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OWA-BASED AGGREGATION OPERATIONS IN MULTI-EXPERT MCDM MODEL

Abstract. *This paper presents an analysis of multi-expert multi-criteria decision making (ME-MCDM) model based on the ordered weighted averaging (OWA) operators. Two methods of modeling the majority opinion are studied as to aggregate the experts' judgments, in which based on the induced OWA operators. Then, an overview of OWA with the inclusion of different degrees of importance is provided for aggregating the criteria. An alternative OWA operator with a new weighting method is proposed which termed as alternative OWAWA (AOWAWA) operator. Some extensions of ME-MCDM model with respect to two-stage aggregation processes are developed based on the classical and alternative schemes. A comparison of results of different decision schemes then is conducted. Moreover, with respect to the alternative scheme, a further comparison is given for different techniques in integrating the degrees of importance. A numerical example in the selection of investment strategy is used as to exemplify the model and for the analysis purpose.*

Keywords: *multi-expert MCDM; OWA operator; IOWA operator; majority concept; weighting methods, financial decision making.*

JEL Classification: C44, D81, G11

1. Introduction

In the past, various multi-criteria decision making models have been developed as tools for modeling human decision making and reasoning (see, Figueira et al., 2005). The models have been extensively used in numerous applications to deal with the ranking and selection of option (or alternative). In complex decision making problems, normally a group of experts (or decision makers) involved in which each of them offsets and/or support the others for an exhaustive judgment. Since then, the expansion of such models to multi-expert MCDM (ME-MCDM) problems has become the main focus in the literature (see, for example, in Taib et al., 2016).

Central to the ME-MCDM problems, aggregation process plays a crucial role in obtaining the final decision, either to synthesize the criteria or to fuse the overall judgment of experts. An overview of the main aggregation operators and their properties can be referred, for instance, in Beliakov et al. (2007) and Grabisch et al. (2009). The weighted arithmetic mean (WA) and the ordered weighted averaging (OWA) operators are among the most widely used aggregation operators in the decision making models. The OWA (Yager, 1988) provides a general class of mean-type aggregation operators which can be ranged from two extreme cases, i.e., '*and*' (min) and '*or*' (max) operators. It modifies the basic aggregation process used in decision making model by applying the concept of fuzzy set theory, precisely, using the fuzzy linguistic quantifiers (Zadeh, 1983) for a soft aggregation process. In comparison to the WA which represents the degrees of importance associated with particular criteria, the weights in OWA reflect the importance or satisfaction of values with respect to ordering. By appropriately selecting the weighting vector, different kinds of relationships between the criteria can be modeled. In certain cases, the WA is necessary in representing the MCDM problems. For example, some experts may prefer to associate a specific weight for each criterion based on its degree of importance. Hence, considering the advantages of both WA and OWA in modeling the real applications, Yager (1988) then proposed the inclusion of unequal degrees of importance in OWA as an integrated approach. Consequently, a number of other techniques to deal with the same problem have been developed. The integration of these weighting methods has been formalized in two different approaches. In the first approach, the relative weights are only used to modify the argument values to be aggregated, specifically without the direct integration with ordered weights. Examples in this category include the method based on max-min and product (Yager, 1988), fuzzy system modeling (Yager, 1998) and hybrid weighted average (Xu and Da, 2003). On the other hand, in the second approach, the relative weights and ordered weights are directly integrated as a new set of weights, e.g., method based on linguistic quantifiers (Yager, 1996), weighted OWA (WOWA) (Torra, 1997), OWAWA (Merigó, 2012) and immediate WA (IWA) (Llamazares, 2013).

Another important variant of OWA is the induced OWA (IOWA) operator (Yager and Filev, 1999). Generally, it is an extension of the OWA which involves a pair of values, such as, the additional parameter (order-inducing variables) used to induce the argument values to be aggregated. Analogously, with respect to a group decision making, the majority agreement among experts can be implemented using the IOWA operators, which synthesizes the opinions of the majority of experts. In this case, the majority opinion refers to a consensual judgment of majority of experts who have similar opinions. In general, the OWA and IOWA operators provide a more flexible model for combining the information in decision making problems, specifically in the complex environment where the attitudinal character of experts is considered.

On the basis of previous discussion, the purpose of this study is on extending and analyzing the ME-MCDM model with respect to two-stage aggregation processes, notably, the fusion of criteria and the aggregation of experts' judgments. Firstly, two models based on majority concept for aggregating the experts' judgments are reviewed. In particular, the methods as introduced by Pasi and Yager (2006) and its extension by Bordogna and Sterlacchini (2014). Pasi and Yager (2006) proposed the method in case of the weights between experts are considered as identical (homogeneous group decision making) and employed a support function based on distance measure to compute the majority agreement between experts. Besides, the support between experts is calculated with respect to the final rankings of options which derived primarily by each expert (classical scheme). On the contrary, Bordogna and Sterlacchini (2014) then extended this idea to include the case where the experts are assigned with different degrees of importance (heterogeneous group decision making) and utilized the similarity measure based on Minkowski OWA (MOWA) to calculate the support between experts. Instead of focusing on the individual ranking on options of each expert, they provide the similarity measure with respect to each specific criterion (alternative scheme). In this study, for the purpose of comparison, some modifications have been made to both methods. In specific, the extension of Pasi-Yager method from the classical scheme to the alternative scheme has been made. Likewise, the Bordogna-Sterlacchini method has been modified to deal with the classical scheme. Hence, these methods with the existing original methods are applied in the ME-MCDM model and then a comparison as to examine the results of different schemes is conducted.

Secondly, some methods based on the integration of OWA and WA for the purpose of aggregating the criteria are presented. In addition, an alternative OWAWA (AOWAWA) operator which combines the characteristics of IWA and OWAWA using the idea of geometric mean is proposed. As a comparison, the ME-MCDM model with respect to Bordogna-Sterlacchini approach on the alternative scheme is applied as to observe the results of distinct weighting techniques in the aggregation process. The outline of this paper is as follows. In Section 2 the definitions of OWA, IOWA and MOWAD operators are presented. In Section 3 the aggregation techniques for modeling the majority opinion are discussed. Then, Section 4 reviews the integrated weighting methods based on WA and OWA as well as the proposed AOWAWA operator. In Section 5, the general frameworks of ME-MCDM model based on classical and alternative schemes are outlined. Then, a numerical example in a selection of investment strategy is provided in section 6.

2. Preliminaries

This section provides the definitions and basic concepts related to OWA, IOWA and MOWAD aggregation operators that will be used throughout the study.

2.1 OWA operator

Definition 1. (Yager, 1988). An OWA operator of dimension n is a mapping $OWA: \mathbb{R}^n \to \mathbb{R}$ that has an associated weighting vector $W = (w_1, w_2, \dots, w_n)$ of dimension n, such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, given by the following formula:

$$OWA_W(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j a_{\sigma(j)} \tag{1}$$

where $a_{\sigma(j)}$ is the argument value a_j being ordered in non-increasing order $a_{\sigma(1)} \geq a_{\sigma(2)} \geq \cdots \geq a_{\sigma(n)}$.

Note that, the reordering process makes the OWA operator is no longer a standard linear combination of weighted arguments, but it is rather a piecewise linear function (Beliakov and James, 2011).

Given that a function $Q: [0,1] \to [0,1]$ as a regular monotonically non-decreasing fuzzy quantifier and it satisfies: i) $Q(0) = 0$, ii) $Q(1) = 1$, iii) $a > b$ implies $Q(a) \geq Q(b)$, then the associated OWA weights can be derived using this function as follows (Yager, 1988):

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right), \qquad j = 1,2, \dots, n \tag{2}$$

such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

The linguistic quantifier Q(Zadeh, 1983) can be presented in the form of $Q(r) = r^\gamma$, $\gamma > 0$ with the main characteristics such that: $\gamma \to 0$, then $W = W^*$, where $W^* = (1,0, \dots, 0)$; $\gamma = 1$ then $W = W_{1/n}$, where $W_{1/n} = (1/n, 1/n, \dots, 1/n)$; and $\gamma \to \infty$ then $W = W_*$, where $W_* = (0,0, \dots, 1)$.

2.2 IOWA operator

Definition 2. (Yager and Filev, 1999). An IOWA operator of dimension n is mapping $IOWA: \mathbb{R}^n \to \mathbb{R}$ that has an associated weighting vector W such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, given by the following formula:

$$IOWA_W(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j a_{\sigma(j)} \tag{3}$$

where $a_{\sigma(j)}$ is the argument value of pair $\langle u_j, a_j \rangle$ of order-inducing variable u_j, reordered such that $u_{\sigma(1)} \geq u_{\sigma(2)} \geq \cdots \geq u_{\sigma(n)}$ and the convention that if$u_{\sigma(j)}$ are tied, i.e., $u_{\sigma(j)} = u_{\sigma(j+1)}$, then, the value $a_{\sigma(j)}$ is given as their average (see, Yager and Filev, 1999; Beliakov and James, 2011).

2.3 Minkowski OWA distance

Definition 3. (Merigó and Gil-Lafuente, 2008). A MOWAD operator of dimension n is a mapping $MOWAD: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ that has an associated weighting vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$ with $w_j \in [0,1]$ and the distance between two sets A and B is given as follows:

$$MOWAD_W(d_1, d_2, \dots, d_n) = \left(\sum_{j=1}^{n} w_j d_{\sigma(j)}^{\lambda}\right)^{1/\lambda}, \tag{4}$$

where $d_{\sigma(j)}$ is the component of d_j being ordered in non-increasing order $d_{\sigma(1)} \geq d_{\sigma(2)} \geq \cdots \geq d_{\sigma(n)}$ and $d_j$is the individual distance between A and B, such that $d_j = |a_j - b_j|$ with λ is a parameter in a range $\lambda \in \mathbb{R} \setminus \{0\}$.

By setting different values for the norm parameter λ, some special distance measures can be derived. For example, if $\lambda = 1$, then the Manhattan OWA distance can be obtained, if$\lambda = 2$ then the Euclidean OWA distance can be acquired, $\lambda = \infty$ then Tchebycheff OWA is derived, etc. Equivalently, OWA and IOWA operators can be generalized in the similar way (see, Merigó and Gil-Lafuente, 2009; Merigó and Yager, 2013; Yager, 2004).The OWA, IOWA and MOWAD operators are all satisfying commutative, monotonic, bounded and idempotent properties.

3. Aggregation Methods based on Majority Concept

In this section, the methods for aggregating experts' judgments by the inclusion of majority concept are presented. In particular, the method by Pasi and Yager (2006) and its extension by Bordogna and Sterlacchini (2014) are studied.

3.1 Pasi-Yager approach

In the following, a brief description of the mentioned methods is given. Two fundamental steps in both methods are on determining the order-inducing variable and on deriving the associated weights of experts. The methodology used to obtain the majority opinion based on Pasi and Yager (2006) can be expressed as the following.

Suppose that a set of individual opinions of $h$experts $(h = 1,2, \dots, k)$ is given as the vector $P_i^h = (p_i^1, p_i^2, \dots, p_i^k)$, i.e., with respect to each option $i, (i = 1,2, \dots, m)$. For a simple notation, P_h can be used instead ofp_i^h since each option can be evaluated independently using the same formulation. For a single option, the similarity of each expert can be calculated using the support function as follows:

$$supp(p_l, p_h) = \begin{cases} 1 & if\ |p_l - p_h| < \beta, \\ 0 & otherwise. \end{cases} \tag{5}$$

The support function represents the similarity or dissimilarity between expert l with each of the other experts $h, (h = 1,2, \dots, k)$ (not include himself/herself), such that $l \in h$. Then the overall support for each individual expert l can be given as:

$$u_l = \sum_{\substack{h=1 \\ h \neq l}}^{k} supp(p_l, p_h), \tag{6}$$

where u_l constitute the values of order-inducing variable $U = \left(u_{\sigma(1)}, \dots, u_{\sigma(k)}\right)$ which ordered in non-decreasing order, such that $u_{\sigma(1)} \leq u_{\sigma(2)} \leq \cdots \leq u_{\sigma(k)}$.

In consequence, to compute the weights of the weighting vector, define the values t_l based on an adjustment of the u_l values, such that: $t_l = u_l + 1$ (including himself/herself:$supp(p_l, p_l) = 1$). The t_l values are in non-decreasing order, $t_1 \leq t_2 \leq \cdots \leq t_k$. On the basis of t_l values, the weights are computed as follows:

$$w_l = \frac{Q(t_l/k)}{\sum_{l=1}^{k} Q(t_l/k)}. \tag{7}$$

The value $Q(t_l/k)$ denotes the degree to which a given member of the considered set of values represents the majority. The quantifier Q with semantic '*most*' for the majority opinion of experts can be given as follows:

$$Q(r) = \begin{cases} 1 & if\ r \geq 0.9, \\ 2r - 0.8 & if\ 0.4 < r < 0.9, \\ 0 & if\ r \leq 0.4, \end{cases} \tag{8}$$

where $r = t_l/k$. As can be seen, the weight of experts here is derived based on the arithmetic mean (AM) where each expert is considered as having an equal degree of importance or trust, e.g., reflect the average of the most of the similar values..Then, the final evaluation is determined using the IOWA operators. Note that, here the values of order-inducing variable are reordered in non-decreasing order instead of non-increasing order as in the original IOWA, such in Eq. (3). This type of ordering reflects the conformity of quantifier '*most*' as to model the majority concept (see, Pasi and Yager, 2006) for detailed explanation. Note also that, the quantifier Q here is an alternative representation of $Q(r) = r^{\gamma}$. For representing the majority opinion of experts, this type of quantifier will be used throughout the study.

However, the vector $P_i^h = \left(p_i^1, p_i^2, \dots, p_i^k\right)$, that derived after the first stage of aggregation process shows a slight different between its values due to the normalization process. This condition then leads to the values of $|p_l - p_h|$ less differentiable and cause a difficulty in assigning a value for β. Hence, in this study, a slight modification to the support function in Eq. (6) is suggested and the formulation is given as follows:

$$supp(p_l, p_h) = \begin{cases} 1 & if \ \dfrac{|p_l - p_h|}{\max\limits_{l \in h}|p_l - p_h|} < \beta, \\ 0 & otherwise. \end{cases} \tag{9}$$

where $\max\limits_{l \in h}|p_l - p_h|$ is the maximum distance between all experts.

Example 1: Suppose that a set of individual opinion of experts is given as$P_h = (p_1, p_2, \dots, p_5) = (0.7, 0.86, 0.76, 0.72, 0.6)$ with respect to each option, A_i. Then, the final majority opinion of experts can be computed as the following.

A_i / P_i^h	E_1 0.7	E_2 0.86	E_3 0.76	E_4 0.72	E_5 0.6		E_1 0.7	E_2 0.86	E_3 0.76	E_4 0.72	E_5 0.6
$supp_{1,h}$	-	0.16	0.06	0.02	0.1	divided by	-	0.62	0.23	0.08	0.39
$supp_{2,h}$	0.16	-	0.1	0.14	**0.26**	$\xrightarrow{}$	0.62	-	0.39	0.54	1
$supp_{3,h}$	0.06	0.1	-	0.04	0.16	$\max\limits_{h}\lvert p_l - p_h\rvert$	0.23	0.39	-	0.15	0.62
$supp_{4,h}$	0.02	0.14	0.04	-	0.12		0.08	0.54	0.15	-	0.46
$supp_{5,h}$	0.1	0.26	0.16	0.12	-		0.39	1	0.62	0.46	-

By setting $\beta = 0.4$, the overall support for each expert can be obtained, such as: $s_1 = 3$, $s_2 = 1$, $s_3 = 3$, $s_4 = 2$, and $s_5 = 1$. In case of 'ties', the stricter β can be imposed ($\beta = 0.1$,in this example), to order the p_h values. The vector of order-inducing variable then can be given as $U = (u_{\sigma(1)}, \dots, u_{\sigma(5)}) = (1,1,2,3,3)$ and the weighting vector can be obtained as $W^{Maj} = (w_1, \dots, w_5) = (0, 0\ , 0.2, 0.4, 0.4)$. The final majority opinion of experts can be calculated as follows:

$$IOWA(\langle 1, 0.6\rangle,\ \langle 1, 0.86\rangle,\ \langle 2, 0.72\rangle,\ \langle 3, 0.76\rangle,\ \langle 3, 0.7\rangle) = (0 \times 0.6) + (0 \times 0.86) + (0.2 \times 0.72) + (0.4 \times 0.76) + (0.4 \times 0.7) = 0.73.$$

3.2 Bordogna-Sterlacchini approach

In the following, the method based on Bordogna and Sterlacchini (2014) is presented. Contrary to the previous method, here the majority opinion of experts with respect to each specific criterion is considered. Suppose that a collection of judgment of $h$experts is given as vector $P_j^h = (p_j^1, p_j^2, \dots, p_j^k)$ for criterion $j, (j = 1,2, \dots, n)$. In this method, instead of using the support function based on distance measure, they used the Minkowski OWA-based similarity measure to obtain the $Q_{coherence}$ for the order-inducing variable. The $Q_{coherence}$ of each expert l can be defined as follows:

$$u_l = Q_{coherence}(P_l, P_h) = MOWA(s_1, \dots, s_k) = \left(\sum_{h=1}^{k} \omega_h s_{\sigma(h)}^{\lambda} \right)^{1/\lambda}, \qquad (10)$$

where $s_l = s(p_l, p_h) = 1 - |p_l - p_h|$ is a similarity measure between expert l with each of the other experts h (includes himself), given that $l \in h$ and $s_{\sigma(h)}$are ordering of $(s_1, \dots, s_k)$ in non-increasing order $\left(s_{\sigma(1)} \geq s_{\sigma(2)} \geq \cdots \geq s_{\sigma(k)}\right)$. Meanwhile ω_h are the ordered weights with the inclusion of importance degrees of experts $t_h, h = 1,2, \dots, k$, given as $\omega_h = Q\left(\sum_{i=1}^{h} t_{\sigma(i)}\right) - Q\left(\sum_{i=0}^{h-1} t_{\sigma(i)}\right)$, such that$\omega_h, t_h \in [0,1]$ and $\left(\sum_{h=1}^{k} \omega_h = \sum_{h=1}^{k} t_h = 1\right)$. The norm parameter $\lambda \in \mathbb{R} \setminus \{0\}$ provides a generalization of the model. Here the quantifier $Q(r) = r^{\gamma}$ is employed. The OWA weights ω_h will be explained in great detail in the next section.

With respect to the Eq. (10), the order inducing vector can be given as:

$$U = (u_1, \dots, u_k) = \left(Q_{coherence}(P_1, P_h), \dots, Q_{coherence}(P_k, P_h)\right), \qquad (11)$$

Moreover, Q as the generalized quantifiers can take any semantics to modify the weights of experts (or trust degrees) for different strategies. When $Q(t_h) = t_h$ as for $(\gamma = 1)$, then $Q_{coherence}$ is reduced to:

$$u_l = coherence(P_l, P_h) = \left(\sum_{h=1}^{k} t_h s_h^{\lambda} \right)^{1/\lambda}, \qquad (12)$$

which is the Minkowski WA-based similarity measure. Formally, $Q_{coherence}$ can be ranged in between $Q_*(t_h)$ for $\gamma \to 0$, to $Q^*(t_h)$for $\gamma \to \infty$.

Afterwards, the weights for the IOWA operator can be derived using the following formula:

$$m_h = \frac{argmin_h(u_1 \cdot t_1, \dots, u_k \cdot t_k)}{\sum_{h=1}^{k} argmin_i(u_1 \cdot t_1, \dots, u_k \cdot t_k)}, \qquad (13)$$

where m_h are reordered in non-decreasing order. Analogously, given the quantifier Q as in Eq. (8) for the majority opinion, the weighting vector $W^{Maj} = (w_1, \dots, w_k)$ can be computed as follows:

$$w_h = \frac{Q(m_h)}{\sum_{h=1}^{k} Q(m_h)}. \qquad (14)$$

Note that, the general weights w_h represent the quantification of majority of experts for the final agreement on each criterion, whilst the weights ω_h reflect $Q_{coherence}$ for deriving the order-inducing values.

Next, the overall aggregation process can be computed using the IOWA such in Eq. (3). Similarly, the non-decreasing inputs $\langle u_h, p_h \rangle$ is implemented as explained in previous sub-section. It can be shown that, the coherence function Eq. (12) can be represented as the dual of similarity measure, which is the distance measure:

$$\begin{aligned} coherence(P_l, P_h) &= \left(\sum_{h=1}^{k} t_h (1 - |p_l - p_h|)^{\lambda} \right)^{1/\lambda} \\ &= 1 - \left(\sum_{h=1}^{k} t_h |p_l - p_h|^{\lambda} \right)^{1/\lambda}, \end{aligned} \tag{15}$$

such that for any p_l and p_h with $s(p_l, p_h) \in [0,1]$, the properties: i)$s(p_l, p_l) = 1$ (reflexive) and, ii) $s(p_l, p_h) = s(p_h, p_l)$ (symmetric) are fulfilled for each single value of l and h.

Analogously, to more differentiate between the values and to avoid the 'ties' problem, a simple modification to the similarity measure is suggested as follows:

$$s(p_l, p_h) = 1 - \left(\frac{|p_l - p_h|}{\max\limits_{l \in h} |p_l - p_h|} \right), \tag{16}$$

where $\max\limits_{l \in h} |p_l - p_h|$ is the maximum distance between all experts.

Correspondingly, the weights for IOWA aggregation process Eq. (13) can also be modified to the following formula:

$$m_h = \frac{argmin_h(u_1 \cdot t_1, \ldots, u_k \cdot t_k)}{Max_h(u_1 \cdot t_1, \ldots, u_k \cdot t_k)}. \tag{17}$$

Example 2: Suppose that a set of opinion of experts on a single criterion $C_j$is given as$P_j^h = (p_1, p_2, \ldots, p_k) = (0.31, 0.34, 0.30, 0.28, 0.11)$. The majority agreement of experts can be calculated as follows:

C_j / P_j^h	E_1 0.31	E_2 0.34	E_3 0.3	E_4 0.28	E_5 0.11		t_1	t_2	t_3	t_4	t_5	U
$supp_{1h}$	1	0.85	0.96	0.90	0.15		0.3	0.3	0.2	0.1	0.1	0.85
$supp_{2,h}$	0.85	1	0.87	0.75	0	$\xrightarrow{s_h \times t_h}$	0.3	0.3	0.2	0.1	0.1	0.79
$supp_{3,h}$	0.96	0.81		0.94	0.19		0.3	0.3	0.2	0.1	0.1	0.84
$supp_{4,h}$	0.9	0.75	0.94	1	0.26		0.3	0.3	0.2	0.1	0.1	0.81
$supp_{5,h}$	0.15	0	0.19	0.26	1		0.3	0.3	0.2	0.1	0.1	0.21

where $U = \sum_{h=1}^{k} s_h t_h$. In this case, for $Q(t_h) = t_h$ and by setting $\lambda = 1$, the vector of order-inducing variables can be determined, specifically$U = (s_{\sigma(1)}, \ldots, s_{\sigma(5)}) = (0.21, 0.79, 0.81, 0.84, 0.85)$. Next, by using the quantifier Q with semantics '*most*' for majority, the weighting vector $W^{Maj} = (w_1, \ldots, w_5) =$

$(0, 0, 0.20, 0.40, 0.40)$ can be obtained. The final majority opinion of experts can be given as the following:

$$IOWA(\langle 0.21, 0.11\rangle, \langle 0.79, 0.34\rangle, \langle 0.81, 0.28\rangle, \langle 0.84, 0.30\rangle, \langle 0.85, 0.31\rangle) = 0.30.$$

4. OWA Operators with inclusion of the Degrees of Importance

In this section, some OWA aggregation operators with their weighting methods are reviewed, in particular, the weighting methods based on the inclusion of WA. In addition, an alternative weighting method with its respective aggregation operator called as alternative OWAWA operator is proposed.

4.1 Some of the existing methods

Prior to the definition of integrated weighting methods, the general definition of WA is given as the following.

Definition 4. Let $V = (v_1, v_2, \dots, v_n)$ be a weighting vector (degrees of importance) of dimension n such that $v_j \in [0,1]$ and $\sum_{j=1}^{n} v_j = 1$, then a mapping $WA: \mathbb{R}^n \to \mathbb{R}$ is a weighted arithmetic mean (WA) if $WA_V(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} v_j a_j$.

The WA satisfies monotonic, idempotent and bounded properties, but it is not commutative (Beliakov et al., 2007; Grabisch et al., 2009; Torra, 1997).

There are a number of methods in the literature which have been proposed for obtaining weights for the OWA aggregation operators (see, Xu, 2005). One of them is by using the linguistic quantifiers as defined in the preliminaries section, refer to Eq. (2). Throughout the study, the OWA weighting vector $W$is exclusively referred to this type of weights, specifically to be integrated with the weighting vector, V (except for the methods in Definitions 8 and 9 as will be explained later).

Definition 5. (Yager, 1988). Let V and W be two weighting vectors of dimension n, then a mappingOWA: $\mathbb{R}^n \to \mathbb{R}$ isan OWA-MP operator of dimension n if:

$$OWA_{V,W}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j \breve{a}_{\sigma(j)}, \tag{18}$$

where $\breve{a}_{\sigma(j)}$is the value$\breve{a}_j$being ordered in non-increasing order $\breve{a}_{\sigma(1)} \geq \breve{a}_{\sigma(2)} \geq \cdots \geq \breve{a}_{\sigma(n)}$ such that $\breve{a}_j = H(a_j, v_j) = (v_j \vee \bar{\alpha}) \cdot (a_j)^{v_j \vee \alpha}$ and α is the orness measure and $\bar{\alpha} = 1 - \alpha$ is its complement.

This is the unified formulation of the methods which proposed earlier in Yager (1978) and Yager (1987), specifically based on the max-min and product approaches. In this study, it is denoted as OWA-MP. Notice that in the special cases: if $\alpha = 0$, then it can be reduced to a pure '*and*' operator. Specifically, given that $\breve{a}_j = a_j^{v_j}$ with $W = W_*$, then $\breve{a}_{\sigma(n)}$ is generated, which is the smallest value of

$\breve{a}_{\sigma(j)}$. Conversely, if $\alpha = 1$, then it can be reduced to a pure '*or*' operator. Given that $\breve{a}_j = v_j a_j$ with $W = W^*$, then $\breve{a}_{\sigma(1)}$ is generated, which is the largest value of $\breve{a}_{\sigma(j)}$. The OWA-MP operators meet monotonic and idempotent properties, however they are not commutative as involve WA. Moreover they are also not bounded, as in the case of argument value, $a_j \in [0,1]$, the modified argument values $\breve{a}_j$ are always greater than or equal to the argument values, a_j.

Definition 6. (Yager, 1998). Let V and W be two weighting vectors of dimension n, then a mappingOWA: $\mathbb{R}^n \to \mathbb{R}$ is an OWA-FSM operator of dimension n if:

$$OWA_{V,W}(a_1, a_2, \dots, a_n) = \sum\nolimits_{j=1}^{n} w_j \hat{a}_{\sigma(j)}, \tag{19}$$

where $\hat{a}_{\sigma(j)}$is the value of $\hat{a}_j$being ordered in non-increasing order $\hat{a}_{\sigma(1)} \geq \hat{a}_{\sigma(2)} \geq \cdots \geq \hat{a}_{\sigma(n)}$ given that $\hat{a}_j = H(a_j, v_j) = \bar{\alpha}\bar{v}_j + v_j a_j$ and $\bar{\alpha} = 1 - \alpha$, that is the complement of orness.

This method is based on fuzzy system modeling and is termed as OWA-FSM in this study. Notice that in the special cases: if $\alpha = 0$, then it reduces to a pure '*and*' operator. Specifically, given that $\hat{a}_j = \bar{v}_j + v_j a_j$ and $w_n = 1$, then $\hat{a}_{\sigma(n)}$ is generated, which is the smallest value of $\hat{a}_{\sigma(j)}$. Whilst, if $\alpha = 1$, then it is a pure '*or*' operator. Given that $\hat{a}_j = v_j a_j$ and $w_1 = 1$, then $\hat{a}_{\sigma(1)}$ is generated, which is the largest value of $\hat{a}_{\sigma(j)}$. The OWA-FSM operators meet monotonic and idempotent properties, but, they are not commutative as involve WA. Moreover, they are also not bounded, as in the case of $a_j \in [0,1]$, then $\hat{a}_j \geq a_j$.

Definition 7. (Xu and Da, 2003). Let V and W be two weighting vectors of dimension n, then a mappingHA: $\mathbb{R}^n \to \mathbb{R}$ is a hybrid averaging operator of dimension n if:

$$HA_{V,W}(a_1, a_2, \dots, a_n) = \sum\nolimits_{j=1}^{n} w_j \acute{a}_{\sigma(j)}, \tag{20}$$

where $\acute{a}_{\sigma(j)}$ is the argument value $\acute{a}_j$being ordered in non-increasing order $\acute{a}_{\sigma(1)} \geq \acute{a}_{\sigma(2)} \geq \cdots \geq \acute{a}_{\sigma(n)}$ given that $\acute{a}_j = n v_j a_j$ and n is the balancing coefficient.

It can be shown that when $W = (1/n, 1/n, \dots, 1/n)$, then the HA operator reduces to the WA, whilst when $V = (1/n, 1/n, \dots, 1/n)$, the HA operator reduces to the OWA. The HA operators meet monotonic property, however, they are neither idempotent nor bounded. As can be seen, the Definitions 5-7 are based on the approach where the degrees of importance, v_j are used to modify the argument values to be aggregated. In the following, the approaches based on the direct integration between v_j and w_j are presented.

Definition 8. (Torra, 1997). Let V and W be two weighting vectors of dimension n, then a mapping $WOWA: \mathbb{R}^n \to \mathbb{R}$ is a weighted ordered weighted averaging (WOWA) operator of dimension n if:

$$WOWA_{V,W}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \omega_j a_{\sigma(j)}, \tag{21}$$

where $a_{\sigma(j)}$ is the argument value of $a_j$being ordered in non-increasing order $a_{\sigma(1)} \geq a_{\sigma(2)} \geq \cdots \geq a_{\sigma(n)}$and $\omega_j = f\left(\sum_{k=1}^{j} v_{\sigma(k)}\right) - f\left(\sum_{k=0}^{j-1} v_{\sigma(j)}\right)$with f being a monotonic non-decreasing function that interpolates the points $\left((j/n), \sum_{k=1}^{j} w_j\right)$ together with the point $(0,0)$. The function f required to be a straight line when the points interpolated in this way.

It can be demonstrated that when $W = (1/n, 1/n, \dots, 1/n)$, then WOWA operator reduces to WA, whilst when $V = (1/n, 1/n, \dots, 1/n)$, WOWA operator reduces to OWA. Moreover, they are monotonic, idempotent, and bounded. Equivalently, the WOWA operator can be transformed to the OWA operator with the inclusion of degrees of importance (Yager, 1996), if a regular monotonically non-decreasing fuzzy quantifier Q is used as the function f and it can be defined as the following.

Definition 9. (Yager, 1996). Let$V$ and W be two weighting vectors of dimension n, then a mapping $OWA: \mathbb{R}^n \to \mathbb{R}$ is an OWA operator of dimension n if :

$$OWA_{V,W}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \omega_j a_{\sigma(j)}, \tag{22}$$

where $a_{\sigma(j)}$ is the argument value a_j being ordered in non-increasing order $a_{\sigma(1)} \geq a_{\sigma(2)} \geq \cdots \geq a_{\sigma(n)}$and $\omega_j = Q\left(\sum_{k=1}^{j} v_{\sigma(k)}\right) - Q\left(\sum_{k=0}^{j-1} v_{\sigma(k)}\right)$ such that $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Definition 10.(Llamazares, 2013). Let V and W be two weighting vectors of dimension n, then a mapping$IWA: \mathbb{R}^n \to \mathbb{R}$ is an immediate weighted averaging (IWA) operator of dimension n if:

$$IWA_{V,W}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \pi_j a_{\sigma(j)}, \tag{23}$$

where $a_{\sigma(j)}$ is the argument value $a_j$being ordered in non-increasing order $a_{\sigma(1)} \geq a_{\sigma(2)} \geq \cdots \geq a_{\sigma(n)}$and $\pi_j = w_j v_j / \sum_{j=1}^{n} w_j v_j$.

As can be seen, the IWA is a manipulation of immediate probability (Engemann et al., 1996; Merigó, 2012; Yager et al., 1995) by using the WA instead of the probability distribution. The IWA operators satisfy the generalization properties as $V = (1/n, 1/n, \dots, 1/n)$, it reduces to OWA and when $W = (1/n, 1/n, \dots, 1/n)$, the IWA reduces to the WA (Llamazares, 2013). The IWA operators meet monotonic, idempotent, bounded properties.

Definition 11. (Merigó, 2012). Let V and W be two weighting vectors of dimension n, then a mapping $OWAWA: \mathbb{R}^n \to \mathbb{R}$ is an ordered weighted averaging-weighted average (OWAWA) operator of dimension n if:

$$OWAWA_{V,W}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \varphi_j a_{\sigma(j)}, \tag{24}$$

where $a_{\sigma(j)}$ is the argument value of a_j being ordered in non-increasing order $a_{\sigma(1)} \geq a_{\sigma(2)} \geq \cdots \geq a_{\sigma(n)}$ and $\varphi_j = \beta w_j + (1-\beta) v_{\sigma(j)}$ with $\beta \in [0,1]$.

OWAWA operators satisfy monotonic, idempotent, bounded properties. Moreover, the value returned by the OWAWA operator lies between the values returned by the WA and OWA, and coincides with them when both are equal.

In addition, by taking the advantages of the IWA and the OWAWA operators, a new weighting method can be derived as in the next sub-section.

4.2 Alternative OWAWA operator

Definition 12. Let V and W be two weighting vectors of dimension n, then a mapping $AOWAWA: \mathbb{R}^n \to \mathbb{R}$ is an alternative ordered weighted averaging-weighted average (AOWAWA) operator of dimension n if:

$$AOWAWA_{V,W}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \hat{\varphi}_j a_{\sigma(j)}, \tag{25}$$

where $a_{\sigma(j)}$ is the argument value of a_j being ordered in non-increasing order $a_{\sigma(1)} \geq \cdots \geq a_{\sigma(n)}$ and $\hat{\varphi}_j = \left(w_j{}^{\beta} \cdot v_{\sigma(j)}{}^{(1-\beta)}\right) / \sum_{j=1}^{n} \left(w_j{}^{\beta} \cdot v_{\sigma(j)}{}^{(1-\beta)}\right)$ with $\beta \in [0,1]$, by convention that $(0^0 = 0)$.

The AOWAWA operator are monotonic, bounded, idempotent. However, it is not commutative because the AOWAWA operator includes the WA. The AOWAWA operators generalized to WA and OWA when $\beta = 0$ and $\beta = 1$, respectively.

Theorem 1(Monotonicity) Assume that f is the AOWAWA operator, let $A = (a_1, a_2, \dots, a_n)$ and $B = (b_1, b_2, \dots, b_n)$ be two sets of arguments. If $a_j \geq b_j$, $\forall j \in (1,2,\dots,n)$, then:

$$f(a_1, a_2, \dots, a_n) \geq f(b_1, b_2, \dots, b_n).$$

Proof. It is straightforward and thus omitted.

Theorem 2 (Idempotency) Assume f is the AOWAWA operator, if $a_j = a$, $\forall j \in (1,2,\dots,n)$, then:

$$f(a_1, a_2, \dots, a_n) = a.$$

Proof. It is straightforward and thus omitted.

Theorem 3(Bounded) Assume f is the AOWAWA operator, then:

$$Min\{a_j\} \leq f(a_1, a_2, \dots, a_n) \leq Max\{a_j\}$$

Proof. It is straightforward and thus omitted.

5. ME-MCDM Model based on Different Decision Schemes

In this section, the general frameworks of ME-MCDM model based on the classical and alternative schemes are presented. In addition to the original methods by Pasi and Yager (2006) and Bordogna and Sterlacchini (2014), some extensions have been made as the following. First, the majority concept of Pasi-Yager method which is originally based on the classical scheme is extended to the case of alternative scheme. Secondly, the Bordogna-Sterlacchini method which is based on the alternative scheme is modified to the case of the classical scheme. These methods are used for the comparison purpose in the next section. The algorithms for the model are structured as in the following.

5.1 Classical scheme

- *Stage I: Internal aggregation* (*Local aggregation*)

Step 1: First, a decision matrix for each expert $D^h, h = 1,2, \dots, k$, is constructed as follows:

$$D^h = \begin{array}{c c} & \begin{array}{ccc} C_1 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} a_{11}^h & \cdots & a_{1n}^h \\ \vdots & \ddots & \vdots \\ a_{m1}^h & \cdots & a_{mn}^h \end{pmatrix} \end{array}, \tag{26}$$

where A_i indicates the option/alternative $i (i = 1,2, \dots, m)$ and C_j denotes the criterion j $(j = 1,2, \dots, n)$. Meanwhile the a_{ij}^h represents the preference for option A_i with respect to criterion C_j, such that $a_{ij}^h \in [0,1]$.

Step 2: Next, determine the weighting vector for all the expert using one of the available methods, such as in Eqs. (18-25). Note that, in this case, the proportion of criteria to be considered is subject to the attitudinal character of individual experts. Hence, each expert can provide distinct decision strategies separately.

Step 3: Aggregate the judgment matrix of each expert by the weighting vector as determined in *Step 2*. At this stage, each expert derives the ranking of all options individually.

- *Stage II: External aggregation* (*Global aggregation*)

With respect to the type of aggregation methods, the consensus measure for the majority of experts can be calculated as follows:

(P-Y)* *The Pasi-Yager method (Homogeneous group decision making):*

Step 4: Determine the order-inducing variable using the Eqs. (5-6) or in the case where the argument values are very close to each other, use the modified support function such in Eq. (9).

Step 5: Calculate the weighting vector which represents the majority of experts using the Eq. (7) based on quantifier '*most*' as in Eq. (8). In this case, the weight of each expert is considered as equal.

(B-S)* *The modified version of Bordogna-Sterlacchini method (Heterogeneous group decision making):*

Step 4: Determine the order-inducing variable using the Eqs. (10-12) or in the case where the argument values are very close to each other, then use the modified similarity measure such in Eq. (16).

Step 5: Calculate the weighting vector using the Eq. (14) and Eq. (17). In this case, the weight or trust degree is associated to each expert.

5.2 Alternative scheme

- *Stage I: External aggregation*

Step 1: By the similar way, a decision matrix for each expert is constructed such in Eq. (26). Then, the aggregation based on majority concept can be implemented using one of the following methods:

*(B-S**)* *The Bordogna-Sterlacchini method (Heterogeneous GDM):*

Step 2: Determine the order-inducing variable such in *Step 4(B-S*)* of the classical scheme. But, instead of aggregate the opinion of experts with respect to each option, here, the aggregation process is conducted on each criterion.

Step 3: Calculate the weighting vector such in *Step5(B-S*)* of the classical scheme using the values of the order-inducing variable in the previous step.

*(P-Y**)* *The extension of Pasi-Yager method (Homogeneous GDM):*

Step 2: Determine the order-inducing variable as in *Step4(P-Y*)* of the classical scheme. But, instead of aggregate the opinion of experts with respect to each option, here, the aggregation process is conducted on each criterion.

Step 3: Calculate the weighting vector such in *Step5(P-Y*)* of the classical scheme using the order-inducing variable derived in the previous step.

- *Stage II: Internal aggregation* (*Global aggregation*)

Step 4: Determine the weighting vector using one of the methods as shown in Eqs. (18-25).

Step 5: Finally, aggregate the judgment matrix of the majority of experts with respect to the weighting vector derived in *Step 4*. Note that here, the proportion of criteria is subject to the attitudinal character of the majority of experts.

6. Numerical Example

In this section, an investment selection problem is studied where a group of experts or analysts are assigned for the selection of an optimal strategy. Assume that a company plans to invest some money in a region. Primarily, they consider five possible investment options as follows: $A_1 =$ invest in the European market, $A_2 =$American market, $A_3 =$Asian market, $A_4 =$African market, $A_5 =$ do not invest money. In order to evaluate these investments, the investor has brought together a group of experts. This group considers that each of investment options can be described with the following characteristics: $C_1 =$ benefits in the short term, $C_2 =$ benefits in the mid-term, $C_3 =$ benefits in the long term, $C_4 =$ risk of the investment, $C_5 =$ other variables. The available investment strategies depending on the characteristic C_j and the option A_i for each expert are shown in Table 1.

Table 1. Available investment strategies of each expert, E_h

E_1				
C_1	C_2	C_3	C_4	C_5
0.7	0.6	0.7	0.6	0.9
0.8	1	0.2	1	0.6
0.6	0.7	0.6	0.6	0.5
0.9	0.6	0.8	1	0.9
0.3	0.7	0.7	0.8	0.9

E_2				
C_1	C_2	C_3	C_4	C_5
0.6	0.9	1	0.9	0.9
1	0.7	0.1	1	0.8
0.4	0.9	0.8	0.7	0.6
0.9	0.5	0.7	1	0.9
0.7	0.7	0.9	0.9	0.9

E_3				
C_1	C_2	C_3	C_4	C_5
0.5	0.7	0.9	0.8	0.9
0.9	0.9	0.2	1	0.7
0.8	0.8	0.7	0.7	0.6
0.9	0.5	0.8	1	0.7
0.8	0.7	0.8	0.9	0.8

	E_4				
	C_1	C_2	C_3	C_4	C_5
A_1	0.4	0.7	0.9	0.8	0.8
A_2	0.9	0.7	0.1	0.9	0.6
A_3	0.6	0.6	0.5	0.8	0.4
A_4	0.7	0.5	0.7	0.7	0.9
A_5	0.4	0.6	0.7	0.8	0.9

	E_5				
	C_1	C_2	C_3	C_4	C_5
A_1	0.5	0.6	0.7	0.6	0.8
A_2	0.9	0.8	0.4	0.9	0.5
A_3	0.6	0.6	0.5	0.8	0.7
A_4	0.8	0.7	0.6	0.9	0.8
A_5	0.2	0.6	0.8	0.6	0.8

In this study, two analyses are conducted. First is to analyze the effect of different decision schemes for the homogeneous and heterogeneous cases. The aggregated results of analysis are presented in Table2. Note that, for the heterogeneous case, the weights(0.3, 0.1, 0.1, 0.4, 0.1)represent the expert E_1, E_2, E_3, E_4 and E_5, respectively. As can be seen, there is a slight difference between the results that derived from both majority aggregation approaches with respect to different decision schemes. The majority opinion of experts with respect to the classical scheme provides A_4, A_2, A_1, A_5 and A_3as the final ranking for both methods. While the majority opinion of experts computed with respect to alternative scheme exhibits the ranking of A_4, A_1, A_5, A_2 and A_3 (also for both methods). Hence, the aggregated results demonstrated the effect on different decision schemes in ranking the options.

Table 2. The aggregated results

	Homogeneous case, $t_h = 1/n$		Heterogeneous case, $t_h \neq 1/n$	
	ME-MCDM-PY*	ME-MCDM-PY**	ME-MCDM-BS*	ME-MCDM-BS**
A_1	0.7143 (R3)	0.7726 (R2)	0.7169 (R3)	0.7989 (R2)
A_2	0.7178 (R2)	0.6992 (R4)	0.7200 (R2)	0.6580 (R4)
A_3	0.6280 (R5)	0.6361 (R5)	0.5952 (R5)	0.6057 (R5)
A_4	0.7886 (R1)	0.8027 (R1)	0.7800 (R1)	0.8000 (R1)
A_5	0.7029 (R4)	0.7225 (R3)	0.6800 (R4)	0.6969 (R3)

Note: '*' refers to the classical scheme and '**' refers to the alternative scheme; R = ranking.

Secondly, as a further analysis, the method of ME-MCDM-BS** based on the integration of WA and OWA weights is conducted. Table 3 shows the aggregated results of the model based on different weighting techniques.

Table 3. The aggregated results with respect to ME-MCDM-BS model**

OWA (Q)	WOWA	IWA	OWA-WA	AOWA-WA	OWA (FSM)	OWA (MP)	HA
0.6957	0.6992	0.6972	0.7526	0.7076	0.9177	0.9053	0.3598
0.1543	0.1147	0.1207	0.3866	0.2124	0.7325	0.5319	0.1672
0.4837	0.5080	0.4988	0.5547	0.5158	0.8564	0.8279	0.2455
0.5227	0.5217	0.5302	0.6563	0.5736	0.8791	0.8493	0.4504
0.4185	0.4946	0.4472	0.5685	0.5085	0.8926	0.8742	0.2215

The weights $v_j$for the criteria are given as 0.1, 0.2, 0.3, 0.3, 0.1 and the ordered weights, w_j are represented as '*most*' $(\gamma = 10)$, i.e., "most of the criteria have to be satisfied". As can be noticed, the proposed AOWAWA operator with $\beta = 0.5$indicates the similar ranking as the WOWA and IWA methods, A_1, A_4, A_3, A_5 and A_2. Concurrently, the rest weighting techniques show slightly different results.

Note that in this case, the decision strategy is subject to the attitudinal character of the majority of experts. By selecting any parameter γ to represent the linguistic quantifier, various decision strategies can be derived. Specifically for $\gamma \to 0$ (at least one criteria is considered), $\gamma = 1$ (averagely all) and $\gamma \to \infty$ (all criteria are considered). The aggregated results of AOWAWA operator with different decision strategies are presented in Tables 4.

Table 4. Decision strategies based on AOWAWA operator

At least one $\gamma \to 0$	Few $\gamma = 0.1$	Some $\gamma = 0.5$	Half (average) $\gamma = 1$	Many $\gamma = 2$	Most $\gamma = 10$	All $\gamma \to \infty$
0.8989	0.8463	0.8202	0.8038	0.7801	0.7076	0.6945
0.9976	0.8397	0.7188	0.6375	0.5249	0.2124	0.1000
0.6994	0.6628	0.6354	0.6169	0.5908	0.5158	0.4727
0.9986	0.9071	0.8379	0.7926	0.7320	0.5736	0.5000
0.8976	0.7871	0.7367	0.7084	0.6695	0.5085	0.3846

In addition, the rankings of AOWAWA operator with different values of β can be seen in Table 5. These values show the effect of the selection WA and OWA in the final evaluation process. For example, if only WA is applied, then $\beta = 0$, whilst $\beta = 1$ implies only OWA is used.

Table 5. Aggregated results of AOWAWA operator based on β values

$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1$
0.6957	0.6975	0.7024	0.7160	0.7493	0.8094
0.1543	0.1699	0.1942	0.2376	0.3338	0.6190
0.4837	0.4925	0.5065	0.5270	0.5590	0.6257
0.5227	0.5370	0.5589	0.5916	0.6459	0.7900
0.4185	0.4426	0.4816	0.5410	0.6223	0.7185

7. Conclusion

In this paper, the analysis on extensions of ME-MCDM model based on the OWA operators has been conducted. The focus is given on the aggregation operation, specifically with respect to the fusions of criteria and experts' judgments. The majority concept based on the IOWA and linguistic quantifiers to aggregate the experts' judgments is analyzed, in which concentrated on the classical and alternative schemes of group decision making model. Then, a review on the weighting methods related to the integration of WA and OWA is provided. Correspondingly, the alternative weighting technique is proposed which is called as the AOWAWA operator. The ME-MCDM model based on two-stage aggregation processes then is developed. A comparison is conducted to see the effect of different weighting techniques in aggregating the criteria and the results of using different decision schemes for the fusion of majority opinion of experts. A numerical example in the selection of investments then has been used for the comparison purpose. In general, the conclusion which can be made from the analysis and comparison is that, the selection of decision schemes as well as the weighting methods employed in the aggregation process shown different rankings for the options. Moreover, each of the decision schemes represents the decision strategy (i.e., with respect to criteria) in a different way, whether as an individual expert decision strategy or as group/majority decision strategy. Hence, the selection of both approaches reflects different results.

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