and EEMD in non-recursive mode decomposition, avoiding the envelope estimation error caused by recursive mode decomposition. Accumulate and overcome the end effect (Dragomiretskiy & Zosso, 2014). Since its introduction, it has been rapidly applied to the field of signal analysis and fault diagnosis of rotating machinery and achieved good results. Liu X Y et al proposed a bearing fault feature extraction method based on VMD and ICA. This method improves the decomposition efficiency, solves the problem that the signal is susceptible to noise interference, and realizes the accurate diagnosis of bearing fault (Liu et al., 2017). Wang Xin and Yan Wenyuan proposed a fault diagnosis method for rolling bearing based on VMD and support vector machine (SVM). This method can effectively classify the working state and fault type of bearing in a small number of samples (Wang & Yan, 2017). Jianwei E, Yanling Bao and Jimin Ye proposed a combination method, which including VMD, ICA and ARIMA. This method is used to analyze the influence factors of crude oil price and predict the future crude oil price. It also proved that the proposed method can forecast the crude oil price more accurately (Jianwei et al., 2017). Therefore, In light of the above work, we propose a hybrid methodology, which combines the variational mode decomposition method with LASSO regression (VMD-based Lasso regression).

The structure of this present paper is as follows: In Section2, we recall basic theory about VMD and LASSO. Empirical analysis is applied to stock index data from the Chinese market to test the modeling process, and compare its performance with that of three existing methods is performing in Section 3. Section 4 is the discussion.

## **2. Methodology**

In the process of modeling, a binary time series is constructed firstly. Then, one of the time series is decomposed by VMD, and the resulting IMFs are treated as the independent variable X. Finally, take another time series as the dependent variable Y, using LASSO to select effective variables, and establish LASSO

regression model between independent variables and dependent variables.

The modeling process:

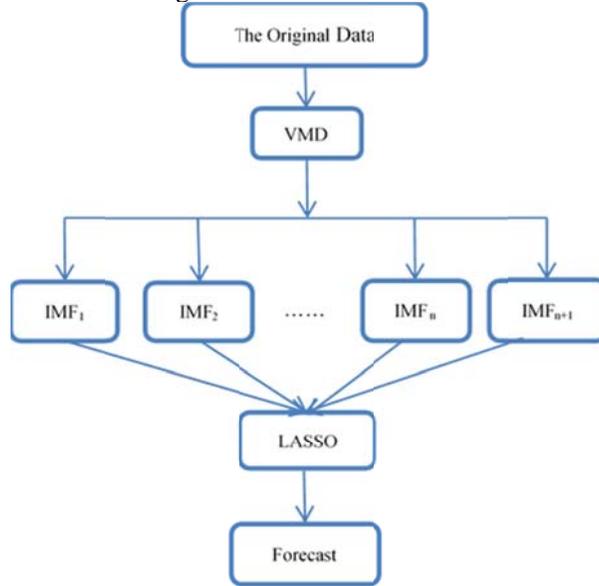
Step 1: Decomposing one time series into several IMFs,  $u_k (k = 1, 2, \dots, k-1, k)$ .

$$\sum_{k=1}^k u_k(t) = f(t)$$

Step 2: Establishing LASSO regression model between independent variables  $u_k$  and dependent variables  $Y(t)$ .

$$\arg \min \left\{ \sum_{k=1}^k \left( Y(t) - \beta_0 - \sum_{i=1}^p u_k(t) \beta_i \right)^2 \right\} \quad s.t. \sum_{i=1}^p |\beta_i| \leq s$$

The flowchart of the Lasso regression based on VMD is demonstrated in Figure1.



**Figure 1. Flowchart of the Lasso regression based on VMD**

### 2.1 Variational mode decomposition (VMD)

The VMD algorithm is a new, adaptive and quasi-orthogonal signal decomposition method. The goal of VMD is to decompose a real valued input signal  $f$  into a discrete number of subsignals (modes),  $u_k$ , that have specific sparsity properties while reproducing the input, and all sub- signals is mostly compact around a center pulsation  $\omega_k$ . Therefore, VMD can be regarded as a constrained variational problem is the following Eq. (1):

$$\min_{\{u_k\}, \{\omega_k\}} \left\{ \sum_k \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k(t)} \right\|_2^2 \right\} \quad s.t. \quad \sum_k u_k = f \quad (1)$$

Where  $f$  is the original signal,  $\delta$  is the Dirac distribution,  $k$  is the number of modes,  $u_k = \{u_1, \dots, u_k\}$  denotes each mode function,  $\omega_k = \{\omega_1, \dots, \omega_k\}$  indicates each center frequency,  $\sum_k = \sum_{k=1}^k$  represents the sum of all mode function,  $*$  denotes convolution.

In order to obtain the optimal solution of the variational model, the quadratic penalty term and Lagrangian multipliers are brought in, the function is constructed as Eq.(2):

$$L(\{u_k\}, \{\omega_k\}, \lambda) = \alpha \sum_k \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 + \|f(t) - \sum_k u_k(t)\| + \langle \lambda(t), f(t) - \sum_k u_k(t) \rangle \quad (2)$$

The Lagrange function is transformed from the time domain to the frequency domain, and the extremum is obtained. The frequency domain expressions of the modal component and the center frequency are obtained respectively by Eq. (3) and Eq. (4):

$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i \neq k} \hat{u}_i^{n+1}(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k)^2} \quad (3)$$

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega |\hat{u}_k(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_k(\omega)|^2 d\omega} \quad (4)$$

Then, the alternating direction multiplier algorithm (ADMM) is used to find the optimal solution of the constrained variational model, so that the original signal can be decomposed into  $K$  narrow mode components.

In summary, the steps of the VMD algorithm are as follows:

Step 1. Initialize  $\{u_k^1\}, \{\omega_k^1\}, \lambda^1$  and  $n$  to 0;

Step 2. The value of  $u_k$ ,  $\omega_k$ , and  $\lambda$  is updated respectively according to Eq.(3), Eq.(4) and the following formula Eq. (5):

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^n(\omega) + \tau \left[ \hat{f}(\omega) - \sum_k \hat{u}_k^{n+1}(\omega) \right] \quad (5)$$

Step 3. Repeat step 2 until the iteration termination condition is satisfied:

$$\sum_k \left\| \hat{u}_k^{n+1} - \hat{u}_k^n \right\|_2^2 / \left\| \hat{u}_k^n \right\|_2^2 < \varepsilon, \varepsilon > 0 \quad (6)$$

Where  $\varepsilon$  is a given accuracy requirement.

### 2.2 Improved LASSO algorithm based on LARS

In order to automatically select variables simultaneously, Tibshirani (1996) proposed a new method called LASSO, which preserves the good features of

subset selection and ridge regression (Tibshirani R,1996 ). LASSO adds penalty items on the basis of least squares estimation.

The LASSO estimate is the solution to

$$\arg \min_{\beta} \left\{ \sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p x_{ij} \beta_j \right)^2 \right\} \text{ s.t. } \sum_{j=1}^p |\beta_j| \leq s \quad \lambda \in [0, \infty] \quad (7)$$

The penalty term of LASSO is put into the objective function through the Lagrange multiplier method, which constitutes a goal optimization problem. The objective function is as follows:

$$\arg \min_{\beta} \left\{ \sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p x_{ij} \beta_j \right)^2 \right\} + \lambda \sum_{j=1}^p |\beta_j| \quad (8)$$

where  $\lambda$  is a tuning parameter,  $\lambda \in [0, \infty]$ , which will affect the degree of compression. The smaller  $\lambda$ , the smaller the degree of compression, the more variables are retained; the larger  $\lambda$ , the greater the degree of compression, the less variables are retained.

The essence of the LASSO variable selection is to use the absolute value of the model coefficients as a penalty term to limit the coefficients so that the model coefficients with small absolute values are compressed to zero. The two purposes of variable selection and parameter estimation are completed at the same time, however, the variable selection and parameter estimation of the conventional method are performed separately. LASSO better overcomes the shortcomings of traditional variable selection, and balances the accuracy and interpretability of the regression model. LASSO is a biased estimation method for dealing with multicollinearity. By increasing the deviation and reducing the partial variance, the mean square error is minimized.

Since LASSO has successfully overcome the shortcomings of traditional variable selection methods, this method has attracted a lot of attention in the field of regression and classification. Many researchers have tried to design effective algorithms so as to take advantages of this approach. Including the shooting algorithm (Fu, 1998), the homotopy algorithm (Osborne & Turlach, 2000), and the LARS algorithm (Bradley et al., 2004). The LARS algorithm provides an especially effective solution to computational problems in LASSO. It is based on the forward selection algorithm and the forward gradient algorithm. It gradually improves the computational complexity and reduces the computational complexity while preserving the information correlation as much as possible.

The basic steps of the LARS algorithm are as follows:

Step 1. Begin with  $\beta_1, \beta_2, \dots, \beta_p = 0$ , and  $r = y - \bar{y}$ .

Step 2. Determining the correlation between covariates and the residual, and find the predictor  $x_j$  most correlated with  $r$ .

Step 3. Moving  $\beta_j$  from 0 towards its least-squares coefficient  $\langle x_j, r \rangle$ , until some

other covariate  $x_k$  has as much correlation with the current residual as  $x_j$ .

Step 4. Moving  $\beta_j$  and  $\beta_k$  in the direction defined by their joint least squares coefficient of the current residual on  $(x_j, x_k)$ , until some other covariate  $x_l$  has as much correlation with the current residual.

Step 5. Continue in this way until all  $p$  predictors have been entered. After  $\min(N-1, p)$  steps, we arrive at the full least-squares solution.

### 2.3 Evaluation criteria of forecasting accuracy

The present study performed other model performance metrics, including the Relative Error (RE), Mean Absolute Deviation (MAE), Mean-Square Error (MSE), Root Mean Square Error (RMSE), Mean-Square Error adjusted for heteroskedasticity (HMAE), and Mean Absolute Deviation adjusted for heteroskedasticity (HMSE), to confirm the superiority of the VMD-LASSO model (see Equations 9–14). In fact, the forecasting performance evaluation criteria used in this paper are also well employed in a number of previous studies (Brailsford & Faff, 1996; Lopez, 2001; Marcucci, 2005; Wei et al., 2010). In all metrics,  $y_t$  represents the observed values at time  $t$ ,  $\hat{y}_t$  represents the forecasting values at time  $t$ .

$$RE = \frac{|y_t - \hat{y}_t|}{y_t} \times 100\% \quad (9)$$

$$MSE = \sum_{t=1}^n (y_t - \hat{y}_t)^2 / n \quad (10)$$

$$MAE = \sum_{t=1}^n |y_t - \hat{y}_t| / n \quad (11)$$

$$HMSE = \sum_{t=1}^n (1 - y_t / \hat{y}_t)^2 / n \quad (12)$$

$$HMAE = \sum_{t=1}^n |1 - y_t / \hat{y}_t| / n \quad (13)$$

$$RMSE = \sqrt{\sum_{t=1}^n (y_t - \hat{y}_t)^2 / n} \quad (14)$$

### 3. Correlation analysis and prediction of the stock market

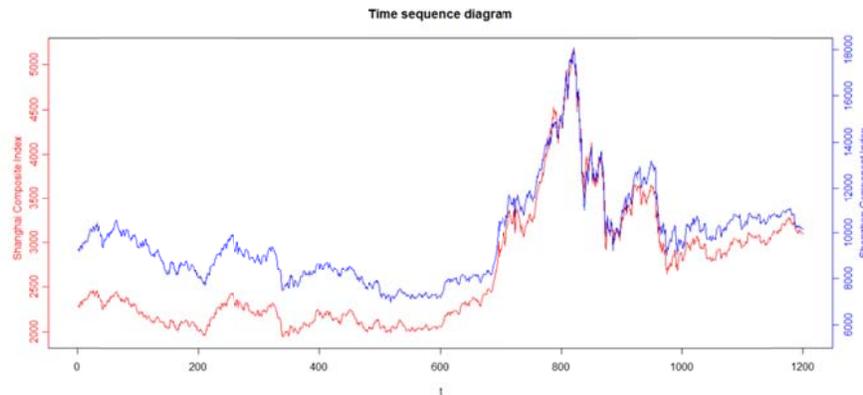
The work aims to use the proposed method to build a model, and finding the interaction between the two stock markets. The experimental contents are mainly divided into three parts. In the first subsection, the two series are decomposed by VMD; In the second subsection, each component of one of them is used as the dependent variable and all components of the other sequence are subjected to LASSO regression; the third subsection, decomposition one of the two variables, the undecomposed variable is used as the dependent variable, and the

decomposed component is used as the independent variable. All the different experimental analysis processes are used to reveal the true relationship between independent variables and dependent variables, which proves that the modeling process of the proposed method is reasonable.

### 3.1 Data sources and decomposition

The data used is the daily closing price of the Shanghai Composite Index (stock code: 000001) and Shenzhen Securities Index (stock code: 399001) from January 30, 2012 to December 30, 2016. All data came from Flush websites, and there were 1201 observations in the data set. The sequence diagram of the data set is plotted firstly, and the result is shown in Figure 2.

In the Figure 2, the ordinate on the left is the Shanghai Composite Index, and its polyline is red; the right coordinate is the Shenzhen Stock Index, and its polyline is blue. It can be seen from the figure that there is a long-term sequence correlation between the two stock index.



**Figure 2. Daily closing prices for the Shanghai Composite Index and the Shenzhen Component Index are plotted over time**

The K value of VMD decomposition is determined according to a threshold method. Through a lot of experiments, it is found that as the increase of decomposition number K, the square of error  $\gamma$  between original data and decomposed data will gradually decrease. At the same time, different K values will produce different iteration termination conditions  $\varepsilon$ . With the increase of K values,  $\varepsilon$  values show a process of first decreasing and then increasing. The threshold method combines  $\gamma$  and  $\varepsilon$  to determine the K value. Therefore, the K value of VMD decomposition is determined to be 10.

The time series of the Shanghai composite index is recorded as  $F$ , the components after decomposition are recorded as  $f_i (i=1, \dots, 10)$ ; the Shenzhen Securities Index is  $G$ , the components after decomposition are recorded as

$g_i (i=1, \dots, 10)$ . Using VMD to decompose the two stock time series into 10 components, the trend of each component after decomposition is shown in Figure 3 and 4.

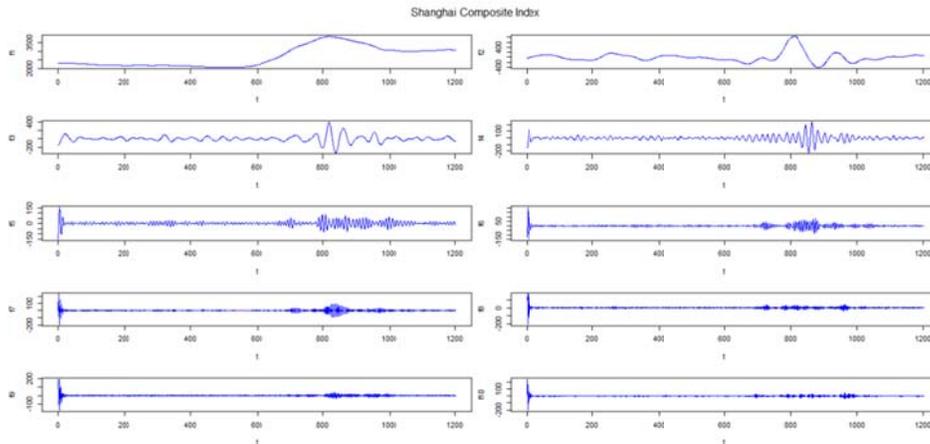


Figure 3. VMD Components of the Shanghai Composite Index

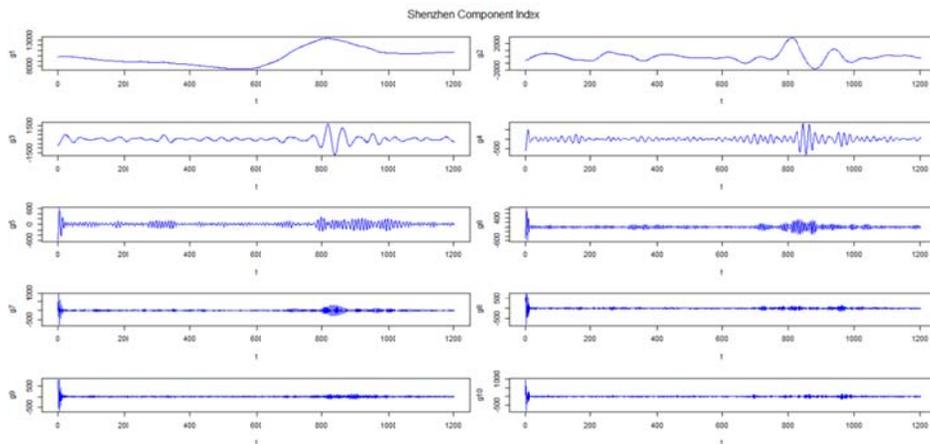


Figure 4. VMD Components of the Shenzhen Component Index

### 3.2 Correlation analysis of two stocks

Let  $f_i (i=1, \dots, 10)$  to be the dependent variable separately, and take all the sequences of  $g_i (i=1, \dots, 10)$  as the independent variables. Then, performing 10 times of LASSO regression, the order of the independent variables  $g_i (i=1, \dots, 10)$  entering the model is shown in Figure 5. Similarly, the sequence of  $g_i (i=1, \dots, 10)$  is used as the dependent variable, and take all the sequences of  $f_i (i=1, \dots, 10)$  as

the independent variables. Performing 10 times of LASSO regression, the order of the independent variables  $f_i (i=1, \dots, 10)$  entering the model is shown in Figure 6.

It can be found from Figure 5 that when  $f_1$  is a dependent variable, only the first component  $g_1$  of all components of the Shenzhen Component Index is selected; when  $f_2$  is the dependent variable, only the second component  $g_2$  is selected. The same is true for the results of the remaining 8 variables. Similarly, as can be seen from Figure 6 that only the corresponding components are selected when LASSO regression is established between one component of Shenzhen Composite Index and all components of Shanghai Composite Index.

Therefore, the empirical results show that the mutual influence between the two stock markets is symmetrical and one-to-one.

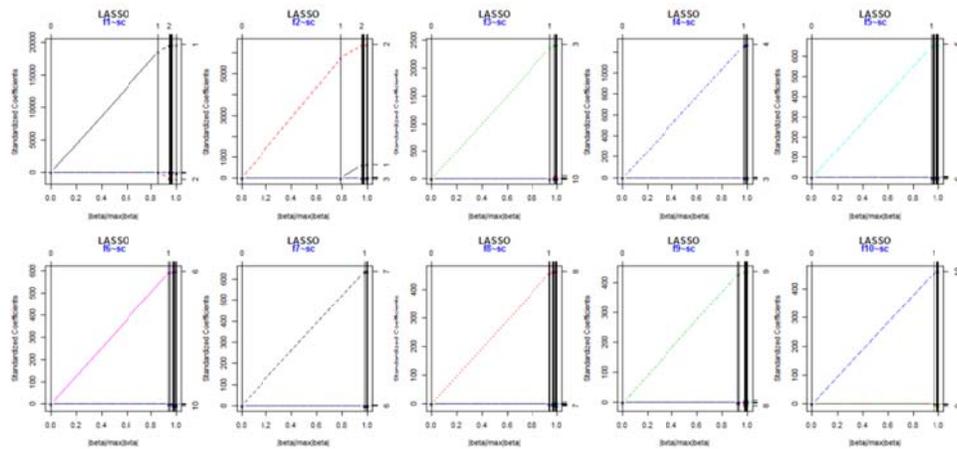


Figure 5. The order of  $f_i$  entering lasso regression mode

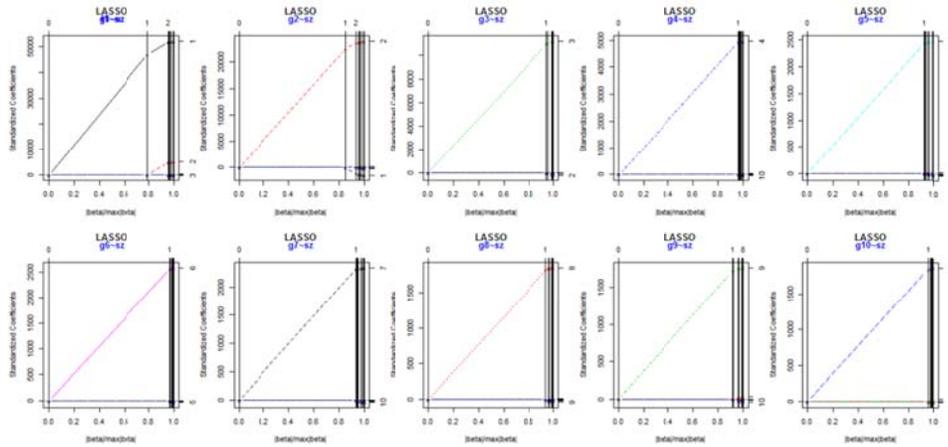
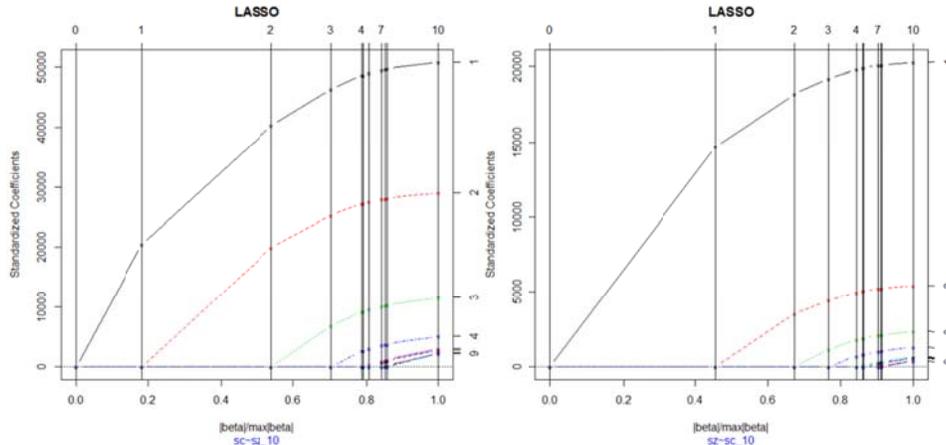


Figure 6. The order of  $g_i$  entering lasso regression mode

### 3.3 VMD-LASSO model

Since the research in Section 3.2 shows that there is a long-term interaction between the Shanghai Composite Index and Shenzhen Composite Index, VMD-LASSO models are constructed for these two sequences respectively. First, let the Shanghai Composite Index  $F$  to be the dependent variable,  $g_i (i=1, \dots, 10)$  are the independent variables to construct the VMD-LASSO model, and the process of variable selection is shown in the figure; then the Shenzhen Composite Index sequence  $G$  is used as the dependent variable,  $f_i (i=1, \dots, 10)$  are the independent variables to establish the VMD-LASSO model, and the process of variable selection is shown in the Figure 7.

The sequence of variables entering the VMD-LASSO model can be seen from the Figure 7, and 10 variables in each model are selected.



**Figure 7. LASSO coefficients for the two models that describe the Shanghai Composite Index and the Shenzhen Component Index and their decomposed components**

### 3.4 Prediction results of regression model

Next, We compared the prediction accuracy of the proposed method with three traditional methods used in the literature: (1) linear least-squares regression between the response variable,  $F$ , and the covariate  $G$ , with intercept; (2) linear least-squares regression between the response variable  $F$ , and all the decomposed components of covariate  $g_i (i=1, \dots, 10)$  without intercept; and (3) ridge regression between the response variable  $F$ , and all the decomposed components of covariate  $g_i (i=1, \dots, 10)$ . The four models are constructed to predict the next three phases. Taking the Shenzhen stock index as an example, the prediction values corresponding to the four models are aggregated to the Table 1. The relative errors of the predicted values of the four models are calculated and summarized into the Table 2.

**Table 1. Prediction Values Corresponding to the Four Models**

real	VMD-LASSO method	Least square method	VMD-Least square method	VMD-Ridge method
10262.85	10961.94	11080.67	11195.01	11059.16
10384.87	11182.7	11145.06	11516.82	11373.51
10371.47	10395.2	11163.7	9781.056	10397.02

**Table 2. Relative Errors**

VMD-LASSO method	Least square method	VMD-Least square method	VMD- Ridge method
6.81%	7.97%	9.08%	7.76%
7.68%	7.32%	10.9%	9.52%
0.23%	7.63%	5.69%	0.25%

The result of prediction shows in the Table 1 and Table 2, it is found that the prediction accuracy of the proposed model is higher than that of the other three basic methods. The prediction accuracy between our method and the third method is very similar, but obviously superior to this method. In summary, the method proposed in this paper finds real relationships between variables and provides highly accurate predictions.

**Table 3. Evaluation Criteria for Forecasting Accuracy**

Method	MSE	MAE	RMSE	HMSE	HMAE
VMD-LASSO method	375274.2	506.88	612.596	0.003	0.046
Least square method	624782.3	790.08	790.432	0.005	0.071
VMD-Least square method	832940.6	884.84	912.656	0.006	0.081
VMD- Ridge method	537390.5	603.50	733.069	0.004	0.054

The evaluation criteria for the prediction accuracy of the four models are listed in table 3, including MSE, MAE, RMSE, HMSE and HMAE. It can be found easily from table 3 that five values of VMD-LASSO model are smaller than those of the others. Therefore, it also verifies the view that the model proposed in this paper has higher prediction accuracy and better prediction effect.

#### 4. Conclusion

This paper presents a LASSO regression process based on VMD method, which identifies the relationship between two variables through the time-frequency structure of data sets. Data analysis shows that the proposed modeling process can accurately find the real independent variables (decomposition components) affecting the dependent variables, and the analysis process finds that there is a long-term interaction between the two Chinese markets. The model established by this method can produce higher precision prediction value.

The modeling process proposed in this paper has several advantages over ordinary methods: (1) VMD method can decompose time-frequency data into multiple components, which makes it possible to select variables between two columns of data; (2) Since the inherent modal components obtained by VMD are orthogonal, it is reasonable to select variables through LASSO; (3) LASSO regression can effectively select the independent variable components with the greatest influence on dependent variables; (4) The proposed modeling method can give better prediction accuracy than the traditional model.

#### ACKNOWLEDGEMENTS

*We are grateful to the referees for their valuable comments. This work is financially supported by the National Natural Science Foundation of China (No. 11301036 and 11226335) and the Scientific Project of Education Department of Jilin Province.*

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