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THE MULTI-CRITERIA HESITANT FUZZY GROUP DECISION MAKING WITH MULTIMOORA METHOD

Abstract. *As a generalization of fuzzy set, hesitant fuzzy set permits the membership degree of an element to a set being represented by several possible values and thus can be considered as a powerful tool to express uncertain information in the process of group decision making. The aim of this study is to present a multi-criteria decision making method, namely MULTIMOORA-HF, to tackle hesitant fuzzy information. Being an extension of the crisp MULTIMOORA method, the MULTIMOORA-HF is designed to facilitate group decision making with uncertain information. The proposed methods consists of the three parts—the Ratio System, the Reference Point, and the Full Multiplicative Form. The proposed method thus provides the means for multi-criteria decision making related to uncertain and complex assessments. Utilization of aggregation operators also enables to reduce subjectivity and bias in the group multi-criteria decision making. The numerical simulation exhibits the possibilities for application of the proposed method.*

Keywords: *MCDM, MULTIMOORA, hesitant fuzzy set, personnel management, group decision making, OWA operator***.**

JEL Classification: C44.

1. Introduction

In multi-criteria decision making (MCDM) problems, due to the increasing complexity of the socioeconomic environment and the lack of knowledge or data about the problem domain, crisp data are sometimes unavailable. Thus, the input arguments may be vague or fuzzy in nature. Besides fuzzy set (FS) by Zadeh (1965), several extensions of this concept have been introduced in the literature, for example, type 2 fuzzy set (Mizumoto and Tanaka, 1976) and intuitionistic fuzzy set (IFS) (Atanassov, 1986). The IFS is equivalent to interval-valued fuzzy sets (Atanassov and Gargov, 1989), and the prominent characteristic of IFS is that it assigns to each element a membership degree and a non-membership degree. The membership of an element to a type 2 fuzzy set is defined in terms of a fuzzy set on the domain of memberships. However, these extensions cannot deal with the situation when decision makers have certain hesitancy in providing their preferences over objects in the process of decision making, which permits the membership degree of an element to a set presented as several possible values between 0 and 1. To deal with such cases, Torra (2010) introduced another generalization of fuzzy set, viz. hesitant fuzzy set, allowing the membership degree having a set of possible values. Torra (2010) also discussed the relationships between hesitant fuzzy set and other three kinds of fuzzy sets (intuitionistic fuzzy set, type-2 fuzzy set, and fuzzy multiset). They showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set, all hesitant fuzzy sets are type-2 fuzzy set, and hesitant fuzzy set and fuzzy multiset have the same form, but their operations are different. The hesitant fuzzy set (HFS) can deal with the situation where the evaluation of an alternative under each criterion is represented by several possible values, not by a margin of error, or some possibility distribution on the possible values. For example, three decision makers give the membership of x into A, and they want to assign 0.5, 0.6, and 0.8 as these values, which, in turn, can be considered as a hesitant fuzzy element $(0.5, 0.6, 0.8)$ rather than the convex of 0.5 and 0.8, or the interval between 0.5 and 0.8. More and more multiple attribute decision making theories and methods under hesitant fuzzy environment have been developed. For example, Xia and Xu (2011) proposed some aggregation operators for hesitant fuzzy information, investigated the connections of these operators and applied them to the multi-criteria decision making. Xu and Xia (2011) gave a detailed study on distance and similarity measures for hesitant fuzzy sets and hesitant fuzzy elements respectively. Zhu et al. (2012) defined the hesitant fuzzy geometric Bonferroni mean (HFGBM) and the hesitant fuzzy Choquet geometric Bonferroni mean (HFCGBM). Wei (2012) developed the hesitant fuzzy prioritized operators and applied them to multiple attribute decision making.

Multi-criteria decision making (MCDM) problems are very common in the literature (Gil-Lafuente & Merigó, 2010). There are different ways and methods for solving the decision process (Hwang & Yoon, 1981; Merigó & Gil-Lafuente, 2010; Zavadskas & Turskis, 2011; Zeng & Su, 2012). One of the most popular one is the MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus the Full Multiplicative From) method, which originated from MOORA (Multi-Objective Optimization by Ratio Analysis) proposed by Brauers and Zavadskas (2006). By supplementing MOORA with the Full Multiplicative Form, Brauers and Zavadskas (2010) offered the MULTIMOORA method for MCDM. Brauers and Zavadskas (2011) discussed the issues of aggregation and normalization in MULTIMOORA. The MOORA method has been studied and applied in a wide range of problems (Chakraborty, 2011; Karande & Chakraborty, 2012). Noteworthy, Brauers et al. (2011) updated MULTIMOORA with triangular fuzzy numbers, whereas Baležentis et al. (2012) proposed fuzzy MULTIMOORA for group decision making. This study aims at updating the crisp MULTIMOORA method with hesitant fuzzy information and thus offering the MULTIMOORA-HF.

The extended MULTIMOORA-HF is applied for solving personnel selection. Therefore the remaining part of the paper is organized in the following way. Section 2 discusses preliminaries for hesitant fuzzy set and aggregation thereof. The following Section 3 presents the crisp MULTIMOORA. Section 4 treats the MULTIMOORA method updated with hesitant fuzzy set. Finally, Section 6 presents an application of the proposed method in group decision making.

2. Preliminaries

In the following, we briefly describe some basic concepts and basic operational laws related to hesitant fuzzy sets.

2. 1. Hesitant fuzzy set

Zadeh (1965) introduced the use of fuzzy set theory when dealing with problems involving fuzzy phenomena.

Definition 1. Let a set X be fixed, a fuzzy set A in X is given by Zadeh (1965) as follows:

$$A = \{< x, \mu_A(x) > | \, x \in X\} \tag{1}$$

where $\mu_A : X \to [0,1]$, and $\mu_A(x)$ denotes the degree of membership of the element x to the set A.

Consequently, Atanassov (1986) introduced a generalized fuzzy set called intuitionistic fuzzy set (IFS), shown as follows:

Definition 2. Let a set X be fixed, an intuitionistic fuzzy set A in X is defined as

$$A = \{< x, \mu_A(x), v_A(x) > | \, x \in X\} \tag{2}$$

where functions $\mu_A : X \to [0,1], x \in X \to \mu_A(x) \in [0,1]$ and $v_A : X \to [0,1], \quad x \in X \to v_A(x) \in [0,1]$ satisfy the condition $0 \le \mu_A(x) + v_A(x) \le 1$, for all $x \in X$. Moreover, here $\mu_A(x)$ and $v_A(x)$ denote the degree of membership and the degree of non-membership of the element x to the set A, respectively. For convenience of computation, Xu (2007) named $\alpha = (\mu_\alpha, v_\alpha)$ an intuitionistic fuzzy value (IFV), In addition, $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy of x to A or the degree of hesitancy of x to A. Noteworthy, if $\pi_A(x) = 0$ for all $x \in X$, then the intuitionistic fuzzy set A is reduced to a common fuzzy set.

The IFS is highly useful in depicting uncertainty and vagueness of an object, and thus can be used as a powerful tool to express data information under various different fuzzy environments. However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra (2010) proposed another generation of fuzzy set.

Definition 3. Given a fixed set X, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of $[0,1]$.

To be easily understood, Xia and Xu (2011) express the HFS by mathematical symbol:

$$E = \left(\langle x, h_E(x) \rangle \middle| x \in X \right) \quad (3)$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xia and Xu (2011) called $h = h_E(x)$ a hesitant fuzzy element (HFE) and H the set of all HFEs.

Given three HFEs represented by h, h_1 and h_2, Torra (2010) defined some operations on them, which can be described as:

(1) $h^c = \bigcup_{\gamma \in h} \{1-\gamma\}$

(2) $h_1 \bigcup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$

(3) $h_1 \bigcap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$

Torra (2010) showed that the envelope of a HFE is an IFV, expressed in the following definition:

Definition 4. Given a HFE h, we define the IFV $A_{env}(h)$ as the envelope of h, where $A_{env}(h)$ can be represented as $\left(h^-, 1-h^+\right)$, with $h^- = \min\left\{\gamma \middle| \gamma \in h\right\}$ and $h^+ = \max\left\{\gamma \middle| \gamma \in h\right\}$.

Then, he gave the further study of the relationship between HFEs and IFVs:

(1) $A_{env}(h^c) = \left(A_{env}(h)\right)^c$

(2) $A_{env}(h_1 \bigcup h_2) = A_{env}(h_1) \bigcup A_{env}(h_2)$

(3) $A_{env}(h_1 \bigcap h_2) = A_{env}(h_1) \bigcap A_{env}(h_2)$

To compare the HFEs, Xia and Xu (2011) defined the following comparison laws:

Definition 5. For a HFE h, s $s(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) < s(h_2)$, then $h_1 < h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu (2011) define some new operations on the HFEs h, h_1 and h_2:

(1) $h^{\lambda} = \bigcup_{\gamma \in h} \{\gamma^{\lambda}\}$

(2) $\lambda h = \bigcup_{\gamma \in h} \{1-(1-\gamma)^{\lambda}\}$

(3) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$

(4) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$

Note that the number of values in different HFEs may be different, to compute the distance measure between two HFSs h_1 and h_2, Xu and Xia (2011) give the following regulation: let $l = \max\{\#h_1, \#h_2\}$, where $\#h_1$ and $\#h_2$ is the number of the elements in h_1 and h_2, respectively. Then we shall arrange the elements in h_1 and h_2 in decreasing order, and let $h_1^{\rho(i)} (i = 1, 2, ..., \#h_1)$ and $h_2^{\rho(i)} (i = 1, 2, ..., \#h_2)$ be the *i*-th smallest value in h_1 and h_2, respectively. If $\#h_1 < \#h_2$, then h_1 should be extended by adding the minimum value in it until it has the same length with h_2; if $\#h_1 > \#h_2$, then h_2 should be extended by adding the minimum value in it until it has the same length with h_1. Based on these regulations, Xu and Xia (2011) gave the distance measure between h_1 and h_2 as following:

$$d(h_1, h_2) = \frac{1}{l}\sum_{i=1}^{l}\left|h_1^{\rho(i)} - h_2^{\rho(i)}\right| \tag{4}$$

2. 2. Hesitant fuzzy OWA operator

The ordered weighted averaging (OWA), initially proposed by Yager (1988), is a prevailing operator to aggregate decision-makers' opinions in group decision making problems. The OWA operator provides an aggregation which lies in between the ''and'' requiring all the criteria to be satisfied, and the ''or'' requiring at least one of the criteria to be satisfied. It is a common generalization of the three basic aggregation operators, i.e. max, min, and the arithmetic mean. This operator differs from the classical weighted mean in that coefficients are not associated directly with a particular attribute but rather to an ordered position. It encompasses several operators since it can implement different aggregation rules by changing the order weights. It can be defined as follows:

Definition 6. An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting $w = (w_1, w_2, ..., w_n)^T$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1, a_2 ..., a_n) = \sum_{j=1}^{n} w_j b_j \tag{5}$$

where b_j is the j th largest of the a_i

A key characteristic of the OWA operator is the reordering of the arguments based upon their values, in particular an argument ai is not associated with a specific weight w_i but rather a weight w_i is associated with a specific ordered position i of the arguments.

The OWA operator provides a unified framework for decision making under uncertainty, in which different decision criteria such as maximax, maximin, equally likely (Laplace) and Hurwicz criteria are characterized by different OWA operator weights. To apply the OWA operator for decision making, a crucial issue is to determine its weights. We calculate weights of the OWA operator using fuzzy linguistic quantifiers, which for a non-decreasing relative quantifier Q, are given by

$$w_i = Q(i/n) - Q((i-1)/n) \ , \ i = 1, 2, ..., n . \tag{6}$$

The non-decreasing relative quantifier, Q, is defined as (Herrera, Herrera-Viedma, & Martínez, 2000)

$$Q(y) = \begin{cases} 0, & y < a \\ \dfrac{y-a}{b-a} & a \le y \le b \\ 1 & y > b \end{cases} \tag{7}$$

with $a, b, y \in [0,1]$, and $Q(y)$ indicating the degree to which the proportion y is compatible with the meaning of the quantifier it represents. Some non-decreasing relative quantifiers are identified by terms "most", "at least half", and "as many as possible", with parameters (a,b) given as $(0.3, 0.8)$, $(0, 0.5)$, and $(0.5,1)$, respectively.

The OWA operator, however, has usually been used in situations where the input arguments are the exact values. Xia and Xu (2011) extended the OWA operator to accommodate the situations where the input arguments are hesitant fuzzy information, obtaining the hesitant fuzzy ordered weighted averaging (HFOWA) operator.

Definition 7. Let $h_j\,(j=1,2,...,n)$ be a collection of HFEs, if

$$HFOWA(h_1,h_2,...,h_n)=\bigoplus_{j=1}^{n}(w_j h_{\sigma(j)})=\bigcup_{\gamma_1\in h_{\sigma(1)},...,\gamma_n\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}(1-\gamma_{\sigma(j)})^{w_j}\right\} \tag{8}$$

then the FHOWA is called the hesitant fuzzy ordered weighted averaging (HFOWA) operator, where $(\sigma(1),...,\sigma(n))$ is a permutation of $(1,2,...,n)$, such that $h_{\sigma(j-1)} \le h_{\sigma(j)}$ for all $j=2,3,...,n$ and $w=(w_1,w_2,...,w_n)^T$ is the aggregation-associated weight vector such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Note that we can also use the fuzzy linguistic quantifiers given by (6) and (7) to computer the HFOWA weights.

3. The crisp MULTIMOORA method

As already said earlier, Multi-Objective Optimization by Ratio Analysis (MOORA) method was introduced by Brauers and Zavadskas (2006) on the basis of previous research. Brauers and Zavadskas (2010) extended the method and in this way it became more robust as MULTIMOORA (MOORA plus the full multiplicative form).

MOORA method begins with matrix X where its elements x_{ij} denote ith alternative of jth objective ($i=1,2,...,m$, $j=1,2,....,n$). MOORA method consists of two parts: the Ratio System and the Reference Point approach. The MULTIMOORA method includes internal normalization and treats originally all the objectives equally important. In principle all stakeholders interested in the issue only could give more importance to an objective. Therefore they could either multiply the dimensionless number representing the response on an objective with a significance coefficient or they could decide beforehand to split an objective into different sub-objectives.

The Ratio System of MOORA. Ratio system defines data normalization by comparing alternative of an objective to all values of the objective:

$$x_{ij}^{*}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \tag{9}$$

where x^*_{ij} denotes *i*th alternative of *j*th objective. Usually these numbers belong to the interval [0, 1]. These indicators are added (if desirable value of indicator is maximum) or subtracted (if desirable value is minimum), thus, the summarizing index of each alternative is derived in this way:

$$y_i^* = \sum_{j=1}^{g} x_{ij}^* - \sum_{j=g+1}^{n} x_{ij}^*, \tag{10}$$

where $g = 1, 2, ..., n$ denotes number of objectives to be maximized. Then every ratio is given the rank: the higher the index, the higher the rank.

The Reference Point of MOORA. Reference point approach is based on the Ratio System. The Maximal Objective Reference Point (vector) is found according to ratios found by employing Eq. (9). The *j*th coordinate of the reference point can be described as $r_j = \max_i x^*_{ij}$ in case of maximization. Every coordinate of this vector represents maximum or minimum of certain objective (indicator). Then every element of normalized response matrix is recalculated and final rank is given according to deviation from the reference point and the Min-Max Metric of Tchebycheff:

$$\min_i \left(\max_j \left| r_j - x_{ij}^* \right| \right). \tag{11}$$

The Full Multiplicative Form and MULTIMOORA. Brauers and Zavadskas (2010) proposed MOORA to be updated by the Full Multiplicative Form method embodying maximization as well as minimization of purely multiplicative utility function. Overall utility of the *i*th alternative can be expressed as dimensionless number:

$$U_i^{'} = \frac{A_i}{B_i}, \tag{12}$$

where $A_i = \prod_{j=1}^{g} x_{ij}$ ($i = 1, 2, ..., m$) denotes the product of objectives of the *i*th alternative to be maximized with $g = 1, 2, ..., n$ being the number of objectives to be maximized and where $B_i = \prod_{j=g+1}^{g} x_{ij}$ denotes the product of objectives of the *i*th alternative to be minimized with $n - g$ being the number of objectives (indicators) to be minimized. Thus MULTIMOORA summarizes MOORA (i.e. Ratio System and Reference point) and the Full Multiplicative Form. Ameliorated Nominal Group and Delphi techniques can also be used to reduce remaining subjectivity (Brauers & Zavadskas, 2010). The Dominance theory was proposed to summarize three ranks provided by respective parts of MULTIMOORA into a single one (Brauers & Zavadskas, 2011).

4. The MULTIMOORA–HF method

This section describes the proposed extension of the MULTIMOORA method, namely MULTIMOORA–HF, which enables to tackle uncertainty and vagueness in decision making. Let us assume that a group of experts (decision makers) is about to choose the most compromising alternative with respect to multiple criteria. These criteria are grouped into cost criteria ($j \in C$, C is the set thereof) and benefit criteria ($j \in B$).

Step 1. Each expert evaluates the alternatives under consideration in terms of the defined criteria. The assessments are expressed in HFEs h_{ij}^{k} , where $i = 1,2,..., m$ stands for respective alternatives, $j = 1,2,..., n$ denotes the *j*-th criterion, and $k = 1,2,..., K$ indicates the *k*-th decision maker. Thus K response matrices are defined.

Step 2. Employ the HFOWA operator to aggregate the expert decision matrices into a single response matrix:

$$HFOWA(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{K}) = h_{ij}, \forall i, j \ , \tag{13}$$

where $h_{ij} = \bigcup_{r_{ij} \in h_{ij}} \{r_{ij}\}$ denotes the value given for the *i*-th alternative according to the *j*-th criterion.

Step 3. The transformed response matrix $\beta_{m \times n}$ is defined by transforming cost criteria into benefit ones. We can transform the $\beta_{m \times n}$ by the method given by Zhu et al. (2012), where

$$\beta_{ij} = \bigcup_{t_{ij} \in \beta_{ij}} = \begin{cases} \{r_{ij}\}, j \in B \\ \{1 - r_{ij}\}, j \in C \end{cases}. \tag{14}$$

where $\bigcup_{r_{ij} \in h_{ij}} \{1 - r_{ij}\} = h^{c}$ is the complement of h . These values are bounded to the interval of [0, 1] and therefore do not require an additional normalization.

Step 4. *The Ratio System of MULTIMOORA–IFN*. The assessments of a certain alternative are aggregated across the criteria:

$$y_i = \sum_{j=1}^{n} \beta_{ij} \ . \tag{15}$$

Accordingly, alternatives with higher values of y_i are attributed with higher ranks (cf. Definition 6).

Step 5. *The Reference Point of MULTIMOORA-HF*. Generally, the two types of the reference point might be chosen: (i) the Maximal Objective Reference Point, and (ii) the Utopian Reference Point. In case of the Maximal Objective Reference Point the maximum for every criterion is defined according to Definition 5: $\beta_j = \max_i \beta_{ij}$. In case of the Utopian Reference Point one may set $\beta_j = 1$. Then, distances from the reference point are calculated for each of the alternatives (Eq. (4)):

$$\max_j \{d(\beta_{ij}, \beta_j)\}. \tag{16}$$

The alternatives with larger distances from the Reference are attributed with lower ranks.

Step 6. *The Full Multiplicative Form of MULTIMOORA–HF*. The overall utility of a certain alternative is determined:

$$U_i = \prod_{j=1}^{n} \beta_{ij} . \tag{17}$$

The alternatives are ranked in descending order of the overall utility.

Step 7. The three ranks obtained in Steps 4–6 are summarized by applying the Dominance theory (Brauers & Zavadskas, 2011), see Annex A.

5. Numerical example

Suppose that a telecommunication company intends to choose a manager for R&D department from four volunteers named A_1, A_2, A_3 and A_4 (adopted from Liu & Jin (2012)). The decision making committee assess the four concerned volunteers based on five attributes shown as follows:

(1) Proficiency in identifying research areas (C_1);
(2) Proficiency in administration (C_2);
(3) Personality (C_3);
(4) Past experience (C_4);
(5) Self-confidence (C_5).

The number of the committee members is four, with decision makers labeled as DM_1, DM_2, DM_3 and DM_4, respectively. Each decision maker has presented his (her) evaluation information for four volunteers in Tables 1. Note that these evaluation values are expressed in HFEs. Specifically, the values of zero and unity denote complete incompliance and compliance, respectively, of a certain alternative against a certain criterion.

Table 1: The ratings provided by the decision makers (DM_1–DM_4) to the candidates (A_1–A_4) in terms of the multiple criteria (C_1–C_5).

		C_1	C_2	C_3	C_4	C_5
DM_1	A_1	(0.5)	(0.5, 0.8)	(0.6, 0.9)	(0.6)	(0.3, 0.5)
	A_2	(0.4, 0.6)	(0.7, 0.9)	(0.7)	(0.6, 0.8)	(0.6, 0.7)
	A_3	(0.5, 0.8)	(0.8)	(0.6, 0.8)	(0.5, 0.7)	(0.2, 0.5)
	A_4	(0.7, 0.8)	(0.6)	(0.2, 0.4)	(0.7)	(0.8)
DM_2	A_1	(0.2, 0.7)	(0.5, 0.6)	(0.5, 0.8)	(0.3, 0.6)	(0.7)
	A_2	(0.6, 0.8)	(0.9)	(0.5, 0.7)	(0.1, 0.4)	(0.6, 0.8)
	A_3	(0.4, 0.7)	(0.2, 0.5)	(0.6, 0.7)	(0.5, 0.9)	(0.4, 0.5)
	A_4	(0.6, 0.7)	(0.8)	(0.5, 0.8)	(0.4, 0.8)	(0.2, 0.6)
DM_3	A_1	(0.6, 0.8)	(0.7)	(0.6, 0.7)	(0.4, 0.7)	(0.6, 0.8)
	A_2	(0.8)	(0.7, 0.8)	(0.3, 0.4)	(0.4, 0.8)	(0.5)
	A_3	(0.5, 0.6)	(0.4, 0.7)	(0.6, 0.9)	(0.8)	(0.7)
	A_4	(0.1, 0.5)	(0.7, 0.9)	(0.6, 0.8)	(0.5, 0.6)	(0.3, 0.7)
DM_4	A1	(0.5, 0.8)	(0.5, 0.7)	(0.5)	(0.7)	(0.7, 0.8)
	A2	(0.5, 0.6)	(0.7)	(0.2, 0.6)	(0.6)	(0.3, 0.8)
	A3	(0.5)	(0.4, 0.8)	(0.4)	(0.3, 0.6)	(0.6)
	A4	(0.7)	(0.7, 0.8)	(0.3, 0.6)	(0.4)	(0.2, 0.5)

In this example, we calculate the HFOWA weights for four decision-makers as $w = (0, 0.4, 0.5, 0.1)^T$ by using the linguistic quantifier "most" and Eqs. (6) and (7). Then, employ the HFOWA operator to aggregate the expert decision matrices into a single response matrix (Table 2).

Table 2: The aggregated decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.48, 0.48, 0.52, 0.52, 0.64, 0.64, 0.67, 0.67)	(0.5, 0.51, 0.61, 0.62, 0.65, 0.66, 0.73, 0.74)	(0.55, 0.57, 0.72, 0.73, 0.74, 0.75, 0.84, 0.84)	(0.48, 0.51, 0.63, 0.65)	(0.62, 0.642, 0.64, 0.64, 0.73, 0.73, 0.74, 0.74)
A_2	(0.53, 0.55, 0.58, 0.6, 0.65, 0.66, 0.68, 0.70)	(0.7, 0.76, 0.81, 0.84)	(0.35, 0.36, 0.47, 0.47, 0.54, 0.54, 0.62, 0.63)	(0.49, 0.49, 0.51, 0.51, 0.67, 0.67, 0.68, 0.68)	(0.46, 0.46, 0.52, 0.52, 0.71, 0.71, 0.74, 0.74)
A_3	(0.46, 0.46, 0.52, 0.52, 0.59, 0.59, 0.64, 0.64)	(0.38, 0.42, 0.56, 0.58, 0.60, 0.62, 0.72, 0.73)	(0.58, 0.58, 0.64, 0.64, 0.68, 0.68, 0.73, 0.73)	(0.48, 0.51, 0.60, 0.62, 0.73, 0.74, 0.79, 0.80)	(0.47, 0.50, 0.52, 0.54)
A_4	(0.61, 0.61, 0.64, 0.64, 0.67, 0.67, 0.68, 0.68)	(0.69, 0.75, 0.80, 0.84)	(0.48, 0.48, 0.51, 0.51, 0.64, 0.64, 0.66, 0.66)	(0.45, 0.51, 0.65, 0.68)	(0.24, 0.27, 0.46, 0.46, 0.48, 0.49, 0.62, 0.64)

All of the criteria are benefit ones and expressed in the HFEs, therefore we do not need to normalize them. The Maximal Utopian Reference Point in employed for MCDM. According to Steps 3-7, we get the following results. Note that $s(\cdot)$ represents the score of the overall hesitant fuzzy values.

Table 3. Ranking of the candidates according to MULTIMOORA.

	The System	Ratio	The Reference Point		The Full Multiplicative Form		MULTIMOORA (Final rank)
	$s(y_i)$	Rank	$\max_j \{d(\beta_{ij}, \beta_j)\}$	Rank	$s(U_i)$	Rank	
A_1	0.68	1	0.34	1	0.51	2	1
A_2	0.65	2	0.42	2	0.53	1	2
A_3	0.59	4	0.51	4	0.46	4	4
A_4	0.64	3	0.48	3	0.49	3	3

According to the multi-criteria evaluation, the fourth candidate (A_1) should be recruited, whereas the second candidate (A_2) is the second-best option. At the other end of spectrum, candidates A_4 and A_3 are the last two.

6. Conclusion

The proposed method MULTIMOORA-HF enables to tackle vague and ambiguous ratings express in HFEs thanks to the power ordered weighted average operators employed when aggregating the opinions of the decision makers. Furthermore, the three parts of MULTIMOORA-HF prioritizes the alternatives in terms of different techniques and thus provides one with a more robust ranking. A decision maker can choose the linguistic quantifier for the aggregation operator as well as the type of the operator and thus test the sensitivity of the results. Furthermore, the reference point can also be defined in various ways.

The numerical simulation of personnel selection was implemented to test the MULTIMOORA-HF. Further extensions of the MULTIMOORA method aimed at facilitating soft computing with various types of information would contribute to the area of the imprecise decision making. One can mention interval-valued hesitant fuzzy sets as a prospective direction for the development of MULTIMOORA.

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