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FEEDBACK INFLUENCE ON THE BEHAVIOR OF FINANCIAL SYSTEM

Abstract: *When modeling difficult economical problems, we are very often challenged by the fact that relations between different values are fluid over time. This article deals with formulation of dynamical financial system model, and analyzing the impact of changing one of the parameters on the whole system. Our goal is to create dynamical financial system model that would express real economic situation and would respect even impact of history of factors considered. One such possibility how to properly express the dynamics of processes in the model is to describe the dynamic model using differential equations with delayed argument. Created financial system model is expressed as system of three delayed differential equations. The article presents graphical solution of the model, on which the influence of feedback on its behavior is tracked. Original, classical model is thus supplanted with new, that is closer to real economic situation, due to respecting the influence of history, using the so-called "Delayed" differential equations.*

Keywords: *financial model, differential equations with delay, interest rate, investment demand, price index.*

JEL Classification: C02, C69

1 Introduction

It is a natural trend in theoretical basis of scientific disciplines to study various model situations using increasingly precise models which, among other things, enable us to analyze more accurately the simulated processes, to search for more exact meaning of the circumstances under which they run, and to derive practical conclusions, bases, optimum solutions, etc. from the findings. Measurement of economic quantities then corresponds to use of quantitative methods, i.e. methods based on findings of mathematical disciplines.

One way to give a true picture of the dynamics of processes in a model is to describe dynamical models using differential equations. In that case time must be perceived as a continuous quantity. Transition to continuous quantities allows us to use a sophisticated mathematical apparatus of differential calculus and integral calculus and the outcome is not an estimate of parameters of a pre-defined function type but a function itself whose shape demonstrates the character of examined quantities.

For several decades research into differential equation has been helping us to understand and deal with practical problems in a range of scientific fields. The past decades have seen increasingly often complex practical problems which cannot be adequately described by ordinary differential equations. They are primarily models whose solution leads to so-called differential equations with delay or, in general, differential equations with deviating argument. The aim of this article is to present a model of a financial system which analyses the impact of a change in input parameter on the development of interest rate, the investment demand and the price index curve by the use of analytic and synthetic methods, dynamical modeling and solving the system of delay differential equations. The new model of financial system allows for the influence of previous periods, thus leading to the system of differential equations with delay.

2 Differential equations with delay

2.1 Mathematical problem formulation

Among frequently applied mathematical models in economics are differential equations and their systems, the solution of which, if meeting certain conditions, can model the behaviour of economic attributes in time *t*. This covers, in general, problems regarding solvability, characteristics and solution of systems of non-linear ordinary differential equations,

(1)

meeting so-called boundary value conditions

(2)

where functions $f_1, \ldots, f_n$ a so-called functionals $h_1, \ldots, h_n$ meet well-founded requirements in the theory of such problems. At the same time, assumptions concerning the given functions and functionals determine the attributes of the problem's solution (1), (2) ranging from „classic " (all components are continuous and continuously differentiable functions of independent variable *t*), through the first half of the 20th century's so-called Carathéodory's solution (components of the solution are continuous but its derivation do not have to necessarily exist in the "zero level set"), to a so-called generalized solution whose components can be only partly Carathéodory’s ones (examined since the second half of the 20th century).

To clarify – the graph of the classic solution is a „smooth“ curve (a tangent can be drawn in each point of the curve), the graph of Carathéodory’s solutions is a continuous curve with points where a tangent cannot be drawn (points, tips) and the graph of the generalized solution is a curve made up of “broken” curves of the previous types.

As the theory of differential equations claims, every higher-order differential equation (system) can be represented by an equivalent system (1) and boundary value conditions for a specific solution in the form (2). Studying problems for higher-order differential equations (systems) in the original form may lead to „easier “ and „more detailed“ solution’s description. Problems (1), (2) will be considered for our purposes.

It should also not be forgotten that specific examples of system (1) are linear systems

(3)

and specific examples of boundary value conditions (2) are initial conditions

(4)

or 2-point linear

(5)

the specific example of which are both initial conditions (for), and so-called periodic conditions (for .

Mathematical models used in order to describe economic processes have long used and still extensively use the „classic“ model of a problem (1), (2). As new methods of „Carathéodory’s theory“ of differential equations developed and as the general theory of so-called functional differential equations emerged and developed, new mathematical models taking into account even the real economic relationships have been emerging since the second half of the 20th century.

Let’s focus now on specific cases, practically applied and proven in economic practice, of systems of functional differential equations, on so-called systems of differential equations with constant delay (also with more and non-constant delay or, in general, with „deviating “ arguments).

(6)

The specific example of which are so-called linear systems of differential equations with constant delay

(7)

possibly on systems comprising both solution's components with delay and without delay. Given the necessity to know the solution in this situation "before" current time *t*, the boundary value conditions of solution's behaviour must be complemented by relevant data on the solution's „history", for for example in the form:

(8)

where functions are continuous in the given interval.

The available literature on solvability of systems of differential equations with delay arguments and corresponding problems contains a whole range of findings applicable in economic practice. Solvability conditions (i.e. existence and uniquenesse) of given problems, both general and specific, are studied in detail as well as correctness conditions (i.e. small dependence of a solution on "slight" changes in initial conditions and parameters required for a numerical solution), conditions of non-negativeness of a solution, etc., including description of the method of a solution.

2.2 Differential equations with delay in economy

Study of various model situations, focus on simulating conditions, searching for outcomes, optimal solutions and so on are among the most important current trends. For example, McNelis (2003) in his paper applies neural network methodology to inflation forecasting in the Euro-area and the USA. The paper Ioana *et al*. (2010) presents a new concept for Fuzzy Logic in economic processes. In his paper Zhang (2012), investigated the sensitivity of estimated technical efficiency scores from different methods including stochastic distance function frontier. In related literature Boucekkine *et al*. (2004) also studied the two-stage optimal control problem involving two deterministic AK models, and Harada(2010) examined the switch from the Solow to AK economies using a similar technique. Many studies have been conducted in order to validate the hysteresis hypothesis, e.g. Alonzo (2011).

Particular specific sets of tasks concerning differential equations with dynamic argument were being solved by L. Euler and M. Kondors, but their systematic study only began in the 20th century.

A comprehensive and complex analysis of dynamic argument equations was made by attendants of the Permu seminar Azbelev (2001, 2003), the results were then systematized in monographs Azbelev *et al.* (1991, 2002).

In their monograph, Kobrinskij and Kuzmin (1981) pointed out the necessity of using variables of historic type in dynamic economic models that have impact on system development and leads to major changes in the character of the entire process. In Simonovs studies (2002, 2003) has modified existing micro and macro economical models, such as Walras-Evans-Samuelson (WEC) model with regard to delay between offer and demand, Allen's model on the single-commodity market, with regard to delay of deliveries and dependence of demand and offer on the price and speed of price changes Alen, Vidal-Wolf's model of single-product sale Dychta and Camsonjuk (2003) etc.

3 Dynamical model of financial system

3.1 Financial system modeling

It is very important to understand how private investments react to changes in fiscal politics. Financial system conditions and its' potential are one of deciding factors in the economy (for example Sinevičienė and Vasiliauskaitė (2010), Lakstutiene *et al*. (2011) or Snieska and Venckuviene (2011)). Verbose analysis of cause and effect between agrarian investments and financial development of the country is provided in Huang (2011). The work of Asante (2000), confirms that macroeconomical and political instability is closely related to negative changes in private investments. A New Keynesian model with fiscal and monetary policies interactions is tested for Romanian economy in article Caraiani (2012).

Currently, there is a number of mathematical means available for constructing mathematical models of economical systems. For example the study Onoja *et al.* (2012) analyzed trends of agriculture loan behavior during pre-a post financial reform. The goal of Albulescu (2010) was to create a general stability index for Romanian financial system. Relations between investments and development of financial system is engaged in Simplice (2012), using possibilities offered by VAR models. In Zivot and Wang (2006) introduces some typical nonlinear time series models, that were found out effective when modeling nonlinear behavior of economical and financial time series.

Kochetkov points out in his article Kochetkov (2012) the possibilities of using contemporary mathematical regressive models for analyzing macroeconomical indicators. The document Eslamloueyan (2004) deals with impacts of foresighted and unforeseen shocks of official exchange rate.

The economy can be perceived as large open system that is influenced by both inside and outside fluctuations. Because economy as a system contains nonlinear relations, its behavior may shift up to point of chaos Yu (2004). In recent years, complex systems approach has been raised to an alternative scientific methodology to understand the highly complex dynamics of real financial and economic systems. There is growing interest in applying nonlinear dynamic to economic modeling, examples are the IS-LM model (for example Fanti and Manfredi (2007); Mihaela *et al*. (2007)), the comprehensive national strength model in Xing (2007), the Kaldor-Kalecki business cycle model and other models proposed in various references (Wu and Wang (2010); Dumitru and Opris (2009)). Thus, the Chaos theory didn't elude the economy. It was first observed by Benoit Mandelbrot, when studying the price of wool. Nonlinear chaotic systems were studied in many scientific, technical and mathematical works (for example Lu and Chen (2002), Ma and Chen (2001), Zhang (2012)). On this line of research, there have been many works on nonlinear modeling for nonlinear dynamical model on finance system (for example Chen (2008) or Gao and Ma (2009)). Over time it become very important that inner structural characteristics of complex economical systems are to be studied deeply.

3.2 Dynamical model

Under the term of dynamic economic models we can understand such economic and mathematical models, the structure of which includes behavior of the analyzed system over time. In article McNelis (2003) the author presents a dynamic model of finance, composed of three first-order differential equations. The author has come up with a financial model which describes the time variation of three static variables: the interest rate *x*, the investment demand *y*, and the price index *z*. The model is represented by three-dimensional ordinary differential equations:

$$\begin{aligned} x'(t) &= z(t) + (y(t) - a)x(t) \\ y'(t) &= 1 - by(t) - x^2(t) \\ z'(t) &= -x(t) - cz(t) \end{aligned} \tag{1}$$

where *a* is the saving amount, *b* is the cost per investment, and *c* is the elasticity of demand of commercial markets. It is obvious that all three constants (a, b, and c), in model (1) are non-negative.

3.3 Dynamical model with delay

By adding time-delayed feedbacks to system (1) we can obtain the following new system

$$\begin{aligned} x'(t) &= z(t) + (y(t) - a)x(t) + k_1[z(t) + (y(t) - a)x(t) - (z(t-\Delta_3) + (y(t-\Delta_2) - a)x(t-\Delta_1))] \\ y'(t) &= 1 - by(t) - x^2(t) + k_2[1 - by(t) - x^2(t) - (1 - by(t-\Delta_2) - x^2(t-\Delta_1))] \\ z'(t) &= -x(t) - cz(t) + k_3[-x(t) - cz(t) - (-x(t-\Delta_1) - cz(t-\Delta_3))] \end{aligned} \tag{2}$$

The modified system (2) is described by differential equations with delay where k_i (i = 1; 2; 3) is the feedback strengths and Δ_i (i = 1; 2; 3) is the delay times. For the $k_i = 0$ or $\Delta_i = 0$ (i = 1; 2; 3) system (2) is equivalent to the system (1). Following are the results of our investigations of various cases studied.

3.4 Analysis of the solution

Let us now analyze the above mentioned problem using the modern theory of so-called functional differential equations, a very special part of which is also the theory of linear difference equations with delayed argument. In order to numerically solve the problem in question for the system of differential equations with delayed argument, a method was used, which has been, in current studies, derived for solving marginal problems for systems of so-called functional differential equations – see Kuchyňková and Maňásek (2006).

General theory, which makes it possible to solve not only the above mentioned problems, but also others, can be found in a monograph Kiguradze and Půža (2003), and its application on the above mentioned types of differential equations with delay, including the description of the way the desired solution was designed can be found in for example Kuchyňková and Maňásek (2006) and there cited bibliography.

Calculations were made using system Maple, which is used as mathematical software because of the possibility to deal with calculations symbolically. It is very similar to programs Mathematica and Maxima, which offer much fewer functions. An indisputable advantage of Maple is that not only can it make analytic calculations with formulae, but it can equally provide a numerical calculation or graphic representation of results. Therefore, it is a system with a very attractive and user-friendly environment which offers a range of options regarding the application of quantitative methods in practice, application problems, and scientific calculations for many disciplines, etc.

Numerical procedures for solving ordinary differential equations used in Maple are transferred, via the above mentioned theory, to solution of differential equations with delayed argument.

4 Illustrative example

To illustrate the possibilities of a new approach to solving the original problem, let us assume that 'historical development' before t=0 can be simulated using function $y=\sin(\pi t/18)$. The parameters were chosen to be a = 3.0, b = 0.1, and c = 1.0, with an initial state (x(0),y(0),z(0)) = (0.0, 0.0, 0.0), $\Delta_i = 9$ (i = 1; 2; 3). We used equal time delay for easier comparing of results for different values of the parameter k_i (i = 1; 2; 3). For following variants and henceforth noted down as k.

If choosing k=0 we can obtain the solution of the system with no feedback influence, and as apparent from comparing graphical denotation (see Fig 1.) with the results of (McNelis, 2003) the results are corresponding to the formerly published.

If choosing k_i=0.1 the problem considered is solved using “small” feedback factor and as obvious in graphical representation (see Fig.2.), the result is already different from the first. Figure 3 shows the same system as three phase portrait projections in 2D. The graph makes apparent that our modeled financial system can be considered “nonlinear financial chaotic system”.

Parameter value k=0

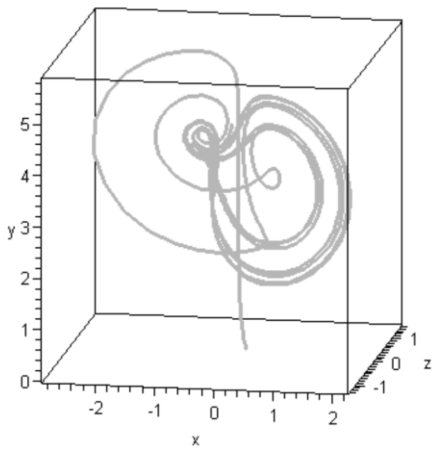

Figure 1. Zero-strength feedback
Source: Authors ’

At the same time we see that the graph contains a part corresponding to long-term stabilized system behavior, that we call the ‘attractor’. The chaotic attractor bears many similarities with butterfly wings, in similar manner to Lorenz’s attractor. Shifts appearing in all solutions are significantly different when compared to non-delayed system in the beginning phase - that is for the initial time period - though eventually the solution behaves similarly to the previous case and stabilizes in the butterfly shape.

Parameter value k=0.1

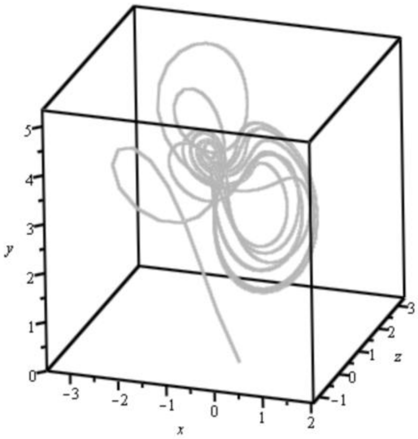

Figure 2. Small feedback factor
Source: Authors'

Parameter value k=0.1

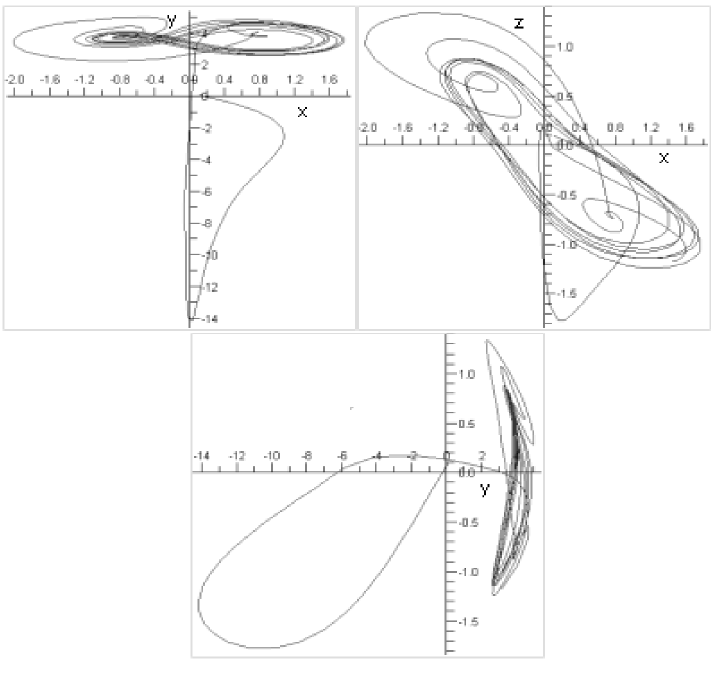

Figure 3. Phase portraits projections in 2Ds
Source: Authors'

By further strengthening of the parameter value to k=0.5 (see Fig.4.), the difference between values of individual components for the solutions from solutions of non-feedback systems deepens, while similarly to the previous case, the difference is more marginal then in the last period, gradually decreases and in the end of measured time period the geometrical solution is once again close to the “butterfly” model, which means that the system stabilizes itself. Similarity of the solution shape (see Fig.4.) within sufficient range of time period (butterfly model) can be observed by adjusting the measure for graphical representation.

This phenomenon of growing influence of the feedback on solving the system is apparent even for k=0.9. Common point for all those models with different choices of *k* parameter with fixed delta is that the feedback strength is apparent in distinct increase of solution values during the beginning of observed time period (viz. Fig. 5). Also common is that in the end of time period, the solutions stabilize themselves to curves reminiscent of models without delay, that is the “butterfly” shape.

Parameter value k=0.5

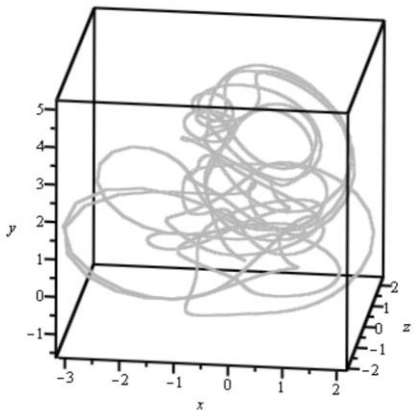

Figure 4. Marginal feedback strength
Source: Authors’

Parameter value k=0.9

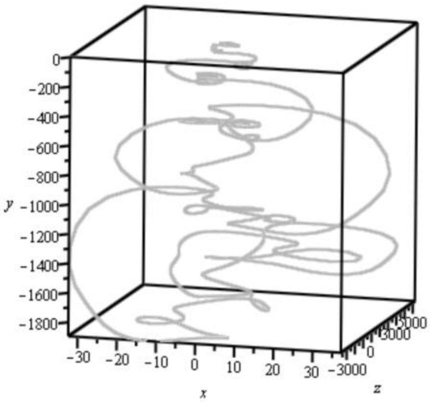

Figure 5. Strong feedback
Source: Authors'

The last illustrative graph (see Fig.6.) demonstrates the influence of so-called "inhibited feedback influence", meaning that the delay influence is gradually decreasing to zero during observed time period. This effect corresponds to the situations described earlier. In reality, the inhibited feedback influence is most likely to be expected.

Parameter value k=1/(1+t)

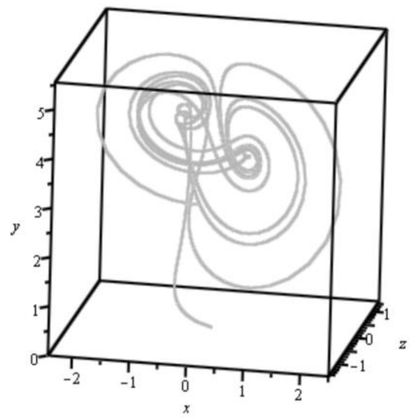

Figure 5. Inhibited feedback influence
Source: Authors'

To sum up, we can establish that the observed system certainly has complex dynamic behaviour. Based on the analysis of presented solutions of delayed differential equation systems, we can confirm that the strength of feedback influence has important effect on the total system stabilisation. Provided we can estimate the value of this parameter, we can make assumptions about further behaviour of the system and possibilities of its stabilisation.

A new methodology used, such as numerical solving of delayed differential equation system using the solid point method, allows for successful solving of the aforementioned problem and to analyse the impact of changes to individual parameters. For illustrations, only changes of the delay impact coefficient on the behaviour of system were used. The applied methodology and the software allow us to investigate general models with big of freedom.

5 Conclusion

When modelling complex economic issues we often have to face the fact trade-offs between variables are changes in time. The dynamic character can be captured by including delay exogenous and endogenous variables in specifying the structure of a model.

Another way to include dynamic processes in models is to see time as a continuous variable and to describe dynamic models by means of differential equations.

We studied the finance system with the distributed time delay. Mathematical modelling of the finance system can deepen our understanding of sudden major changes of economic variables often encountered in economic system. The new model of financial system allows for the influence of previous periods, thus leading to the system of differential equations with delay. Its solution required use of modern methods of the theory of functional differential equations.

This paper indicates that the complex dynamic behaviour in such an economic system can be controlled under appropriate strength feedback and delay times, the feedbacks either suppress or enhance the dynamic behaviour. It is to be expected that the procedure for solving the dynamic economic model, described above, that uses contemporary mathematic methods of so-called “Differential Equations Theory” with delayed argument, can be successfully used for both modelling further concrete economic relations and for economic models in general.

REFERENCES

[1]Albulescu, C. (2010), ***Forecasting the Romanian Financial System Stability Using a Stochastic Simulation Model*****;** Romanian Journal of Economic Forecasting, 13(1), 81-98;

[2]Alonzo, P., Di Sanzo, S. (2011), ***Unemployment and Hysteresis: A Nonlinear Unobserved Components Approach;*** Studies in Nonlinear Dynamics and Econometrics, Vol.15(2);

[3]Asante, Y. (2000), ***Determinants of Private Investment Behaviour;*** AERC Research Paper 100. Available from: http://www.aercafrica.org/ ;

[4]Asea, P. and Zak, P. (1999), Time-to-build and Cycles, Journal of Economic Dynamics and Control, Vol. 23;

[5]Azbelev, N.V. (2001), ***K 25-letnju Permskogo ceminara po funkcionalno-differencialnym upravnenijam,*** Differenc. Upravnenija,Vol. 37(8);

[6]Azbelev, N.V. (2003), ***Kak eto bylo (ob osnovnych etapach pazvitija sovremnnoj teorii funkcionalno-differencialnych upravnenij)***, Problemy nelinejnnogo analyza v inženernych sistemach, Vol.9 (17);

[7]Azbelev, N.V., Maximov, V.T., Rachmatullina, L.F. (1991), ***Vvedenie v teoriuju funkcionalno-differencialnych upravnenij***, Moskva: Nauka;

[8]Azbelev, N.V., Maximov, V.T., Rachmatullina, L.F. (2002), ***Elementy sovremnnoj teorii funkcionalno-differencialnych upravnenij. Metody I prilozenija***, In-t kompjuter. Issled. Vol.2002;

[9]Boucekkine R., C. Saglam, T. Vallee, (2004), ***Technology Adoption under Umbodiment: a two-stage optimal control approach,*** Macroeconomic Dynamics, vol. 8, pp. 250–271

[10]Caraiani, P. (2012), ***Monetary and fiscal policies interactions in an estimated New Keynesian Model for Romania****;* Economic Computation and Economic Cybernetics Studies And Research, Vol. 1;

[11]Chen, W. C. (2008), ***Nonlinear Dynamics and Chaos in a Fractional-Order Financial System***, Chaos, Solitons & Fractals

[12]Dumitru, L., Opris D. **(2009),** ***Neimark-sacker bifurcation for the discrete-delay kaldorkalecki model,*** Chaos, Solitons and Fractals 41, pp. 2405–2413;

[13]Dychta,B.A., Camsonjuk, O.N. (2003), ***Optimalnoe impulsnoe upravlenie c priloženijami***. Moskva: Fizmatlit;

[14]Eslamloueyan, K. (2004), ***Impact of Shocks to Official Exchange Rate on Black Market Premium in Iran: A Two-Stage Model****;* Quarterly Iranian Economic Research, 191-29;

[15]Fanti L, Manfredi P. **(2007),** ***Chaotic Business Cycles and Fiscal Policy: An is-lm Model with Distributed Tax Collection Lags;*** Chaos Solitons Fractals 32, pp. 736–744;

[16]Gao Q., Ma J.H. (2009), ***Chaos and hopf bifurcation of a finance system,*** Nonlinear Dynamics, 58, pp. 209–216;

[17]Harada T. (2010), ***Path-dependent Economic Growth with Technological Trajectory***, Economics of Innovation and New Technology, Vol. 19 pp. 521–538;
[18]Hicks, J.R (1949), ***Harrod's dynamic theory***, Economica, Vol. 16;
[19]Huang, Y. (2011), ***Private Investment and Financial Development in a Globalized Word****;* Empirical Economics, Physica Verlag, An Imprint of Springer-Verlag GmbH, Vol. 41 (1), pp. 43-56;
[20]Kiguradze, I., Půža, B. (2003), ***Boundary Value Problems for Systems of Linear Functional Differential Equations****;* Folia Fac.Sci.Mat. Masarykianae Brunensis, Mathematica, vol. 12, Brno ;
[21]Kobrinskij, N.E., Kuzmin, V.I. (1981), ***Točnost ekonomiko-matematičeskich modelej***, Moskva: Progress;
[22]Kochetkov, Z. (2012), ***Modern Model of Interconnection of Inflation and Unemployment in Latvia;*** Inzinerine Ekonomika-Engineering Economics, 2012, 23(2), 117-124;
[23]Kuchyňková, M., Maňásek, L. (2006), ***On Constructing a Solution of a Multi-point Boundary Value Problem for the Pantograph Equation***; Folia Fac.Sci.Mat.Masarykianae Brunensis, Mathematica, Vol. 16;
[24]Lakstutiene, A., Krusinskas, R., Platenkoviene, J. (2011), ***Economic Cycle and Credit Volume Interaction: Case of Lithuania;*** Inzinerine Ekonomika-Engineering Economics, 22(5), 468-476;
[25]Lu J., Chen G. (2002), ***A New Chaotic Attractor Coined;*** Int. J. Bifurcation Chaos. 12, 659-661;
[26]Ma, J. H., Chen, Y.S. (2001), ***Study for the Bifurcation Topological Structure and the Global Complicated Character of a Kind of Nonlinear Finance System (I);*** Appl Math Mech (English ed.);
[27]Maplesoft, (2012), ***Math Software for Engineers, Educators & Students;*** Available from: http://www.maplesoft.com ;
[28]Mihaela N, Dumitru O, Constantin C. (2007), ***Hopf Bifurcation in a Dynamic is-lm Model with Time Delay,*** Solutions and Fractals 34, pp. 519–530;
[29]McNelis, P.D. (2003), ***Nonlinear Phillips Curves in the Euro area and USA? Evidence from linear and neural network models*** Proceedings. 2003 IEEE International Conference, pp. 145- 149, 20-23;
[30]Onoja, A., Onu, M., Ajodo-Ohiemi, S. (2012), ***Financial Sector Reforms and Credit Supply to Nigerian Agricultural Sector before and after the Reforms (1978-2009)***; Science Education Development Institute, Vol.2 (5) 176 – 183;
[31]Simonov, P.M. (2002), ***O nekotorykh dinamicheskich modeljach makroekonomiki,*** Perm: Un.T. Perm;
[32]Simonov, P.M. (2003), ***Issledovanie ustoichivosti rešenij někotorych dinamičeskich modelyach mikro - i makroekonomik,*** Vestnik PGTU 2003;
[33]Simplice A. (2012), ***Linkages between Investment Flows and Financial Development: Causality Evidence from Selected African countries*** **;**MPRA Paper, University Library of Munich, Germany. Available from: http://mpra.ub.unimuenchen.de/38719/;

[34]Sineviciene, L., Vasiliauskaite, A. (2010), ***Fiskalines politikos ipatumai ciklinio ekonomikos vystymosi kontekste,*** Economics & Management, 1024-1030;
[35]Snieska, V., Venckuviene, V. (2011)*, Hybrid Venture Capital Funds in Lithuania: Motives, Factors and Present State of Development*; Inzinerine Ekonomika-Engineering Economics, 22(2), 157-164;
[36]Wu X.P., Wang L.C. (2010), ***Multi-parameter Bifurcations of the Kaldor-kalecki Model of Business Cycles with Delay*;** Nonlinear Analysis 11, pp. 869–887;
[37]Xing, X.M. (2007), ***An Analysis of the hopf Bifurcation for Comprehensive National Strength Model with Dela;,*** Journal of Yili Normal University (Natural Science Edition) 3, pp. 1–4;
[38]Yu, Y., Zhang, S. (2004), ***Hopf Bifurcation Analysis of the LÄusystem*;** Chaos, Silutions & Fractals. 21, 1215- 1220;
[39]Zhang, T. (2012), ***A Monte Carlo Analysis for Stochastic Distance Function Frontier***; Inzinerine Ekonomika-Engineering Economics, 2012, 23(3), 250-255;
[40]Zivot, E., Wang, J. (2006), ***Modeling Financial Time Series with S-PLUS®;*** *Springer,* New York, pp. 653-712.