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EXTENDED VIKOR METHOD BASED ON INDUCED AGGREGATION OPERATORS FOR INTUITIONISTIC FUZZY FINANCIAL DECISION MAKING

Abstract*. In this paper, we develop a new method for intuitionistic fuzzy multiple criteria decision making (MCDM) by integrating induced aggregation operators into VIKOR approach. For doing so, we develop a new intuitionistic fuzzy aggregation operator called the intuitionistic fuzzy induced ordered weighted averaging standardized distance (IFIOWASD) operator, which provides a wide range of intuitionistic fuzzy standardized distance measures between the maximum and the minimum. The main advantage of the IFIOWASD is that it is able to reflect the complex attitudinal character of the decision maker by using order inducing variables and provide much more complete information for decision making. Moreover, it is able to deal with uncertain environments where the information is very imprecise that can be assessed with intuitionistic fuzzy information. We study some of the IFIOWASD's different particular case. Finally, we apply the integrated IFIOWASD method in an intuitionistic fuzzy multiple criteria decision making problem.*

Keywords*: financial decision making, intuitionistic fuzzy set, induced aggregation operators, VIKOR.*

JEL Classification: D81, M12, M51

1. Introduction

As extension of Zadeh's fuzzy set (Zadeh, 1965) whose basic component is only a membership function, the intuitionistic fuzzy set (IFS) introduced by Atanassov (1986) has been proven to be highly useful to deal with uncertainty and vagueness, and a lot of work has been done to develop and enrich the IFS theory (Boran and Akay, 2014; Peng et al., 2014; Wan et al., 2016; Wei, 2010; Xu and Wang, 2012; Xu, 2007; Yu, 2014, 2015; Yue, 2014).

The VIKOR method was developed as a MCDM method to solve a discrete decision problem with non-commensurable and conflicting criteria

(Opricovic and Tzeng, 2002, 2004). This method focuses on ranking and selecting from a set of alternatives, and determines compromise solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision. Here, the compromise solution is a feasible solution which is the closest to the ideal, and a compromise means an agreement established by mutual concessions. In the recent years, the VIKOR method has been studied and applied in a wide range of problems (Girubha and Vinodh, 2012; Liu et al., 2012; Kim et L., 2015; Wang and Tzeng, 2012).

The ordered weighted averaging (OWA) operator (Yager, 1988) is one of the most common aggregation operators found in the literature. It provides a parameterized family of aggregation operators that range from the maximum to the minimum. An interesting generalization of the OWA operator is the induced OWA (IOWA) operator (Yager and Filev, 1999). Its main advantage is that it deals with complex reordering processes in the aggregation by using order inducing variables. Since its introduction, the IOWA operator has been studied by a lot of authors (Liu et al., 2013; Li et al., 2014; Meng et al., 2015; Merigó and Casanovas, 2009, 2011; Xu and Xia, 2011; Zeng, 2013; Zeng and Su, 2012). In particular, Merigó and Casanovas (2011) presented the induced ordered weighted averaging distance (IOWAD) operator, which extends the OWA operator with the use of distance measures and a reordering of arguments that depends on order-inducing variables. Liu et al. (2013) developed an IOWA-based VIKOR (IOWA-VIKOR) method for multiple criteria decision making. Zeng and Su (2012) developed the linguistic induced generalized ordered weighted averaging distance (LIGOWAD) operator, which is an extension of the IOWA operator by using distance measures and uncertain information represented in the form of linguistic variables.

The objective of this paper is to present a new method for intuitionistic fuzzy MCDM by using induced aggregation operators in the VIKOR method. For doing so, we shall develop the intuitionistic fuzzy induced ordered weighted averaging standardized distance (IFIOWASD) operator. The IFIOWASD is a new aggregation operator that includes a parameterized family of intuitionistic fuzzy standardized distance aggregation operators in its formulation that ranges from the minimum to the maximum standardized distance. Moreover, this operator is able to deal with complex attitudinal characters (or complex degrees of orness) of decision maker and provide a more complete picture of the decision making process.

To do so, the remainder of this paper is organized as follows. In Section 2, some basic concepts that are used throughout the paper are briefly reviewed. In Section 3, we introduce the classical VIKOR method for MCMD problems. Section 4 presents the IFIOWASD operator and analyzes different types of IFIOWASD operators. Section 5 briefly describes the decision making process based on developed approach and we give a numerical example in Section 6. Section 7 summarizes the main conclusions of the paper.

2. Preliminaries

In this Section we briefly describe the intuitionistic fuzzy set (IFS), the IOWA operator and the IOWAD operator.

2.1 Intuitionistic Fuzzy Sets

The purpose of clustering methods is to group similar elements together. The similarity is established through specific distance metrics, based on which similarity or distance matrix are computer (Aggarwal, 2013). Afterward, clustering algorithms interpret the matrix and create clusters. There are three main clustering methods categories: partitional methods, hierarchical methods and quartet methods.

Intuitionistic fuzzy set (IFS) introduced by Atanassov (1986) is an extension of the classical fuzzy set, which is a suitable way to deal with vagueness. It can be defined as follows.

Definition 1. Let a set $X = \{x_1, x_2, ..., x_n\}$ be fixed, an IFS A in X is given as following:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\}, \quad (1)$$

The numbers $\mu_A(x)$ and $v_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A, $0 \le \mu_A(x) + v_A(x) \le 1$, for all $x \in X$. The pair $(\mu_A(x), v_A(x))$ is called an intuitionistic fuzzy number (IFN) (Xu, 2007) and each IFN can be simply denoted as $\alpha = (\mu_\alpha, v_\alpha)$, where $\mu_\alpha \in [0,1]$, $v_\alpha \in [0,1]$, $\mu_\alpha + v_\alpha \le 1$.

Additionally, $S(\alpha) = \mu_\alpha - v_\alpha$ and $H(\alpha) = \mu_\alpha + v_\alpha$ are called the score and accuracy degree of α respectively. For any three IFNs $\alpha = (\mu_\alpha, v_\alpha), \alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$, the following operational laws are valid (Xu, 2007).

(1) $\alpha_1 + \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1} \cdot \mu_{\alpha_2}, v_{\alpha_1} \cdot v_{\alpha_2})$;

(2) $\lambda\alpha = (1-(1-\mu_\alpha)^\lambda, v_\alpha{}^\lambda)$, $\lambda > 0$.

To compare any two IFNs α_1 and α_2, Xu (2007) introduced a simple method as below:

- If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;
- If $S(\alpha_1) = S(\alpha_2)$, then

(1) If $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 < \alpha_2$;

(2) If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$.

Let $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$ be two IFNs, Xu (2010) defined an intuitionistic fuzzy distance as following:

Definition 2. Let $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$ be two IFNs, then

$$d(\alpha_1, \alpha_2) = \frac{1}{2}\left(\left|\mu_{\alpha_1} - \mu_{\alpha_2}\right| + \left|v_{\alpha_1} - v_{\alpha_2}\right|\right), \tag{2}$$

2.2 The Induced Ordered Weighted Averaging (IOWA) Operator

The IOWA operator is an extension of the OWA operator. The main difference between the two is that in the IOWA operator, the reordering step is not performed with the values of the a_i arguments. In this case, the reordering step is developed with order-inducing variables that reflect a more complex reordering process. The IOWA operator also includes the maximum, the minimum and the average criteria as specific cases. It can be defined as follows:

Definition 3. An IOWA operator of dimension n is a mapping IOWA: $R^n \times R^n \to R$, which has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, ..., \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is a_i value of the IOWA pair $\langle u_i, a_i\rangle$ with the j th largest u_i, u_i is the order-inducing variable and a_i is the argument variable.

2.3 The Induced Ordered Weighted Averaging Distance (IOWAD) Operator

The IOWAD operator introduced by Merigó and Casanovas (2011) is a distance measure that uses the IOWA operator in the process of normalization of the Hamming distance. For two sets $A=\{a_1,a_2,...,a_n\}$ and $B=\{b_1,b_2,...,b_n\}$, the IOWAD operator can be defined as follows:

Definition 4. An IOWAD operator of dimension n is a mapping IOWAD: $R^n \times R^n \times R^n \to R$, which has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IOWAD\left(\left(u_1,a_1,b_1\right),...,\left(u_n,a_n,b_n\right)\right)=\sum_{j=1}^{n} w_j d_j , \tag{4}$$

where d_j is the $|a_i - b_i|$ value of the IOWAD triplet (u_i, a_i, b_i) possessing the j th largest u_i, u_i is the order-inducing variable, and $|a_i - b_i|$ is the argument variable, represented in the form of individual distances.

3. The VIKOR Method

The VIKOR method was introduced as one applicable technique to be implemented within MCDM problem and it was developed as a multi criteria decision making method to solve a discrete decision making problem with non-commensurable (different units) and conflicting criteria. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria, which can help the decision makers to reach a final solution. The multi-criteria measure for compromise ranking is developed from the L_p–metric used as an aggregating function in a compromise programming method (Zeleny, 1982).

Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The various *m* alternatives are denoted as. For an $A_1, A_2,...,A_m$ alternative A_i, the rating of the j th aspect is denoted by f_{ij}, i.e., f_{ij} is the value of j th criterion function for the alternative A_i; n is the number of criteria. The VIKOR method was developed with the following form of L_p–metric:

$$L_{p,i} = \left\{ \sum_{j=1}^{n} \left[\frac{w_j \left(f_j^* - f_{ij} \right)}{f_j^* - f_j^-} \right]^p \right\}^{1/p}, 1 \le p \le \infty, i = 1, 2, ..., m, \tag{5}$$

In the VIKOR method, $L_{1,i}$ (as S_i in Eq. (6)) and $L_{\infty,i}$ (as R_i in Eq. (7)) are used to formulate ranking measurements. The solution gained by min S_i is with a maximum group utility ("majority" rule), and the solution gained by min R_i is with a minimum individual regret of the "opponent".

Step 1: Determine the best f_j^* and the worst f_j^- values of all criterion ratings,

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij}, \quad i = 1,2,...,m \tag{6}$$

Step 2: Compute the values S_i and R_i, $i = 1, 2, ..., m$, by the relations

$$S_i = \sum_{j=1}^{n} \frac{w_j \left(f_j^* - f_{ij} \right)}{f_j^* - f_j^-}, \tag{7}$$

$$R_i = \max_j \left(\frac{w_j \left(f_j^* - f_{ij} \right)}{f_j^* - f_j^-} \right), \tag{8}$$

where w_j are the weights of criteria, expressing their relative importance.

Step 3: Compute the values Q_i, $i = 1, 2, ..., m$, by the relation

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1 - v) \frac{R_i - R^*}{R^- - R^*}, \tag{9}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$ and v is introduced as a weight for the strategy of the maximum group utility, whereas $1 - v$ is the weight of the individual regret. Usually, the value of v is taken as 0.5.

Step 4: Rank the alternatives, sorting by the values S, R and Q in increasing order. The results obtained are three ranking lists.

Step 5: Propose a compromise solution, the alternative ($A^{(1)}$), which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

C1. Acceptable advantage: $Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \ge DQ$, where $A^{(2)}$ is the alternative with second position in the ranking list by Q, $DQ = 1/(m-1)$.

C2. Acceptable stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" ($v < 0.5$).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied or
- Alternatives $A^{(1)}, A^{(2)}, ..., A^{(M)}$ if the condition C1 is not satisfied: $A^{(M)}$ is determined by the relation $Q\left(A^{(M)}\right) - Q\left(A^{(1)}\right) < DQ$ for maximum M (the positions of these alternatives are "in closeness").

4. Extended VIKOR Method with the IOWA Operator for the Decision Making Method with Intuitionistic Fuzzy Set

In many complex decision making problems, the decision information provided by the decision maker is often imprecise or uncertain due to time pressure, lack of data, or the decision maker's limited attention and information processing capabilities. The IFS is a very suitable tool to be used to describe imprecise or uncertain decision information, which allow decision makers to assign the membership and non-membership degree to each alternative. Therefore, in this paper, we should extend the VIKOR method with the IOWA operator to solve MCDM problem with the intuitionistic fuzzy information, and develop the intuitionistic fuzzy induced ordered weighted averaging standardized distance (IFIOWASD) operator. Let Ω be the set of all IFNs, $F^* = \left\{ f_1^*, f_2^*, ..., f_n^* \right\}$, $R_i = \left\{ f_{i1}, f_{i2}, ..., f_{in} \right\}$ and $F^- = \left\{ f_1^-, f_2^-, ..., f_n^- \right\}$ be three sets of IFNs, then the IFIOWASD operator can be defined as follows.

Definition 5. An IFIOWASD operator of dimension n is a mapping IFIOWASD: $R^n \times \Omega^n \times \Omega^n \to R$ that has an associated weighting vector $w = \left(w_1, w_2, ..., w_n\right)^T$, with $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$, such that:

$$IFIOWASD\left(\left\langle u_1, f_1^*, f_{i1} \right\rangle, ..., \left\langle u_n, f_n^*, f_{in} \right\rangle\right) = \sum_{k=1}^{n} w_k \tilde{d}_k, \qquad (10)$$

where $\left| f_j^* - f_{ij} \right|$ and $\left| f_j^* - f_j^- \right|$ are the distances between intuitionistic fuzzy numbers given in Definition 2, $\tilde{d}_k$ represents the $\frac{\left| f_j^* - f_{ij} \right|}{\left| f_j^* - f_j^- \right|}$ value of the IFIOWASD triplet $\left\langle u_j, f_j^*, f_{ij} \right\rangle$ having the k th largest u_j, u_j is the order inducing variable, $\frac{\left|f_j^* - f_{ij}\right|}{\left|f_j^* - f_j^-\right|}$ is the argument variable represented in the form of individual normalized distances, f_j^* and f_j^- are the best value and the worst value of the j th criterion, respectively, and f_{ij} is the assessment of i th alternative with respect to C_j, $i = 1, 2, ..., m$.

The IFIOWASD operator is an extension of the IOWAD operator to solve MCDM problems with conflicting and non-commensurable criteria. The main difference is that in this case, we reorder the arguments of the individual standardized distances according to order inducing variables. Moreover, it is able to deal with uncertain environments where the information is very imprecise and can be assessed with intuitionistic fuzzy information.

An interesting issue is to consider the measures for characterizing the weighting vector W of the IFIOWASD operator such as the attitudinal character, the entropy of dispersion, the divergence of W and the balance operator. As this feature does not depend upon the linguistic arguments, the formulation is the same than the IOWAD operator. The entropy of dispersion is defined as follows:

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j), \tag{12}$$

The balance operator can be defined as:

$$BAL(W) = \sum_{j=1}^{n} w_j \left(\frac{n+1-2j}{n-1} \right), \tag{13}$$

And the divergence of W:

$$Div(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W) \right)^2, \tag{14}$$

The degree of orness can be defined as follows:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} \right), \tag{15}$$

Similar to the IOWAD operator, the IFIOWASD operator is commutative, monotonic, idempotent, bounded, nonnegative and reflexive. Moreover, by using a different manifestation of the weighting vector, we are able to obtain different types of IFIOWASD operators, for example,

- The intuitionistic fuzzy maximum standardized distance (IFMAXSD) is obtained if $w_p = 1$, $w_j = 0$, for all $j \neq p$, and $u_p = \max\left\{ \frac{\left| f_j^* - f_{ij} \right|}{\left| f_j^* - f_j^- \right|} \right\}$.
- The intuitionistic fuzzy minimum standardized distance (IFMINSD) is obtained if $w_p = 1$, $w_j = 0$, for all $j \neq p$, and $u_p = \min\left\{ \frac{\left| f_j^* - f_{ij} \right|}{\left| f_j^* - f_j^- \right|} \right\}$.
- More generally, if $w_k = 1$ and $w_j = 0$ for all $j \neq k$, we get the step-IFIOWASD operator.
- The intuitionistic fuzzy normalized Hamming standardized distance (IFNHSD) is formed when $w_j = 1/n$ for all j.
- The intuitionistic fuzzy weighted Hamming standardized distance (IFWHSD) is obtained when the ordered position of the u_i is the same as $\frac{\left| f_j^* - f_{ij} \right|}{\left| f_j^* - f_j^- \right|}$.
- The intuitionistic fuzzy ordered weighted averaging standardized distance (IFOWASD) operator is obtained if the ordered position of u_i is the same as the ordered position $\frac{\left| f_j^* - f_{ij} \right|}{\left| f_j^* - f_j^- \right|}$.

5. An Approach to Intuitionistic Fuzzy MCDM with the IFIOWASD Operator

In what follows, we are going to present an IFIOWASD method for solving MCDM problems with conflicting and non-commensurable criteria. Suppose that a MCDM problem contains m alternatives $A_i\left(i = 1, 2, ..., m\right)$, and n decision criteria $C_j\left(j = 1, 2, ..., n\right)$. Each alternative is evaluated with respect to the n criteria, and the compromise ranking could be performed by comparing the measure of closeness to the ideal solution F^* (the best values of criteria). All the performance ratings assigned to the alternatives with respect to each criterion form a decision matrix denoted by $R = \left(f_{ij}\right)_{m \times n}$. Then, the main steps of the proposed IFIOWASD algorithm can be described as follows:

Step 1: Determine the best f_j^* and the worst f_j^- values of all criterion ratings,

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij}, \ i=1,2,...,m, \tag{16}$$

Step 2: Compute the values S_i and R_i, $i = 1, 2, ..., m$, by the relations

$$S_i = IFIOWASD\left(\left\langle u_1, f_1^*, f_{i1}\right\rangle, ..., \left\langle u_n, f_n^*, f_{in}\right\rangle\right) = \sum_{k=1}^{n} w_k \tilde{d}_k, \tag{17}$$

$$R_i = \max_k \left(w_k \tilde{d}_k\right), \tag{18}$$

where ω_k are the ordered weights of criteria, expressing the relative importance of their ordered positions. Note that it is possible to consider a wide range of distance aggregation operators such as those described in the previous section.

Step 3: Compute the values Q_i, $i = 1, 2, ..., m$, by the relation

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1-v) \frac{R_i - R^*}{R^- - R^*}, \tag{19}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$ and v is introduced as a weight for the strategy of the maximum group utility, whereas $1-v$ is the weight of the individual regret. Usually, the value of v is taken as 0.5.

Step 4: Rank the alternatives, sorting by the values S, R and Q in increasing order. The results obtained are three ranking lists.

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C1. Acceptable advantage: $Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \geq DQ$, where $A^{(2)}$ is the alternative with second position in the ranking list by Q: $DQ = 1/(m-1)$.

C2. Acceptable stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" ($v < 0.5$).

6. Numerical Example

In this section we consider an example where the enterprise's board of directors, which includes 5 members, is to plan the development of large projects (strategy initiatives) for the following 5 years consistent with the example introduced in Parreiras et al. (2010). Suppose there are four possible projects $A_i (i = 1, 2, 3, 4)$ to be evaluated. It is necessary to compare these projects to select the most important of them as well as order them from the point of view of their importance, taking into account four criteria suggested by the Balanced Score card methodology (it should be noted that all of them are of the maximization type):

C_1: financial perspective, C_2: the customer satisfaction, C_3: internal business process perspective, and C_4: learning and growth perspective.

In the following, we use the developed methods to get the optimal project. In order to avoid influencing each other, the decision makers are required to provide their preferences in anonymity and the decision matrix $R=\left(f_{ij}\right)_{m\times n}$ is presented in Table 1, where $f_{ij}\ (i, j=1,2,3,4)$ are in the form of IFNs.

Table 1: Intuitionistic fuzzy decision matrix

	A_1	A_2	A_3	A_4
C_1	(0.5,0.4)	(0.5,0.3)	(0.2,0.6)	(0.4,0.4)
C_2	(0.7,0.2)	(0.7,0.3)	(0.5,0.5)	(0.6,0.2)
C_3	(0.5,0.4)	(0.6,0.4)	(0.6,0.2)	(0.5,0.3)
C_4	(0.8,0.1)	(0.7,0.2)	(0.4,0.5)	(0.5,0.2)

Due to the fact that the attitudinal character depends upon the opinion of several members of the board of directors, it is very complex. Therefore, they need to use order inducing variables in the reordering process. The results are shown in Table 2.

Table 2: Order inducing variables.

	A_1	A_2	A_3	A_4
C_1	13	10	8	20
C_2	12	8	15	18
C_3	16	18	22	28
C_4	8	14	20	26

With this information, it is now possible to develop different VIKOR methods for selecting a material based on the IFIOWASD operator. In this example, we are going to consider the IFNHSD, the IFWHSD, the IFOWASD and the IFIOWASD operators. We will assume the following weighting vector $w=\left(0.2, 0.2, 0.3, 0.3\right)$. The results are shown in Table 3.

Table 3: Aggregated results

		A_1	A_2	A_3	A_4
IFNHSD	Q	0.502	0.504	0.25	0.5
IFWHSD	Q	0.927	1	0.5	0.786
IFOWASD	Q	0	1	0.54	0.258
IFIOWASD	Q	0	0.29	1	0.258

As we can see, depending on the particular type of aggregation operator used, the values Q are different. Note that the optimal choice would be the alternative with the lowest value of Q in each method. If we want to rank the alternative materials, a typical situation when we want to consider more than one alternative, we can get the ranking of the alternatives for each particular case as shown in Table 4.

Table 4: Ordering of the Strategies

	Ordering
IFNHSD	$A_3 \succ A_4 \succ A_1 \succ A_2$
IFWHSD	$A_3 \succ A_4 \succ A_1 \succ A_2$
IFOWASD	$A_1 \succ A_4 \succ A_3 \succ A_2$
IFIOWASD	$A_1 \succ A_4 \succ A_2 \succ A_3$

As a general conclusion for the example, we can see that depending on the distance aggregation operator used, the rankings of the alternatives may be dissimilar and the decision maker may select a different material. It should be noted that the method used has to be in accordance with the interests of the decision maker. Therefore, by using the IOWA operator in the VIKOR method, we can represent complex reordering processes in the aggregation in order to consider more complex information in the decision making problem.

7. Conclusions

In real world, decision makers' attitudes are blended with some amount of uncertainty degree due to the lack of enough knowledge and information about alternatives. This situation can be completely dealt with in the best way using the intuitionistic fuzzy concept. In this paper, we extend the concept of VIKOR method with the IOWA operator to develop a methodology for solving MCDM problems with intuitionistic fuzzy information. We develop the IFIOWASD operator. Then, the principles and steps of the proposed IFIOWASD method are presented in this paper. Finally, a numerical example illustrates an application of the IFIOWASD method. By the illustrative example, we found that the IFIOWASD method provides a more complete picture of the decision making process, enabling the decision maker to select the alternative that it is more in accordance with his/her interests.

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