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THE ULTIMATUM GAME WITH EXTERNALITIES

Abstract. *This paper examines the equilibrium properties of an ultimatum game model with externalities. Unlike the equilibrium of the traditional version of this game and even recent extensions of it in a similar direction as ours, three novel results may emerge: (i) a negotiation breakdown, (ii) a perfectly equitable sharing solution, and (iii) a solution in which the responder gets a higher fraction of the pie under division than the proposer. It is shown that whereas the first result depends on the externality level suffered by both players, the two last, conditional on that an agreement exists, only depend on the responder's externality level. It is further argued that these results can be especially relevant in negotiations involving interpersonal interactions and resolution of highly polarized conflicts.*

Keywords: *bargaining, externalities, ultimatum game, envy, polarized negotiation.*

JEL Code: C72, C78, D62, D74

1 Introduction

One of the most puzzling results in bargaining theory is the abundant experimental evidence collected on the so-called *ultimatum game* (Güth et al., 1982). The puzzling nature of such evidence lies in two findings: (i) why the second-mover player (the *responder*) tends to reject small offers, and (ii) why the first-mover player (the *proposer*) tends to make offers that are more generous than those predicted for a pure utility-maximizing economic agent.

In this paper we provide an explanation to these two phenomena based upon the role played by externalities. Specifically, we propose a modified version of the ultimatum game in which each party suffers a negative externality proportional to the surplus captured by his rival.

The equilibrium of our modified game reveals the following results. First, as opposed to the subgame perfect equilibrium (SPE) of the classical ultimatum game without externalities, a negotiation breakdown is now possible. This occurs if the level of externalities experienced by both parties is high enough so that relative bargaining powers become *incompatible*.

Second, if bargaining powers are *compatible*, the agreement outcome is quite different from that characterized by the SPE of the traditional version of the ultimatum game. In particular, there is a more equitable distribution that differs from the classic virtual corner solution in which the proposer appropriates almost all the surplus at stake. Interestingly, this distribution depends *only* on the externality level suffered by the responder, which works as a counterbalance of bargaining powers in the game. Accordingly, whereas the responder's surplus fraction is increasing with this level of externality, the proposer's one is decreasing. This equilibrium property implies that two implausible results for the SPE of the classic ultimatum game are, by contrast, possible in our modified version of the game: (i) a fully equitable division of the surplus, and (ii) an asymmetric distribution in which the winning party becomes the responder.

Our article is directly related to previous theoretical research that has also proposed modifications of the ultimatum game in order to account for agreements different from the above discussed corner solution. In this research line, social (other-regarding) arguments have been included in the agents' preferences, such as fairness (Kahneman et al., 1986; Nowak et al., 2000; Xie et al., 2012), envy (Kirchsteiger, 1994), inequity-aversion (Fehr and Schmidt, 1999), reciprocity (Falk and Fischbacher, 2006), and trust (Berg et al., 1995). Despite of the closeness of these approaches and ours, it is remarkable that none of these attempts has considered negative externalities as we do here.

The closest work to ours is Kirchsteiger (1994), who proposes an ultimatum game in which both players are envious, as each suffers a negative effect if his counterpart's surplus increases. The one-period version of the game proposed by Kirchsteiger (1994) assumes more general utility functions than we do. That model, however, is more restrictive on the maximum potential degree of envy (the externality in our case), which implies that our setup delivers insights into the outcome of the ultimatum game that such framework is not able to characterize. As a consequence, our model contributes to the most related extant literature on ultimatum games by stating specific conditions, in terms of externality parameters, for three novel outcomes: (i) a responder's rejection, and thus, a disagreement, (ii) a perfectly equitable solution, and (iii) a solution in which the responder gets a higher surplus than the proposer's.

In addition, this article has connections with a vivid experimental research contrasting the implications of various of the theoretical ultimatum–game models above cited. That literature considers experiments testing the role played by feelings and emotional states such as anger, envy or a displeasing sentiment in face of unfair or unkind offers (Fischbacher et al., 2013; Kagel and Wolfe, 2001; Pfister and Böhm, 2012; Pillutla and Murnighan, 1996; Sanfey et al., 2003; van 't Wout et al., 2006). In a work particularly related to ours, Pfister and Böhm (2012) conducts a three-player ultimatum-game experiment in order to disentangle responder's rejections motivated by anger from those driven by envy. Their results suggest that although responders experience dissatisfaction when sharing solutions are biased in favor of either the proposer (anger) or a third passive player (envy), only the first emotion seems to influence the decision to accept or reject an offer.

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Lastly, the present paper is also related to research trying to find a negotiation breakdown in a distributive bargaining game. The closest work of this literature is perhaps Laengle and Loyola (2012), which although also considers externalities, construct a *static* setting based on the classic Nash demand game. Whereas Laengle and Loyola (2012) finds similar conditions for a negotiation breakdown to ours, that framework delivers, notwithstanding, multiple agreement equilibria in which externality parameters of *both* parties play a role.

The rest of this article proceeds as follows. Section 2 proposes an ultimatum game with externalities and presents its equilibrium. Section 3 discusses the properties of such equilibrium, stressing the influence of externalities suffered by both players over the characteristics of disagreement and agreement solutions. A brief comparative analysis with the most related ultimatum game models of previous literature is also performed. Section 4 presents concluding remarks, limitations and implications of this work. The proof of our main result is contained in the Appendix.

2 The Game

Consider a distributive negotiation game in which two players, Emile (E) and Frances (F), want to divide a pie of size 1. Under this bargaining scheme, the distributive process has two stages. In the first one, player E (the *proposer*) makes a take-it-or-leave-it offer x to player F (the *responder*). In the second stage, player F can accept or reject that offer. In the first case, player E appropriates a fraction x of the pie under negotiation and player F the remaining one. Otherwise, if player F rejects his counterpart's offer, both players get nothing.

The utility functions of Emile and Frances are given by $U_E(x, y) = x - \gamma_E(1 - x)$ and $U_F(x, y) = (1 - x) - \gamma_F x$, respectively. The constants γ_E and γ_F are non-negative ($\gamma_E, \gamma_F \geq 0$) and represent the marginal externality that each party suffers when some surplus is appropriated by his counterpart.

In order to isolate the effect of externalities on the bargaining outcome from other asymmetric preferences-based elements, we assume a zero discount rate for both players. Outside opportunities of both parties are normalized to $\underline{U} = 0$.

The following proposition characterizes the Subgame Perfect Equilibrium (SPE) of this game.

Proposition 2.1 *The SPE of the ultimatum game with externalities is given by:*

(a) If $\gamma_E \gamma_F > 1$, the SPE is so that player E chooses

$$x^* = \frac{1}{1 + \gamma_F} + \varepsilon,$$

for some $\varepsilon > 0$, and player F accepts the offer x if $x \leq \frac{1}{1 + \gamma_F}$ and rejects it otherwise.

(b) If $\gamma_E \gamma_F \leq 1$, the SPE is so that player E chooses

$$x^* = \frac{1}{1+\gamma_F},$$

and player F accepts the offer x if $x \leq \frac{1}{1+\gamma_F}$ *and rejects it otherwise.*

Proof : See Appendix.

3 Properties of the equilibrium

From Proposition 2.1 we can see that depending on the level of global externalities (the product $\gamma_E\gamma_F$), the following two bargaining outcomes are possible.

Case 1. On the one hand, when both parties experience sufficiently high externality levels, their bargaining powers become *incompatible* and negotiation breaks down. According to Proposition 2.1 (a), this occurs when $\boldsymbol{\gamma_E\gamma_F > 1}$.

Case 2. On the other hand, a *compatible* bargaining power relationship does exist as long as $\boldsymbol{\gamma_E\gamma_F \leq 1}$, which in turn guarantees that an equilibrium exists and that both players reach an *agreement*. In that situation, the surplus fractions appropriated by *E* and *F* are, respectively, given by:

$$x^* = \frac{1}{1+\gamma_F},$$

and

$$1 - x^* = \frac{\gamma_F}{1+\gamma_F}.$$

This bargaining agreement exhibits various properties quite different from that attained in the classic ultimatum game. Note that, conditional on reaching an agreement, the surplus distribution depends only on the responder's externality level γ_F, but not on the other player's externality. Further, this externality works as an element that *balances* bargaining powers in the game: it counterbalances the traditional first-mover advantage in the classic ultimatum game, as the outcome is different from a corner solution. The responder now gets a positive surplus fraction as $\gamma_F > 0$, and the division benefits (damages) the responder (proposer) as this parameter increases. This property is so that it is even possible that the responder's fraction be larger than the proposer's one (as long as $\gamma_F > 1$), which can be understood as an extreme level of envy: the absolute value of the utility impact of an increase in his rival's share is larger than that produced by an increase in his own share.

Notice that the proposed game nests a linear formulation of the one-period version of the so-called *ultimatum game with envy* (Kirchsteiger, 1994), whose set of possible agreement solutions is replicated by our model as long as $\gamma_E, \gamma_F \in (0,1)$. In addition, our framework extends this set of solutions to situations in which either $\gamma_E \geq 1$ or $\gamma_F \geq 1$, conditional on that $\gamma_E\gamma_F \leq 1$. This extension allows our model, in contrast to Kirchsteiger (1994), to deliver a perfectly equitable equilibrium as long as $\gamma_F = 1$, or an equilibrium more favorable to the responder than the proposer as long as $\gamma_F > 1$. Lastly, our setup

also extends the results of Kirchsteiger (1994) by allowing for a region of disagreement outcomes whenever $\gamma_E \gamma_F > 1$. Thus, the main theoretical contribution of the present work is to characterize a larger set of solutions than that stated by the most related previous literature.

Note finally that the ultimatum game with externalities yields the same corner solution to that attained in the classic version of the game when $\gamma_F = 0$, irrespective of γ_E.

All of these comparative results are explained below with the help of Fig. 1.
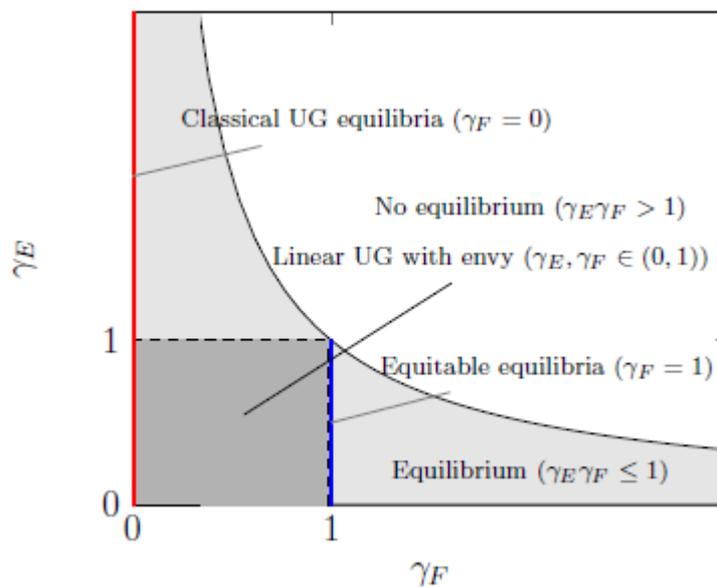

Figure 1. Equilibria of the Ultimatum Game (UG) with Externalities

A compatible bargaining power relationship lies on the curve ($\gamma_E \gamma_F = 1$) and in both gray regions ($\gamma_E \gamma_F < 1$). A particular perfectly equitable solution exists when $\gamma_F = 1$ (in blue). An incompatible bargaining power relationship takes place if the level of global externalities is too high ($\gamma_E \gamma_F > 1$); in such case a negotiation breakdown emerges. A corner solution similar to the classical UG follows when $\gamma_F = 0$ (in red). The set of equilibria for a linear version of the UG with envy is nested by our model whenever $\gamma_E, \gamma_F \in (0,1)$ (darker shaded region).

4 Conclusions

We have proposed a version of the ultimatum game in which each party suffers an externality proportional to the surplus captured by his counterpart. Under this formulation, three new results can emerge: (i) a negotiation breakdown, (ii) a fully equitable agreement, and (iii) an agreement biased in favor of the responder. Whereas the first result

depends on the externality level suffered by both parties, the two last only depend on the responder's externality provided that an agreement exists.

In light of these novel findings, we argue that the framework here proposed offers many contributions to the practice of negotiation and other dispute resolution activities. First, our theoretical findings stress that negative sentiments (through externalities) can indeed give a bargaining advantage to the parties in a negotiation process. This result seems to be at odds with practical negotiation approaches that emphasize the importance of separating clearly the negotiators from the negotiation itself ('separate the people from the problem').[1] In contrast, our model suggests that in distributive negotiations the party that plays the role of responder may improve his bargaining power by delegating the protection of his interests to a negotiator with a reputation of being a 'tough' bargainer, that is, a negotiator willing to take retaliations if he considers received offers as unfair or extremely unequal.

Second, our results may be useful to understand negotiations in which social and interpersonal interactions are involved, as for instance, dissolution of partnerships such as a marriage or a consulting partnership formed by few members. In fact, these dissolutions often take place after a tense (even stormy) relationship. Thus, one may expect that negative feelings (externalities in our terminology) stemming from concerns about the fairness or kindness of an agreement may arise and condition, to some extent, the bargaining process conducted to distribute assets or claims among the partners.

Third, the insights of the negotiation game here studied may be especially applicable to highly polarized bargaining processes, where profound ideological, historical, ethnic or religious differences lead each party to suffer a kind of envy whether his counterpart obtains something in a potential agreement. In a context in which negotiators act on behalf of voters or supporters, the externality parameter γ of our model could take the form of a penalty each party imposes on his representative negotiator when some surplus is 'given up' to his counterpart. Under this setting, frequent negotiation breakdown outcomes in this class of conflicts may be explained on the basis of a too high level of polarization (the condition $\gamma_E \gamma_F > 1$). These polarized situations include, among others, peacemaking processes, disarming agreements, and political transitions (from dictatorship to democracy).

We end by identifying some limitations and possible extensions of our analysis. First, since our game has only one round of negotiations, we are not able to examine the role played by neither impatience nor the impact of previous offers on externality parameters over the time. This may be tackled by adopting a model of offers and counteroffers *à la Rubinstein* (Rubinstein, 1982). Second, as our model assumes complete information, the sufficient condition for a negotiation breakdown involves a level of externality, at least on the part of one player, too high. On the contrary, the adoption of a setup with private information on the responder's externality parameter may require a condition less demanding and more realistic, as the underestimation of such parameter on the part of the

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[1] Consider, for instance, the *principled negotiation* method developed at the Harvard Negotiation Project (Fisher et al., 2011).

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proposer may yield an *ex post* rational disagreement. Lastly, recent experimental evidence for the ultimatum game suggests the importance of separating anger and envy as drivers behind rejecting decisions of responders. A natural extension may then be to construct a model that accounts for this difference, by assuming for instance a utility function that punishes both the action itself taken by the proposer (anger) and the consequences of this action over the final wealth of players (envy).

5 Appendix

Proof of Proposition 2.1. Applying backward induction, let us start with the last stage. At this point, player F accepts the player E's offer if his payoff from accepting is equal or greater than his reservation utility and rejects otherwise. That is, he accepts if and only if

$$(1 - x) - \gamma_F x \geq 0$$
$$\Leftrightarrow$$
$$x \leq \frac{1}{1 + \gamma_F},$$

and rejects otherwise.

Then, at the first stage, player E chooses x so that

$$\max_x x - \gamma_E(1 - x)$$
$$\text{s.t.}$$
$$0 \leq x \leq \frac{1}{1 + \gamma_F} \tag{1}$$
$$x \geq \frac{\gamma_E}{1 + \gamma_E}, \tag{2}$$

which is equivalent to the following program:

$$\max_x x(1 + \gamma_E)$$
$$\text{s.t.}$$
$$\frac{\gamma_E}{1 + \gamma_E} \leq x \leq \frac{1}{1 + \gamma_F}. \tag{3}$$

Notice that the interval defined for x by constraint (3) is non-empty as long as $\gamma_E \gamma_F \leq 1$. Otherwise, there is a negotiation breakdown:

Case 1. $\gamma_E \gamma_F \leq 1$. The previous program has a corner solution so that

$$x^* = \frac{1}{1+\gamma_F},$$

and thus, the SPE is given by:

$$SPE(G) = \left\{ x^* = \frac{1}{1+\gamma_F}, \text{ player } F \begin{cases} accepts & if\ x \le \frac{1}{1+\gamma_F} \\ rejects & otherwise \end{cases} \right\}.$$

Case 2. $\gamma_E\gamma_F > 1$. There is no x that satisfies constraint (3), and thus, a negotiation breakdown occurs. The SPE is then described by:

$$SPE(G) = \left\{ x^* = \frac{1}{1+\gamma_F} + \varepsilon, \text{ player } F \begin{cases} accepts & if\ x \le \frac{1}{1+\gamma_F} \\ rejects & otherwise \end{cases} \right\},$$

where $\varepsilon > 0$.

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