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EXISTENCE OF A GENERAL EQUILIBRIUM IN A MULTILEVEL ECONOMY

Abstract. This paper studies different approaches to defining micro- and macroeconomic equilibrium, specifies conditions under which from the existence of a microeconomic equilibrium follows a macroeconomic equilibrium. It introduces the concept of a general equilibrium in a multilevel economy, proposes a more robust definition of microequilibrium as compared to the traditional definition. When such microequilibrium is achieved, micro- and macroeconomic equilibrium conditions are fulfilled in all traditional senses.
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1. Introduction

According to Joseph Schumpeter, a giant in the history of economic thought, the general equilibrium theory is the only concept of economics 'that will stand comparison with the achievements of theoretical physics'(Schumpeter, 1996). As is known a theoretical foundation for the general equilibrium analysis, i.e. simultaneous equilibria in multiple markets, was initially laid down by Leon Walras (Walras, 2014).

The Walrasian model describes rational individuals that optimize their objective functions; all necessary information consists of prices and is uniformly accessible for all participants; markets are perfectly competitive; changes take place immediately. Mathematically a demand function and budget constraint are given for each consumer; each producer is described by a factor demand function and zero-profit condition. A general equilibrium is found as a nonnegative vector of prices and quantities of goods satisfying the condition of equality of market demand and market supply functions.

Abraham Wald (Wald, 1951) was the first to formulate strictly mathematically the conditions of the existence and uniqueness of a general equilibrium (weak axiom of revealed preference for demand function and condition of substitution of all goods). On that basis Kenneth Arrow (Arrow, Block, Hurwicz, 1959; Arrow, Hurwicz, 1958), Gerard Debreu (Debreu, 1952; Arrow, Debreu, 1954; Debreu, 1954), Lionel McKenzie (McKenzie, 2010) developed the modern general economic equilibrium theory and all necessary apparatus to prove the existence, uniqueness, and optimality of an equilibrium.

In the Arrow-Debreu model the production is described by technology sets instead of fixed coefficients of production and preference functions are introduced instead of utility functions. Firms maximize their profits at given market prices while households maximize utility at given prices and shares of firms profits. Then equilibrium is nonnegative prices vector by which demand doesn't exceed supply for each commodity (prices are equal to zero in case of excess supply).

In our analysis we will focus on this static general equilibrium model and will investigate influence between equilibrium at micro- and macroeconomic levels.

Models like the Walras-Arrow-Debreu one provide a microeconomic view on equilibrium i.e. describe the state of simultaneous equilibria in all micromarkets.

Thus let there be M consumers and K firms producing N commodities in the economy. Each ith commodity is characterized by a nonnegative price $p_i \geq 0$ and inventories W_i.

The behavior of economic agents is described using individual demand functions for jth consumer and individual supply function for kth firm for ith commodity as functions only of the price vector $p = (p_1, ..., p_N)$:

$$d_{ij} = d_{ij}(p), j = \overline{1, M}, i = \overline{1, N}, \qquad (1)$$

$$s_{ik} = s_{ik}(p), k = \overline{1, K}, i = \overline{1, N}, \qquad (2)$$

For each ith commodity market demand and supply functions can be expressed as:

$$d_i(p) = \sum_{j=1}^{M} d_{ij}(p), i = \overline{1, N}, \qquad (3)$$

$$s_i(p) = \sum_{k=1}^{K} s_{ik}(p), i = \overline{1, N}. \qquad (4)$$

and the excess demand function $e_i(p)$ (McKenzie, 2002, P.46)[1]:

$$e_i(p) = \sum_{j=1}^{M} d_{ij}(p) - \sum_{k=1}^{K} s_{ik}(p) - W_i, \quad i = \overline{1, N}. \qquad (5)$$

2. Equilibrium definitions

A partial equilibrium in the market of an individual commodity or factor is defined as an equation of demand and supply taking into account inventories. The general economic equilibrium is defined as the state of simultaneous equilibria (at nonnegative prices) in all commodity and service markets and can be represented as the condition (Shoven, Whalley, 2003, P.11):

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[1] The excess demand function may not include inventories and can be expressed as $e_i(p) = \sum_{j}^{M} d_{ij}(p) - \sum_{k=1}^{K} s_{ik}(p)$ (Shoven, Whalley, 2003, P.10).

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$$e_i(p) \le 0, i = \overline{1, N}. \tag{6}$$

It being understood that either the commodity is not 'economic', i.e. it is freely accessible and its price $p_i = 0$, or this commodity market with demand and supply exists, i.e. the commodity is sold at price $p_i > 0$. Mathematically this condition is expressed as follows (McKenzie, 2002, P.50):

$$\left[\begin{array}{l} \begin{cases} e_i(p) < 0 \\ p_i = 0 \end{cases} \\ \begin{cases} e_i(p) = 0 \\ p_i > 0. \end{cases} \end{array} \right. \tag{7}$$

The so called Walras' Law implies no excess demand (in money terms) in the economy as a whole under perfect competition (Jehle, Reny, 2000, P.190; McKenzie, 2002, P.46; Shoven, Whalley, 2003, P.10):

$$\sum_{i=1}^{N} p_i e_i(p) = 0 \tag{8}$$

In this case the fulfillment of the condition (8) does not mean that all markets are in equilibrium, – surplus at one market might compensate for deficits at other markets. From the condition (7) necessarily follows (8), the converse is not true in general. So the expression (7) is a sufficient condition of general economic equilibrium while the expression (8) is a necessary condition.

What is of interest to us is only the case of economic goods market equilibrium (equation of demand and supply) in the form:

$$e_i(p) = 0, i = \overline{1, N} \tag{9}$$

under the condition of nonnegative prices $p_i \ge 0$.

Thus there are four ways of the market equilibrium definition: (6)-(9).

Finding the equilibrium reduces to iterative adjustments of the price vector based on mapping of $e_i(p) \to p_i$ using the following iteration algorithm (Shoven, Whalley, 2003, P.15):

$$p_i \leftarrow \frac{p_i + \max\{0; e_i(p)\}}{1 + \sum_i \max\{0; e_i(p)\}}, i = \overline{1, N}. \tag{10}$$

The ideas of the general economic equilibrium theory that we have just presented refer to Walras's microeconomic approach. Such model represents an example of microapproach under which the state of a system is described via the state of each of its elements. Macroeconomic general equilibrium models are also known, for example IS-LM, DSGE models (Goodfriend, King, 1997). In these models an equilibrium is described using aggregate variables (total revenue, price level and so on) and represents simultaneous equilibria of several aggregate sectors and markets.

Aggregate demand and supply as functions of aggregate prices *P*, and aggregate inventory level take the form (let us leave aside the question of arguments of these functions for a while):

$$AD(P) = \sum_{i=1}^{N} d_i(p), \tag{11}$$

$$AS(P) = \sum_{i=1}^{N} s_i(p), \tag{12}$$

$$AW = \sum_{i=1}^{N} W_i. \tag{13}$$

Aggregate excess demand *E* can be defined similarly to microeconomic excess demand functions (5):

$$E(P) = AD(P) - AS(P) - AW, \tag{14}$$

or by aggregating microeconomic excess demand functions:

$$E(P) = \sum_{i=1}^{N} e_i(p). \tag{15}$$

It is important to note that in expressions (11)-(13), (15) different prices appear: microeconomic price vector *p* on the right side (as microeconomic function arguments) and aggregate prices *P* on the left side (as aggregate function arguments).

As it is known, consistent aggregation is possible only under very strict restrictions imposed on microeconomic functions (linearity and homogeneity (same parameters) for all system elements) (Felipe, Fisher, 2003, P.133). Then let us assume that these strict assumptions hold true and aggregate functions *AD(P)*, *AS(P)* and *E(P)* exist.

Let us consider a question of how equivalent are two aggregate excess demand definitions (14) and (15). In other words let us see whether the equation holds:

$$AD(P) - AS(P) - AW = \sum_{i=1}^{N} e_i(p) \ ? \tag{16}$$

Under condition (5), (11)-(13) equation (16) holds automatically:

$$E(P) = \sum_{i=1}^{N} e_i(p) = \sum_{i=1}^{N} (\sum_{j=1}^{M} d_{ij}(p) - \sum_{k=1}^{K} s_{ik}(p) - W_i) =$$
$$= AD(P) - AS(P) - AW = E(P)$$

In other words aggregate excess demand definitions (14) and (15) can be deemed equivalent.

Now similarly to the microeconomic equilibrium let us try to find a macroeconomic equilibrium.

The first microeconomic equilibrium definition (6) corresponds to the following macroeconomic equilibrium definition (aggregate demand does not exceed aggregate supply):

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$$E(P) \leq 0. \tag{17}$$

If we exclude from the consideration a situation when aggregate price level in the economy is zero then definitions (7) and (9) correspond to a single macroeconomic equilibrium definition:

$$E(P) = 0. \tag{18}$$

Finally the following macroeconomic equilibrium condition corresponds to Walras' Law (8):

$$P \cdot E(P) = 0. \tag{19}$$

The expression (19) reflects the aggregate market equilibrium characterized by aggregate price level P and aggregate excess demand $E(P)$. Since we excluded the case of zero aggregate prices the condition (19) is equivalent to condition (18).

Thus we can say that four microeconomic equilibrium conditions correspond to two macroeconomic equilibrium conditions.

3. Equilibrium in hierarchical system

The hierarchical analysis raises the question as to whether micro- and macroeconomic general equilibrium models are consistent. In other words whether the existence of an equilibrium at one level means the existence of an equilibrium at the other level?

The second question concerns the fact that micro- and macroeconomic theories study equilibrium at separate levels of the hierarchy. The hierarchical approach treats both micro and macro levels as a system. Therefore the hierarchical (system) approach allows us to introduce one more definition of equilibrium in the whole hierarchical system as a simultaneous equilibria at micro and macro levels. In other words, a hierarchical equilibrium is such vector of micro- and macro-prices *(p   P)* that micro- and macro-level equilibria are achieved simultaneously.

We can pose a more interesting question: if macroeconomics is treated not just as an aggregate and passive reflection of microeconomics, but as a system with its own regularities, does a micro-level equilibrium mean an equilibrium in a two-level economy? We will leave this question outside the scope of this paper.

Obviously the first two questions are interrelated: if from a microeconomic equilibrium follows a macroeconomic equilibrium then simultaneous equilibrium in the whole hierarchical system is achieved. Therefore let us focus on the first question.

If a microeconomic equilibrium is achieved in the sense of (6) then such an equilibrium price vector p^* exists that demand for each commodity does not exceed its supply: $e_i(p^*) \leq 0$ for $i = \overline{1, N}$. Hence it follows that macroeconomic equilibrium will be achieved in the sense of (17):

$$E(P^*) \equiv \underbrace{e_1(p^*)}_{\leq 0} + \underbrace{e_2(p^*)}_{\leq 0} + \ldots + \underbrace{e_N(p^*)}_{\leq 0} \leq 0.$$

However a macroeconomic equilibrium might not be achieved in the sense of (18): $E(P^*) = 0$. Obviously macroequilibrium (17) does not mean the achievement of microeconomic equilibrium (6).

The microeconomic equilibrium in the sense of (7) is equivalent to the same situation. This means that the macroeconomic equilibrium in the sense of (17) holds.

Let us review situation (9) i.e. when there exists such a positive vector p^* that demand and supply for each commodity are in equilibrium $e_i(p^*) = 0$. In this case the macroeconomic equilibrium is achieved in the sense of (17) and (18) since

$$E(P^*) \equiv \underbrace{e_1(p^*)}_{=0} + \underbrace{e_2(p^*)}_{=0} + ... + \underbrace{e_N(p^*)}_{=0} = 0.$$ Moreover since $E(P^*) = 0$, then

the macroeconomic equilibrium condition is met in the form of Walras' Law (19) $P^* E(P^*) = 0$.

As in the previous case the microeconomic equilibrium does not follow form the macroeconomic equilibrium since there might be surpluses in one markets and deficits in the others when the aggregate excess demand function is equal to 0.

The situation with Walras' Law (8) (when the condition $p_1^* e_1(p^*) + p_2^* e_2(p^*) + \cdots + p_N^* e_N(p^*) = 0$ is fulfilled) is not trivial. It is evident that from the condition (8) does not follow macroeconomic equilibrium either in the sense of (17) or in the sense of (18).

Let us investigate the question under which conditions from the fulfillment of Walras' Law at the micro level follows its fulfillment at the macro level. In other words how expressions (8) and (19) are related. For this type of analysis it is not sufficient to know quantitative parameters of equilibrium in the form of excess demand functions $e_i(p)$. It is also necessary to bring in data about nominal values (prices).

First let us examine a simple situation when aggregate prices is a simple average of microeconomic prices:

$$P \equiv \frac{1}{N} \sum_{i=1}^{N} p_i \tag{20}$$

In this case, condition (19) takes the form of:

$$P^* E(P^*) \equiv \frac{1}{N} \sum_{i=1}^{N} p_i^* \sum_{i=1}^{N} e_i(p^*) = \frac{1}{N} \underbrace{\sum_{i=1}^{N} p_i^* e_i(p^*)}_{=0} + \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq i} p_i^* e_j(p^*) \neq 0$$

The second sum reflects correlation between prices and excess demands for different commodities which is nonzero in general. Hence, if Walras' Law is true at the micro level in general, it does not mean that it is true at the macro level. Walras' Law is true at the macro level only under condition of equilibrium (9).

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It may be argued that it is more correct to use a weighted mean instead of arithmetic mean (20). Let us analyze whether the obtained results depend on the way prices are aggregated? Let us examine whether Walras' Law is fulfilled in a two-level economy, if more correct aggregation – weighted mean – is used. Then the price level in the economy is defined as:

$$P = \sum_{i=1}^{N} w_i p_i, \qquad (21)$$

where w_i are weight coefficients $\sum_{i=1}^{N} w_i = 1$.

Once again if Walras' Law is fulfilled at the microeconomic level then:

$$P^* E(P^*) \equiv \sum_{i=1}^{N} w_i p_i^* \sum_{i=1}^{N} e_i(p^*) = \sum_{i=1}^{N} w_i p_i^* e_i(p^*) + \sum_{i=1}^{N} \sum_{j \neq 1} w_i p_i^* e_j(p^*) \neq 0$$

Obviously in the general case when weight coefficients are randomly selected Walras' Law is not fulfilled at the macro level and in a hierarchical system as a whole.

However such weight coefficients can be selected that the fulfillment of Walras' Law at the micro level follows its fulfillment at the macro level. Let use excess demand for weight coefficients:

$$w_i = \frac{e_i(p^*)}{\sum_{j=1}^{N} e_j(p^*)}, \qquad (22)$$

on condition $\sum_{j=1}^{N} e_j(p^*) \neq 0$ then

$$P^* E(P^*) \equiv \sum_{i=1}^{N} w_i p_i^* \sum_{i=1}^{N} e_i(p^*) = \sum_{i=1}^{N} \frac{e_i(p^*) p_i^*}{\sum_{j=1}^{N} e_j(p^*)} \cdot \sum_{i=1}^{N} e_i(p^*) = \sum_{i=1}^{N} e_i(p^*) p_i^* = 0$$

This means that if weight coefficients are selected correctly Walras' Law is fulfilled both at the micro- and macro levels.

It should be noted that weight coefficients in formula (22) have sense only in case $\sum_{j=1}^{N} e_j \neq 0$. This is a noteworthy condition. To fulfill Walras' Law we need that neither a microeconomic equilibrium in the sense of (9) nor a macroeconomic equilibrium in the sense of (18) is achieved. On the other hand if $\sum_{j=1}^{N} e_j = 0$ then there is aggregate excess demand $E(P)=0$ and Walras' Law at the macro level $P \cdot E(P) = 0$ is fulfilled automatically.

Another restriction concerns an attempt to use not volumes of excess demand $e_i(p^*)$ as in (22) but nominal values $e_i(p^*) p_i^*$ for defining weight coefficients. In this case denominator $\sum_{j=1}^{N} p_i^* e_i(p^*) = 0$ because of fulfillment of Walras' Law at the micro level.

Thus we see that the correct selection of weight coefficients is not a simple task and is crucial for the economic theory.

Let us return to the important note that we have made above. We have said that macroeconomic function $E(P)$ itself exists (can be exactly derived from microeconomic functions) only when microeconomic functions $e_i(p)$ are linear in all variables and are homogeneous in the sense that function parameters are the same for all elements. Then excess demand functions for all commodities will be identical.

$$e_i(p) = e(p). \tag{23}$$

In this case Walras' Law at the micro level takes the following form $p_1^* e(p^*) + p_2^* e(p^*) + \ldots + p_N^* e(p^*) = e(p^*)\sum_{i=1}^{N} p_i^* = 0$. Either the sum of prices is equal to zero which is impossible or there must be no excess demand $e(p^*)$ for each individual commodity. Namely we get into the microeconomic equilibrium situation in the sense of (9). In this case Walras' Law at the macro level is fulfilled automatically regardless of price aggregation method:

$$P^* E(P^*) \equiv P^* \sum_{i=1}^{N} e_i(p^*) = P^* N \underbrace{e(p^*)}_{=0} = 0.$$

4. Conclusions and generalization

The obtained results can be tabulated as follows:

Microeconomic equilibrium conditions	Macroeconomic equilibrium conditions		
	$E(P) \leq 0$	$E(P) = 0$	$P \cdot E(P) = 0$
$e_i(p) \leq 0$	+	-	-
$\begin{cases} e_i(p) < 0 \\ p_i = 0 \end{cases}$ $\begin{cases} e_i(p) = 0 \\ p_i > 0 \end{cases}$	+	-	-
$\sum_{i=1}^{N} p_i e_i(p) = 0$	-	-	$\pm$
$e_i(p) = 0$	+	+	+

Let us formulate the main conclusions.

First, different definitions of micro- and macroeconomic equilibriums are interrelated. Macroeconomic equilibrium does not follow from any microeconomic equilibrium condition in general.

Second, the condition (9) that we have proposed is the most strict, but is

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universal (its fulfillment results in the fulfillment of the remaining equilibrium conditions) and invariant (its fulfillment at the micro level means achievement of macroeconomic equilibrium regardless of the way how macroeconomic equilibrium is defined).

Third, Walras' Law is specifically distinguished – its fulfillment at the micro level does not result in its fulfillment at the macro level in general and depends on the price aggregation method.

Fourth, the method itself for statistical aggregation of prices cannot be arbitrary. It must be supported by the tenets of the micro- and macroeconomic theory. In turn, the macroeconomic theory must be derived from microeconomics (however it does not mean reductionism, where macroeconomics reduces to microeconomics) taking into account achievements in the statistical theory, since, it is the fifth conclusion, in all situations examined microeconomic equilibrium cannot be postulated from macroeconomic equilibrium.

Above we have reviewed consistent descriptions of micro- and macroeconomic levels from the general equilibrium viewpoint. Let us summarize our conclusions for the economics in general.

Suppose that the microeconomic level is described using vector-function $f : R^n \rightarrow R^n$, which defines vector $y = (y_1 \ldots y_n)$ of outputs corresponding to vector of inputs $x = (x_1 \ldots x_n)$.

In the economy as a whole, correlation between aggregate resources consumed X and aggregate output Y is described by means of a macroeconomic production function $F : R \rightarrow R$.

Relations $\chi : R^n \rightarrow R$ and $\varphi : R^n \rightarrow R$ also exist between variables x and X, y and Y.

Using algebraic apparatus one can define a problem of consistency of a two-level system description as a diagram (Figure 1).
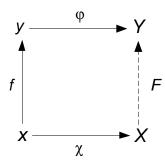

Figure 1. Relations of microeconomic function *f* and macroeconomic function *F* and aggregation operators χ and φ

For our problem the condition $\varphi \circ f = F \circ \chi$ must be fulfilled, i.e. there must

be a commutative diagram.

As Lawrence Klein stated (Klein, 1946, P.94), a solution can be found in two ways. First of all, macrofunction *F* should be based on microdescription. Macroeconomic regularities must be derived and the whole theory of macroeconomics must be constructed based on the microeconomics and statistical aggregation rules. In our paper we used this approach.

The alternative approach proposed by Lawrence Klein is that it is necessary to construct statistical aggregates ('bridges') which are consistent with micro- and macrodescriptions. To this end, it is necessary to have solid foundations of micro- and macroeconomics. But as of today, economists themselves call their science a 'schizophrenic' discipline, two main branches (micro- and macroeconomics) of which have drastically different views of the world (Sargent, Lucas, 1984, P.295-296).

There may be a third approach as well: we are given aggregation operators and a macro theory, and we need to find a microeconomic theory matching them. That was the path by which economic theory had been evolving in the second half of the 20th century when Keynesian and New Classical schools were building their own microfoundation for their macroeconomic doctrines.

5. Example

In conclusion as an illustration let us give an example of a simplest economy consisting of two commodities with linear excess demand functions. Suppose that excess demand functions are linear functions and are equal to $e_1(p_1, p_2) = a_1 - b_1 p_1$ and $e_2(p_1, p_2) = a_2 - b_2 p_2$ respectively. In this case a microeconomic equilibrium is defined by prices:

$$p_1^* = \frac{a_1}{b_1},$$
$$p_2^* = \frac{a_2}{b_2}. \tag{24}$$

This is a microdescription of the economy. For the purpose of the macrodescription let us define excess demand function at the macro level as *E(P)=A-BP*. It is obvious that equilibrium level of aggregate prices in the economy will be equal to *P*=A/B*.

The question arises as to how equilibrium prices at the micro- and macro levels correlate to each other. If we calculate average prices in the economy,

$$P^{**} = \frac{1}{2}(p_1^* + p_2^*) = \frac{a_1 b_2 + a_2 b_1}{2 b_1 b_2}, \tag{25}$$

will they be equal to macroeconomic equilibrium prices *P**? If a microeconomic equilibrium is achieved, does aggregate-price level *P*** provide a macroeconomic equilibrium as well? How A, B, a_1, a_2, b_1, b_2 should correlate to each other so as to

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satisfy the relation $P^*=P^{**}$?

Obviously, to satisfy the equation $P^*=P^{**}$ it is necessary to meet the following condition

$$\frac{A}{B} = \frac{a_1 b_2 + a_2 b_1}{2 b_1 b_2}.$$ (26)

However, intuitively, it feels like coefficients should be aggregated in a different way. Since a_1 and a_2 reflect excess demand quantity at zero prices, then it is logical to calculate $A = a_1 + a_2$. Coefficients b_1 and b_2 reflect the extent of excess demand response on prices, and again intuitively 'average' response of aggregate excess demand on aggregate price level, i.e. $B = \frac{1}{2}(b_1 + b_2)$. Based on similar reasoning we could get a pseudo-aggregated price level:

$$P^{***} = 2\frac{a_1 + a_2}{b_1 + b_2}.$$ (27)

In the assumption of homogeneity of microeconomic excess demand functions ($a_1 = a_2 = a$ and $b_1 = b_2 = b$) the aggregate excess demand function exists and has the form of $E(P) = e_1(p) + e_2(p) = 2a - b(p_1 + p_2) = 2a - 2bP$. We can establish a correlation between parameters of macro- and microeconomic excess demand functions: $A = 2a$ and $B = 2b$. Then the equation (25) holds, and more importantly

$$P^* = P^{**} = p_1^* = p_2^* = \frac{a}{b}$$ (28)

The latter means that identity of micro- and macrodescriptions is achieved only in case of absolute sameness. This means that only in case of price equality in all commodity markets, there will be no inconsistency between macroeconomic equilibrium P^* and aggregate from macroeconomic equilibrium P^{**}. This also means that if microeconomic equilibrium $p^*=(p_1^*;\ p_2^*)$ is achieved, macroeconomic equilibrium and simultaneous equilibria in two-level economy are achieved respectively.

In economics the principle of 'representative agent' is often used when certain average representations of economic agent and/or market behavior are extrapolated to the economy as a whole. The above example shows that the representative agent approach can be justified only in the assumption of absolute identity of commodities, agents, and markets. In more general and realistic cases, more valid research methods are needed.

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