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AN IMPROVED METHOD OF GRANGER CAUSALITY TEST AND APPLICATION ON THE STOCK MARKET RISK TRANSMISSION

***Abstract**: Granger causality analysis has been actively adopted in the field of economics, operations research, finance, statistics, data mining and decision making. The current Granger causality analysis method can just perform effectively in dealing with the two-variable causality test, and thus it could lead to the confusion between direct cause and indirect cause and also pseudo-cause problems due to the homologous data. In order to solve the flaws considered above, an improved method is here proposed by introducing additional variables to the original Granger test equation and defining a new test statistic through Monte Carlo method. Furthermore, a simulation analysis for validation and an empirical analysis in stock market risk transmission for application were both made. Both the results demonstrate that the improved method is able to avoid the confusions caused by the current method and also capable of finding the risk conduction path clearly.*

Keywords: *Causality analysis, Multiple variables, Simulation, Monte Carlo, Stock market, Risk transmission, Decision Analysis.*

JEL Classification: C25, C53, C81, G32

1. Introduction

Causality analysis has been a widely-used method in the field of Economics and Management Science (Ruxanda & Muraru, 2011; Ismail & Rashid, 2013). The causality analysis method used by most papers was raised by Granger in the 1970s, who was awarded by the Nobel Prize (Granger, 1969, 1980; Anderson & Vastag, 2004; Athanasenas, 2010; Ghosh, et al, 2010). Numerous researchers have discussed the validity and practicability of this method, such as Toda & Phillipp (1993) and He & Koichi (2001), who both studied the factors affecting the validity of the Granger causality test. Also, Bult, et al (1997) discussed the method's practicability in small samples; Caporate & Pittis (1997) interpreted the method's usage in an incomplete system, and so forth. The original Granger causality test is mainly suitable for short-time dependent and stationary time series data, accordingly there is a research trend in the past few years to enlarge the applying range, for example, Hassapis, et al (1999) dealt with the non-stationary data's test problem, and Ray & Tsay (2000) improved the existing method to adapt to the large range data.

Although the methods have been improved with time going by in the existing literatures as above, the existing Granger causality analysis can only deal with the test between two variables, and thus the so-called multiple variables causality analysis was all based on the combinations of two variables test's results (Warr & Ayres, 2010; Lean & Smyth, 2010). Our paper points out two defects that may be caused by the original method when multiple variables are concerned with. One is the confusion between direct cause and indirect cause, and the other is the pseudo-cause problem due to the homologous data. These two defects can possibly cause significant misleading influences on the result of causality analysis (the example can be found in part 4 of this paper). Therefore this paper focuses on the improvement of the current methods for causality analysis in order to make it effective in handling the causality analysis of multiple variables within a network structure.

To further test the validity and practicability of the improved method proposed in this paper, we designed a network structure including multiple variables given with their causalities, and further validated whether the improved method can identify the causality relationships contained in the network structure correctly. Such analysis is of significance in testing the validity of the improved method. This method, if capable of identifying network structure as shown in the above analysis, can be widely used in many fields, such as the stock market risk transmission, which has been discussed in numerous studies. For example, Blasco, et al, (2005) studied the causality among bad news, Dow Jones and Spanish stock markets by means of the two variables method; Johansson & Ljungwall (2009) applied the Granger Causality analysis method to discuss Stock Risk transmission among Shanghai, Shenzhen and Hong Kong stock markets from the point of view of information overflow; Qiao & Lam (2011) focused on China's stock and studied its risk transmission based on the theory of Granger causality test and nonlinear science; King & Wadhwani (1990), King, et al (1994) and Duca & Ruxanda (2013) have done similar research. In the above listed literatures, some work was left by the researchers of overcoming the limitations of the traditional causality test methods in our opinion. In order to enrich the empirical study of this field and to make a clear application in this field, we selected the stock risk transmission study for empirical analysis as well. It is of both theoretical and practical significance to verify the validity of the model and to be a guidance of the investment strategy.

The main contributions of this paper are as follows: (1) The paper proposes a new method for causality test of multiple variables that are appropriate for network structure analysis, and makes a simulation test and an empirical analysis for the method's validation and application. (2) The method proposed in this paper overcomes the limitation of the current Granger causality analysis method, which is not only proper for the two variables' test problem, but also is feasible in identifying the network structure as a new tool for data mining and knowledge discovery. (3) The improved method of this paper is suitable for the analysis of stock market volatility risk transmission and has better credibility as a guide for investment.

The study proceeds as follows: Section 2 reviews the current Granger causality analysis method and discusses its limitations. Section 3 improves the current Granger causality analysis method to make it capable of handling the test of causalities between multiple variables so as to overcome the defects caused by the traditional method. Section 4 verifies the effectiveness of the new method through a simulation. Section 5 applies the new causality analysis method to study stock market volatility risk transmission among the world's four main stock markets, and compares its result with that of the old method. Section 6 concludes.

2. Review of The traditional Granger causality test

Here, we provide a brief review of the two variable Granger causality test as follows. The traditional Granger causality test involves only two variables. Taking two time series x_t and y_t as an example, if causality exists between sequence x_t and y_t, there are three possible situations: x_t is the cause of y_t, y_t is the cause of x_t, and each one is the cause of the other. The consequent test procedure is as follows:

Firstly, we set up the benchmark equations

$$x_t = \sum_{i=1}^{\infty} a_{1i} x_{t-i} + \varepsilon_{1t},\ y_t = \sum_{i=1}^{\infty} b_{1i} y_{t-i} + \eta_{1t}\ . \tag{1}$$

where, ε_{1t} and η_{1t} represent for the white noise, and a_{1i} and b_{1i} are coefficients. Note that the formula (1) is just a general expression by defining the lagged item as infinite; in fact, the lagged item is generally of finite order in practice. So, if the first lagged item in the equation is of p order, which just means that $a_{1i} = 0$ ($i > p$). Usually the order number of lagged items are determined by using the AIC rules and the SC rules comprehensively (Lee, 2009**)**. Furthermore, the variance-covariance matrix H_1 of residual $\hat{\varepsilon}_{1t}$ and $\hat{\eta}_{1t}$ can be obtained by fitting formula (1) as

$$\mathrm{H}_1 = \begin{pmatrix} \Sigma_{xx}^J & \Sigma_{xy}^J \\ \Sigma_{xy}^J & \Sigma_{yy}^J \end{pmatrix}. \tag{2}$$

Secondly, we set up the contrast equations as follows:

$$x_t = \sum_{i=1}^{\infty} a_{2i} x_{t-i} + \sum_{i=1}^{\infty} b_{2i} y_{t-i} + \varepsilon_{2t}, \; y_t = \sum_{i=1}^{\infty} c_{2i} x_{t-i} + \sum_{i=1}^{\infty} d_{2i} y_{t-i} + \eta_{2t}. \tag{3}$$

Meanings of variables and parameters in formula (3) can be deduced from the formula (1). Thus we can obtain the variance-covariance matrix H_2 of residuals of formula (3) as

$$\mathrm{H}_2 = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{xy} & \Sigma_{yy} \end{pmatrix}. \tag{4}$$

Thirdly, to testify that x_t is the cause of y_t, the statistic $\Delta_{x\to y}$ is defined

$$\Delta_{x\to y} = \frac{(\Sigma_{yy}^J - \Sigma_{yy})/p}{\Sigma_{yy}/(n-p-q)} \; . \tag{5}$$

where, n is the sample size, p and q represent the lag intervals of x_t and y_t respectively in the formula (3). Pierc, et al (1997) have proved that the statistic Δ complies with the distribution $F(p, n-p-q)$ when given the null hypothesis that y_t is not the cause of x_t. By referring to the F distribution table, we can judge whether Δ is significant in some confidence level. Likewise, to testify that y_t is the cause of x_t, we can define the statistic $\Delta_{y\to x}$ as

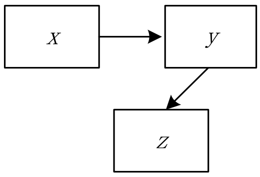
$$\Delta_{y\to x} = \frac{(\Sigma_{xx}^J - \Sigma_{xx})/q'}{\Sigma_{xx}/(n-p'-q')}. \tag{6}$$

where, n is the sample size, q' represents lag order number of y_t and p' represents that of x_t in formula (3). Finally, when each sequence is the cause of the other, we just need to test the statistic defined in the equation (5) and (6) at the same time.

However, limitations exist in the above mentioned two variables' Granger causality test. Traditional Granger causality method is suitable for two-variable causality test problem, but not able to handle the pseudo-cause problems caused by two kinds of limitations. Next, we list the two kinds of limitations and then provide the exemplification of these limitations in **Section 4**.

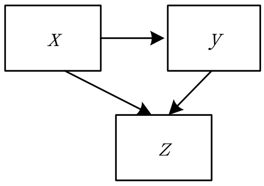
Limitation 1. Traditional methods cannot distinguish direct causality from indirect causality.

As showed in Figure 1, there is no direct causality from x to z. The relationship between them is indirect and formed by the intervening variable y. However we are likely to get the causality result among the three variables showed in Figure 2 by applying the traditional causal analysis method. It is obvious to find the difference between Figure1 and Figure 2. Figure 2 gives an incorrect result in which it shows direct causality from x to z .



x is the cause of *y*, *y* is the cause of *z*, and there is no direct causality between *x* and *z* , the relation between them is caused by *y*.

Figure 1. Causality among three variables (*y* as the intervening variable)

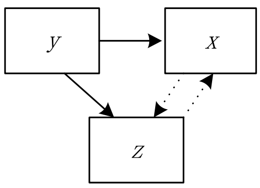


x is the cause of *y*, *y* is the cause of *z*, and *x* is also the cause of *z*.

Figure 2. Result of causal analysis by the traditional method

Limitation 2. The traditional method cannot distinguish a pseudo-cause problem caused by the homologous data.

In Figure 3, solid arrow represents real causality, whereas dotted arrows may be rested to exist based on the traditional method. This phenomenon is caused mainly due to the disability of the traditional method in analyzing the causality disturbed by the homologous data. Therefore, the original method needs improvement to enable identify the indirect causality caused by intermediary transfer and the pseudo-cause problem caused by the homologous data, in order to accurately identify the causality relationships among more than two sequences.



y is the cause of *x* and *z*, *x* and *z* have no causality originally, but the causality may be tested to exist because of the influence of *y* according to the traditional method

Figure 3. Pseudo-cause problem caused by homologous data

3. Research methodology

First of all, we discuss the principle and the process of the improved method. In order to overcome the limitations of the traditional method, we set up the method as follows. Meanings of variables can be deduced from the formula (1).

Firstly, we set up the benchmark equations

$$y_t = \sum_{i=1}^{\infty} a_{3i} y_{t-i} + \sum_{i=1}^{\infty} b_{3i} z_{t-i} + \eta_{3t}, \quad z_t = \sum_{i=1}^{\infty} c_{3i} y_{t-i} + \sum_{i=1}^{\infty} d_{3i} z_{t-i} + \gamma_{3t} . \quad (7)$$

From which, we can obtain the variance-covariance matrix H_3 of residuals as

$$\mathrm{H}_3 = \begin{pmatrix} \Sigma_{yy}^J & \Sigma_{yz}^J \\ \Sigma_{yz}^J & \Sigma_{zz}^J \end{pmatrix}. \tag{8}$$

Then, we set up the contrast equations as follows,

$$\begin{aligned} x_t &= \sum\nolimits_{i=1}^{\infty} a_{4i} x_{t-i} + \sum\nolimits_{i=1}^{\infty} b_{4i} y_{t-i} + \sum\nolimits_{i=1}^{\infty} c_{4i} z_{t-i} + \varepsilon_{4t} \\ y_t &= \sum\nolimits_{i=1}^{\infty} d_{4i} x_{t-i} + \sum\nolimits_{i=1}^{\infty} e_{4i} y_{t-i} + \sum\nolimits_{i=1}^{\infty} g_{4i} z_{t-i} + \eta_{4t} \ . \\ z_t &= \sum\nolimits_{i=1}^{\infty} h_{4i} x_{t-i} + \sum\nolimits_{i=1}^{\infty} m_{4i} y_{t-i} + \sum\nolimits_{i=1}^{\infty} n_{4i} z_{t-i} + \gamma_{4t} \end{aligned} \tag{9}$$

Next, the variance-covariance matrix H_4 of residuals is

$$\mathrm{H}_4 = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} & \Sigma_{xz} \\ \Sigma_{xy} & \Sigma_{yy} & \Sigma_{yz} \\ \Sigma_{xz} & \Sigma_{yz} & \Sigma_{zz} \end{pmatrix}. \tag{10}$$

To testify that x_t is the cause of y_t, when regarding y_t as an intervening variable, the statistic $\Delta_{x \to z|y}$ is defined as follows,

$$\Delta_{x \to z|y} = (\Sigma_{zz}^J - \Sigma_{zz}) / \Sigma_{zz} \ . \tag{11}$$

When the relationship between x and z depends entirely on the intervening effect from y as showed in Figure 1, the two z_t 's equations in formula (1) and (3) should be the same because x_{t-i} $(i = 1, 2, \cdots)$ does not provide additional information to explain the variable z. If so, it holds that $\Sigma_{zz} = \Sigma_{zz}^J$ and $\Delta_{x \to z|y} = 0$. Otherwise, x_{t-i} $(i = 1, 2, \cdots)$ must provide additional information, and thus more information included in z_t can be explained by adding x_{t-i} $(i = 1, 2, \cdots)$ to be its explanatory variables. In this case, it holds that $\Sigma_{zz} < \Sigma_{zz}^J$ and $\Delta_{x \to z|y} > 0$. Likewise, when y is both the information source of x and z as described in Figure 3, formation of the causality between x and z may relies on the shared information belonging to y. When y_{t-i} $(i = 1, 2, \cdots)$ are used as the explanatory variables, x_{t-i} $(i = 1, 2, \cdots)$ in the formula (3) does not provide additional information to explain the variable z, thus in this case, the null hypothesis $h_{4i} = 0$ $(i = 1, 2, \cdots)$ are tenable. As a result, we can also obtain the causal statistic $\Delta_{x \to z|y}$ used for testifying that x is the cause of z when y is the homologous data, which takes the same form of equation (11) as

$$\Delta_{x \to z|y} = (\Sigma_{zz}^J - \Sigma_{zz}) / \Sigma_{zz} \ . \tag{12}$$

Through the comparison between Figure 1 and Figure 3, we can discover that the statistics shares the same form but the structure is different. One takes y as an intervening variable but the other takes y as the homologous variable. Given the same formula, the computational cost of multivariate causal analysis is reduced markedly, since calculation only once is now sufficient.

Further, the existence of randomness made it worthy to pay attention to the degree of $\Delta_{x\to z|y}$ above zero, which means that it can be considered as statistically significant instead of being caused by random disturbances. There are two basic methods to discuss the problem: one is the inference method based on the statistical hypothesis, and the other based on a Monte Carlo simulation that can obtain the threshold value at some confidence level as the standard of judgment. With the development of computer technology, the second method has become popular, so this paper wants to acquire the threshold value at the 0.95 confidence level through the second method and set it as the standard of judgment.

Following the above discussions, we continue to acquire statistic threshold value through Monte Carlo simulation. This paper takes the statistic defined in equation (11) as an example to describe the method of acquiring statistic threshold value through Monte Carlo simulation.

First, we disturb the order of existing x_t in the equation of z_t in formula (9), and then make a random arrangement of x_t to obtain a new order and mark it as $\hat{x}_t$.

Second, we calculate the value of equation (11).

Third, we repeat the above process for 500 times, and sort the values of equation (11) from the small to the large.

Last, we take the 475$^{\text{th}}$ data as the threshold value because this data is larger than 95% of all data, which indicates that it is the statistic at the 0.95 confidence level.

Once the statistic obtained from the actual data, which has been defined in equation (11), is larger than the threshold value just calculated, the null hypothesis showed in Figure 1 is rejected and the structure showed in Figure 2 is accepted, and vice versa. The method also works to deal with the homologous case showed in Figure 3.

In detail, the principle of using the Monte Carlo simulation method in this paper is as follows: If the null hypothesis that $h_{4i}=0$ $(i=1,2,\cdots)$ as implicated in Figure 1 is accepted, then the rearrangement of x_t does not change the value of $\Delta_{x\to z|y}$ defined in formula (5). However, the existence of randomness will make the distribution of $\Delta_{x\to z|y}$ to be the normal distribution based on the null hypothesis or rearranging x_t s. Accordingly, the threshold value can be achieved by finding the 0.95 quantile of the statistic's distribution which inferred from repeated simulations. The detailed corresponding steps are shown in Figure 4. After Step 5, if the real x_t 's statistic is larger than the threshold value, the null hypothesis that $h_{4i} = 0$ $(i = 1, 2, \cdots)$ is rejected and the causality among the three variables is the same with the structure showed in Figure 2, while if the real x_t 's statistic is smaller than the threshold value, the null hypothesis is accepted and the causality among the three variables is the same with the structure showed in Figure 1. It is noted that the method also works to deal with the homologous case showed in Figure 3.

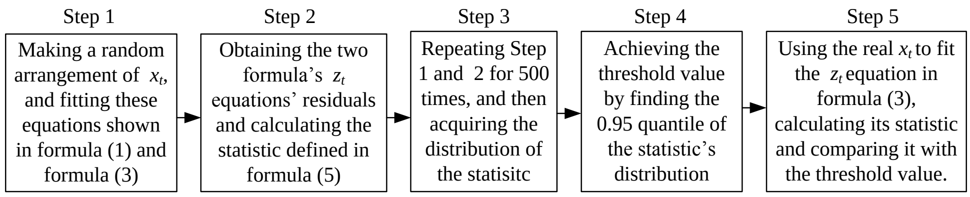


Figure 4. Steps of obtaining threshold value of the statistic defined in formula (11)

To sum up, implementation steps of the new method is provided as follows. The new method is proper for short-time dependent and stationary time series data, which inherits the existing Granger causality test method, and expands its causality test to make it not only work between two variables but also functional among three variables. The new method is proved to be effective in analyzing indirect causality caused by intervening variables and pseudo-cause problems caused by the homologous variables, so it is a milestone to handle problems concerned with three modules. In fact, when there are more than three modules, analyzing each three-module part by using the new method will be enough, and it will be just a matter of volume and time but no qualitative change. Implementation steps are as follows:

Step 1. Examine whether the existing time-series data is stationary. Transform it into the proper time-series data that satisfy the preconditions of causality test;

Step 2. Apply the current Granger causality analysis method to test the causality between each two sequences and obtain a system structure containing redundancy causality (pseudo-cause problem illustrated in two cases);

Step 3. Find the structure as showed in Figure 2 or Figure 3 on the basis of the relationship graph acquired in step 2, and clear away the wrong connections produced by indirect causality or pseudo-cause problems caused by the homologous data by using the new method. Repeat this process until all pseudo-causal connections are eliminated.

4. Simulation-based Validations

To testify the validity of the method and exemplify the two limitations listed in Section 2, we put forward a four-variable network system with the causality showed in Figure 5.

Meanwhile, we suppose and set up these four variables' relationships artificially as follows. Among them, $\varepsilon_i(t)$ $(i=1,2,3,4)$ comply with the normal distribution with the mean of zero and variances supposed in order as 0.16, 0.25, 0.04, 0.09. All initial values of $x_i(t)$ $(i=1,2,3,4)$ are set as 1. Then we acquire 100 time-series data of each equation described above, and then testify whether these variables have the causality showed in Figure 5.

$$x_1(t)=0.5x_1(t-1)+\varepsilon_1(t)$$
$$x_2(t)=0.4x_1(t-1)+0.9x_4(t-1)+\varepsilon_2(t)$$
$$x_3(t)=0.8x_2(t-1)+0.3x_3(t-1)+\varepsilon_3(t)$$
$$x_4(t)=0.7x_1(t-1)+0.6x_4(t-1)+\varepsilon_4(t)$$

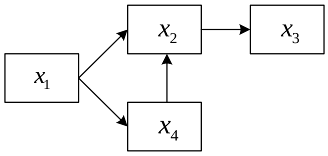


Figure 5. Network graph of the causality among four variables

Step 1. Test of the sequences' stationarity. We apply KPSS method with intercept term and trend term into this problem, which has the null hypothesis that the test sequence is stationary. The results are showed in Table 1.

Table 1. Results of stationarity test of the four time series data

	sequence	LM-value	LM statistic at 0.95 confidence level	result
1	x_1	0.1169	0.1460	stationary
2	x_2	0.1391	0.1460	stationary
3	x_3	0.1447	0.1460	stationary
4	x_4	0.1418	0.1460	stationary

The results of this test demonstrate that the data are stationary and the method of this paper can be directly applied.

Step 2. Examine causality between each two sequences using the traditional Granger causality method and obtain a graphical representation of the system structure containing redundancy causality. The null hypothesis is that the causality does not exist; accordingly, we can get the result in Table 2.

As a result, a graphical representation (Figure 6) of the system structure containing redundancy causality is shown based on the above test results in Table 2.

Table 2. Results of two-variable Granger causality test

	causality	F-value	P-value	result at 0.95 confidence level
1	$x_1 \to x_2$	89.51	0.000	tenable
2	$x_2 \to x_1$	0.15	0.698	untenable
3	$x_1 \to x_3$	28.93	0.000	tenable
4	$x_3 \to x_1$	0.61	0.437	untenable
5	$x_1 \to x_4$	534.44	0.000	tenable
6	$x_4 \to x_1$	1.01	0.315	untenable
7	$x_2 \to x_3$	3773.69	0.000	tenable
8	$x_3 \to x_2$	9.95	0.002	tenable
9	$x_2 \to x_4$	0.06	0.815	untenable
10	$x_4 \to x_2$	401.06	0.000	tenable
11	$x_3 \to x_4$	3.67	0.059	untenable
12	$x_4 \to x_3$	230.70	0.000	tenable

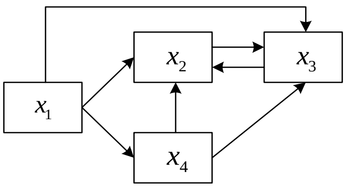


Figure 6. The system structure containing redundancy causality

Step 3. Apply the improved method to find triples and obtain the ultimate graphical representation. It easy to find that (x_2, x_3, x_4), (x_1, x_2, x_3), (x_1, x_2, x_4) and (x_1, x_3, x_4) are four triples in the Figure 6 and they need to be analyzed one by one. Taking the analysis of (x_2, x_3, x_4) as an example, we first calculate the causality statistic $\Delta_{x_3 \to x_2 | x_4}$ which represents x_3 is the cause of x_2 when x_4 is the shared information source.

$$\Delta_{x_3 \to x_2 | x_4} = (132.25 - 131.73) / 131.73 = 3.9 \times 10^{-3},$$

In the equation, 132.25 is the residual sum of squares obtained from the equation of x_2 like the z_t in formula (1) when x_3 and its lag terms are the independent variables, and 131.73 is the residual sum of squares obtained from the equation of x_2 like the z_t in formula (3) when x_3, x_4 and their lag terms are the independent variables. Thus the value of equation (5) can be calculated. The threshold value at the 0.95 confidence level gained by the Monte Carlo method are 0.041, which comes from 500 times stochastic simulation; then we have the fact that $3.9 \times 10^{-3} < 0.041$, therefore the null hypothesis is accepted, which is that the causality from x_3 to x_2 relies on the homologous effect of x_4, so the arrow from

x_3 to x_2 in Figure 6 should be deleted. Next we calculate the causality statistic which represents that x_2 is the cause of x_3 when x_4 is the shared information source.

$$\Delta_{x_2 \to x_3 | x_4} = (119.82 - 20.31) / 20.31 = 4.90 .$$

The threshold value gained by the Monte Carlo method is 0.037, then we have the fact that $4.90 > 0.037$; therefore the null hypothesis is rejected, which is that the causality from x_2 to x_3 is not caused by the homologous effect of x_4, so the arrow from x_3 to x_2 in Figure 6 can not be deleted. Moreover, the causality statistic which represents x_4 is the cause of x_3 when x_2 is the intervening variable is

$$\Delta_{x_4 \to x_3 | x_2} = (20.39 - 20.31) / 20.31 = 3.9 \times 10^{-3} .$$

The threshold value gained by the Monte Carlo method is 0.047, then we have the fact that $3.9 \times 10^{-3} < 0.047$, therefore the null hypothesis is accepted, which is that the causality from x_4 to x_3 is caused by the intervening effect of x_2, so the arrow from x_4 to x_3 in Figure 6 should be deleted.

So far, through the analysis above, causality among three sequences (x_2, x_3, x_4) are consistent with Figure 5. Relationships among the rest of data can be analyzed in the same way and the relations as showed is Figure 5 can be acquired. Therefore, the simulation results indicate that the traditional method can cause two kinds of limitations listed in part 1.1 and the improved method presented in this paper is effective for providing the correct causality among the multiple variables.

5. Application on risk transmission of stock market

The improved method illustrated in this paper is different from the existing Granger causality test method in two aspects. Formally the new method is capable of observing the causality among multiple variables, and substantially it can analyze the structure of causality that tells the process mechanism of formation of causality among variables, so it is applicable to analyze risk transmission. Observation of the transmission mechanism is essential in analysis of stock risk linkage effects because it can obtain the transmission path mechanism.

Firstly, we introduce the sample selection and data preparation. To make our empirical analysis more representatives, we selected China's Shanghai composite index, Japan's nikkei index, Britain's financial times index, and the Dow Jones index of the United States as samples which are the major stock markets in the world. The representativeness lies in that (1) China's Shanghai stock market is one of the biggest emerging stock markets and also an important national stock market; (2) Japan and China both belong to Asia region and their financial markets are closely connected, and Japan's market is the world's second largest stock market and its nikkei index can fully reflect the Japan's stock market situation; (3) Britain and American stock markets are mature and have a long history which can stand for the stock market in Europe, America, and even the whole world; thus, the financial times index and Dow Jones index have strong representativeness respectively. We choose the everyday’s closing indexes of four stock markets during the whole year of 2012.

Note that two points of data processing details need special attention. One is the difference of stock closing time according to different time tone; the other is that different markets have different trade dates; namely, when China’s stock market closes on some day, the US stock market may open on the same day. In the Granger causality test, the data’s orders and mutual correspondences are important and so is the paper's improved model. For the first question, the order of the four nations is Japan, China, Britain, and America according to time zone. In the view of information utilization, the first three nations' stock market information of the day can be made good use of America’s because America's market opens the latest. And the last three nations' stock markets’ information in the day before can be used for Japan because Japan's market opens the earliest. Such dispose is important for arranging lagged item when analyzing the causality between them. For the second question, we settle the data by deleting the date which deals with no trade in all four stock markets, and setting closing price data as the index in the last open day when it is on the date on which the individual market is closed. The dispose implies that the volatility, after logarithm calculation, is zero. In fact it is reasonable because volatility on the day is definitely zero if the market closes; moreover, the deleting date, which deals with no trade in all four stock markets, is precise for analyzing correlationships for their volatilities, because the influences caused by missing data are reduced by the dispose. After the data preparation, we obtain the closing index P_t in the t th day of the four stock indexes as foundations for further measuring the volatility risk.

Secondly, Modeling and measuring of the volatility risk should be prepared as follows. Many of the GARCH models are capable of modeling and measuring the risk of volatility, so this paper intends to choose the threshold ARCH model, which can reflect the asymmetric effects of different information impacts from the GARCH-kind models. The chosen model has been applied by many researchers and is proved to be of good goodness-of-fit ability (Li, 2010; Wang, et al, 2012; Tokmakcioglu, & Tas, 2012; Li, 2014).

This paper considers it appropriate to set the threshold ARCH(1,1) model for the nikkei index and the Shanghai index, and to set the threshold ARCH(2,1) model for financial times index and the Dow Jones index through the comparisons and repeated experiments of both the AIC and the SC rules. The threshold ARCH model consists of a mean equation and a variance equation showed as follows in the formula (13), (14), and (15).

The mean equation is

$$\ln(P_t) = \gamma \ln(P_{t-1}) + \mu_t \,, \tag{13}$$

The variance equation of the threshold ARCH(1,1) model is

$$\sigma_t^2 = \omega + \alpha \cdot \mu_{t-1}^2 + \theta \cdot \mu_{t-1}^2 I_{t-1}^- + \beta \cdot \sigma_{t-1}^2 \,, \tag{14}$$

The variance equation of the threshold ARCH(2,1) model is

$$\sigma_t^2 = \omega + \alpha \cdot \mu_{t-1}^2 + \theta \cdot \mu_{t-1}^2 I_{t-1}^- + \eta \cdot \mu_{t-2}^2 + \beta \cdot \sigma_{t-1}^2 \,. \tag{15}$$

where, I_{t-1}^- is a virtual variable, of $I_{t-1}^- = 1$ if $\mu_{t-1} < 0$, or else, $I_{t-1}^- = 0$. Therefore the term $\theta \cdot \mu_{t-1}^2 I_{t-1}^-$ reflects the asymmetric effects of different information. The σ_t is volatility risk to be calculated, the μ_t is the residual term of the mean equation, and other terms such as γ, ω, α, θ, η and β are the parameters to be determined. By fitting the model in Eviews 5.1, we get the results in Table 3.

Table 3. Results of threshold ARCH model

		Japan's nikkei index	Shanghai composite index	Britain's financial times index	Dow Jones index
1	γ	0.999** (12973)	0.999** (11067)	0.999** (13100)	1.000** (16650)
2	ω	4.51E-5** (3.288)	2.37E-5 (1.303)	1.04E-6** (2.696)	4.60E-6** (2.537)
3	α	-0.126** (-1.707)	-0.077** (-2.236)	-0.097 (-1.415)	-0.102** (-3.203)
4	θ	0.565** (5.056)	0.073* (1.605)	0.332** (3.895)	0.202** (3.046)
5	η	—	—	0.119** (2.191)	0.132** (2.781)
6	β	0.565** (5.921)	0.851** (6.214)	0.765** (51.672)	0.830** (17.962)
7	R^2	0.966	0.984	0.945	0.902

As seen in Table 3, The value in parenthesis are t statistics, ** means significance at the 0.95 confidence level, * means significance at the 0.99 confidence level. It can be found that the threshold ARCH model has a good performance in describing volatility risks of the four stocks. All parameters pass the test at the 0.95 confidence level except only two of them, and the goodness-of-fit are more than 0.90 for all the four models. Thus an important foundation has been laid for further descriptions of the transmission of volatility risk. The volatility risk σ_t s of the four indexes by the models is shown in Figure 7 to Figure

10. The volatility risks, as shown in the four figures (Figure 7 to Figure 10), are of no obvious unit root process, which is the necessary condition of the stationary time series.

Lastly, Before proceeding analysis of risk transmission of the four stock indexes according to these steps given in section 3. Firstly we conduct the stationary test of volatility risk σ_t based on the above results.

In Table 4, ** means significant at the 0.95 confidence level. From the results, we can see that the four sequences of volatility risks are all stationary at the 0.95 confidence level by the test of three common methods as above. Thus, we can infer that the four sequences are stationary with sufficient evidences, which is the premise of traditional and the improved Granger causality test.

Table 4. Stationary test of volatility risk of the four stock indexes

	Test methods	Japan's nikkei index	Shanghai composite index	Britain's financial times index	Dow Jones index
1	ADF	-7.457**	-5.304**	-4.374**	-3.278**
2	PP	-6.816**	-5.348**	-3.864**	-2.879**
3	KPSS	0.136**	0.273**	0.656**	0.644**

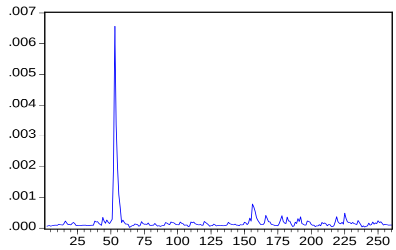


Figure 7. Volatility risk of Nikkei Index

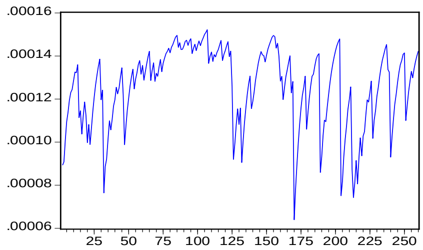


Figure 8. Volatility risk of Shanghai Stock Index

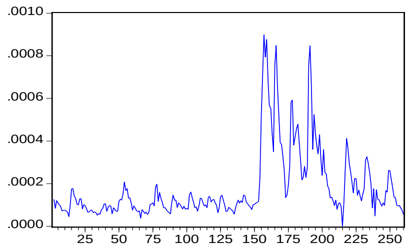


Figure 9. Volatility risk of Financial Times Index

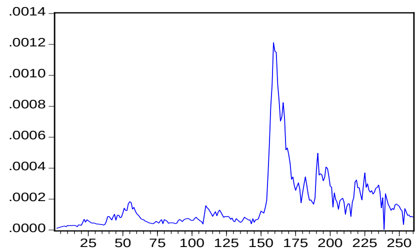


Figure 10. Volatility risk of Dow Jones Index

In line with step 2 presented in section 3, we apply the traditional Granger causality test method to conduct a causality test between each two variables firstly, and from there orders of the lagged items are selected on the basis of many indicators such as LR，FPE，AIC，SC and HQ. We pick the order of the lagged items that contain the most optimal indexes. Then we gain the results of the causality test between each two variables.

In Table 5, g1, g2, g3, g4 represent in order for risk volatility sequences of the nikkei index, the shanghai index, Britain's financial times index and the Dow Jones index. According to Table 5, the volatility risk transmission relations generated by the traditional Granger causality test are summarized in Figure 11. As Figure 11 shows, the causality may fail to show the actual transmission path of risk because the causal structures shown in Figure 2 and Figure 3 appear. So we apply the improved method introduced by this paper to compute the statistic $\Delta_{g3\to g2|g4}$. Two fittings still need to be done.

Table 5. The results of traditional Granger causality test

	Causality expressions	order of lagged items	P-value	results
1	g1 is not the cause of g2	1	0.890	accepted
2	g2 is not the cause of g1		0.890	accepted
3	g1 is not the cause of g3	2	0.372	accepted
4	g3 is not the cause of g1		0.332	accepted
5	g1 is not the cause of g4	3	0.308	accepted
6	g4 is not the cause of g1		0.411	accepted
7	g2 is not the cause of g3	5	0.138	accepted
8	g3 is not the cause of g2		0.002	rejected
9	g2 is not the cause of g4	3	0.386	accepted
10	g4 is not the cause of g2		0.004	rejected
11	g3 is not the cause of g4	5	0.087	rejected
12	g4 is not the cause of g3		3.7E-6	rejected

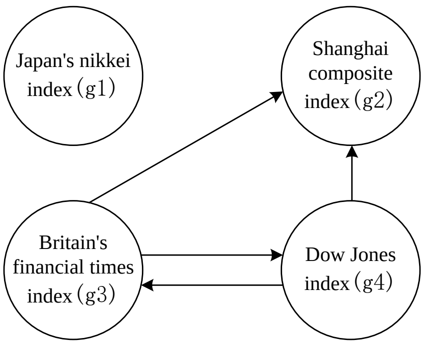


Figure 11. Volatility risk transmission relations from the traditional method

Firstly, we build the time series equation treating g2 as dependent variable and lagged items of g2 and g4 as independent variables, and from there the orders of lagged items are determined by using AIC and SC rules. The fitting result is as follows:

$$g2_t = 2.79\times10^{-5} + 0.787g2_{t-1} + 0.035g4_{t-1} - 0.040g4_{t-3}$$
$$(5.895^{**}) \quad (21.587^{**}) \quad (3.055^{**}) \quad (-3.541^{**})$$

whose residual sum of squares Σ^J_{g2g2} is 2.74×10^{-8}, the value in parenthesis are t-statistics from which we can find coefficients are significant at the 0.95 confidence level of the t-test.

Secondly, we continue to build the other time series equation treating g2 as a dependent variable, and lagged items of g2, g3 and g4 as independent variables, and from there the orders of lagged items are determined by using AIC and SC rules. The fitting result is as follows:

$$g2_t = 2.79\times10^{-5} + 0.787\cdot g2_{t-1} - 0.056g3_{t-2} + 0.054g3_{t-3} + 0.035\cdot g4_{t-1} - 0.040\cdot g4_{t-3}$$
$$(5.476^{**}) \quad (21.142^{**}) \quad (-2.448^{**}) \quad (2.211^{**}) \quad (3.917^{**}) \quad (-3.668^{**})$$

whose residual sum of squares is Σ_{g2g2} is 2.67×10^{-8}, The value in parenthesis are t statistics from which we can find coefficients are significant at the 0.95 confidence level of the t-test. By applying equation (11), we get

$$\Delta_{g3\to g2|g4} = (\Sigma^J_{g2g2} - \Sigma_{g2g2}) / \Sigma_{g2g2} = 0.026.$$

After running 500 times a Monte Carlo simulation, we found that the threshold value of the value is 0.037 at the 0.95 confidence level. And the fact 0.026<0.037 makes the null hypothesis accepted, which means the causality from g3 to g2 is caused by the intervening effect of g4 , so the arrow from g3 to g2 in Figure 8 can be deleted. Likewise, we get

$$\Delta_{g4\to g2|g3} = (2.79\times10^{-8} - 2.67\times10^{-8}) / 2.67\times10^{-8} = 0.045.$$

After 500 times Monte Carlo simulation, we noticed that the critical value of the value is 0.033 at the 0.95 confidence level. And the fact 0.045>0.033 makes the null hypothesis rejected, which means the causality from g4 to g2 is not caused by the homologous effect or the intervening effect of g3 , so the arrow from g4 to g2 in Figure 11 cannot be deleted. Based on the above results comprehensively, we delete indirect causality and get the transmission paths in Figure 12.

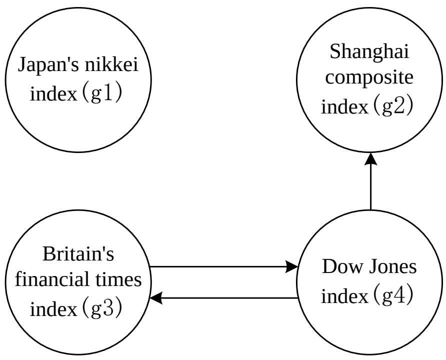


Figure 12. Volatility risk transmission relations from the improved method

It can be seen, from the comparison between Figure11 and Figure 12, the correct transmission path is $g_3 \to g_4 \to g_2$, and g3 has mutual transmission with g4, but we can not clearly find the above transmission path in Figure 10 because that many paths exist in that figure. Thus, the analysis and the above graphs illustrate the merits of the improved method.

6. Conclusions

This paper introduces a new method for performing a causality test of multiple variables on the basis of improving the traditional two-variable Granger causality test method, validates the correctness of the improved method by one typical simulation analysis, and applies the new method in analysis of stock risk transmission and conducts the comparison between the two methods. Theoretical analysis, simulation analysis and empirical research show that the improved method has such features as follows:

(1) It is proper for short-time dependent and stationary time series data;

(2) It is objective by using the statistical inference to set the threshold value by means of Monte Carlo simulation;

(3) It gets rid of indirect causality caused by intervening variables and pseudo-cause problems caused by the homologous data;

(4) It shows precise transmission path clearly beyond the existing two-variable Granger causality test method;

(5) It proves to be practical and effective in finding the proper network structure and uncovering the stock risk transmission in the real-world stock markets

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