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IDEAL AND ANTI-IDEAL COMPROMISE PROGRAMMING FOR ROBUST Bi-OBJECTIVE PORTFOLIO SELECTION PROBLEM

Abstract: *This paper proposes a robust optimization approach for bi-objective portfolio selection problem. We propose mean-CVaR as a bi-objective model and in this model we consider parameter uncertainty. We use Bertsimas and sim approach to consider uncertainty in the model and we try to use ideal and anti-ideal compromise programming to solve model. This solving approach is better than compromising and goal programming approach for portfolio selection problem because this approach tries to increase the distance of the solution and anti-ideal criteria that in our model is CVaR and on the other hand tries to decrease the distance between expected rate of return of portfolio and the solution. Efficiency of this model is tested by real life data.*

Keywords: *Portfolio Selection, Robust Optimization, Bi-Objective, Mean-CVaR, Ideal and Anti-Ideal Compromise Programming*

JEL Classification: C61, G11

1. Introduction

Portfolio selection problem is one of the basic issues in finance. Decision maker in portfolio selection problem try to maximize return and minimize risk of portfolio. Markowitz (Markowitz, 1952) proposed the first portfolio selection problem. In the original Markowitz model assume that the return follow from normal distribution and risk measured by variance.

The original Markowitz model is a quadratic programming but in real life data because of the size of variance-covariance matrix is unpractical.
There are many researches that try to develop portfolio selection problem such as (Chiodi et al., 2003; Kellerer et al., 2000; Konno and Yamazaki, 1991; Laniado et al., 2012; Mansini et al., 2003; Michalowski and Ogryczak, 2001; Papahristodoulou and Dotzauer, 2004; Rockafellar and Uryasev, 2000; Xie et al., 2008).
In 90's decade new risk measure has been introduced. This risk measure is called value at risk (VaR). But VaR has some bad mathematical fecures. In order to overcome these features Rockafellar and Uryasev proposed a new risk measure which is derived from VaR and this new risk measure is called Conditional Value at Risk (CVaR). In this paper we use CVaR as a risk measure(Rockafellar and Uryasev, 2000).
In portfolio selection modeling there are some papers that consider multiple criteria to modeling such as (Alexander and Resnick, 1985; Azmi and Tamiz, 2010; Ghahtarani and Najafi, 2013a; Gupta et al., 2013; Kumar and Philippatos, 1979; Wu et al., 2007). In this paper we use bi-objective programming to model portfolio selection problem.
In some mathematics models, there are some coefficients subject to uncertainty. In recent years, researchers try to develop a new approach to consider uncertainty in mathematical models.
This new approach is called robust optimization. The first step in this approach was taken by Soyster (Soyster, 1973) but this approach was too conservative, it means this approach ignores optimality instead of feasibility. Another step in this concept was taken by Ben-tal and Nemirovski (Ben-Tal and Nemirovski, 2000). They proposed an approach that can control the conservatism of the solutions but their model is nonlinear,. The final step in robust optimization was taken by Bertsimas and Sim (Bertsimas and Sim, 2004). This new approach is linear programming and also decision maker can control the conservatism of the solutions. There are some practical robust optimization models in portfolio selection problem such as: (Chen and Tan, 2009; El Ghaoui et al., 1998; Ghahtarani and Najafi, 2013b; Kawas and Thiele, 2011; Ling and Xu, 2012; Moon and Yao, 2011; Pınar and Burak Paç, 2014; Quaranta and Zaffaroni, 2008; Tütüncü and Koenig, 2004).
Quaranta and Zaffarani (Quaranta and Zaffaroni, 2008) developed a robust optimization in portfolio selection problem by used of Ben-tal and Nemirowski approach. Risk measure in their model is CVaR and their model is single objective, But in this paper we use Bertsimas and Sim approach in a multi-objective model. We apply CVaR as risk measure in our model. Furthermore, we use ideal and anti-ideal compromise programming to solve model.
The first contribution of this paper is developing a bi-objective portfolio selection problem. This model has two criteria that these criteria have conflict together, the second contribution of this paper is developing robust optimization to consider parameters uncertainty. Parameter uncertainty is one of the basic features of

financial market and without considering it, the financial model aren't adapted to the real word. The last contribution of this paper is a solving approach; this solving approach is adapted to portfolio selection problem better than other solving approach. This approach tries to increase the distance between the solution and anti-ideal criteria and decrease the distance between solution and ideal criteria. In portfolio selection problem risk is anti-ideal criteria that DMs try to increase the distance from it and return is ideal criteria that DMs try to decrease the distance from it.

The structure of this paper is as follow, next selection explain CVaR and a multi-objective model for portfolio selection problem. Third section proposes robust optimization for multi-objective portfolio selection problem that proposed in section two. Forth section proposes ideal and anti-ideal compromise programming to solve the model and in fifth section the results of historical data are discussed. In final section, the conclusions are proposed.

2. Bi-Objective (mean-CVaR) Programming for Portfolio Section Problem

This section proposes a Multi-objective model for portfolio selection problem. Portfolio selection problem is a basic problem in finance where decision makers want to invest their money in financial asset.

In this paper, we use CvaR as risk, at first we define VaR and CvaR.

Definition 1.1 (Value at Risk (VaR)). Let K be a random variable and let F be its distribution function, that is $F(h) = P\{k \leq h\}$.

Let $F(w) = \min\{h : F(h) \geq w\}$ be it's $\alpha - \mathrm{VaR}$ the $\alpha -$quantile of k :

$$\alpha - \mathrm{VaR}(k) = F^{-1}(\alpha) \tag{1}$$

*Definition 1.2 (*Conditional Value at Risk (CVaR)). Let $x \in X \subset R^N$ be a decision vector representing a portfolio, $y \in T \subset R^N$ be a vector representing the future value of a number of variables.

For each x that denote by $\psi(x,0)$ the distribution function of the loss $Z = f(x, y)$ i.e .

$$\psi(x,\alpha) = P\{y \mid f(x, y) \leq \alpha\} \tag{2}$$

For given $\alpha > 0$, the $\alpha - \mathrm{CVaR}$ of loss associated with x is the mean of the $\alpha - \mathrm{tail}$ distribution of the loss function, that is the mean of distribution function $\psi_\alpha(x,0)$ defined by:

$$\psi_\alpha(x,a) = \begin{cases} 0 \quad \text{if} \quad a < a_\alpha(x) \\ \dfrac{\psi(x,a)-\alpha}{1-\alpha} \quad \text{if} \quad a \geq a_\alpha(x) \end{cases} \tag{3}$$

and $a_\alpha(x)$ is the $\alpha - \mathrm{VaR}$ of the loss associated with X.

The model of min $\mathrm{CVaR}_\alpha(x)$ based on Rockafellar and Uryasev is as follow (Rockafellar and Uryasev, 2000):

$$\min a + \frac{1}{m(1-\alpha)} \sum_{k=1}^{m} U_k \tag{4}$$

S.t

$$X_r^T + a + U_k \geq 0 \tag{5}$$

$$U_k \geq 0 \qquad \forall k = 1,\dots,m \tag{6}$$

$$\sum_{i=1}^{m} x_i = 1 \tag{7}$$

$$0 \leq x_i \leq 1 \qquad \forall i = 1,\dots,m \tag{8}$$

The model mentioned above is CVaR that we use it for risk measure.
The bi-objective portfolio selection problem is as follow:

$$\max \sum_{i=1}^{n} X_i r_i \tag{9}$$

$$\min CVaR \tag{10}$$

S.t

$$\sum_{i=1}^{m} x_i = 1 \tag{11}$$

$$0 \leq x_i \leq 1 \qquad \forall i = 1,\dots,m \tag{12}$$

The above formulation rewrite as follow:

$$\max \sum_{i=1}^{n} X_i r_i \tag{13}$$

$$\min a + \frac{1}{m(1-\alpha)} \sum_{k=1}^{m} u_k \tag{14}$$

S.t

$$\sum_{i=1}^{n} X_i r_{it} + a + u_k \geq 0 \tag{15}$$

$$u_k \geq 0 \qquad \forall k = 1,\ldots,m \tag{16}$$

$$\sum_{i=1}^{m} x_i = 1 \tag{17}$$

$$0 \leq x_i \leq 1 \qquad \forall i = 1,\ldots,m \tag{18}$$

Above formulation is a bi-objective portfolio selection problem that considers both return and risk as criteria.

3. Robust Optimization for Mean-CVaR Portfolio Selection Problem

This section proposes a robust optimization for portfolio selection problem. We use Bertsimas and Sim approach to deal robust optimization.

In this model r_i is subject to uncertainty. This parameter is cited in objective function. At first we change that objective function (objective function with r_i) to a constraint.

The model reformulate as follow:

$$\max w \tag{19}$$

$$\min a + \frac{1}{m(1-\alpha)} \sum_{k=1}^{m} u_k \tag{20}$$

S.t

$$\sum_{i=1}^{n} X_i r_{it} + a + u_k \geq 0 \tag{21}$$

$$\sum_{i=1}^{m} r_i x_i \geq w \tag{22}$$

$$u_k \geq 0 \qquad \forall k = 1,\ldots,m \tag{23}$$

$$\sum_{i=1}^{m} x_i = 1 \tag{24}$$

$$0 \leq x_i \leq 1 \qquad \forall i = 1,\ldots,m \tag{25}$$

To deal with Bertsimas and Sim approach we have to change the uncertain constraint as follow (Bertsimas and Sim, 2004) :

$$-\sum_{i=1}^{m} r_i x_i \leq -w \tag{26}$$

In above formulation, $\mathrm{r_i}$ is uncertain. Let j be the set of coefficient $\mathrm{r_i}$. $\tilde{r}_j$ $i \in J$ take values according to a c distribution with a mean $\mathrm{r_i}$ in the intervals $\left[\mathrm{r_i} - \hat{\mathrm{r}}_\mathrm{i}, \mathrm{r_i} + \hat{\mathrm{r}}_\mathrm{i}\right]$. We introduce a parameter Γ that is called price of robustness. Γ Takes values in the interval $[0,|J|]$.

Based on Bertsimas and Sim, formulation is as follow (Bertsimas and Sim, 2004):

$$\max w \tag{27}$$

$$\min a + \frac{1}{m(1-\alpha)} \sum_{k=1}^{m} u_k \tag{28}$$

S.t

$$-\sum_{i=1}^{m} r_i x_i + \max_{\{S \cup \{t\} | S \subseteq J, |S| = \lfloor \Gamma \rfloor, t \in J/S\}} \left\{ \sum_{i \in S} \hat{r}_i y_i + (\Gamma - \lfloor \Gamma \rfloor) \hat{r}_t y_t \right\} \leq -w \tag{29}$$

$$\mathrm{u_k} \geq 0 \tag{30}$$

$$\sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{x_i} = 1 \tag{31}$$

$$0 \leq \mathrm{x_i} \leq 1 \tag{32}$$

$$-\mathrm{y_i} \leq \mathrm{x_i} \leq \mathrm{y_i} \tag{33}$$

If Γ is integer then:

$$B(x, \Gamma) = \max_{\{S | S \subseteq J, |S| = \Gamma\}} \left\{ \sum_{i \in S} \hat{r}_i |x_i| \right\} \tag{34}$$

To reformulated (27) to (33) as a linear programming, we need the following proposition. $X*$ is optional value:

$$B(x^*, \Gamma) = \max_{\{S \cup \{t\} | S \subseteq J, |S| = \lfloor \Gamma \rfloor, t \in J/S\}} \left\{ \sum_{i \in S} \hat{r}_i |x^*_i| + (\Gamma - \lfloor \Gamma \rfloor) \hat{r}_t |x_i^*| \right\} \tag{35}$$

Above constraint is equal to the objective function of following models:

$$B(x^*, \Gamma) = \max \sum_{i \in J} \hat{r}_i |x_i^*| z_i \tag{36}$$

$$\sum_{i \in J} Z_i \leq \Gamma \tag{37}$$

$$0 \leq Z_i \leq 1 \qquad \forall i \in J \tag{38}$$

The dual problems (36) to (38) are as follow:

$$\min \sum_{i \in J} P_i + \Gamma z \tag{39}$$

$$Z + P_i \geq \hat{r}_i \mid X_i^* \mid \qquad \forall i \in J \tag{40}$$

$$P_i \geq 0 \qquad \forall i \in J \tag{41}$$

$$Z \geq 0 \tag{42}$$

By strong duality, it is provable that (36) to (38) are feasible and bounded, because (39) to (42) are feasible and bounded for all $\Gamma \in [0, |J|]$. The robust optimization of this model is as follow:

$$\max w \tag{43}$$

$$\min a + \frac{1}{m(1-\alpha)} \sum u_k \tag{44}$$

S.t

$$-\sum_{i=1}^{m} r_i x_i + z\Gamma + \sum_{i=1}^{m} P_i \leq -w \tag{45}$$

$$\sum_{i=1}^{m} r_{it} x_i + a + u_k \geq 0 \tag{46}$$

$$\mathrm{u_k} \geq 0 \qquad \forall \mathrm{k} = 1, \ldots, \mathrm{m} \tag{47}$$

$$Z + P_i \geq \hat{r}_i y_i \tag{48}$$

$$\sum_{i=1}^{n} X_i = 1 \tag{49}$$

$$-\mathrm{y_i} \leq \mathrm{x_i} \leq \mathrm{y_i} \tag{50}$$

$$P_i \geq 0 \qquad y_i \geq 0 \qquad Z \geq 0 \tag{51}$$

$$0 \leq \mathrm{x_i} \leq 1 \tag{52}$$

Above formulation is a robust optimization model that considers parameter uncertainty. The contribution of this model is the use of robust optimization in a bi-objective model with CVaR as risk measure. In the next section we propose a solving approach as another contribution in this paper.

4. Ideal and Anti-Ideal Compromise Programming for Mean-CVaR Portfolio Selection Problem

In this section we propose a L-P metrics approach to deal with robust bi-objective portfolio selection problem. In this paper we use ideal and anti-ideal compromise programming to solve model. This model has two objective functions, mean rate of return and risk measure, that this two objectives are conflict. Triantaphyllou introduced utopia approach for multi-objective problem(Triantaphyllou, 2000). This approach is a L-P metrics that try to minimize the distance between utopia and the solution of the model. In this approach for every objective function the problem solved and $f_{i(x)}$ is the result.

$$\min/\max f_{j(x)} \tag{53}$$

S.t

$$g_{i(x)} \leq 0 \tag{54}$$

$$x \in E^{n} \tag{55}$$

The optimal solution is $\{f_{(x^{*1})},\ldots f_{(x^{*k})}\}$ and x^{*j} is the set of x that can optimize $f_j(x^{*j})$ if $x^{*1} = x^{*2} = \ldots = x^{*k}$ then the ideal solution is the optimal solution but this situation usually not happen, so the DM try to minimize the distance between ideal solution and the solution of the model as follow:

$$L-P=\{\sum_{j=1}^{k}\gamma_j[f_j(x^{*j})-f_j(x)]^p\}^{\frac{1}{p}} \tag{56}$$

γ_j is the weight of each objective function. Zelany proposed the anti-ideal approach (Zelany, 1974). In this approach the DM try to increase the distance between anti-ideal criteria and the solution of the model.in this approach for each objective function the model solved and if the general objective function is max (min) the single objective problem change based on min (max). If the solution of this models are $\{f_1(x^1),\ldots, f_k(x^k)\}$ then the DM try to increase the distance between anti-ideal solution and the solution of the model as follow:

$$L-P=\{\sum_{j=1}^{k}\gamma_j[f_j(x)-f_j(x^j)]^p\}^{\frac{1}{p}} \tag{57}$$

The model of this paper has two objective functions. The first is mean rate of return that is ideal and we can use utopia approach to consider it and the risk measure (CVaR) that is anti-ideal. We can combine these two approaches to solve model.

At first in order to find ideal and anti-ideal we should solve two single objective problems as follow:

$$\max w = f_1(x^*) \tag{58}$$

S.t

$$-\sum_{i=1}^{m} r_i x_i + z\Gamma + \sum_{i=1}^{m} P_i \le -w \tag{59}$$

$$Z + P_i \ge \hat{r}_i y_i \tag{60}$$

$$\sum_{i=1}^{n} X_i = 1 \tag{61}$$

$$-\mathrm{y_i} \le \mathrm{x_i} \le \mathrm{y_i} \tag{62}$$

$$0 \le \mathrm{x_i} \le 1 \tag{63}$$

$$\min a + \frac{1}{m(1-\alpha)} \sum u_k = f_2(x) \tag{64}$$

S.t

$$\sum_{i=1}^{m} r_{it} x_i + a + u_k \ge 0 \tag{65}$$

$$\sum_{i=1}^{n} X_i = 1 \tag{66}$$

$$\mathrm{u_k} \ge 0 \qquad \forall \mathrm{k} = 1,\ldots,\mathrm{m} \tag{67}$$

$$0 \le \mathrm{x_i} \le 1 \tag{68}$$

Then we should solve the following model:

$$\min \gamma_1 [f_1(x^*) - f_1(x)]^p + \gamma_2 [f_2(x) - f_2(x)]^p \tag{69}$$

S.t

$$-\sum_{i=1}^{m} r_i x_i + z\Gamma + \sum_{i=1}^{m} P_i \le -w \tag{70}$$

$$Z + P_i \geq \hat{r}_i y_i \tag{71}$$

$$\sum_{i=1}^{n} X_i = 1 \tag{72}$$

$$\sum_{i=1}^{m} r_{it} x_i + a + u_k \geq 0 \tag{73}$$

$$u_k \geq 0 \qquad \forall k = 1,...,m \tag{74}$$

$$0 \leq x_i \leq 1 \tag{75}$$

Above formulation is based on the ideal and anti-ideal compromise programming that consider different objective function and consider uncertainty based on robust optimization.

5. Numerical Results

This section illustrates the numerical result for robust ideal and anti-ideal compromise programming. We use New York stock exchange data to deal with the model. The data extracted from 1/10/2012 to 1/10/2013 for one year. The data is monthly for 12 periods and 15 stocks. Solving approach in this paper consist of 3 models. The first model consider on rate of return without other criteria, in this model rate of return is subject to uncertainty that we consider this uncertainty with robust optimization approach. This model runs for different price of robustness. The result of this model is as follow:

Table 1: The result of first model

Price of robustness	Objective function
0	0.09504300
0.1	0.09314210
0.2	0.09124120
0.3	0.08934030
0.4	0.08743940
0.5	0.08553850
0.6	0.08363760
0.7	0.08173670
0.8	0.07983580
0.9	0.07793490
1	0.07603400

The data of table 1 use in ideal section of our solving approach. This result is refer to $f_1(x^*)$. As shown in table 1by increase of price of robustness the result gone worse that it is a predictable result in robust approach. Price of robustness can vary to 15 but in this case the result after 1 is equal, the cause of this result is that after $\Gamma = 1$ the robust approach doesn't allow to parameter to volatile because the volatility after this quantity may result infeasibility, but robust optimization approach wouldn't let the infeasibility to occurred. But in other data for this formulation may this situation not happen.

In second step, the DM should run the second model to calculate the anti-ideal criteria. Then the DM can run the essential model. γ_i and p in this model are equal to one. The results of essential model are as follow:

Table 2: Results of the model

Γ	Objective function	Portfolio rate of return	CVaR
0	0.02353960	0.08570698	0.2491090
0.1	0.02310736	0.08423832	0.2491090
0.2	0.02267512	0.08276966	0.2491090
0.3	0.02222768	0.07391269	0.1192627
0.4	0.02145953	0.07277993	0.1192627
0.5	0.02069139	0.07164717	0.1192627
0.6	0.01992325	0.07051442	0.1192627
0.7	0.01915510	0.06938166	0.1192627
0.8	0.01838696	0.06824890	0.1192627
0.9	0.01761882	0.06711615	0.1192627
1	0.01685067	0.06598339	0.1192627

The above table shows the result of essential ideal and anti-ideal compromise programming with robust approach to consider uncertainty. The different results for portfolio rate of return and objective function is because of parameter uncertainty. By increase of price of robustness the portfolio rate of return gone worse that is because of uncertainty. The above results are optimal values under worst uncertainty parameters.

We summarize the results of the model in two charts; the first chart shows the variation of objective function against price of robustness and the second chart shows the variation of portfolio rate of return against price of robustness. Bothe chart illustrate that by increase of price of robustness the model gone conservative and the result gone worse.

The objective function in this model gone well but these phenomena means by increase of price of robustness the distance between criteria in first models and final model are decrease. This improvement in objective function at first glance is against of the fundamental of robust optimization. Because in robust optimization by increase of price of robustness the objective function gone worse. In this model objective function is the distance between criteria and solutions. Since the rate of decrease of portfolio rate of return in first model is more than final model so the objective function gone well but in reality rate of return of portfolio gone worse.

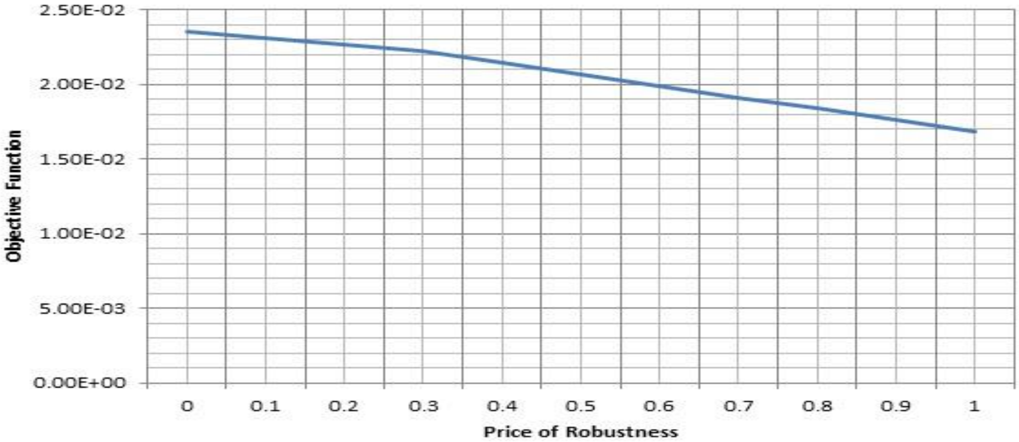

Chart 1: Objective function vs. price of robustness

The chart number one shows that the objective function decreases by increase of price of robustness.

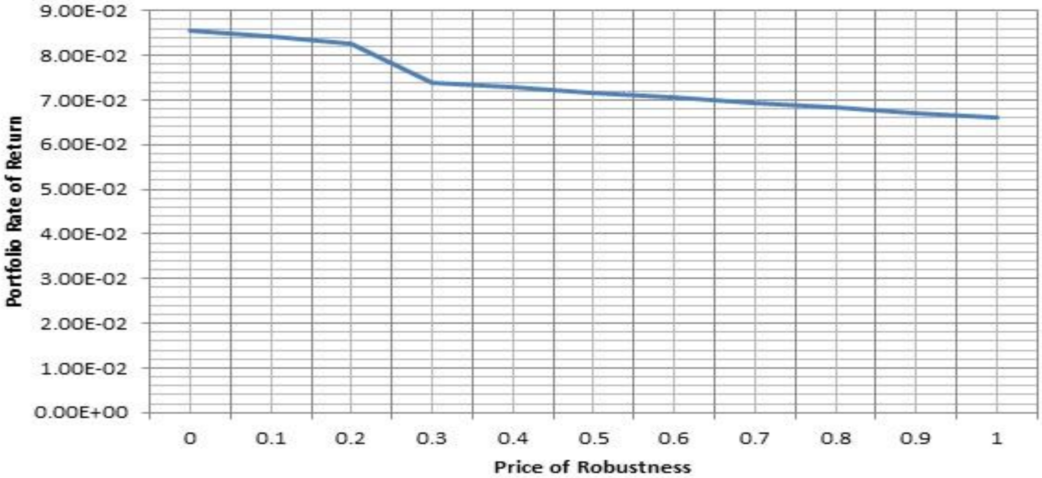

Chart 2: Portfolio rate of return vs. price of robustness

The above chart shows the decrease of portfolio rate of return by increase of price of robustness. These two charts show that the uncertainty of parameters can change the result of portfolio, and the DM should consider the uncertainty of parameters in modeling. Without considering uncertainty the model has less adaption with real word because the uncertainty of parameters are one of the most important features in financial markets.

6. Conclusions

This paper proposes a robust optimization approach for a bi-objective portfolio selection problem. This paper uses Bertsimas and Sim approach to consider uncertainty parameters. The solving approach that use in this paper is ideal and anti-ideal compromising programming that try to decrease the distance between ideal criteria and solution of model and increase the distance between anti-ideal criteria (risk measure) and solution of model. This model shows that the conservatism of the solution increases when the price of robustness increases.

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